Title	ON THE EFFECTS OF COMMODITY TAX IN FREE ENTRY OLIGOPOLY
Sub Title	
Author	TANAKA, Yasuhito
Publisher	Keio Economic Society, Keio University
Publication year	1993
Jtitle	Keio economic studies Vol.30, No.1 (1993.) ,p.43- 52
JaLC DOI	
Abstract	In this paper I analyze the welfare effects of an ad-varolem commodity tax in a free entry Cournot oligopoly. I will show that if the demand function is concave or linear, or not too convex, the tax burden (the loss of consumers' surplus) by a small ad-varolem commodity tax is smaller than an increase in the tax revenue by the tax. I will also show that, with a linear cost function, the tax burden by an ad-varolem tax, which is not small, is smaller than the tax burden by a specific tax which raises the equal tax revenue as the ad-varolem tax.
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19930001-0043

ON THE EFFECTS OF COMMODITY TAX IN FREE ENTRY OLIGOPOLY

Yasuhito TANAKA*

Abstract: In this paper I analyze the welfare effects of an ad-varolem commodity tax in a free entry Cournot oligopoly. I will show that if the demand function is concave or linear, or not too convex, the tax burden (the loss of consumers' surplus) by a small ad-varolem commodity tax is smaller than an increase in the tax revenue by the tax. I will also show that, with a linear cost function, the tax burden by an ad-varolem tax, which is not small, is smaller than the tax burden by a specific tax which raises the equal tax revenue as the ad-varolem tax.

1. INTRODUCTION

Recently several authors, such as Katz and Rosen (1985), Stern (1987) and Dierickx, Matutes and Neven (1988), have theoretically investigated the incidence of indirect taxes in an oligopolistic situation. In particular, Besley (1989), assuming that tax revenues are distributed to consumers in a lump sum fashion, has shown that a small specific commodity tax in a free entry Cournot oligopoly raises the welfare of consumers if and only if the inverse demand function is strictly concave. This is equivalent to that the tax burden (the loss of consumers' surplus) by a small specific commodity tax is smaller than an increase in the tax revenue by the tax if and only if the inverse demand function is strictly concave. In this paper I analyze the welfare effects of an ad-varolem commodity tax in a free entry Cournot oligopoly. I will show that if the inverse demand function is concave or linear, or not too convex, the tax burden by a small ad-varolem commodity tax is samller than an increase in the tax revenue by the tax. This implies that a small ad-varolem tax more likely improve the welfare than a small specific tax. I will also show that, with a linear cost function, the tax burden by an ad-varolem tax, which is not small, is smaller than the tax burden by a specific tax which raises the equal tax revenue as the ad-varolem tax.

In the next section I present the model and consider the equilibrium conditions. In section III I examine the welfare effects of a small ad-varolem tax. In section IV I compare the tax burden by a specific tax and that by an ad-varolem tax in the case where these taxes raise equal tax revenues. The last section concludes this paper.

* I wish to thank an anonymous referee for his valuable comments on an earlier version of this paper. Any remaining errors are, of course, my own.

2. THE MODEL AND EQUILIBRIUM CONDITIONS

I consider a free entry oligopoly. There are n identical firms which produce a homogeneous good. The firms are Nash-Cournot players.

The inverse demand function is represented by

$$p=p(X), \quad X=\sum_{i=1}^{n} x_i$$

where p is the (consumer) price of the good, x_i is the output of Firm i, and X is the total output. I assume $p'<0$, that is, the demand curve is downward sloping. Entry into the industry is free, and n is endogenously determined so that the maximized profits of the firms are zero. I treat n as a continuous variable. An ad-varolem tax, T, is imposed on the consumption of the good. Then the producer price (consumer price net tax) of the good is

$$\bar{p}=\frac{p}{1+T} \tag{1}$$

Firm i has the cost function, $c(x_i)$, which is common to all firms. I assume that the marginal cost and the fixed cost are positive, that is, $c'>0$ and $c(0)>0$. And, throughout this paper, I assume the existence of a unique symmetric equilibrium of a free entry oligopoly. We need that, at an equilibrium, the average cost is decreasing so that there exists an equilibrium in a free entry oligopoly.

The profit of Firm i is represented as

$$\pi_i=\frac{p}{1+T}x_i-c(x_i) \tag{2}$$

Firm i determines its output given the total coutput of the other firms.

