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ASYMMETRY OF MARKET RETURNS AND THE MEAN VARIANCE FRONTIER*

Jati K. SENGUPTA and Hyung S. PARK

Abstract: The hypothesis that the skewness and asymmetry have no significant impact on the mean variance frontier is found to be strongly violated by monthly U.S. data over the period January 1965 through December 1974. This result raises serious doubts whether the common market portifolios such as S&P 500, value weighted and equal weighted returns can serve as suitable proxies for mean-variance efficient portfolios in the CAPM framework. A new test for assessing the impact of skewness on the variance frontier is developed here and empirically applied. This has important implications for models of market volatility characterized by conditional variances of market returns.

1. INTRODUCTION

Over the past years a variety of tests of the two-parameter capital asset pricing model (hereafter CAPM) has been reported in the literature. More recently much attention has been focused on the asymmetry and skewness of market return distributions and other portfolios such as mutual funds. For one thing this issue of asymmetry is important for our understanding of the observed investor behavior. For instance Kraus and Litzenberger (1976), Singleton and Wingender (1986) and Sears and Trennepohl (1986) have found in their empirical studies that the returns' skewness is a major factor in the financial decision models i.e., equilibrium asset returns depend not only upon systematic risk but also upon systematic skewness. The basic point is that, with fixed systematic risk, investors should be rewarded with higher expected returns for any asset portfolio having large systematic skewness if the overall market is *ex ante* negatively skewed. Conversely if the market is positively skewed, then systematic skewness may be deemed highly desirable. Secondly, there is extensive empirical evidence that asset returns exhibit both that fat tailed marginal distributions and volatility clustering (see Merton, 1980; Engle and Bollerslev, 1986; Sengupta and Park, 1990). Thus the time series of monthly returns variances exhibits nonstationarity and in this framework it is important to know if the shocks to volatility of major stock returns are permanent. One class of models, increasingly emphasized in recent times, which recognizes this temporal dependence in the returns variance and also skewness is the

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autoregressive conditional heteroscedastic (ARCH) model and its various generalizations (GARCH) initiated and developed by Engle (1982), Engle and Bollerslev (1986) and many others. Thirdly, the proponents of CAPM have argued that even if the actual returns on stocks are quite volatile over the years, in a conditional sense the mean returns are independent of time and hence can be estimated by standard regression methods. However even this inference fails to hold, as Rothschild (1986) has shown since this has the stringent requirement that the conditional mean of the market portfolio is the same in every state. As the proportion of different assets in the market portfolio, which is determined by demand and supply conditions is bound to vary from period to period, the composition of the market portfolio will surely change as the state changes.

These considerations imply that one has to analyze more closely the asymmetry of the distribution of market returns and the heterogeneity of its variance structure. Our object here is two-fold: to analyze the intertemporal variation of the second moment of the market portfolio and the implications of the skewness parameter. Our investigation of the empirical monthly data of the different market portfolios over the years January 1965 through December 1974 shows that the restrictions imposed by the mean variance efficiency frontier are strongly violated by such data. It is not only that systematic skewness has significant impact on variance, it also contributes to the volatility of the returns process. This seems to present a fundamental puzzle which is not resolved by the existing financial models of investors behavior.

2. A NEW TEST OF MARKET VOLATILITY

The problem of direct measurement, *ex ante*, of volatility of financial asset returns can be looked at in two different ways. One is to compute from the data on market returns some statistic such as the variance and from it make inferences about the future volatility. This generates ARCH type models where the return series $\{y_t\}$ is decomposed into its conditional mean and conditional variance:

$$y_t = E_{t-1}y_t + e_t; \; \sigma_t^2 = E_{t-1}e_t^2 \tag{1}$$

where $E_{t-1}y_t$ is the conditional mean and σ_t^2 is the conditional variance of the return in period t and both depend on the information set available from period $t-1$. The shock to the mean is $y_t - E_{t-1}y_t$ and the shock to the variance is $E_te_t^2 - E_{t-1}e_t^2 = e_t^2 - \sigma_t^2 = v_t$. In a linear system the model for the shock to variance presumed by the GARCH formulation is

$$\sigma_t^2 = \theta(B)v_t + w_t$$

where $\theta(B)$ is a lag polynomial in B, the backshift operator with $\theta(0)=0$ and w_t is a deterministic series. In the simplest case of first order lag this can be simplified to

$$\sigma_t^2 = w_t + \alpha e_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

If the empirical estimates are such that $\hat{w}_t$ is negligible and the sum of the slope coefficients adds up to unity in a statistically significant sense, then the shocks to volatility are permanent.

A second way to analyze the volatility of the return series when its probability distribution changes over time is to use a conditional distribution. Thus suppose we have a return process

$$y_t = \mu_t + \varepsilon_t$$

where μ_t is the mean and ε_t's and identically distributed random variables but not independent of time t. However suppose there is a state variable $s = (s_t)$, such that conditional on s the random variable y_t is independent of t. Suppose also that s_t has a stationary distribution, then the time series data on actual returns can be used to estimate the unconditional means and variances of returns. Two interesting implications of this result readily follow. One is due to Rothschild (1986) who suggests that different values of s could represent different information available to different traders. Thus if there are a finite number (N) of states and that the stationary or ergodic distribution of s is given by $\pi_s = P_r\{s_t = s\}$, then the unconditional means and covariances

$$\mu = \sum_{s=1}^{n} \pi_s \mu_{ts};\ \sigma^2 = \sum_{s=1}^{N} \pi_s (\mu_{ts} - \mu)^2$$

can be estimated from the observed data on actual returns.

