Title	ON DISCRETE DYNAMIC PRODUCER-CONSUMER MARKETS
Sub Title	
Author	SZIDAROVSZKY, Ferenc MOLNAR, Sandor
Publisher	Keio Economic Society, Keio University
Publication year	1994
Jtitle	Keio economic studies Vol.31, No.1 (1994.) ,p.51- 63
JaLC DOI	
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Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19940001-0051

ON DISCRETE DYNAMIC PRODUCER-CONSUMER MARKETS*

Ferenc SZIDAROVSZKY[1] and Sàndor MOLNÀR[2]

Abstract: The global asymptotical stability of discrete single-product dynamic producer-consumer markets is examined under three learning schemes. Namely, static, adaptive, and extrapolative expectations are considered. Stability conditions are derived, interpreted, and compared.

1. INTRODUCTION

The stability of oligopolistic models has been discussed by many researchers. A comprehensive summary on single and multi-product oligopolies is given in Okuguchi (1976) and in Okuguchi and Szidarovszky (1990) for both the discrete and continuous cases. Their discrete model is extended to a multiple producer-consumer market in Szidarovszky and Molnàr (1993), where oligopoly theory is combined with the main idea introduced by Arrow (1960). This new model is based on profit maximizing, competing firms and market prices driven by excess demand. In this paper, the discrete time-scale version of this model is examined. The global asymptotical stability of the resulted dynamic system is discussed under complete information and three types of learning schemes: static, adaptive, and extrapolative expectations.

This paper develops as follows. First, the mathematical models are introduced, and then stability conditions are derived. These conditions are interpreted and compared in the final section.

2. THE MATHEMATICAL MODELS

Consider a market where a commodity or a service is supplied by N competing firms. Let x_k denote the output of firm k ($k=1, 2, \cdots, N$), and it is assumed that $C_k(x_k)=B_k x_k^2+b_k x_k+c_k$ is his cost function, where $B_k>0$, b_k and c_k are given constants. At each time period $t=0, 1, 2\cdots$, firm k selects its output level

* This research was supported by the National Science Foundation (NSF SES 9023055) and by the U.S.-Hungarian Joint Fund (J. F. No. 224). The valuable comments of an anonymous referee are greatly appreciated.

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to maximize his expected profit, given the expected price $p_k^E(t)$. That is, $x_k(t)$ is obtained as

$$x_k(t) = \operatorname{argmax}\{x_k p_k^E(t) - C_k(x_k) \mid x_k \geq 0\}\,. \tag{1}$$

Assuming interior optimum, simple differentiation shows that

$$x_k(t) = -\frac{1}{2B_k}(b_k - p_k^E(t))\,. \tag{2}$$

Notice that in the special case of $b_k = 0$ the profit maximizing output is proportional to the expected price. It is also assumed that the market demand is a function of the expected price: $d(t) = Dp_0^E(t) + d$, where $D < 0$ and d are given constants and $p_0^E(t)$ is the expectation of the market on the price. Following Arrow (1960), let the shortage be denoted by $y(t) = d(t) - \sum_{k=1}^N x_k(t)$, and assume that the price moves as directed by the shortage, rising if the shortage is positive, decreasing if negative and remaining stationary if zero. Assuming linearity, we will use the following price adjustment:

$$p(t+1) = p(t) + Ky(t)\,, \tag{3}$$

where $K > 0$ is a constant.

Equations (2) and (3) form a discrete dynamic system, in which the price expectations $p_k^E(t)$ $(k = 0, 1, 2, \cdots, N)$ have fundamental importance. In this paper, the following expectation types will be examined. The first type is the case of complete information: at each period t, agent k knows the current price level.
Complete information:

$$p_k^E(t) = p(t)\,. \tag{4}$$

The second type is the case of static expectation: at each time period t, agent k expects that the current price is the same level as at period $t-1$.
Static (or Cournot) expectation:

$$p_k^E(t) = p(t-1)\,. \tag{5}$$

The third type is the case of adaptive expectation: at each time period t, agent k forms his expectation as a linear combination of the previous actual and expected prices.
Adaptive expectation:

