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# OPTIMAL COMMODITY TAXATION UNDER MONOPOLISTIC COMPETITION

Yasuhito TANAKA\*

*Abstract:* This paper considers the optimal commodity taxation under monopolistic competition. The optimal commodity taxes are partitioned into three parts. The first is the tax revenue part, which is proportionate to the required tax revenue; the second is the tax shifting part, which depends on differences between industries in the degree of shifting of taxation, and the last is the product variation part which depends on differences between industries in the effects of taxes on the numbers of products.

## 1. INTRODUCTION

Recently several authors have theoretically or empirically investigated incidence of indirect taxes in oligopolistic situations. Dierickx, Matutes and Neven (1988) have shown that shifting of indirect taxes in an oligopoly with a fixed number of firms may exceed 100%. Stern (1987) and Besley (1989) have analyzed a case of free entry.<sup>1</sup> Katz and Rosen (1985) and Besley (1989) have presented some welfare analyses. Karp and Perloff (1989) have reported an empirical analysis about market structure and tax incidence in Japanese television market.<sup>2</sup>

These authors have not studied the problem of the optimal commodity taxes in an oligopoly. On the other hand the optimal commodity taxes under perfect competition have been studied by many authors.<sup>3</sup> Recently Myles (1989) has considered the optimal commodity taxes in an oligopoly with a homogeneous good, and shown that they are dependent on differences between industries in the degree of tax shifting.

This paper analyzes the optimal commodity taxes in the context of monopolistic competition. I consider several independent monopolistically competitive

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<sup>1</sup> Stern uses the term "monopolistic competition" as means a free entry oligopoly with a homogeneous good. In this paper it means an oligopolistic industry under free entry with endogenous number of differentiated goods.

<sup>2</sup> Levin (1985) has studied the role of taxation to control pollution in a Cournot oligopoly with a fixed number of firms. An earlier paper by Bishop (1968) has studied welfare effects of indirect taxes under perfect competition and monopoly, but he did not considered a case of oligopoly.

<sup>3</sup> See, for example, Ramsey (1927), Baumol and Bradford (1970), Lerner (1970), Dixit (1970), Sandmo (1974) and Sandmo (1976).

industries, and assume that the government determines commodity taxes on the monopolistically competitive goods so as to maximize the consumers' welfare subject to the constraint that the tax revenue is equal to a predetermined value. I assume that the government can not utilize taxes on labor income and lump sum taxation. I shall show that the optimal commodity taxes are partitioned into three parts. The first is the tax revenue part, which depends on the required tax revenue; the second is the tax shifting part, which depends on differences between industries in the degree of shifting of taxation, and the last is the product variation part which depends on differences between industries in the effects of taxes on the numbers of products.

The next section presents the model. In section 3 I derive formulas for the optimal commodity taxes under monopolistic competition. The last section concludes this paper. In the Appendix I present an example of the analysis in this paper.

## 2. THE MODEL

Consider an economy with  $n$  monopolistically competitive industries, I call each such industry the  $i$ -th industry,  $i=1, 2, \dots, n$ , in which there are  $m_i$ , which is endogenously determined, differentiated products and firms. For tractability I treat  $m_i$ 's as continuous numbers.

I use the following notation.

$x_{ij}$ : output of the  $j$ -th product in the  $i$ -th industry

$X_i$ : total output of the products in the  $i$ -th industry

$p_{ij}$ : price of the  $j$ -th product in the  $i$ -th industry

$t_i$ : specific commodity tax on the products in the  $i$ -th industry

I ignore the distributive considerations in this paper, and focus attention to the efficiency aspect of the optimal commodity taxes under monopolistic competition.<sup>4</sup> The prices of goods and tax revenues are expressed in units of labor, which serves as the *numeraire*.

Consumers' utility is represented by

$$w = \sum_{i=1}^n \phi_i(U_i) + Y = \sum_{i=1}^n \phi_i \left[ \sum_{j=1}^{m_i} u_i(x_{ij}) \right] + Y \quad (1)$$

where  $Y$  denotes leisure enjoyed by consumers. Each  $\phi_i(U_i)$  is increasing and strictly concave, each  $u_i(\cdot)$  is also increasing and strictly concave,<sup>5</sup> and  $u_i(0)=0$ ,  $\phi_i(0)=0$  for each  $i$ . Strict concavity of  $\phi_i$  implies that the products are substitutes in each industry. This utility function implies that demand for the products in one industry is independent of the products in the other industries. This specification of monopolistic competition is according to Spence (1976) and

<sup>4</sup> For analyses of the optimal commodity taxes under perfect competition which explicitly treat distributive issues, see Mirrlees (1975), Diamond (1975) and Atkinson and Stiglitz (1976).