A second interpretation of the conditioning set $\{y_t \mid s = s_t\}$ is that it introduces a partition in the space of distributions of the random return y_t. This partitioning introduces asymmetry in the sense that for any fixed c the investor's reaction to the event $\{y_t \geq c\}$ may be very different from that of $\{y_t > c\}$. This asymmetry of reactions may vary at different levels of c.e.g., the bull market behavior may significantly differ from the bear market behavior. Some earlier studies (Sengupta and Park, 1990, Dumas and Sengupta, 1989) have found similar contrasting behavior in the case of conservative and aggressive mutual funds.

The partitioning level c may be given a generalized interpretation in terms of the information set underlying conditional mean and conditional variance of returns. Let us define the conditional mean (μ_t) and conditional variance (σ_t^2) as

$$\mu_t = E[y_t \mid I_t],\ \sigma_t^2 = E[(y_t - \mu_t)^2 \mid I_t]$$

where $I_t = \{y_{t-j};\ 1 \leq j \leq \infty;\ x_{t-j};\ 0 \leq j < \infty\}$ is the information set on past levels of returns y_{t-j} and the lagged explanatory variables x_{t-j}. One could replace the regressor variable x_t by a function of the conditional mean, e.g., $\sigma_t^2 = f(\mu_t)$. This yields the variance function model discussed in some detail by Carroll and Ruppert (1988). A standard regression model explains conditonal means in terms of regressor variables, whereas a variance function model is set up to explain the conditional variance in terms of other explanatory variables. In other words, in variance function estimation we attempt to understand the structure of variance as a function of the predictor variables, just as the examination of the structure of the means as a function of predictors. Two useful examples of variance function models, applicable in portfolio models are the so-called Box-Cox transformation and the semi-logarithmic model, e.g.,

$$\log \sigma_t^2 = \theta_0 + \theta_1 x_t + \theta_2 x_t^2$$

where x_t may be replaced by the sample estimates of μ_t. In our empirical applications the sample estimates of μ_t used as the regressor variables are denoted by c_{t-1}; hence the linear variance function model is specified as

$$\sigma_t^2 = \alpha + \sum_{i=1}^{p} \beta_i c_{t-1}^i + e_t$$

where x_t is replaced by the estimated return c_{t-1}. Note that this formulation is closely related to the ARCH models, where lagged variances σ_{t-j}^2 are also introduced as explanatory variables.

The major implication of this partitioning by the truncation level c is that it introduces a decomposition of the variance of return as:

$$\text{Var}[y_t] = E_{z_c}(\text{var}[y_t \mid z_c]) + \text{Var}_{z_c}[E(y_t \mid z_c)]$$

where we define $z_c = 1$ if $y_t \leq c$ and $z_c = 0$ if $y_t > c$. The standard mean variance efficiency frontier characterizes the variance function var$[y_t]$ as a function of c, when the return level c is increased i.e., the higher the level of c, the greater the variance or risk associated with it. But this characterization may fail to hold for the conditional variance var$[y_t \mid z_c]$; also the skewness preference may make higher variance more attractive (for a similar characterization see Bayarri and DeGroot (1987)). It is necessary therefore to characterize the behavior of the conditional variance $V(c) = \text{Var}[y_t \mid y_t > c]$ as c increases in the positive domain. Here we can utilize an important theory proved by Karlin (1982), which says that if the probability density $f(y)$ of y_t is log concave (i.e., $f(y) = \exp(-\phi(y))$, $\phi(y)$ being convex in y), then the conditional variance $V(c)$ is strictly decreasing as c increases; furthermore if $f(y)$ is log convex on (a, ∞), $a \geq 0$ then $V(c)$ is increasing as c traverses (a, ∞). Although the class of log concave densities which includes the normal density, all Gamma densities with nonnegative parameters, the double exponential and all Polya frequency densities is very wide, it is strictly an empirical question how $V(c)$ changes with the truncation level c when the class of densities $f(y)$ is not known.

The overall market is influenced by two groups of traders: the active traders who seek returns $(y_t > c)$ higher than the past mean level and the passive traders who are more conservative $(y_t \leq c)$ and therefore more slow to change. It is clear that in the bullish market the active traders would dominate whereas in bearish markets the passive traders would dominate. Hence the hypothesis of Karlin's theorem is likely to hold more strongly in the bullish periods when the return variance would exhibit a declining behavior with respect to the mean.

3. DATA SET ON MARKET RETURNS

For empirical investigation of the pattern of market volatility measured by conditional variance of raturns we have used monthly data for five market indexes are drawn form Chicago's Center for Research in Security Prices (CRSP) for the period of January 1965 through December 1974. The rates of return for these five indexes include the dividends paid per share and are transformed by taking the natural logarithm of one plus monthly return. Hence, the rate of return, y_{it}, for market index i in month t is a continuously compounded rate of the change in the tatal value of the market index over a month per dollar of initial investment. The five market indexes used in empirical analysis are as follows:

a. Value-weighted market return including all NYSE securities with dividends reinvested.
b. Value-weighted market return including all NYSE and AMEX securities with dividends reinvested.
c. Equally-weighted market return including all NYSE securities with dividends reinvested.
d. Equally-weighted market return including all NYSE and AMEX securities with dividends reinvested.
e. Standard & Poor Composite 500 Index return.