$$p_k^E(t) = M_k p(t-1) + (1 - M_k)p_k^E(t-1)\,, \tag{6}$$

where $0 < M_k < 2$ is a given constant. Since this equation can be rewritten as

$$p_k^E(t) = p_k^E(t-1) + M_k(p(t-1) - p_k^E(t-1))\,, \tag{7}$$

adaptive expectations are interpreted as a certain part of the expectation error is added to the previous expectation. In the special case of $M_k = 1$, adaptive expectations reduce to static expectations. If $0 < M_k < 1$, then $p_k^E(t)$ is between

$p(t-1)$ and $p_k^E(t-1)$. In the case of $1<M_k<2$, the value of $p_k^E(t)$ is between $p(t-1)$ and $2p(t-1)-p_k^E(t-1)$, allowing the expectation adjustment in equation (7) to be larger than the expectation error at time period $t-1$. Many authors assume only that $0<M_k\leq 1$. In this study we, howeveer, consider the slightly more general case. The fourth type is the case of extrapolative expectation: at each time period t, agent k forms his expectation as a linear combination of the two preceding actual prices.

Extrapolative expectation:

$$p_k^E(t)=M_k p(t-1)+(1-M_k)p(t-2)\,, \tag{8}$$

where M_k is a positive constant. Notice again that the case of $M_k=1$ corresponds to static expectations. The above expectation schemes are the same as those introduced earlier for multi-product oligopolies (Okuguchi and Szidarovszky, 1990).

Before analyzing the global asymptotical stability of the resulting dynamic systems, the relation of our models to those introduced by Arrow (1960) is discussed. Consider first the special case, when all producers have complete information on the market price. Then equation (2) implies that the supply is given as

$$\sum_{k=1}^{N} x_k(t)=-\sum_{k=1}^{N}\frac{1}{2B_k}(b_k-p(t))=\left(\frac{1}{2}\sum_{k=1}^{N}\frac{1}{B_k}\right)p(t)-\left(\frac{1}{2}\sum_{k=1}^{N}\frac{b_k}{B_k}\right),$$

which is a linear function of the actual price. Therefore, in this case our model is the discrete counterpart of Arrow's model. In the case of any other expectation types, the supply depends on expected price and/or earlier actual prices. Arrow did not consider such cases as well as extrapolative expectations were not discussed by him.

3. STABILITY ANALYSIS

Consider first the case of complete information. Equations (2), (3), (4) can be summarized as

$$\begin{aligned} p(t+1)&=p(t)+K\left[Dp(t)+d+\sum_{k=1}^{N}\frac{1}{2B_k}(b_k-p(t))\right] \\ &=\left(1+KD-K\sum_{k=1}^{N}\frac{1}{2B_k}\right)p(t)+\left(Kd+K\sum_{k=1}^{N}\frac{b_k}{2B_k}\right). \end{aligned} \tag{9}$$

This system is globally asymptotically stable if and only if

$$-1<1+KD-K\sum_{k=1}^{N}\frac{1}{2B_k}<1\,. \tag{10}$$

Hence we proved the following:

PROPOSITION 2.1. *Suppose that both the demand side and all producers have complete information on the price. Then the price adjustment system is globally asymptotically stable if and only if*

$$0>D>\frac{1}{2K}\left(K\sum_{k=1}^{N}\frac{1}{B_k}-4\right). \tag{11}$$

In the case of static expectations, equations (2), (3), and (5) give the system

$$\begin{aligned} p(t+1)&=p(t)+K\left[Dp(t-1)+d+\sum_{k=1}^{N}\frac{1}{2B_k}(b_k-p(t-1))\right]\\ &=p(t)+\left(KD-K\sum_{k=1}^{N}\frac{1}{2B_k}\right)p(t-1)+\left(Kd+K\sum_{k=1}^{N}\frac{b_k}{2B_k}\right). \end{aligned} \tag{12}$$