<sup>5</sup> Strict concavity of  $u_i$  excludes the case where the products are homogeneous.

Mankiw and Whinston (1986).<sup>6</sup>

Consumers' budget constraint is written as

$$\sum_{i=1}^n \sum_{j=1}^{m_i} p_{ij} x_{ij} + Y = L \quad (2)$$

where  $L$  is the maximum labor supply, so  $L - Y$  is the actual labor supply, and is equal to labor income.

From (1) and (2), consumers' utility maximization yields

$$p_{ij} = \phi'_i u'_i(x_{ij}), \quad i = 1, 2, \dots, n \quad \text{and} \quad j = 1, 2, \dots, m_i \quad (3)$$

This is the inverse demand function for the  $j$ -th product in the  $i$ -th industry.

The cost function of the firms in the  $i$ -th industry is  $c_i(x_{ij})$  which is twice differentiable. I assume that all firms have the same cost function in each industry. The cost function has the increasing returns to scale property, that is, the average cost of each firm is decreasing in its output, which implies that the marginal cost is always smaller than the average cost. In an equilibrium under monopolistic competition, each firm sets its output so that the marginal revenue, which is smaller than the price, is equal to the marginal cost. On the other hand the price should be equal to the average cost from free entry. Thus the marginal cost must be smaller than the average cost in an equilibrium.

The profit of the  $j$ -th firm in the  $i$ -th industry is represented as

$$\pi_{ij} = (p_{ij} - t_i)x_{ij} - c_i(x_{ij}) = [\phi'_i u'_i(x_{ij}) - t_i]x_{ij} - c_i(x_{ij}), \\ i = 1, 2, \dots, n \quad \text{and} \quad j = 1, 2, \dots, m_i$$

Each firm selects its quantity,  $x_{ij}$ , so as to maximize its profit given  $\phi'_i(U_i)$ . Under the assumption that  $m_i$  is large for each  $i$ ,  $U_i$  is given for each firm in the  $i$ -th industry. The first order conditions for profit maximization are

$$\phi'_i(u'_i + x_{ij}u''_i) - c'_i - t_i = 0, \quad i = 1, 2, \dots, n, \quad \text{and} \quad j = 1, 2, \dots, m_i \quad (4)$$

The second order conditions are

$$\phi'_i(2u''_i + x_{ij}u'''_i) - c''_i < 0, \quad i = 1, 2, \dots, n, \quad \text{and} \quad j = 1, 2, \dots, m_i \quad (5)$$

Under free entry we have the following zero profit conditions for the firms in the  $i$ -th industry,

$$[\phi'_i u'_i(x_{ij}) - t_i]x_{ij} - c_i(x_{ij}) = 0, \quad i = 1, 2, \dots, n \quad \text{and} \quad j = 1, 2, \dots, m_i \quad (6)$$

(4), (5) and (6) define the equilibrium of the  $i$ -th industry.

The firms are identical in each industry, hence we have a symmetric equilibrium in each industry. In a symmetric equilibrium we have  $\phi_i(U_i) = \phi_i[m_i u_i(x_i)]$  where

<sup>6</sup> Spence (1976) calls this specification of monopolistic competition "the generalized CES case". Dixit and Stiglitz (1977) presents a similar model of monopolistic competition. In the international economics literature, Krugman (1979), (1980) and Gros (1987) have used a simplified version of this specification.

$x_i$  is the output per firm (or equivalently per product) in the  $i$ -th industry, and the prices are equal over all products in each industry. I denote the equilibrium price of the products in the  $i$ -th industry by  $p_i$ . Selection of  $t_i$  for each  $i$  by the government determines  $x_i$  and  $m_i$ . Then the equilibrium price is obtained by

$$p_i = \phi'_i[m_i u_i(x_i)] u'_i(x_i).$$

Differentiating (4) and (6) with  $x_{ij} = x_i$  and  $\phi_i(U_i) = \phi_i[m_i u_i(x_i)]$  with respect to  $t_i$ ,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} dx_i \\ dm_i \end{pmatrix} = \begin{pmatrix} 1 \\ x_i \end{pmatrix} dt_i, \quad i = 1, 2, \dots, n \quad (7)$$