Then the first order condition for profit maximization for Firm i is obtained as follows,

$$\frac{p}{1+T}+\frac{1}{1+T}x_ip'-c'(x_i)=0 \tag{3}$$

Under free entry, the profit of the firms are drivent to zero. Therefore we have

$$\frac{p}{1+T}x_i-c(x_i)=0 \tag{4}$$

The second order condition for Firm i is as follows,

$$\frac{1}{1+T}(2p'+x_ip'')-c''(x_i)<0 \tag{5}$$

Since all firms are identical, all x_i are equal in an equilibrium. (3) and (4) define the equilibrium of this industry with two unknowns, x_i and n. p is determined by the inverse demand frnction as $p=p(nx_i)$. I denote the equilibrium output per firm by x.

3. THE WELFARE EFFECTS OF A SMALL AD-VAROLEM TAX

In this section I examine the welfare effects of a small ad-varolem tax. Differentiation of (3) and (4) with respect to T yields

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} dx \\ dn \end{pmatrix}=\frac{1}{1+T}\begin{pmatrix} E \\ F \end{pmatrix}dT \tag{6}$$

where

$$A=(n+1)p'+nxp''-(1+T)c''(x)$$
$$B=x(p'+xp'')$$
$$C=(n-1)xp'$$
$$D=x^2p'$$
$$E=p+xp'$$

and

$$F=xp$$

In derivation of (6) I use $p-(1+T)c'(x)=-xp'$ from (3). From (6), we obtain

$$\frac{dx}{dT}=\frac{1}{\Delta}\left(\frac{1}{1+T}\right)(DE-BF)=\frac{1}{\Delta}\left(\frac{1}{1+T}\right)x^3[(p')^2-pp''] \tag{7}$$

and

$$\begin{aligned}\frac{dn}{dT}&=\frac{1}{\Delta}\left(\frac{1}{1+T}\right)(AF-CE)\\&=\frac{1}{\Delta}\left(\frac{1}{1+T}\right)x\{2pp'+x[npp''-(n-1)(p')^2]-p(1+T)c''(x)\}\\&=\frac{1}{\Delta}\left(\frac{1}{1+T}\right)x\{p[2p'+xp''-(1+T)c''(x)]+x(n\text{-}1)[pp''-(p')^2]\}\end{aligned} \tag{8}$$

where

$$\Delta=AD-BC=x^2p'[2p'+xp''-(1+T)c''(x)]$$

Δ is positive from (5) and $p'<0$. (7) represents the effect of an increase in an ad-varolem tax on the output per firm. It is positive when the inverse demand function is concave or linear ($p''\leqq 0$), and may be postive even when it is convex ($p''>0$) if it is not too convex. (8) represents the effect of an increase in an ad-varolem tax on the equilibrium number of firms in the industry. The first term in the parentheses of (8), $p[2p'+xp''-(1+T)c''(x)]$, is negative from (5), but the second term, $x(n-1)[pp''-(p')^2]$, may be either positive or negative depending on p''.

Now I assume the following relations:

$$A=(n+1)p'+nxp''-(1+T)c''(x)<0 \tag{9}$$

and

$$p'-(1+T)c''(x)<0 \tag{9$'$}$$

(9) is derived from the stability conditions for an oligopoly which are obtained by Dixit (1986)[1]. (9) and (9)′ imply that the total output increases when the number of firms increases as we shall see below[2].

Differentiating (3) with respect to n, the response of the output per firm to an exogenous change in the number of firms is obtained as follows:

$$\frac{dx}{dn}=-\frac{B}{A}=-\frac{x(p'+xp'')}{(n+1)p'+nxp''-(1+T)c''(x)}$$

Then we obtain the response of the total output to a change in the number of firms as follows:

$$\frac{dX}{dn}=x+n\frac{dx}{dn}=\frac{x[p'-(1+T)c''(x)]}{(n+1)p'+nxp''-(1+T)c''(x)}$$

From (9) and (9)′ this is positive, that is, the total output increases when the number of firms increases.

The effect of an increase in an ad-varolem tax on the (consumer) price is

$$\begin{aligned}\frac{dp}{dT}&=p'\left(n\frac{dx}{dT}+x\frac{dn}{dT}\right)=\frac{1}{\Delta}\left(\frac{1}{1+T}\right)x^2p'[p'(2p+xp')-(1+T)pc''(x)]\\&=\frac{1}{\Delta}x^2p'\{p'c'(x)+\frac{1}{1+T}p[p'-(1+T)c''(x)]\}\end{aligned} \tag{10}$$

where the last equality is due to (3). From (9)′ this is unambiguously positive. Thus an ad-varolem tax is shifted to consumers. Each of the output per firm and the number of firms may rise or fall in the face of a tax, but $dp/dT>0$ means that the aggregate output unambiguously falls.