The characteristic equation of this system is:

$$\lambda^2-\lambda-\left(KD-K\sum_{k=1}^{N}\frac{1}{2B_k}\right)=0. \tag{13}$$

In order to find conditions which guarantee that the roots of this quadratic equation are inside the unit circle, we will use the following:

LEMMA 1. *Equation $\lambda^2+a_1\lambda+a_2=0$ (a_1, a_2 are real) has roots inside the unit circle if and only if*

$$a_2<1,\ a_2>-a_1-1,\ a_2>a_1-1. \tag{14}$$

The proof of this result can be found, for example, in Okuguchi and Szidarovszky (1990). Apply condition (14) for equation (13) to get the relations

$$\begin{aligned} &-K\left(D-\sum_{k=1}^{N}\frac{1}{2B_k}\right)<1\\ &-K\left(D-\sum_{k=1}^{N}\frac{1}{2B_k}\right)>1-1\\ &-K\left(D-\sum_{k=1}^{N}\frac{1}{2B_k}\right)>-1-1, \end{aligned} \tag{15}$$

which can be summarized as

PROPOSITION 2.2. *Suppose that both the demand side and all producers form static expectations. Then the price adjustment system is globally asymptotically stable if and only if*

$$0>D>\frac{1}{2K}\left(K\sum_{k=1}^{N}\frac{1}{B_k}-2\right). \tag{16}$$

A simple modification of the above model is obtained by assuming that the market has a complete knowledge of the price and all producers still form static expectations. In this case system (12) is modified as

$$\begin{aligned} p(t+1) &= p(t) + K\left[Dp(t) + d + \sum_{k=1}^{N} \frac{1}{2B_k}(b_k - p(t-1))\right] \\ &= (1+KD)p(t) - \left(K\sum_{k=1}^{N}\frac{1}{2B_k}\right)p(t-1) + \left(Kd + K\sum_{k=1}^{N}\frac{b_k}{2B_k}\right) \end{aligned} \tag{17}$$

with characteristic equation

$$\lambda^2 - (1+KD)\lambda + K\sum_{k=1}^{N}\frac{1}{2B_k} = 0\,. \tag{18}$$

By Lemma 1, this equation has roots inside the unit circle if and only if

$$\begin{aligned} &K\sum_{k=1}^{N}\frac{1}{2B_k} < 1 \\ &K\sum_{k=1}^{N}\frac{1}{2B_k} > 1 + KD - 1 \\ &K\sum_{k=1}^{N}\frac{1}{2B_k} > -1 - KD - 1\,, \end{aligned} \tag{19}$$

which can be summarized as

PROPOSITION 2.3. *Assume that the demand side has complete information and all producers form static expectations. Then the price adjustment system is globally asymptotically stable if and only if*

$$K\sum_{k=1}^{N}\frac{1}{B_k} < 2\,, \tag{20}$$

and

$$0 > D > \frac{1}{2K}\left(-K\sum_{k=1}^{N}\frac{1}{B_k} - 4\right).$$

Consider next the case of adaptive expectations. If all producers and the market form adaptive expectations, then equations (2), (3), and (6) imply the dynamic equations

$$\begin{aligned} p(t+1) &= p(t) + K\left[Dp_0^E(t) + d + \sum_{k=1}^{N}\frac{1}{2B_k}(b_k - p_k^E(t))\right] \\ p_k^E(t+1) &= M_k p(t) + (1-M_k)p_k^E(t) \qquad (k=0, 1, 2, \cdots, N) \end{aligned} \tag{21}$$

with coefficient matrix

$$\begin{pmatrix} 1 & KD & -\dfrac{K}{2B_1} & \cdots & -\dfrac{K}{2B_N} \\ M_0 & 1-M_0 & & & \\ M_1 & & 1-M_1 & & \\ \vdots & & & \ddots & \\ M_N & & & & 1-M_N \end{pmatrix}. \tag{22}$$