where

$$A = \phi'_i(2u'_i + x_i u''_i) + m_i \phi''_i u'_i(u'_i + x_i u''_i) - c''_i$$

$$B = \phi''_i u_i(u'_i + x_i u''_i)$$

$$C = m_i x_i \phi''_i (u'_i)^2$$

and

$$D = x_i \phi''_i u_i u'_i$$

Then we obtain

$$\frac{dx_i}{dt_i} = \frac{1}{\Delta_i} (D - x_i B) = -\frac{1}{\Delta_i} x_i^2 \phi''_i u_i u''_i, \quad i = 1, 2, \dots, n \quad (8)$$

and

$$\frac{dm_i}{dt_i} = \frac{1}{\Delta_i} (x_i A - C) = \frac{1}{\Delta_i} \{x_i [\phi'_i(2u'_i + x_i u''_i) - c''_i] + m_i x_i^2 \phi''_i u'_i u''_i\}, \quad i = 1, 2, \dots, n \quad (9)$$

where

$$\Delta_i = AD - BC = x_i \phi''_i u_i u'_i [\phi'_i(2u'_i + x_i u''_i) - c''_i]$$

This is positive from (5) and  $\phi''_i < 0$ , hence (8) is negative since  $u''_i < 0$ , so the output per firm in each industry is decreased by an increase in tax. On the other hand, the sign of (9) is ambiguous since the second term in the parentheses is positive. If (9) is negative (or positive), the number of products in the  $i$ -th industry is decreased (or increased) by an increase in tax.

From (8) and (9) the responses of the equilibrium price and the total output in the  $i$ -th industry to a change in tax are obtained as

$$\frac{dp_i}{dt_i} = [\phi'_i u''_i + m_i \phi''_i (u'_i)^2] \frac{dx_i}{dt_i} + \phi''_i u_i u'_i \frac{dm_i}{dt_i}$$

$$= \frac{1}{\Delta_i} \{x_i \phi_i'' u_i u_i' [\phi_i'(2u_i'' + x_i u_i''') - c_i''] - x_i^2 \phi_i' \phi_i'' u_i (u_i'')^2\},$$

$$i = 1, 2, \dots, n \quad (10)$$

and

$$\frac{dX_i}{dt_i} = m_i \frac{dx_i}{dt_i} + x_i \frac{dm_i}{dt_i} = \frac{1}{\Delta_i} \{x_i^2 [\phi_i'(2u_i'' + x_i u_i''') - c_i''] - m_i x_i^2 \phi_i'' u_i'' (u_i - x_i u_i')\}, \quad i = 1, 2, \dots, n \quad (11)$$

(10) is obtained from  $p_i = \phi_i'[m_i u_i(x_i)] u_i'(x_i)$ . It is unambiguously positive since  $\phi_i'' < 0$ , so the commodity tax is unambiguously shifted to consumers. From strict concavity of  $u_i$  and  $u_i(0) = 0$ , we have  $u_i - x_i u_i'(x_i) > 0$ .<sup>7</sup> Thus (11) is unambiguously negative, and hence the total output in each industry is decreased by an increase in tax.

### 3. THE OPTIMAL COMMODITY TAXES

The government determines the commodity tax,  $t_i$  for each  $i$ ,<sup>8</sup> so as to maximize the consumers' welfare subject to the constraint that the tax revenue is equal to a predetermined value,  $T$ . Formally

$$\begin{aligned} \max w &= \sum_{i=1}^n \phi_i[m_i u_i(x)] + Y \\ \text{s.t. } &\sum_{i=1}^n t_i m_i x_i = T \end{aligned}$$

Writing this problem by a Lagrange function,

$$\Psi = \sum_{i=1}^n \phi_i[m_i u_i(x)] + Y + \lambda \left( \sum_{i=1}^n t_i m_i x_i - T \right)$$

$\lambda$  is a Lagrange multiplier. Substituting the consumers' budget constraint, (2), into this,

$$\Psi = \sum_{i=1}^n \phi_i[m_i u_i(x)] - \sum_{i=1}^n p_i m_i x_i + L + \lambda \left( \sum_{i=1}^n t_i m_i x_i - T \right)$$

Then we obtain the first order conditions as follows,

$$\frac{\partial \Psi}{\partial t_i} = \phi_i'(u_i - x_i u_i') \frac{dm_i}{dt_i} - X_i \frac{dp_i}{dt_i} + \lambda \left( X_i + t_i \frac{dX_i}{dt_i} \right) = 0, \quad i = 1, 2, \dots, n \quad (12)$$

<sup>7</sup> For details, see Takayama (1984), Theorem 1.C.3.

