Title	OPTIMUM SUPPLY OF INTERNATIONAL PUBLIC GOODS
Sub Title	
Author	TERASAKI, Katsushi
Publisher	Keio Economic Society, Keio University
Publication year	1992
Jtitle	Keio economic studies Vol.29, No.1 (1992.) ,p.45- 61
JaLC DOI	
Abstract	This paper tries to explain the optimum supply of international public goods and bads between two small open economies, both of which produce two tradable private goods and one non-traded international public good with two kinds of factors the supplies of which are inelastic. Each country can consume domestic and foreign-made public goods by the same amount. Another model deals with international public bads which are good for a providing country but bad for the other country. We show the first-best solution, the optimum tax system, the Nash and Stackelberg equilibria, a free-rider condition and the possibility of immiserizing growth.
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19920001-0045

OPTIMUM SUPPLY OF INTERNATIONAL PUBLIC GOODS

Katsushi TERASAKI

Abstract: This paper tries to explain the optimum supply of international public goods and bads between two small open economies, both of which produce two tradable private goods and one non-traded international public good with two kinds of factors the supplies of which are inelastic. Each country can consume domestic and foreign-made public goods by the same amount. Another model deals with international public bads which are good for a providing country but bad for the other country. We show the first-best solution, the optimum tax system, the Nash and Stackelberg equilibria, a free-rider condition and the possibility of immiserizing growth.

1. INTRODUCTION

In this paper we try to explain the optimum supply and the competitive equilibrium of international public final goods, which are, for example, a fair natural environment (clean air, seas, rivers, lakes and rain, an adequate amount of ozone and forests, etc.), public information (broadcasts, weather forecasts, public research, etc.), public safety (defense, the prevention of international crimes, etc.), public health (the prevention of epidemics, public research for medical treatment of disease, etc.), and so forth. As for defense expenditures, Olson and Zeckhauser (1966) have given a brief description, assuming that two countries have the same cost function which shows a constant average cost and that they produce the same kind of international public goods that are perfect substitutes. The model was reformulated by Hamada (1977) and introduced by Frey (1984). In this field, the main discussion has focused on international macroeconomic policy coordination as Buiter and Marston (1988) and Ohyama (1988) except Connolly (1970, 1972) which have considered externalities arising from international public goods and Markusen (1975) which has showed an optimal tax structure with international externalities.

If there is no alliance between two countries, the defense expenditures in one country constitute a kind of international public bad for the other country. One country might maintain public peace and order by putting those who break the law such as poor, criminals, refuggees, and drug addicts and so forth out of the country. The natural environment of one country might be kept clean by polluting the other country's environment. Nuclear experimentation in one country as one of many defense activities contaminates the environment of that country as well as that of the other. These kinds of public goods for one country are public bads for the other.

This paper tries to contribute to an analysis of international public bads in comparison to international public goods. Our main results in the case of international public bads are that the Stackelberg equilibrium is superior to the Nash equilibrium for each country whichever country is a leader so that one country can be better than of the Nash equilibrium by pretending to be a follower even if she has information about the other's reaction curve or by letting the other know her reaction curve if the other has no information and that there does exist an immiserizing growth where no country gains from one country's economic growth even under the given international commodity prices.

In section 2 we introduce a model with two kinds of private goods and two kinds of public goods, one of which is domestically produced and the other which is foreign-made. There are two small open countries, both of which can consume two kinds of private goods and two kinds of public goods. Each country produces one kind of public good and two kinds of private goods whose prices are given in the world market although both public goods are non-traded. Section 3 gives the first-best solution and section 4 shows the optimum tax system to realize the first-best solution. Section 5 considers the case where both public goods have positive utility for each country whereas section 7 deals with the opposite case where the foreign public goods create a give negative utility to the home country or where the public goods for foreigners are public bads for the domestic population and *vice versa*. In both sections 5 and 7 we explore the Nash and the Stackelberg equilibria and in section 6 and 7 we refer to the possibility of immiserizing growth.

2. THE MODEL

While Connolly (1970, 1972) dealt with two large countries producing one public good and one private good, we consider two small open economies where two tradable private goods (X_j, X_j^*) and one non-traded public final good (X_3, X_3^*) are produced by using two kinds of factor (V_{ij}, V_{ij}^*) such that

$$X_j = X_j(V_{1j}, V_{2j}) \,, \quad X_j^* = X_j^*(V_{1j}^*, V_{2j}^*) \,, \qquad j = 1, 2, 3 \,, \tag{1}$$

where X_j and X_j^* are supposed to be linear homogeneous with respect to the two kinds of input, V_{ij} and V_{ij}^*, respectively, and an asterisk (*) is attached to the foreign variables. We assume that each country produces two kinds of private goods under a relevant amount of public goods and given world prices. Each factor is under a fixed supply (V_i, V_i^*) and stays within each country so that the factor market clearing conditions are given as

$$V_i = \textstyle\sum_j V_{ij} \,, \quad V_i^* = \sum_j V_{ij}^* \,, \qquad i = 1, 2 \,, \; j = 1, 2, 3 \,. \tag{2}$$

Each country can consume both public goods by the same amount so that the national welfare function can be defined as

$$U = U(D_1, D_2, D_3, D_4)\,, \quad U^* = U^*(D_1^*, D_2^*, D_3^*, D_4^*)\,, \tag{3}$$

where D_j and D_j^* denote the social consumption for j-th goods and we assume that each individual has the same preference and its utility function is homothetic with respect to the two private goods.[1] In (3) $D_3 = D_4^* = X_3$ and $D_4 = D_3^* = X_3^*$ by definition of international public good.