The effect of an increase in an ad-varolem tax on the producer price at the zero tax position is

$$\left.\frac{d\bar{p}}{dT}\right|_{T=0}=\left.\frac{dp}{dT}\right|_{T=0}-p=\frac{1}{\Delta}x^3p'[(p')^2-pp'']$$

[1] See the equation (36-i) in Dixit (1986).

[2] For more discussions about this point, see Seade (1980).

The sign of this expression is opposite to the sign of (7). That is, the producer price of the good is raised (lowered) by a small ad-varolem tax when the tax decreases (increases) the output per firm. When (11) is negative, shifting of an ad-varolem tax to consumers is incomplete. On the other hand when it is positive, the tax is excessively (over 100%) shifted.

Consumers' surplus is represented as

$$S=\int_0^X p(z)dz-pX$$

I define the tax burden by an ad-varolem tax under free entry as the magnitude of the loss of consumers' surplus due to the tax as follows,

$$B=-\frac{dS}{dT}=X\frac{dp}{dT}$$

On the other hand the tax revenue from an ad-varolem tax is $(T/(1+T))pX=\bar{p}TX$. Comparing the tax burden and an increase in the tax revenue by a small ad-varolem tax, we obtain

$$\text{(11)}\quad \left[B-\frac{d}{dT}(\bar{p}TX)\right]\Bigg|_{T=0}=X\frac{dp}{dT}\bigg|_{T=0}-\bar{p}X=X\left(\frac{dp}{dT}\bigg|_{T=0}-p\right)=X\frac{d\bar{p}}{dT}\bigg|_{T=0}$$
$$=\frac{1}{\Delta}x^3p'[(p')^2-pp'']$$

where I use the fact that $p=\bar{p}$ at $T=0$. Since $\Delta>0$ and $p'<0$, under the assumptions of (9) and (9)', we obtain the following result:

THEOREM 1. *The tax burden by a small ad-varolem tax is smaller than an increase in the tax revenue by the tax when the following inequality holds*

$$\text{(12)}\quad (p')^2-pp''>0$$

This is satisfied if the inverse demand function is concave or linear, and may be satisfied even if it is convex if it is not too convex.

From (7) we find that the sign of the tax burden minus an increase in the tax revenue by a small ad-varolem tax is opposite to the sign of the effect of the tax on the output per firm. Therefore the tax burden by a small ad-varolem tax is smaller (or larger) than an increase in the tax revenue by the tax if and only if the tax increases (decreases) the output per firm.

In Appendix (1), I present an analysis of the welfare effect of an ad-varolem tax in the case where the number of firms is fixed, and show that a small ad-varolem tax in that case more likely improve the welfare than in the free entry case.

The inverse demand function which makes (11) and the left hand side of (12) be zero is called "negative exponential"[3]. A negative exponential inverse demand

[3] This name is according to Greenhut, Norman and Hung (1987).

function is explicitly written as

$$p = a\exp(-bX), \quad a>0 \quad \text{and} \quad b>0$$

with $p' = ab\exp(-bX)$ and $p'' = ab^2\exp(-bX)$. This is a convex function since $p''>0$. I call that an inverse demand function is more (or less) concave than negative exponential when $(p')^2 - pp''>0$ (or <0). A linear inverse demand function is more concave than negative exponential, and even a convex inverse demand function may be more concave than negative exponential. An example of an inverse demand function which is convex and satisfies $(p')^2 - pp''>0$ is

$$p = a(X-b)^2, \quad a>0, \quad b>0 \quad \text{and} \quad 0 \leqq X \leqq b \quad (\text{or } 0 \leqq p \leqq ab^2)$$

For this inverse demand function, we have $p'' = 2a > 0$ and

$$(p')^2 - pp'' = 4a^2(X-b)^2 - 2a^2(X-b)^2 > 0$$

Using the term "negative exponential", Theorem 1 is rewritten as follows:

THEOREM 1′. *The tax burden by a small ad-varolem tax is smaller* (*or larger*) *than an increase in the tax revenue by the tax when the inverse demand function is more* (*or less*) *concave than negative exponential.*

Besley (1989) has shown that a small specific commodity tax in a free entry Cournot oligopoly raises the welfare of consumers if and only if the inverse demand function is strictly concave, that is, $p''<0$. Therefore we can say that a small ad-varolem tax more likely improve the welfare than a small specific tax.