The monthly data are used instead of the daily series because of two reasons. One is to avoid the presence of special day (e.g,, Monday or Friday) effects and the other is due to the nonsynchronous trading activities. Recently Engle, Ng and Rothschild (1989) have more often used monthly return data in order to avoid these seasonality effects. Also we have used 4-month moving average samples in our estimates of conditional means and conditional variances. This is because we have done previous studies on mutual fund returns, e.g., Sengupta and Park (1990), where annual time series data are utilized with dividends declared once a year. To retain comparability with these earlier studies based on yearly returns series we had to use a 4-month moving average.

The time period covered in our study is from January 1965 through December 1974. Two motivations for selecting this period are as follws: one is that the same period was selected for analyzing the risk return relationships for three types of mutual funds and secondly, this period includes both a bearish period and a bullish period. Hence one could estimate if the estimates of the variance function varies over these optimistic and pessimistic periods.

To compare our variance frontier estimates in the bullish period, we have also considered CRSP data for a more recent bullish period from August 1982 to December 1991. The results of these estimates are reported in Table 3, which have been analyzed in some detail elsewhere by Sengupta and Sfeir (1993).

Since our main objective is to test the impact of the skewness factor on the variance frontier of market returns we use the specification

TABLE 1. Results of Karlin's Test on Variance Monotonicity with Truncation Level.

Market Indices	Intercept	c_{t-1}	c_{t-1}^2	c_{t-1}^3	R^2	DW
Equal Weighted Return:						
NYSE only	0.002	−0.023			0.87	2.29
	(27.32)	(−15.53)				
	0.003	−0.050	0.248		0.98	2.03
	(51.23)	(−22.28)	(12.82)			
	0.003	−0.077	0.828	−3.311	0.99	2.14
	(87.27)	(−29.78)	(16.02)	(−11.39)		
	0.002			−1.316	0.49	1.75
	(14.62)			(−5.95)		
NYSE plus AMEX	0.003	−0.025			0.87	1.69
	(28.12)	(−15.41)				
	0.004	−0.054	0.221		0.97	2.06
	(48.25)	(−20.39)	(11.58)			
	0.004	−0.086	0.797	−2.750	0.99	2.31
	(88.06)	(−30.28)	(16.85)	(−12.37)		
	0.002			−1.008	0.48	1.59
	(15.27)			(−5.87)		
Value Weighted Return:						
NYSE only	0.001	−0.004			0.10	1.91
	(11.20)	(−2.06)				
	0.002	−0.041	0.383		0.93	2.07
	(41.62)	(−21.64)	(20.41)			
	0.002	−0.063	0.925	−3.442	0.99	2.69
	80.07	(−32.84)	(21.50)	(−12.83)		
	0.001			0.229	0.03	1.41
	(14.14)			(1.04)		
NYSE plus AMEX	0.001	−0.006			0.18	1.79
	(12.13)	(−2.86)				
	0.002	−0.040	0.357		0.90	2.01
	(35.88)	(−18.25)	(16.47)			
	0.001	−0.066	1.000	−4.056	0.99	2.42
	(82.54)	(−34.88)	(23.55)	(−15.42)		
	0.001			0.062	0.002	1.51
	(144.06)			(0.28)		
S&P 500 Index Return:						
	0.001	−0.002			0.03	1.95
	(10.78)	(−1.05)				
	0.001	−0.036	0.384		0.90	2.03
	(36.75)	(−18.01)	(17.98)			
	0.002	−0.057	0.961	−3.938	0.98	2.29
	(67.60)	(−26.53)	(18.04)	(−11.03)		
	0.001			0.484	0.11	1.24
	(15.02)			(2.09)		

Note: t-values are in parenthesis, R^2 is the squared multiple correlation coefficient and DW is Durbin-Watson statistic.

$$V(c)=\alpha+\beta_1 c_{t-1}+\beta_2 c_{t-1}^2+\beta_3 c_{t-1}^3$$

in our estimation for the five market portfolios. Three types of sampling framework were considered, e.g., (a) fixed sample size, (b) moving average samples and (c) increasing sample sizes. For the fixed sample size case the entire 10 year period is divided into 30 four-month subperiods, so that the first subperiod is January 1965 through April 1965, the second from May 1965 through August 1965 and so on. For the moving average samples the first sample runs from January 1965 through April 1965, the second from February 1965 through May 1965 and so forth. Finally, the increasing sample size test is based on the means and standerd deviations computed by gradually increasing the sample sizes by 4 months beginning with January 1965. This provides a direct test of the mean variance relationship when the sample size is gradually increased in terms of the statistical criteria such as R^2 and t-values of the coefficients, the increasing sample size test performed the best. Hence we use this framework in all our estimation results.

TABLE 2. RESULTS OF KARLIN'S TEST ON VARIANCE MONOTONICITY WITH TRUNCATION Level.