For simplicity, assume that $M_k = M$ for $k = 0, 1, \cdots, N$. The eigenvalue equation of this matrix can be written as

$$u + KDv_0 - \sum_{k=1}^{N} \frac{K}{2B_k} v_k = \lambda u \tag{23}$$

$$Mu + (1-M)v_k = \lambda v_k \qquad (k = 0, 1, 2, \cdots, N).$$

From the second equation

$$v_k = \frac{M}{\lambda - 1 + M} u, \tag{24}$$

where we assume that $\lambda \neq -M+1$. (Since $-1 < -M+1 < 1$, the special case of $\lambda = -M+1$ does not affect global asymptotical stability.) Substitute relation (24) into the first equation of (23) to get the quadratic equation

$$\lambda^2 + \lambda(M-2) + \left(1 - M - KDM + KM \sum_{k=1}^{N} \frac{1}{2B_k}\right) = 0. \tag{25}$$

Lemma 1 implies that this equation has roots only inside the unit circle if and only if

$$\begin{aligned} &1 - M - KDM + KM \sum_{k=1}^{N} \frac{1}{2B_k} < 1, \\ &1 - M - KDM + KM \sum_{k=1}^{N} \frac{1}{2B_k} > -M + 2 - 1, \\ &1 - M - KDM + KM \sum_{k=1}^{N} \frac{1}{2B_k} > M - 2 - 1. \end{aligned} \tag{26}$$

Easy calculation shows that under our assumptions, these relations are equivalent to inequality (16). Hence we proved the following:

PROPOSITION 2.4. *Suppose that both the demand side and all producers form adaptive expectations. Then the price adjustment system is globally asymptotically stable if and only if relation* (16) *holds.*

We illustrate the conditions of Propositions 2.1, 2.2, 2.3, and 2.4 in Figure 1. The special case of $\sum_{k=1}^{N} (1/B_k) = 1$ is selected.

If the market has complete information on the price, but all producers still form

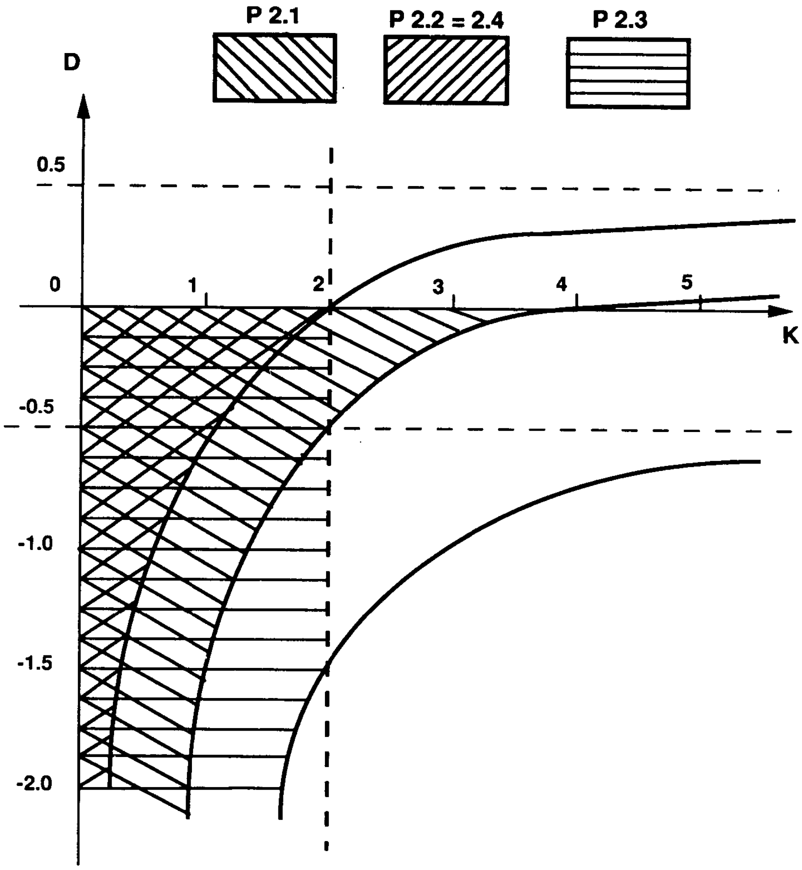

Fig. 1. Comparison of Propositions 2.1, 2.2 = 2.4, and 2.3.