4. COMPARISON OF AN AD-VAROLEM TAX AND A SPECIFIC TAX

I call that an ad-varolem tax and a specific tax, which are not small, are equivalent, if they raise equal tax revenues. In this section I compare the tax burden under an ad-varolem tax and that under a specific tax which is equivalent to the ad-varolem tax.

ASSUMPTION 1. An increase in a tax rate, either an ad-varolem tax or a specific tax, increases the tax revenue.

We know that at an extremely high tax rate, either an ad-varolem tax or a specific tax, an increase in a tax rate may reduce the tax revenue. But it is an unrealistic situation, so I assume this assumption. In Appendix (2) I present some discussions about this assumption.

ASSUMPTION 2. The cost function of the firms is linear with a constant marginal cost and a positive fixed cost as follows:

$$c(x) = cx + f$$

For this cost function the average cost $c + f/x$ is decreasing in x. Then, under Assumptions 1 and 2, we can show the following result.

THEOREM 2. *The tax burden under an ad-varolem tax, which is not small, is smaller than the tax burden under a specific tax which is equivalent to the ad-varolem tax.*

Proof. Suppose that a specific tax, t^*, and an ad-varolem tax, T^*, give rise to the equal consumer price of the good, p^*, and hence the equal total output, X^*. Denote the tax revenue from t^* and T^*, respectively, by $R_1 = t^*X^*$ and $R_2 = (T^*/(1+T^*))p^*X^*$. I show that R_2 is larger than R_1. Rewriting the first order and the zero profit conditions for the firms under an ad-varolem tax,

$$\frac{p^*}{1+T^*} + \frac{1}{1+T^*}xp'(X^*) - c = 0 \tag{13}$$

and

$$\left(\frac{p^*}{1+T^*} - c\right)x - f = 0 \tag{14}$$

Combining these two expressions we get

$$\left(\frac{p^*}{1+T^*} - c\right)^2 = -\frac{1}{1+T^*}p'(X^*)f \tag{15}$$

On the other hand under a specific tax we obtain the followin expressions which are the counterparts of (13) and (14),

$$p^* + xp'(X^*) - c - t^* = 0 \tag{16}$$

and

$$(p^* - c - t^*)x - f = 0 \tag{17}$$

Combining these two expressions we get

$$(p^* - c - t^*)^2 = -p'(X^*)f \tag{18}$$

Since p^* and X^* in (15) are equal to those in (18), $p'(X^*)$ in (15) is equal to $p'(X^*)$ in (18). Then, so long as $T^* > 0$, the right hand side of (15) is smaller than the right hand side of (18). Thus $p^*/(1+T^*)$ in (15) must be smaller than $p^* - t^*$ in (18) so that both of (15) and (18) should be satisfied, and hence $t^* < (T^*/(1+T^*))p^*$. Thus we have completed the proof that R_2 is larger than R_1. Since I assume that an increase in a tax rate increases the tax revenue, we must lower the rate of an ad-varolem tax so that R_1 and R_2 should become equal. Then the consumer price of the good falls since $dp/dT > 0$, consumers' surplus is increased, and the tax burden become smaller. (Q.E.D.)

Theorem 2 implies that an ad-varolem tax is more desirable than a specific tax in a free entry Cournot oligopoly from the point of view of consumers' welfare.

From (13) and (14) we obtain the following relation under an ad-varolem tax,

$$-\frac{1}{1+T^*}p'(X^*)x^2=f \tag{19}$$

On the other hand under a specific tax, from (16) and (17) we obtain

$$-p'(X^*)x^2=f \tag{20}$$

With equal $p'(X^*)$ in (19) and (20) the output per firm, x, in (19) must be larger than the output per firm in (20) so long as $T^*>0$. This implies that, with the equal price and total output, the number of firms under an ad-varolem tax is samller than the number of firms under a specific tax. Then, from the increasing returns to scale (decreasing average cost), we have a lower average cost under an ad-varolem tax than under a specific tax. Since the average cost is equal to the producer price of the good under free entry, with the equal consumer price the producer price under an ad-varolem tax is lower than the producer price under a specific tax. In other words, the tax margin under an ad-varolem tax is larger than the tax margin under a specific tax. And hence we have $R_2>R_1$.