Market Indices	Intercept	c_{t-1}	c_{t-1}^2	c_{t-1}^3	R^2	DW
Equal Weighted Return:						
NYSE only						
Simple Var.	0.003	−0.077	0.828	−3.311	0.99	2.14
	(87.27)	(−29.78)	(16.02)	(−11.39)		
Log Var.	−5.727	−33.038	244.888	−1590.5	0.99	2.34
	(−117.4)	(9.31)	(3.47)	(−4.00)		
NYSE plus AMEX						
Simple Var.	0.004	−0.086	0.797	−2.750	0.99	2.31
	(88.06)	(30.28)	(16.85)	(−12.37)		
Log Var.	−5.417	−31.578	261.192	−1369.8	0.99	2.42
	(−201.2)	(−19.2)	(9.54)	(−10.6)		
Value Weighted Return:						
NYSE only						
Simple Var.	0.002	−0.063	0.925	−3.442	0.99	2.69
	(80.07)	(−32.84)	(21.50)	(−12.83)		
Log Var.	−6.158	−59.098	832.92	−2682.3	0.98	2.51
	(−191.7)	(−22.15)	(13.88)	(−7.65)		
NYSE plus AMEX						
Simple Var.	0.001	−0.066	1.000	−4.056	0.99	2.42
	(82.54)	(−34.88)	(23.55)	(−15.42)		
Log Var.	−6.122	−60.653	855.78	−3094.3	0.97	2.19
	(−164.8)	(−19.79)	(12.49)	(−7.29)		
S&P 500 Index Return:						
Simple Var.	0.002	−0.057	0.961	−3.938	0.98	2.29
	(67.60)	(−26.53)	(18.04)	(−11.03)		
Log Var.	−6.387	−58.03	973.90	−3998.4	0.97	2.06
	(−215.78)	(−21.15)	(14.30)	(−8.76)		

Note: t-values are in parenthesis.

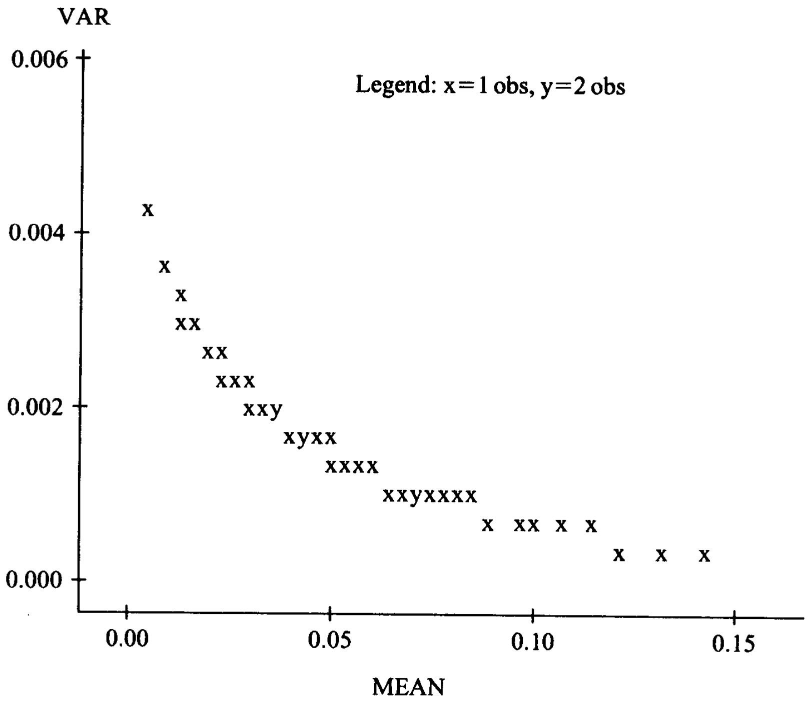

Fig. 1. Plot of EWRET with Karlin's method.

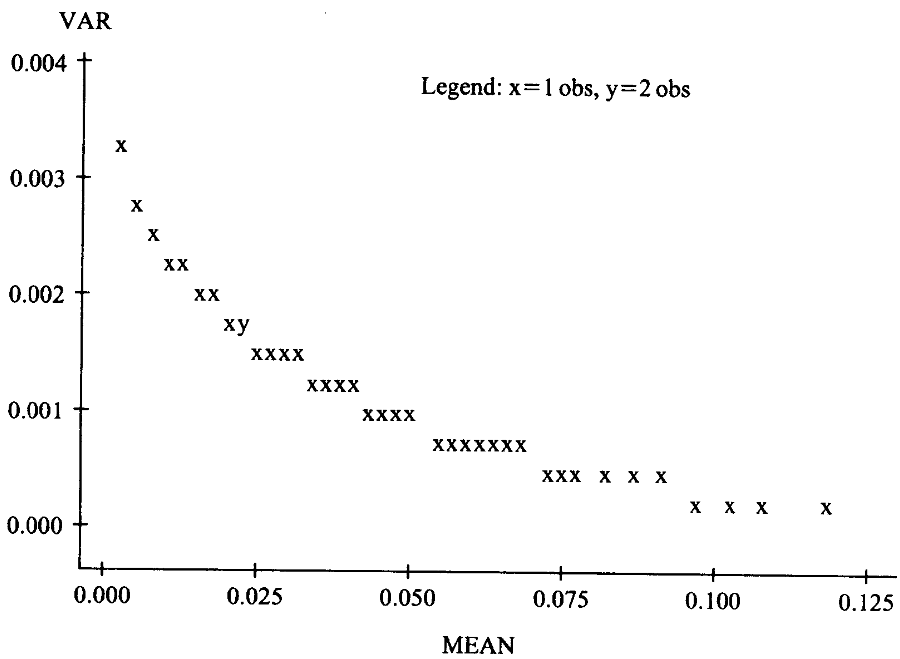

Fig. 2. Plot of EWRETNY with Karlin's method.