The conjoined budget constraint of the two countries for the first-best solution is given as

$$\sum p_k(X_k + X_k^*) = \sum p_k(D_k + D_k^*)\,, \qquad k = 1, 2\,, \tag{4}$$

where p_k is the world market price of k-th private commodity, which is given for each small open economy. The conjoined income constraint (4) for the first-best solution referred to in the next section implies international trade between two countries and the rest of the world as well as international transfer between two countries.

3. THE FIRST-BEST SOLUTION

Now, if both countries agree with maximization of the conjoined welfare function, W, then the optimum supply of public final good is given as a cooperative solution of the following maximization problem:

$$\max W[U(D_1, D_2, X_3, X_3^*)\,,\; U^*(D_1^*, D_2^*, X_3^*, X_3)]$$

with respect to X_j, X_j^*, D_k, D_k^*, V_{ij}, V_{ij}^* and subject to production functions, (1), factor endowments, (2), and conjoined income constraint, (4). After defining multipliers for each of the constraint and setting up the Lagrangian, the problem becomes:

$$\begin{aligned} \max L = {} & W(U, U^*) + \sum \lambda_j[X_j(V_{1j}, V_{2j}) - X_j] + \sum \lambda_j^*[X_j^*(V_{1j}^*, V_{2j}^*) - X_j^*] \\ & + \sum \gamma_i(V_i - \textstyle\sum_j V_{ij}) + \sum \gamma_i^*(V_i^* - \textstyle\sum_j V_{ij}^*) + \mu \sum p_k(X_k - D_k + X_k^* - D_k^*)\,, \\ & i, k = 1, 2\,,\; j = 1, 2, 3\,. \end{aligned} \tag{5}$$

The first-order conditions for this model are

$$\partial L/\partial X_k = -\lambda_k + \mu p_k = 0\,, \qquad \partial L/\partial X_k^* = -\lambda_k^* + \mu p_k = 0\,, \qquad k = 1, 2\,, \tag{6}$$

$$\partial L/\partial X_3 = W_1 U_3 + W_2 U_4^* - \lambda_3 = 0\,, \quad \partial L/\partial X_3^* = W_1 U_4 + W_2 U_3^* - \lambda_3^* = 0\,, \tag{7}$$

$$\partial L/\partial V_{ij} = \lambda_j X_{ij} - \gamma_i = 0\,, \qquad \partial L/\partial V_{ij}^* = \lambda_j^* X_{ij}^* - \gamma_i^* = 0\,, \qquad i = 1, 2\,,\; j = 1, 2, 3\,, \tag{8}$$

[1] Markusen (1975) also dealt with an eyesore type of pollution externality which does not affect production functions as we assume in (1) and (3) above.

$$\partial L/\partial D_k = W_1 U_k - \mu p_k = 0\,, \qquad \partial L\ /\partial D_k^* = W_2 U_k^* - \mu p_k = 0\,,$$
$$k = 1, 2\,, \tag{9}$$

and the constraints, (1), (2) and (4), where

$$W_1 \equiv \partial W/\partial U\,, \quad W_2 \equiv \partial W/\partial U^*\,,$$
$$U_k \equiv \partial U/\partial D_k\,, \quad U_k^* \equiv \partial U^*/\partial D_k^*\,, \qquad k = 1, 2, 3, 4\,,$$
$$X_{ij} \equiv \partial X_j/\partial V_{ij}\,, \quad X_{ij}^* \equiv \partial X_j^*/\partial V_{ij}^*\,, \qquad i = 1, 2\,,\ j = 1, 2, 3\,,$$

i.e. W_1 and W_2 are the distributional weights with the conjoined welfare function, U_k and U_k^* the marginal social utilities of k-th good, X_{ij} and X_{ij}^* the marginal productivities of i-th factor in j-th sector. There are thirty-three equations in all, which we assume will generate a unique solution to the thirty-three variables of the model, consisting of the twenty-two economic variables, X_j, X_j^*, D_k, D_k^*, V_{ij}, V_{ij}^*, and eleven Lagrangian multipliers, λ_j, λ_j^*, γ_i, γ_i^*, μ.

The first-order conditions (9) yield:

$$U_1/U_2 = U_1^*/U_2^* = p_1/p_2\,, \quad W_1 U_k = W_2 U_k^*\,, \qquad k = 1, 2\,, \tag{10}$$

which say that the marginal rate of substitution between the two private goods must be the same for two countries and as the price ratio of the goods and that the international distributional weight times marginal utility of k-th private good in the home country equals that of the foreign country.