5. CONCLUSION

This paper has examined the welfare effects of commodity taxes in a free entry Cournot oligopoly. I have established two results. (1) If the demand function is concave or linear, or not too convex, the tax burden (the loss of consumers' surplus) by a small ad-varolem commodity tax is smaller than an increase in the tax revenue by the tax. Comparing with the result in Besley (1989), this implies that a small ad-varolem tax more likely improve the welfare than a small specific tax. (2) With a linear cost function, the tax burden by an ad-varolem tax, which is not small, is smaller than the tax burden by a specific tax which raises the equal tax revenue as the ad-varolem tax.

APPENDIX (1) THE CASE OF FIXED NUMBER OF FIRMS

If the number of firms is fixed, the firms earn positive profits. Thus we should define the tax burden by an ad-varolem tax as the magnitude of the sum of the loss of consumers's surplus and the loss of profits of the firms due to the tax. The total profit of the firms is equal to

$$n[p(X)x-c(x)]$$

Therefore the tax burden is equal to

$$B=-\frac{dS}{dT}-n(p-c')\frac{dx}{dT}-nx\frac{dp}{dT} \tag{A.1}$$

Since $-dS/dT=X(dp/dT)$, (A.1) is rewritten as

$$B = -n(p-c')\frac{dx}{dT}$$

Comparing the tax burden and an increase in the tax revenue by a small ad-varolem tax under the fixed number of firms, we obtain

$$\left[-n(p-c')\frac{dx}{dT} - \frac{d}{dT}(\bar{p}TX)\right]\Bigg|_{T=0} = -n(p-c')\frac{dx}{dT}\Bigg|_{T=0} - pX$$

From (7), this is equal to

$$-\left\{\frac{1}{\Delta}x^3[(p')^2 - pp''] + pX\right\} \tag{A.2}$$

If (A.2) is negative, the tax burden is smaller than an increase in the tax revenue. When $(p')^2 - pp'' > 0$, (A.2) is negative. But, due to the term of pX, even when $(p')^2 - pp'' \leqq 0$, (A.2) may be negative. Thus a small ad-varolem tax in the case where the number of firms is fixed more likely improve the welfare than in the free entry case.

APPENDIX (2)

Denote the tax revenues from a specific tax and an ad-varolem tax, respectively, by $R_1 = tX$ and $R_2 = (T/(1+T))pX$. Differentiating these with respect to, respectively, t and T, we obtain

$$\frac{dR_1}{dt} = X + t\frac{dX}{dt} \tag{A.3}$$

and

$$\frac{dR_2}{dt} = \frac{1}{(1+T)^2}\left[pX + T(1+T)(p+Xp')\frac{dX}{dT}\right] \tag{A.4}$$

We know $dX/dt < 0$ from Besley (1989) and $dX/dT < 0$ from (10) of this paper. So, at $t=0$ and $T=0$, we have $dR_1/dt > 0$ and $dR_2/dt > 0$, that is, small taxes unambiguously increase tax revenues. At a very large t, the second term of (A.3) may be dominant, and we may have $dR_1/dt < 0$. Similarly, at a very large T, the second term of (A. 4) may be dominant, and we may have $dR_2/dT < 0$.

Yamagata University

REFERENCES

Besley, T. (1989), "Commodity Taxation and Imperfect Competition: A Note on the Effects of Entry", Journal of Public Economics, vol. 40, pp, 359–367.

Dierickx, I., C. Matutes and D. Neven (1988), "Indirect Taxation and Cournot Equilibrium", International Journal of Industrial Organization, vol. 6, pp. 385–399.
Dixit, A. K. (1986), "Comparative Statics for Oligopoly", International Economic Review, vol. 27, pp. 107–122.
Greenhut, M. L., G. Norman and C.-S. Hung (1987), The Economics of Imperfect Competition: A Spatial Approach, Cambridge University Press, Cambridge.
Katz, M. and H. Rosen (1985), "Tax Analysis in an Oligopoly Model", Public Finance Quarterly, vol. 13, pp. 3–19.
Seade, J. (1980), "On the Effects of Entry", Econometrica, vol. 48, pp. 479–489.
Stern, N. (1987), "The Effects of Taxation, Price Control, and Government Contracts in Oligopoly and Monopolistic Competition", Journal of Public Economics, vol. 32, pp. 133–158.