The statistical results of this test are reported in Tables 1 and 2 and the Figures 1 and 2 illustrate the plot of the conditional variance $V(c)$ against the mean level c for the equally weighted return index. A number of implications follow from these statistical results. First of all, the linear regression of $V(c)$ on c shows a negative slope consistently for all the five market indices.
All coefficients are significant at 1% level. This result is confirmed when a separate linear regression of the form

$$\sigma(c) = [V(c)]^{1/2} = a + bc_{t-1}$$

is estimated on the basis of an increasing sample size test, starting with the initial subperiod and then adding on the other sample periods consecutively. As Table A shows, the negative impact of the mean on the standard deviation is consistently significant for all the five indices. This empirical result is quite contrary to the positive risk-return relationship of high returns for high risk that is predicted by the standard mean-variance portfolio medel. In other words, additional information as more observations are added does not pay in terms of return and risk trade-off.

One possible reason for the negative relationship between mean and variance is that higher order moments such as skewness and kurtosis are ignored here. In strudies analyzing the performance of British mutual funds (Unit trust), Saunders, Ward, and Woodward (1980) found that the more risky the trust the lower its returns. Also, U.K. investors are found to be less risk averase since, despite the poor return performance of the high-risk trusts, their trading volume did not fall over the period. By employing the stochastic dominance tests, they present evidence that trusts as a group have generally out performed the market.

A second reason is the phenomenon of skewness preference in a bullish market. This phenomenon has been analyzed by Singleton and Wingender (1986), Beedles (1979) and others. It says that if the average investor is optimistic about the future

TABLE A. RESULTS OF INCREASING SAMPLE SIZE TEST.

Market Indices	Intercept	c_{t-1}	R^2	DW
Equal Weight Return:				
NYSE only	0.059	−1.67	0.80	1.89
	(45.25)	(−10.53)		
NYSE plus AMEX	0.070	−0.981	0.76	1.94
	(39.25)	(−9.36)		
Value Weighted Return:				
NYSE only	0.041	−1.022	0.40	1.74
	(26.77)	(−4.33)		
NYSE plus AMEX	0.042	−1.061	0.43	1.91
	(26.90)	(−4.58)		
S&P 500 Index Return:	0.037	−0.827	0.30	1.94
	(40.12)	(−3.45)		

TABLE 3. Variance Frontier Estimates for Market Return over August 1982 Thorough December 1991.

Market Index	Intercept	c_{t-1}	c_{t-1}^2	c_{t-1}^3	R^2	DW
Equal Weighted Return:						
NYSE only	0.0012	−0.0385	3.2361	−43.99	0.58	1.16
	(3.54)	(−2.19)	(8.35)	(−4.85)		
Value Weighted Return:						
NYSE only	0.0015	−0.0628	3.7647	−59.34	0.74	1.25
	(5.92)	(−4.41)	(8.51)	(−4.72)		
S&P 500 Index Return:	0.0015	−0.0441	3.1640	−58.12	0.61	1.18
	(6.14)	(−2.74)	(8.12)	(−4.13)		

Note: t-value in parentheses.

in the sense $\mu_{t+1} > c_t$, then this "good news" effect depresses the conditional variance term σ_t^2. To test if this phenomenon is important, we have performed variance frontier estimates in Table 3 for the more recent period: August 1982 through December 1991, which is considered to be a bullish period by the banking professionals in U.S.

A third possible reason may be that the return distribution has significant departures from a normal distribution and hence the mean variance model based on the normality assumption cannot explain this type of risk return relationship. Since our model is based on Karlin's theorem, it uses a more general framework of distribution of returns than the normal.

A second major finding is that the slope coefficient (β_3) associated with the cubic term (c_{t-1}^3) is consistently negative for all the five market returns and highly significant at 1% level. This pattern is not altered when the logarithm of variance is used as a dependent variable. For the more recent period (August 1982 through December 1991) which is a bullish market period, this pattern is strongly persistent. On replacing the cubic regressor term c_{t-1}^3 by $(c_{t-1} - \mu_T)^3$ where μ_T is viewed as a target return, the asymmetric effect may be captured by dissimilar reactions of the infestor from the events $c_{t-1} > \mu_T$ and $c_{t-1} < \mu_T$. Thus if $c_{t-1} > \mu_T$, i.e., the investor is optimistic, then a negative β_3 coefficient would tend to depress the conditonal variance. A reverse behavior, i.e., inflation of variance is likely in a pessimistic market.