<sup>8</sup> Since the demand functions for the products are symmetric, and all firms have the same cost function in each industry, the optimal commodity taxes for all products should be equal in each industry.

where  $X_i = m_i x_i$ . In derivation of (12) I use  $p_i = \phi'_i u'_i$ .

The consumers' surplus from the products in the  $i$ -th industry is written as

$$CS_i = \phi_i[m_i u(x_i)] - p_i m_i x_i, \quad i = 1, 2, \dots, n$$

Let define

$$\theta_i = \frac{1}{X_i} \frac{\partial CS_i}{\partial m_i} \frac{dm_i}{dt_i} = \frac{1}{X_i} [\phi'_i(u_i - x_i u'_i)] \frac{dm_i}{dt_i}, \quad i = 1, 2, \dots, n$$

and

$$\eta_i = \frac{dCS_i}{dt_i} = \phi'_i(u_i - x_i u'_i) \frac{dm_i}{dt_i} - X_i \frac{dp_i}{dt_i}, \quad i = 1, 2, \dots, n$$

where I use  $p_i = \phi'_i u'_i$ .  $\theta_i$  is negative (or positive) if  $dm_i/dt_i < (\text{or } >) 0$  since  $u_i - x_i u'_i > 0$ . I assume that  $dp_i/dt_i$ , which is positive, is dominant even when  $dm_i/dt_i > 0$ , and  $\eta_i$  is negative for all  $i$ .<sup>9</sup> Now I abbreviate the notations as  $X_{it} = dX_i/dt_i$ ,  $m_{it} = dm_i/dt_i$  and  $p_{it} = dp_i/dt_i$ .

From (12) we obtain

$$\lambda = -\frac{1}{X_i + t_i X_{it}} \eta_i, \quad i = 1, 2, \dots, n \quad (13)$$

The denominator of (13) is the marginal effect of an increase in tax on the tax revenue in the  $i$ -th industry. It must be positive at the optimum. If it is negative, it would be possible to raise more tax revenue and increase consumers' utility by reducing taxes, then initial tax could not have been optimal. (13) implies that the optimal commodity taxes should be set so that a marginal reduction in the consumers' surplus per an increase in the tax revenue in each industry is equal.

Alternatively from (12)

$$t_i = -\frac{1}{X_{it}} \left\{ \frac{1}{\lambda} [\phi'_i(u_i - x_i u'_i) m_{it} - X_i p_{it}] + X_i \right\}, \quad i = 1, 2, \dots, n \quad (14)$$

Substituting (14) into the tax revenue constraint,

$$-\frac{1}{\lambda} \sum_{i=1}^n \left\{ \frac{X_i}{X_{it}} [\phi'_i(u_i - x_i u'_i) m_{it} - X_i p_{it}] \right\} = \sum_{i=1}^n \left( \frac{X_i^2}{X_{it}} \right) + T$$

Then we obtain

$$\frac{1}{\lambda} = -\frac{1}{T} \left[ \sum_{i=1}^n \left( \frac{X_i^2}{X_{it}} \right) + T \right] \quad (15)$$

where

<sup>9</sup>  $\theta_i$  is the marginal effect of an increase in tax on the consumers' surplus in the  $i$ -th industry per output solely through a change in the number of products in that industry.  $\eta_i$  is the total effect of an increase in tax on the consumers' surplus in the  $i$ -th industry.  $\eta_i < 0$  means that an increase in tax reduces the consumers' surplus.

$$\Gamma = \sum_{i=1}^n \left( \frac{X_i}{X_{it}} \right) [\phi'_i(u_i - x_i u'_i) m_{it} - X_i p_{it}] = \sum_{i=1}^n \frac{X_i}{X_{it}} \eta_i$$

This is positive since  $X_{it}$  and  $\eta_i$  are negative.