adaptive expectations, then system (21) is modified as follows:

$$\begin{aligned} p(t+1) &= p(t) + K\left[Dp(t) + d + \sum_{k=1}^{N} \frac{1}{2B_k}\left(b_k - p_k^E(t)\right)\right] \\ p_k^E(t+1) &= M_k p(t) + (1 - M_k) p_k^E(t) \qquad (k = 1, 2, \cdots, N)\,. \end{aligned} \tag{27}$$

The coefficient matrix is the following:

$$\begin{pmatrix} 1+KD & \dfrac{-K}{2B_1} & \cdots & \dfrac{-K}{2B_N} \\ M_1 & 1-M_1 & & \\ \vdots & & \ddots & \\ M_N & & & 1-M_N \end{pmatrix} \tag{28}$$

with eigenvalue equations

$$(1+KD)u - \sum_{k=1}^{N} \frac{K}{2B_k} v_k = \lambda u \tag{29}$$

$$Mu+(1-M)v_k=\lambda v_k\,,$$

where we assume again that $M_k=M\,(k=1, 2, \cdots, N)$. The second equation implies (24), and simple substitution into the first equation gives the quadratic equation

$$\lambda^2+\lambda(M-2-KD)+\left(1-M+KD-KDM+KM\sum_{k=1}^{N}\frac{1}{2B_k}\right)=0\,. \qquad (30)$$

The roots are inside the unit circle if and only if

$$\begin{aligned}
&1-M+KD-KDM+KM\sum_{k=1}^{N}\frac{1}{2B_k}<1\\
&1-M+KD-KDM+KM\sum_{k=1}^{N}\frac{1}{2B_k}>-M+2+KD-1 \qquad (31)\\
&1-M+KD-KDM+KM\sum_{k=1}^{N}\frac{1}{2B_k}>M-2-KD-1\,.
\end{aligned}$$

Simple calculation proves the following

PROPOSITION 2.5. *Suppose that the demand side has complete information and all producers form adaptive expectations. Then the price adjustment system is globally asymptotically stable if and only if*

$$\begin{aligned}
&\textit{for}\quad 0<M<1\,,\quad \frac{M}{2(1-M)K}\left(2-K\sum_{k=1}^{N}\frac{1}{B_k}\right)>D>-\frac{2}{K}-\frac{M}{2(2-M)}\sum_{k=1}^{N}\frac{1}{B_k}\,;\\
&\textit{for}\quad M=1\,,\quad K\sum_{k=1}^{N}\frac{1}{B_k}<2\quad \textit{and}\quad D>\frac{1}{2K}\left(-K\sum_{k=1}^{N}\frac{1}{B_k}-4\right); \qquad (32)\\
&\textit{for}\quad 1<M<2\,,\quad 0>D>\max\left\{\frac{M}{2(M-1)K}\left(K\sum_{k=1}^{N}\frac{1}{B_k}-2\right);\right.\\
&\qquad\qquad\left.-\frac{2}{K}-\frac{M}{2(2-M)}\sum_{k=1}^{N}\frac{1}{B_k}\right\}.
\end{aligned}$$

Consider next the case of extrapolative expectations. If all producers as well as the market form extrapolative expectations on the market price, then equations (2), (3), and (8) can be summarized as the systems equation

$$\begin{aligned}
p(t+1)=p(t)+K\Bigg[&D(M_0p(t-1)+(1-M_0)p(t-2))+d\\
&+\sum_{k=1}^{N}\frac{1}{2B_k}(b_k-M_kp(t-1)-(1-M_k)p(t-2))\Bigg] \qquad (33)
\end{aligned}$$

with characteristic equation

$$\lambda^3-\lambda^2+\lambda\left(-KDM_0+K\sum_{k=1}^{N}\frac{M_k}{2B_k}\right)+\left(-KD(1-M_0)+K\sum_{k=1}^{N}\frac{1-M_k}{2B_k}\right)=0\,. \quad (34)$$