A third important finding is that the signs of the regression coefficients of the variance function estimates alterante, i.e., $\hat{\beta}_1 < 0$, $\hat{\beta}_2 > 0$, $\hat{\beta}_3 < 0$. This is consistent with the expected utility maximization model of von Neumann-Morgenstern, where the utility function is more nonlinear than the quadratic. Fourthly, our estimates of the variance function models for the five market indices show the negative impact of the skewness factor more strongly in a statistical sense, when the bearish subperiod between January 1973 and Decembef 1974 is removed from the data set. The variance frontier estimates for the recent period August 1982 through December 1991 reported in Table 3 confirm the same point, when compared with the earlier period estimates over October 1976 through July 1982, which is not a bearish period. This latter aspect has been discussed in some detail by Sengupta and Sfeir (1993) elsewhere.

Finally, we observe a difference in the variance behavior between the equally weighted (EW) market index and the value weighted (VW) index. While the EW return shows variance to be consistently declining in relation to the lagged mean, the VW return behavior is not that strong or consistent. Two possible reasons may be given for this apparent difference in behavior. One is that the VW return is observed to be less skewed than the EW return, perhaps due to the effect of program trading in high value stocks. Less skewness means the negative impact of variance is less. The second reason is that the EW return follows more closely a gamma-type distribution which yields a monotonic relationship between the mean and variance. The weights of equal proportions tend to preserve this asymmetric distribution pattern.

In general we may thus conclude that the cubic variance function $V(c)$ is not uniformly convex in c_{t-1}, since it depends on the level of c_{t-1} and the value of the coefficient $\hat{\beta}_3$. If we consider however up to the quadratic term, then the variance function is uniformly convex in c_{t-1}, since the coefficient $\hat{\beta}_2$ is uniformly positive. The latter follows directly from the standard mean variance (MW) model which minimizes the variance $\sigma^2 = x'Vx$ of a portfolio (x) subject to a lower bound on the expected return, i.e.,

$$\text{Min } \sigma^2 = x'Vx \text{ s.t. } \bar{r}'x = c, \textstyle\sum x_i = 1$$

where $(\bar{r}, V)$ are the means and variances. The minimal variance σ^{2*} can be easily derived to be a strictly convex function of c, since we have

$$\sigma^{2*} = k_0 - k_1 c + k_2 c^2$$

where $k_0 = \alpha\delta$, $k_1 = 2\beta\delta$, $k_2 = \gamma\delta$, $\delta = (\alpha\delta - \beta^2)^{-1}$, $\alpha = \bar{r}'V^{-1}\bar{r}$, $\beta = \bar{r}'V^{-1}e$, $\gamma = e'V^{-1}e$, e is a vector of ones and prime denotes transpose.

The above results thus suggest the need for generalizing the concept of risk aversion by utilizing nonparametric method such as the stochastic dominance tests (see Sengupta and Park, 1990) or, transformations of the utility function (e.g., Yaari, 1986). However it is clear from the estimated coefficient of the cubic term that the impact of skewness on variance is highly significant and any specification of the mean variance efficiency frontier which ignores this significant impact of the skewness parameter is likely to be heavily biased. Clearly more empirical research is called for (see Hsu (1984), Stambaugh, 1982).

4. TESTS OF PERSISTENCE OF MARKET VOLATILITY

The modelling process in ARCH and GARCH formulations basically estimates how the first two conditional moments of the return distribution depend upon the past information set. For instance in a linear system the estimating equation (2) may be used to test if the shocks to volatility are persistent or permanent when the volatility is measured by the time series $\{\sigma_t^2\}$. This equation may also be understood as a form of variance-function estimation in regression theory, where the predictors are e_{t-1}^2 and σ_{t-1}^2. Regreession analysis is usually understood to be the examination of the structure of the means as a function of the predictors. In variance-function estimation (see Carroll and Rupert, 1988) we try to understand the structure of the variances as a function of predictors. Thus we may apply two formulations of the conditional variance model: one as in (2) related to the ARCH model and the other as

$$\hat{\sigma}_t^2 = k_0 + h_1 \hat{e}_{t-1}^2 + h_2 \sigma_{t-1}^2 \tag{3}$$

where $\{\hat{\sigma}_t^2\}$ is the time series of estimated variances from a given model with additive stationary errors $\{e_t\}$. For the given model we may choose a simple variant of the Karlin's truncation model $V(c) = \alpha + \sum_{i=1}^{3} \beta_i c_{t-1}^i$ and then estimate the volatility series $\{\hat{\sigma}_t^2\}$ and use the linear regression equation (3).

In the first case we apply the linear model (2) to the three market indices: VWRET, EWRET with the combined samples and S&P 500 over the sample period January 1965 through December 1974 using monthly data. The results are reported in Table 4, which may be compared with a similar estimate by Engle and Mustafa (1989), who used the daily CRSP return data from July 1962 through 1985 to estimate market volatility in terms of the S&P 500 index. Their estimates were as follows:

TABLE 4. VARIANCE FUNCTION ESTIMATES OF THE ARCH MODEL.

