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FOLLOWER'S ADVANTAGE IN LABOR-MANAGED DUOPOLY

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Abstract: Follower is more advantagous than leader in both labor-managed Bertrand-Stackelberg and Cournot-Stackelberg duopolies with product differentiation. This result is in sharp contrast with the one for duopoly comprising only profit-maximizing firms. In Bertrand-Stackelberg and Cournot-Stackelberg duopolies, follower and leader have relative advantages, respectively, if they are both profit-maximizers.

1. INTRODUCTION

It is well known for Stackelberg duopoly with symmetric profit-maximizing firms and product differentiation that if firms' strategies are outputs (Cournot-Stackelberg duopoly), leadership is more advantageous than followership; that if firms adopt price strategies (Bertrand-Stackelberg duopoly), relative advantage accrues to follower. In the case of Cournot output-adjusting duopoly with product differentiation, reaction functions are downward-sloping. On the other hand, they are upward-sloping for Bertrand price-adjusting duopoly. The relative advantage of followership in Bertrand-Stackelberg duopoly with product differentiation has been first noticed by Krelle (1971) for a linear model. However, his pioneering contribution has remained unnoticed in non-German speaking countries. Siman and Cruz (1973), and Okuguchi (1976) have given a general proof for the existence of the equilibrium in a Stackelberg leader-follower model. Gal-Or (1985) has established a general result for a symmetric non-cooperative, two-person game that leader (follower) has advantage over follower (leader) if the players' reaction functions are both downward (upward)-sloping.

To the best of knowledge no one has analyzed whether leader (follower) is more advantageous than follower (leader) in the framework of labor-managed duopoly, where two firms are assumed to maximize dividend per unit of labor instead of their profits. The first theoretical analysis of a labor-managed firm has been conducted by Ward (1958). Since then many economists have modified or extended the Ward model. The most comprehensive surveys on labor-managed firms are Ireland and Law (1982), and Bonin and Putterman (1987). Vanek (1970), and Ireland and Law (1982) have observed that reaction functions may slope upward in Cournot duopoly with only labor-managed firms and *without*

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product differentiation. The reaction function for a labor-managed firm may be upward-sloping also in duopoly comprising one profit-maximizing firm and one labor-managed firm (see Miyamoto (1982) and Okuguchi (1993)). The behaviors of some Japanese firms are better understood if they are characterized as labor-managed firms (see Sakai (1993) for the releavant literature for this interpretation).

In this paper we will formulate two models of duopoly with product differentiation and with only labor-managed firms, in one of which two firms' strategies are outputs (labor-managed Cournot duopoly) and prices become strategic variables in the other (labor-managed Bertrand duopoly). We will prove that the reaction functions are upward-sloping under general conditions in both labor-managed Bertrand and Cournot duopolies with product differentiation.[1] Therefore, if two firms are symmetric, followership is more advantageous than leadership in both labor-managed Bertrand-Stackelberg and Cournot-Stackelberg duopolies.

2. LABOR-MANAGED BERTRAND DUOPOLY

In this section we consider Bertrand duopoly where two firms which produce differentiated goods try to maximize their dividends per unit of labor by optimally adjusting their respective prices assuming that their rival's price will not be affected by a change in their own price.

Let firm i $(i=1,2)$ have the following inverse production function (1) and ordinary demand function (2), where l_i, x_i, and p_i are firm i's labor, output, and price, respectively.

$$l_i = h_i(x_i)\,, \quad h_i' > 0\,, \quad h_i'' > 0\,, \qquad i=1,2\,, \tag{1}$$

$$x_i = g^i(p_1, p_2)\,, \qquad i=1,2\,, \tag{2}$$

where

$$\partial g^i/\partial p_i \equiv g_i^i < 0\,, \quad \partial g^i/\partial p_j \equiv g_j^i > 0\,, \qquad i \neq j,\ i,j=1,2\,, \tag{3.1}$$

$$-g_i^i > g_j^i\,, \qquad i \neq j\,, \quad i,j=1,2\,. \tag{3.2}$$

Inequality (3.1) implies that the own-price effect is negative and the two products are substitutes. According to (3.2), the own-price effect dominates the cross one.

If k_i is firm i's fixed cost, its dividend s_i^B (B for Bertrand) per unit of labor is given by

$$\begin{aligned} s_i^B &\equiv (p_i x_i - k_i)/h_i(x_i) \\ &= (p_i g^i(p_1, p_2) - k_i)/h_i(g^i(p_1, p_2))\,, \qquad i=1,2\,. \end{aligned} \tag{4}$$

[1] See Singh and Vives (1984), Vives (1985), Cheng (1985) and Okuguchi (1987) for Cournot vs Bertrand in differentiated oligopoly.