Substituting (15) into (14),

$$\begin{aligned} t_i &= \frac{1}{X_{it} \Gamma} \left\{ \left[ \sum_{k=1}^n \left( \frac{X_k^2}{X_{kt}} \right) + T \right] [\phi'_i(u_i - x_i u'_i) m_{it} - X_i p_{it}] - \Gamma X_i \right\} \\ &= \frac{1}{X_{it} \Gamma} \left\{ T [\phi_i(u_i - x_i u'_i) m_{it} - X_i p_{it}] + X_i \left[ \frac{1}{X_i} \phi_i(u_i - x_i u'_i) m_{it} - p_{it} \right] \sum_{k=1}^n \left( \frac{X_k^2}{X_{kt}} \right) \right. \\ &\quad \left. - X_i \sum_{k=1}^n \left( \frac{X_k^2}{X_{kt}} \right) \left[ \frac{1}{X_k} \phi_k(u_k - x_k u'_k) m_{kt} - p_{kt} \right] \right\} \\ &= \frac{1}{X_{it} \Gamma} \left\{ T \eta_i - X_i \gamma \left[ p_{it} - \frac{1}{\gamma} \sum_{k=1}^n \left( \frac{X_k^2}{X_{kt}} \right) p_{kt} \right] + X_i \gamma \left[ \theta_i - \frac{1}{\gamma} \sum_{k=1}^n \left( \frac{X_k^2}{X_{kt}} \right) \theta_k \right] \right\}, \\ &\quad i = 1, 2, \dots, n \end{aligned} \quad (16)$$

where

$$\gamma = \sum_{k=1}^n \left( \frac{X_k^2}{X_{kt}} \right)$$

This is negative since  $X_{it} < 0$ . From (16) the optimal commodity taxes are partitioned into three parts as follows,

$$t_i^* = t_i^*(r) + t_i^*(s) + t_i^*(v)$$

with

$$t_i^*(r) = \frac{1}{\Gamma} \frac{\eta_i}{X_{it}} T, \quad i = 1, 2, \dots, n$$

$$t_i^*(s) = -\frac{X_i}{X_{it} \Gamma} \gamma [p_{it} - WA(p_{kt})], \quad i = 1, 2, \dots, n$$

and

$$t_i^*(v) = \frac{X_i}{X_{it} \Gamma} \gamma [\theta_i - WA(\theta_k)], \quad i = 1, 2, \dots, n$$

where

$$WA(p_{kt}) = \frac{1}{\gamma} \sum_{k=1}^n \left( \frac{X_k^2}{X_{kt}} \right) p_{kt}$$

is the weighted average of the degrees of tax shifting over all industries, and



$$WA(\theta_k) = \frac{1}{\gamma} \sum_{k=1}^n \left( \frac{X_k^2}{X_{kt}} \right) \theta_k$$

is the weighted average of  $\theta_i$  defined above over all industries. The weight for the  $k$ -th industry in both weighted averages is  $(1/\gamma)(X_k^2/X_{kt})$ .

Now consider the implications of each part.

(1)  $t_i^*(r)$ : the tax revenue part.

This part is non negative, and depends on the required tax revenue. If we confine us to the case of balanced budget ( $T=0$ ), this part vanishes. As we shall see below, the total tax revenue from this part over all industries is equal to the required tax revenue.

(2)  $t_i^*(s)$ : the tax shifting part.

The sign of this part is determined by the relation between  $p_{it}$ , which represents the degree of tax shifting in the  $i$ -th industry, and its weighted average over all industries  $WA(p_{kt})$ . As I have shown above,  $p_{it}$  is positive, so  $WA(p_{kt})$  is also positive. Since  $X_{it}$  and  $\gamma$  are negative, and  $\Gamma$  is positive, the formula of this part implies that the smaller (or larger) the degree of tax shifting in an industry is, the larger (or smaller) the tax rate on the products in that industry should be.

(3)  $t_i^*(v)$ : the product variation part.

This part is unique to monopolistic competition. An increase or decrease in the number of products in the industries directly affects the consumers' welfare under monopolistic competition. The sign of this part is determined by the relation between  $\theta_i$ , which represents the marginal effect of an increase in tax on the consumers' surplus per output through a change in the number of products in the  $i$ -th industry, and its weighted average over all industries  $WA(\theta_k)$ . Since  $X_{it}$  and  $\gamma$  are negative, and  $\Gamma$  is positive, the formula of this part implies that the more (or less) an increase in tax increases the number of products in an industry, the larger (or smaller) the tax rate on the products in that industry should be.