In order to find conditions which guarantee that the roots of this cubic polynomial are inside the unit circle, we will use the following:

LEMMA 2. *Equation $\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0$ (a_1, a_2, a_3 are real) has roots inside the unit circle if and only if*

$$\begin{aligned}&1+a_1+a_2+a_3>0\\&1-a_1+a_2-a_3>0\\&3+a_1-a_2-3a_3>0\\&3-a_1-a_2+3a_3>0\\&1-a_3^2+a_1a_3-a_2>0\,.\end{aligned} \quad (35)$$

The proof of this result can be found, for example, in Okuguchi and Irie (1989). Apply conditions (35) for equation (34) to get the relations

$$\begin{aligned}&K(1-2M)\left(D-\sum_{k=1}^{N}\frac{1}{2B_k}\right)>-2\\&K(3-2M)\left(D-\sum_{k=1}^{N}\frac{1}{2B_k}\right)>-2\\&K(4M-3)\left(D-\sum_{k=1}^{N}\frac{1}{2B_k}\right)>-4\\&K\left(D-\sum_{k=1}^{N}\frac{1}{2B_k}\right)>\frac{1-\sqrt{1+4(1-M)^2}}{2(1-M)^2}\,.\end{aligned} \quad (36)$$

We assume that $M_k=M$ for $k=0, 1, \cdots, N$ and $M\neq 1$. Since we have already discussed the case of $M=1$ (static expectations), we may assume $M\neq 1$ here. Simple calculation shows that the last inequality of (36) is the strongest for all positive values of M; therefore, we have the following:

PROPOSITION 2.6. *Assume that both the demand side and all producers form extrapolative expectations. Then the price adjustment system is globally asymptotically stable if and only if*

$$0>D>\frac{1}{2}\sum_{k=1}^{N}\frac{1}{B_k}+\frac{1-\sqrt{1+4(1-M)^2}}{2K(1-M)^2}\,. \quad (37)$$

In our last case, assume that the market has a complete knowledge of the price, but all producers still form extrapolative expectations. Then equation (33) can be modified as

$$p(t+1)=p(t)+K\left[Dp(t)+d+\sum_{k=1}^{N}\frac{1}{2B_k}(b_k-M_kp(t-1)-(1-M_k)p(t-2))\right] \tag{38}$$

with characteristic equation

$$\lambda^3-\lambda^2(1+KD)+\lambda K\sum_{k=1}^{N}\frac{M_k}{2B_k}+K\sum_{k=1}^{N}\frac{1-M_k}{2B_k}=0\,. \tag{39}$$

The roots are inside the unit circle if and only if

$$\begin{aligned}
&D>-\frac{2}{K}-(2M-1)\sum_{k=1}^{N}\frac{1}{2B_k}\\
&D<\frac{2}{K}+(2M-3)\sum_{k=1}^{N}\frac{1}{2B_k}\\
&D>-\frac{4}{K}-(3-4M)\sum_{k=1}^{N}\frac{1}{2B_k}\\
&(1-M)D<\frac{4-K^2(1-M)^2\left(\sum_{k=1}^{N}\frac{1}{B_k}\right)^2-2K\sum_{k=1}^{N}\frac{1}{B_k}}{2K^2\sum_{k=1}^{N}\frac{1}{B_k}}\,.
\end{aligned} \tag{40}$$

We assume that $M_k=M$ for $k=1,2,\cdots,N$ and $M\neq 1$. These relations can be summarized as:

Proposition 2.7. *Suppose that the demand side has complete information and all producers form extrapolative expectations. Then the price adjustment system is globally asymptotically stable if and only if*

for $M<1$,

$$\begin{aligned}
&\min\left\{\frac{2}{K}+(2M-3)\sum_{k=1}^{N}\frac{1}{2B_k};\ \frac{4-K^2(1-M)^2\left(\sum_{k=1}^{N}\frac{1}{B_k}\right)^2-2K\sum_{k=1}^{N}\frac{1}{B_k}}{2K^2(1-M)\sum_{k=1}^{N}\frac{1}{B_k}}\right\}\\
&\quad>D>\max\left\{-\frac{2}{K}-(2M-1)\sum_{k=1}^{N}\frac{1}{2B_k};\ -\frac{4}{K}-(3-4M)\sum_{k=1}^{N}\frac{1}{2B_k}\right\};
\end{aligned}$$

for $M=1$, $\quad K\sum_{k=1}^{N}\frac{1}{B_k}<2$ *and* $D>-\frac{2}{K}-\frac{1}{2}\sum_{k=1}^{N}\frac{1}{B_k}$; (41)

for $M>1$ $\quad \frac{2}{K}+(2M-3)\sum_{k=1}^{N}\frac{1}{2B_k}>D>\max\left\{-\frac{2}{K}-(2M-1)\sum_{k=1}^{N}\frac{1}{2B_k};\right.$

$$-\frac{4}{K}-(3-4M)\sum_{k=1}^{N}\frac{1}{2B_k};\ \frac{-4+K^2(1-M)^2\left(\sum_{k=1}^{N}\frac{1}{B_k}\right)^2+2K\sum_{k=1}^{N}\frac{1}{B_k}}{2K^2(M-1)\sum_{k=1}^{N}\frac{1}{B_k}}\Bigg\}.$$

Notice that these relations cannot be further simplified since no inequality of (40) is stronger in general than any other one. The conditions of Propositions 2.5, 2.6, and 2.7 are illustrated in Figure 2, where the special values $K=\sum_{k=1}^{N}(1/B_k)=1$ are selected.

4. INTERPRETATIONS AND COMPARISONS

In the case of complete information, $\sum_{k=1}^{N}(1/B_k)$ must be smaller than $4/K$ in order to have negative solutions for D in relation (11). This condition can be interpreted as the B_k's must be large enough or K must be sufficiently small. In this case, inequality (11) requires that the absolute value of the negative slope of