If the maximum is interior, the first order condition for maximization of s_i^B with respect to p_i yields

$$\begin{aligned} \partial s_i^B/\partial p_i &= \{h_i(g^i(p_1, p_2))(g^i(p_1, p_2)+p_i g_i^i(p_1, p_2)) \\ &\quad -(p_i g^i(p_1, p_2)-k_i)h_i'(g^i(p_1, p_2))g_i^i(p_1, p_2)\}/h_i^2(g^i(p_1, p_2)) \\ &\equiv u^{iB}(p_1, p_2)/h_i^2(g^i(p_1, p_2)) \\ &=0, \qquad i=1,2. \end{aligned} \tag{5}$$

Because of (5), the second order condition

$$\partial^2 s_i^B/\partial p_i^2<0, \qquad i=1,2 \tag{6}$$

can be shown to be equivalent to

$$\partial u^{iB}/\partial p_i<0, \qquad i=1,2. \tag{7}$$

We assume that (7) is satisfied. On the other hand, we derive

$$\begin{aligned} \partial u^{iB}/\partial p_j &= h_i' g^i g_j^i+(g_j^i+p_i g_{ij}^i)h_i-(p_i g^i-k_i)h_i'' g_i^i g_j^i \\ &\quad -(p_i g^i-k_i)h_i' g_{ij}^i, \qquad i\neq j, \qquad i,j=1,2. \end{aligned} \tag{8}$$

The sign of the RHS of (8) is, in general, indeterminate even if

$$s_i^B h_i=p_i g^i-k_i\geq 0, \qquad i=1,2 \tag{9}$$

has to hold for labor-managed firms to be viable. We therefore have to impose the following restrictions on the demand functions.

Assumption 1. $g_{ij}^i\leq 0$, $i\neq j$, $i,j=1,2$.

Assumption 2. $g_j^i+p_i g_{ij}^i>0$, $i\neq j$, $i,j=1,2$.[2]

If g^i is linear, both assumptions are satisfied. The Assumption 2 requires two products to be strategic complements for each other. The Assumptions 1 and 2 coupled with (1) and (3.1) lead to

$$\partial u^{iB}/\partial p_j>0, \qquad i\neq j, \qquad i,j=1,2. \tag{8'}$$

Taking into account (5), (7) and (8′), the sign of firm i's reaction function is evaluated as follows.

$$dp_i/dp_j=-\partial u^{iB}/\partial p_j/\partial u^{iB}/\partial p_i>0, \qquad i\neq j, \quad i,j=1,2. \tag{10}$$

[2] Let the demand functions for the two firms be

$$g^1=p_1^{-\alpha_1}p_2^{\beta_1},$$
$$g^2=p_1^{\alpha_2}p_2^{-\beta_2},$$

where α_1, β_1, α_2 and β_2 are all positive constants. We can easily confirm that this assumption is satisfied for firm 1 if $0<\alpha_1<1$ and for firm 2 if $0<\beta_2<1$. Furthermore, these functions satisfy the Assumption 1 with strict inequality. However, the Assumption 3 below holds also with strtict inequality provided $\alpha_1\beta_2-\alpha_2\beta_1>0$ were satisfied.

Hence under our assumptions, the reaction functions are upward-sloping for labor-managed Bertrand duopoly with product differentiation regardless of whether two firms are symmetric or not.

3. LABOR-MANAGED COURNOT DUOPOLY

In this section labor-managed Cournot duopolists are assumed to maximize their dividends per unit of labor adopting outputs as their strategic variables. If the domain of the demand functions (2) is a rectangular region, the assumptions (3.1) and (3.2) enable us to invert them to yield

$$p_i = f^i(x_1, x_2)\,, \qquad i=1,2\,, \tag{11}$$

where

$$\partial f^i/\partial x_j = f^i_j < 0\,, \quad i,j=1,2\,. \tag{12}$$

Firm i's dividens s^{iC} (C for Cournot) per unit of labor is now

$$s^C_i \equiv (x_i f^i(x_1, x_2) - k_i)/h_i(x_i)\,, \qquad i=1,2\,. \tag{13}$$

Maximizing (13) with respect to x_i, the first order condition is derived as

$$\begin{aligned} \partial s^C_i/\partial x_i &= \{h_i(x_i)(f^i(x_1, x_2) + x_i f^i_i(x_1, x_2)) \\ &\qquad - (x_i f^i(x_1, x_2) - k_i)h'_i(x_i)\}/h^2_i(x_i) \\ &\equiv u^{iB}(x_1, x_2)/h^2_i(x_i) \\ &= 0\,, \qquad i=1,2\,. \end{aligned} \tag{14}$$