We find

$$\sum_{i=1}^n t_i^*(s) X_i = - \sum_{i=1}^n \frac{X_i^2}{X_{it} \Gamma} \gamma [p_{it} - WA(p_{kt})] = - \frac{\gamma^2}{\Gamma} \left[ \frac{1}{\gamma} \sum_{i=1}^n \frac{X_i^2}{X_{it}} p_{it} - WA(p_{kt}) \right] = 0$$

and

$$\sum_{i=1}^n t_i^*(v) X_i = \sum_{i=1}^n \frac{X_i^2}{X_{it} \Gamma} \gamma [\theta_i - WA(\theta_k)] = \frac{\gamma^2}{\Gamma} \left[ \frac{1}{\gamma} \sum_{i=1}^n \frac{X_i^2}{X_{it}} \theta_i - WA(\theta_k) \right] = 0$$

Thus the net tax revenues from the tax shifting and the product variation parts are zero. We can also find

$$\sum_{i=1}^n t_i^*(r) X_i = T$$

This means that the total tax revenue from the tax revenue parts is equal to the required tax revenue.

#### 4. CONCLUDING REMARKS

I have obtained the formulas for the optimal commodity taxes under monopolistic competition. The optimal taxes are partitioned into three parts. We find that the smaller (or larger) the degree of tax shifting in an industry is, the larger (or smaller) the tax rate on the products in that industry should be, and that the more (or less) an increase in tax increases the number of products in an industry, the larger (or smaller) the tax rate on the products in that industry should be.

In this paper I have considered only the case where the demand functions for the products in an industry are independent of the products in the other industries. Such assumption clarifies the implications of each part of the optimal commodity taxes.

Ramsey (1927), Lerner (1970) and Dixit (1970) have shown that under perfect competition with decreasing returns to scale and independent demand and supply functions, the rate of the optimal commodity tax for each good is characterized as to be proportional to a combination of the inverse of demand elasticity and the inverse of supply elasticity. Thus the optimal tax for an good depends only on the properties of demand and supply of that good. Contrasting to them I have shown that under monopolistic competition with increasing returns to scale, the tax shifting and the product variation parts of the optimal commodity taxes depend on differences between industries in the effects of taxes on the prices of the products or on the numbers of products in the industries even when the demand and cost functions are independent.

#### APPENDIX: AN EXAMPLE

Consider two industries, and assume the following utility function of consumers and the cost function of firms:

$$\phi_i = \left[ \sum_{j=1}^{m_i} (x_{ij}^{\alpha_j}) \right]^{\beta_i}, \quad 0 < \alpha_i < 1 \quad \text{and} \quad 0 < \beta_i < 1, \quad i = 1, 2 \quad (\text{A.1})$$

and

$$c(x_{ij}) = kx_{ij} + f, \quad i = 1, 2 \quad (\text{A.2})$$

I assume  $\alpha_1 > \alpha_2$ . This means that the goods of the industry 1 are more substitutable than the goods of the industry 2.  $k$  is the common constant marginal cost, and  $f$  is the fixed cost. As an illustration I consider the tax shifting parts of the optimal taxes.

Substituting (A.1) and (A.2) into the equilibrium conditions for the industries, (4) and (6), we derive

$$\beta_i[m_i(x_i^{\alpha_i})]^{\beta_i-1}\alpha_i^2x_i^{\alpha_i-1}-k-t_i=0, \quad i=1, 2 \quad (\text{A.3})$$

$$\{\beta_i[m_i(x_i^{\alpha_i})]^{\beta_i-1}\alpha_i x_i^{\alpha_i-1}-k-t_i\}x_i-f=0, \quad i=1, 2 \quad (\text{A.4})$$

Then we obtain the equilibrium output per firm and the equilibrium price of the good for each industry as follows:

$$x_i = \frac{\alpha_i f}{(1-\alpha_i)(k+t_i)}, \quad i=1, 2$$

and

$$p_i = \frac{1}{\alpha_i} (k+t_i), \quad i=1, 2$$

Then we derive

$$\frac{dp_i}{dt_i} = p_i = \frac{1}{\alpha_i}, \quad i=1, 2 \quad (\text{A.5})$$

This and the assumption,  $\alpha_1 > \alpha_2$ , imply

$$t_1^*(s) > 0 \quad \text{and} \quad t_2^*(s) < 0$$

Therefore the tax to the industry with more substitutable goods should be higher than the tax to the industry with less substitutable goods.

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