Equal Weighted Return:	
NYSE plus AMEX	$y_t = 0.0018 + e_t$; $R^2 = 0007$ (1.02) $\hat{\sigma}_t^2 = 0.00006 - 0.0009\,\hat{e}_{t-1}^2 + 0.986\ \hat{\sigma}_{t-1}^2$ (0.88) (46.65) $R^2 = 0.95$, DW $= 2.05$
Value Weighted Return:	
NYSE plus AMEX	$y_t = 0.00009 + e_t$; $R^2 = 0.004$ (1.08) $\hat{\sigma}_t^2 = 0.00001 + 0.0014\,\hat{e}_{t-1}^2 + 0.993\ \hat{\sigma}_{t-1}^2$ (0.62) (55.67) $R^2 = 0.96$, DW $= 2.51$
S&P 500:	$y_t = -0.0018 + e_t$; $R^2 = 0.0017$ (-0.45) $\hat{\sigma}_t^2 = 0.00002 + 0.0023\,\hat{e}_{t-1}^2 + 0.986\,\hat{\sigma}_{t-1}^2$ (0.80) (1.37) $R^2 = 0.96$, DW $= 1.84$

Note: t-values in parentheses are based on the estimated asymptotic standard errors of the least squares estimates.

$$y_t = \underset{(5.12)}{0.00042} + e_t;\ \hat{\sigma}_t^2 = \underset{(16.13)}{0.073}\ \hat{e}_{t-1}^2 + \underset{(105.68)}{0.925}\ \sigma_{t-1}^2 \qquad R^2 = 0.95$$

Since they used the moving average sampling framework and daily data, their estimates are slightly different from ours. However they both agree in terms of the highly significant coefficient of σ_{t-1}^2 and the insignificant coefficient of $\hat{e}_{t-1}^2$. Several features of our estimates may be commented upon. First of all, the R^2 for the variance equation is quite high and significant, although the mean equation for y_t has disapointingly a low R^2 value. This pattern must also hold for the results of Engle and Mustafa (1989) although they did not report R^2 values. Secondly, if we test the null hypothesis that the sum of the two slope coefficients add up to one by a t-statistic (which may only hold asymptotically if at all), it cannot be rejected at 1% level. This implies that the shocks to market volatility are persistent or permanent. Thirdly, all our estimates find that the slope coefficient of e_{t-1}^2 is statistically not different from zero, which implies that in these formulations the conditional kurtosis of the return distribution is of negligible importance. Finally, Chou, Engle and Kane (1989) have found that nonlinear forms of the mean function and time-varying regression coefficients tend to provide a better empirical fit. To test the plausibility of this hypothesis for our data set we first estimated nonlinear equation

$$y_t = a + b_1 y_{t-1}^2 + b_2 \sigma_t^2 + e_t \tag{4}$$

and then on the basis of estimated residuals $\{\hat{e}_t\}$ and the estimated variance $\{\hat{\sigma}_t^2\}$ we computed the linear regression equation (3). Besides (4) we experimented with other nonlinear forms but this specification produced the best results in terms of goodness of fit. Futhermore this formulation may be viewed as an approximate specificantion of the mean variance trade-off where σ_t^2 is viewed in relation to the mean return and its square. The statistical results for the different market indices now appear as follows:

<u>VWRET</u>

$$y_t = \underset{(36.51)}{7.087} + \underset{(3.16)}{158.03}\, y_{t-1}^2 - \underset{(0.67)}{5464.48}\, \sigma_t^2, \quad R^2 = 0.084; \quad \mathrm{DW} = 0.94$$

$$\hat{\sigma}_t^2 = \underset{(0.45)}{0.00001} - \underset{(-0.07)}{0.0002}\ e_{t-1}^2 + \underset{(52.06)}{0.996}\ \hat{\sigma}_{t-1}^2, \quad R^2 = 0.97; \quad \mathrm{DW} = 1.89$$

<u>EWRET</u>

$$y_t = \underset{(38.47)}{1.74} + \underset{(2.33)}{13.94}\, y_{t-1}^2 - \underset{(0.54)}{561.48}\, \sigma_t^2, \quad R^2 = 0.086; \quad \mathrm{DW} = 0.89$$

$$\hat{\sigma}_t^2 = \underset{(0.11)}{0.0} + \underset{(1.13)}{0.0001}\,\hat{e}_{t-1}^2 + \underset{(43.59)}{0.995}\,\hat{\sigma}_{t-1}^2, \quad R^2 = 0.95; \quad DW = 1.98$$

S&P 500

$$y_t = \underset{(42.47)}{5.77} + \underset{(3.44)}{125.28}\,y_{t-1}^2 - \underset{(0.36)}{4854.37}\,\sigma_t^2, \quad R^2 = 0.10; \quad DW = 0.87$$

$$\hat{\sigma}_t^2 = \underset{(0.33)}{0.0} + \underset{(0.11)}{0.0001}\,\hat{e}_{t-1}^2 + \underset{(42.55)}{0.997}\,\hat{\sigma}_{t-1}^2, \quad R^2 = 0.96; \quad DW = 1.95$$

The sum of the two slope coefficients for the variance function $\hat{\sigma}_t^2$ now adds up to unity more closely in a statistical sense than before, thus implying that the shocks to volatility have more persistence.