The second order condition

$$\partial^2 s^{iC}/\partial x^2_i < 0\,, \qquad i=1,2\,, \tag{15}$$

which is equivalent to

$$\partial u^{iC}/\partial x_i < 0\,, \qquad i=1,2 \tag{16}$$

is assumed to be satisfied. Partially differentiate u^{iC} with respect to x_j and rearrange to yield

$$\partial u^{iC}/\partial x_j = (l_i/x_i - h'_i)x_i f^i_j + h_i x_i f^i_{ij}\,, \qquad i \neq j\,, \quad i,j=1,2\,. \tag{17}$$

Since $h''_i > 0$ and $f^i_j < 0$, the first term on RHS of (17) is positive, but the sign of f^i_{ij} is indeterminate. Thus we introduce the following

Assumption 3. $f^i_{ij} \geq 0$, $i \neq j$, $i,j=1,2$.

This assumption holds if f^i is linear. From (17) we now have

$$\partial u^{iC}/\partial x_j > 0\,, \qquad i \neq j\,, \quad i,j=1,2\,. \tag{17'}$$

Hence (14), (16) and (17) yield

$$dx_i/dx_j = -\partial u^{iC}/\partial x_j / \partial u^{iC}/\partial x_i > 0\,, \qquad i \neq j\,, \quad i, j = 1, 2\,, \tag{18}$$

implying the positively-sloped reaction functions in labor-managed Cournot quantity-adjusting duopoly with product differentiation.

4. CONCLUSION

Since under our assumptions, the reaction functions are positively-sloped in both labor-managed Bertrand and Cournot duopolies with product differentiation, we can conclude advantage of followership on the basis of the result in Gal-Or (1985) for both Bertrand-Stackelberg and Cournot-Stackelberg duopolies with symmetric labor-managed firms and with product differentiation. This result is in sharp contrast with the one known for duopoly comprising only profit-maximizing firms. In Bertrand-Stackelberg and Cournot-Stackelberg duopolies, follower and leader enjoy relative advantage, respectively, if they are both profit-maximizers.

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REFERENCES

Bonin, J. P. and L. Putterman (1987), *Economics of Cooperataion and the Labor-Managed Economy*, Harwoods Academic Publishers, New York.

Cheng, L. (1985), "Comparing Bertrand and Cournot Equilibria: A Geometric Approach," *Rand Journal of Economics*, 16, 146–152.

Ireland, N. J. and P. J. Law (1982), *The Economics of Labour-Managed Enterprises*, Croom Helm, London.

Krelle, W. (1971), *Preistheorie*, 1, J. C. B. Mohr (Paul Siebeck), Tübingen (1. Auflage, 1961).

Miyamoto, Y. (1982), "A Labor-Managed Firm's Reaction Function Reconsidered," *mimeo*, Osaka City University.

Okuguchi, K. (1976), *Expectations and Stability in Oligopoly Models*, Springer-Verlag, Heidelberg and New York.

Okuguchi, K. (1976), "Labor-Managed Bertrand and Cournot Oligoploies," *Journal of Economics* (formerly *Zeitschrift für Nationalokönomie*), 46, 115–122.

Okuguchi, K. (1987), "Equilibrium Prices in the Bertrand and Cournot Oligopolies," *Journal of Economic Theory*, 42, 128–139.

Okuguchi, K. (1993), "Cournot Oligopoly with Profit-Maximizing and Labor-Managed Firms," *Keio Economic Studies*, 30, 27–38.

Sakai, Y. (1993), "The Role of Information in Profit-Maximizing and Labor-Managed Duopoly Models," *Managerial and Decision Economics*, 14, 419–432.

Siman, M. and J. B. Cruz, Jr. (1973), "On the Stackelberg Strategy in Non-Zero Sum Games," *Journal of Optimization Theory and Applications*, 11, 533–555.

Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly," *Rand Journal of Economics*, 15, 546–554.

Vives, X. (1985), "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation," *Journal of Economic Theory*, 36, 166–175.

Ward, B. (1958), "The Firm in Illyria: Market Syndicalism," *American Economic Review*, 68, 566—589.