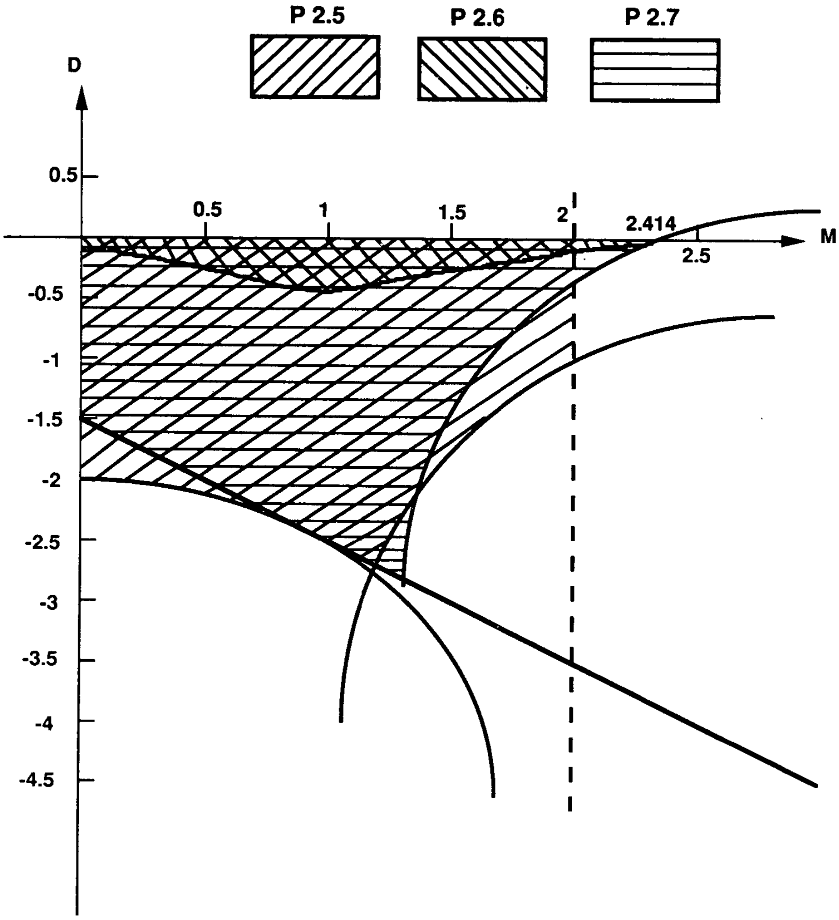

Fig. 2. Comparison of Propositions 2.5, 2.6, and 2.7.

the demand function is sufficiently small. Notice that similar conditions have been found for multi-product oligopolies (Okuguchi and Szidarovszky, 1990, Theorem 4.1.2 and its corollary).

Condition (16) for static expectations and adaptive expectations can be interpreted in the same way as shown above. Notice, however, that condition (16) is more restrictive than (11). Hence, if a system is globally asymptotically stable under static or adaptive expectations, then so is it for the complete information case.

Assume next that the market has complete knowledge of the price, but all producers behave à la Cournot. The condition on the B_k's is more restrictive than that for complete information; it is the same as the one if all producers and the market form expectations à la Cournot. However, the range for D is larger than that for the above two cases, and feasible D always exists.

Consider next the case when the market has complete information and all producers form adaptive expectations. Note that the case of $M=1$ corresponds to Cournot expectations; therefore, the conditions are the same as (20). If $0<M<1$, then the corresponding inequality has negative solutions for D if and only if

$$K\sum_{k=1}^{N}\frac{1}{B_k}\leq 2 \tag{42}$$

or

$$K\sum_{k=1}^{N}\frac{1}{B_k}>2 \quad \text{and} \quad M<\frac{8+K\sum_{k=1}^{N}\frac{1}{B_k}-\sqrt{K^2\left(\sum_{k=1}^{N}\frac{1}{B_k}\right)^2+16K\sum_{k=1}^{N}\frac{1}{B_k}}}{4} \tag{43}$$

That is, feasible D exists if and only if the B_k's are large enough or K is small, or M is sufficiently small. If $1<M<2$, then negative D exists if and only if (42) holds with strict inequality. If (42) holds, then inequality (32) can be interpreted as the absolute value of the negative slope D of the demand function should be small enough. If (42) does not hold, then for $0<M<1$, D must not be too small or too large.

The cases of extrapolative expectations are discussed next. Notice first that inequality (37) has negative solutions for D if and only if the right hand side of inequality (37) is negative. This condition is equivalent to relation

$$\sum_{k=1}^{N}\frac{1}{B_k}<\frac{\sqrt{1+4(1-M)^2}-1}{K(1-M)^2}. \tag{44}$$

That is, the B_k's must be sufficiently large. Observe also that (37) and (44) depend on only $(1-M)^2$, not on M itself. That is, if M is smaller or larger than 1 with the same amount, then these conditions are exactly the same. Hence, the conditions indicate that global asymptotical stability depends on how far the value of M is from unity. If the market has complete knowledge of the price and all producers still form extrapolative expectations, then we have a similar but more complicated situation as before. The case of $M=1$ corresponds again to Cournot expectations, and therefore the corresponding condition is the same as (20). The cases of $M<1$ and $M>1$ can be interpreted as in the previous cases; therefore, the details are omitted.

All models described in this paper can be extended to multi-product markets (Szidarovszky and Molnàr, 1993), and the case of non-equal adjustment constants K_k and M_k can be examined by the special technique developed in Szidarovszky et al. (1993). We also mention that an alternative long-run stability analysis can be performed by replacing the price adjustment process (3) with the market equilibrium condition $y(t)=0$, for all $t \geq 0$. The details will be given in a future paper.

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