If we replace (4) by another nonlinear specification the results appear as follows:

VWRET

$$y_t = \underset{(-1.49)}{-0.0019} + \underset{(3.20)}{2.861}\,\sigma_t^2; \quad R^2 = 0.08; \quad DW = 0.54$$

$$\hat{\sigma}_t^2 = \underset{(0.62)}{0.0} + \underset{(0.63)}{0.093}\,\hat{e}_{t-1}^2 + \underset{(56.39)}{0.994}\,\hat{\sigma}_{t-1}^2, \quad R^2 = 0.97; \quad DW = 2.01$$

EWRET

$$y_t = \underset{(-0.73)}{0.0019} + \underset{(2.46)}{1.930}\,\sigma_t^2; \quad R^2 = 0.05; \quad DW = 0.89$$

$$\hat{\sigma}_t^2 = \underset{(0.91)}{0.0} + \underset{(0.068)}{0.0075}\,\hat{e}_{t-1}^2 + \underset{(47.14)}{0.984}\,\hat{\sigma}_{t-1}^2; \quad R^2 = 0.95; \quad DW = 1.95$$

S&P 500

$$y_t = \underset{(-1.99)}{-0.0027} + \underset{(3.43)}{3.704}\,\sigma_t^2, \quad R^2 = 0.09; \quad DW = 0.91$$

$$\hat{\sigma}_t^2 = \underset{(0.85)}{0.0} + \underset{(1.43)}{0.0020}\,\hat{e}_{t-1}^2 + \underset{(48.54)}{0.987}\,\hat{\sigma}_{t-1}^2, \quad R^2 = 0.96; \quad DW = 1.98$$

It is clear that these models also exhibit the persistence of estimated variance.

5. CONCLUSIONS

This paper analyses some models to test the hypothesis that the skewness and asymmetry have no significant impact on the mean variance trade-off. Two types of models are presented and econometrically estimated for the monthly return data over January 1965 through December 1974. One model presents a truncation model where variance is viewed as a cubic function of the return levels and the impact of the skewness term is found to be highly significant at 1% level. The second model estimates two variants of the ARCH model and finds that the market volatility measured by the conditional variances have a high degree of persistence. This suggests the need for further research to find if the conditional skewness also has a persistence propetry. If it were true that skewness contributes greatly to the variance persistence, then the Arch-type models of market volatility have to be significantly modified in order to allow for the asymmetry due to skewness.

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REFERENCES

Bayarri, M. J. and DeGroot, M. H., "Information and selection models", *Probability and Bayesian Statistics*, New York, Plenum Press (1986).

Beedles, W. L., "On the asymmetry of market returns", *Journal of Financial and Quantitative Analysis*, Vol. 14 (1979), 653–660.

Carroll, R. J. and Ruppert, D., *Transformation and Weighting in Regression*, London, Chapman and Hall (1988).

Chou, R., Engle, R. F., and Kane, A., "Estimating risk aversion with a time-varying price of volatility", Working Paper No. 89–16, University of California, San Diego, (1989).

Dumas, E. B. and Sengupta, J. K., "Nonparametric tests of portfolio efficiency under static and dynamic conditions", Working Paper No. 30–89, Department of Economics, University of California, Santa Barbara, (1989).

Engle, R. F. "Autoregressive conditional heteroskedasticity with estimates of the variances of U.K. inflation", *Econometrica*, Vol. 50 (1982), 987–1008.

Engle, R. F. and Bollerslev, T., "Modelling the persistence of conditional variances", *Econometric Reviews*, Vol. 5 (1986), 1–50.

Engle, R. F., and Mustafa, C., "Implied Arch models from options prices", Working Paper No. 89–29, University of California, San Diego, (1989).

Engle, R. F., Ng, V., and Rothschild, M., "A factor Arch model for stock returns", Working Paper No. 89–31, University of California, San Diego, (1989).

Hsu, D. A., "The behavior of stock returns: is it stationary or evolutionary?", *Journal of Financial and Quantitative Analysis*, Vol. 19 (1984), 11–28.

Karlin, S., "Some results on optimal partitioning of variance and monotonicity with truncation level", in *Statistics and Probability*, Amsterdam, North Holland (1982).

Kraus, A. and Litzenberger, R. H., "Skewness preference and the valuation of risk assets", *Journal of Finance*, Vol. 31 (1976), 1085–1100.

Merton, R. C., "On estimating the expected return on the market: an exploratory investigation", *Journal of Financial Economics*, Vol. 8 (1980), 323–361.

Rothschild, M., "Asset pricing theories", in *Uncertainty Information and Communication*, Cambridge, Cambridge University Press (1986).

Saunders, A., Ward, C., and Woodward, R., "Stochastic dominance and the performance of UK unit trusts", *Journal of Financial and Quantitative Analysis*, Vol. 15 (1980), 323–330.

Sears, R. and Trennepohl, G., "Skewness, sampling risk and the importance of diversification", *Journal of Economics and Business*, Vol. 38 (1986), 77–91.

Sengupta, J. K. and Park, H. S., *Portfolio efficiency tests based on stochastic dominance and cointegration*, Working Paper in Economics, University of California, Santa Barbara, (1990).

Sengupta, J. K. and Sfeir, R. E., "Modeling and testing for market volatility", forthcoming in *International Journal of Systems Science*: Special issue on Keynesianism vs. Monetarism, (1993).

Singleton, J. C. and Wingender, J. R., "Skewness persistence in common stock returns", *Journal of Financial and Quantitative Analysis*, Vol. 21 (1986), 335–352.

Stambaugh, R. F., "On the exclusion of asset from tests of the two-parameter model: a sensitivity analysis", *Journal of Financial Economics*, Vol. 12 (1982), 237–268.

Yaari, M. E., "Univariate and multivariate comparisons of risk aversion: a new approach", in *Uncertainty Information and Communication*, Cambridge, Cambridge University Press (1986).