The first-order conditions (8) yield:

$$X_{1j}/X_{2j} = \gamma_1/\gamma_2\,, \quad X_{1j}^*/X_{2j}^* = \gamma_1^*/\gamma_2^*\,, \qquad j = 1, 2, 3\,, \tag{11}$$

which state that the technical marginal rate of substitution between two factors in production of a good must be equal for all goods, and (8) also yield:

$$X_{ij}/X_{ik} = \lambda_k/\lambda_j\,, \quad X_{ij}^*/X_{ik}^* = \lambda_k^*/\lambda_j^*\,, \qquad i = 1, 2\,,\ j, k = 1, 2, 3\,. \tag{12}$$

The left-hand sides of (12) are the marginal rate of transformation between any two goods obtained by switching i-th factor from one good to the other. Since it holds for all factors switched between one good to the other, it is simply the marginal rate of transformation. The conditions in (12) claim that the marginal rate of transformation must be equal for all factors.

On the other hand, the first-order conditions with respect to each private good, (6), are reduced to

$$\lambda_1/\lambda_2 = \lambda_1^*/\lambda_2^* = p_1/p_2\,,$$

so that (10) combines with (12) to form

$$U_1/U_2 = U_1^*/U_2^* = X_{i2}/X_{i1} = X_{i2}^*/X_{i1}^*\,, \qquad i = 1, 2\,. \tag{13}$$

In other words, the marginal rate of substitution must be the same as the marginal rate of transformation between the two private goods for two countries.

Consider also the first-order conditions with respect to public goods, (7). The first six, (6) and (7), and the last four, (9), conditions and (12) make the following relations:

$$U_3/U_k + U_4^*/U_k^* = X_{ik}/X_{i3}\,, \quad U_3^*/U_k^* + U_4/U_k = X_{ik}^*/X_{i3}^*\,, \qquad i, k = 1, 2\,, \quad (14)$$

which is an international version of the Samuelson (1954) condition. Conditions (14) give the familiar result that the sum of each country's marginal rate of substitution between public goods and any private good equals the marginal rate of transformation between any private good and public good in production.[2]

4. AN OPTIMUM TAX SYSTEM IN MARKET ECONOMY

Whereas Markusen (1975) has developed an optimal tax structure (tariffs, consumption subsidies, production taxes) from one country's point of view because the country cannot tax foreign producers, this section will characterize an optimal tax system like the Lindahl price system as a result of international cooperative agreement in which each country pays respective prices for each public good in the form of taxes.[3]

Non-profit conditions because of perfect competition among producers are shown as:

$$\textstyle\sum_i a_{ij} q_i = p_j\,, \quad \sum_i a_{ij}^* q_i^* = p_j^*\,, \qquad i = 1, 2\,,\ j = 1, 2, 3\,, \quad (15)$$

where $a_{ij} = a_{ij}(q_1/q_2)$, $a_{ij}^* = a_{ij}^*(q_1^*/q_2^*)$, and $p_k = p_k^*$, $k = 1, 2$, *i.e.* a_{ij} and a_{ij}^* stand for the input coefficient of i-th factor in j-th sector which depend upon the relative factor price, q_1/q_2 and q_1^*/q_2^*, respectively, and p_3 and p_3^* denote the unit cost of non-traded public goods. Thus, in (15) q_i, q_i^*, p_3 and p_3^* are uniquely determined by the given world prices, p_k, since we assume that each country produces both types of private goods under given international prices and a relevant output level of public goods.

Let us introduce a tax system where the home country pays $r_i X_3$ for the domestic public goods and $r_2 X_3^*$ for the foreign public goods while the foreign country pays $r_2^* X_3$ and $r_1^* X_3^*$, respectively. Then, the national budget constraints are reduced to

$$y \equiv \textstyle\sum q_i V_i = \sum p_j X_j = \sum p_k D_k + r_1 X_3 + r_2 X_3^*\,, \qquad i, k = 1, 2\,,\ j = 1, 2, 3\,, \quad (16)$$

for the domestic economy and

$$y^* \equiv \textstyle\sum q_i^* V_i^* = \sum p_j^* X_j^* = \sum p_k D_k^* + r_1^* X_3^* + r_2^* X_3\,, \qquad i, k = 1, 2\,,\ j = 1, 2, 3\,, \quad (17)$$

for the foreign economy, where $p_k = p_k^*$, $k = 1, 2$, and the national incomes, y and

[2] Connolly (1970) also derived almost the same conditions as (14) above, taking advantage of the loss coefficients and the spill over coefficients. Provided these coefficients are implicitly included in the utility functions (3), the conditions (14) are more general expressions than Connolly's.

[3] Since we assume that each country is small, an optimal tariff rate is zero.

y^*, are given only by p_1 and p_2 under incomplete specialization in private goods.

As for the public sector's budget constraints, we have

$$(r_1 + r_2^*)X_3 = p_3 X_3 \quad \text{or} \quad r_1 + r_2^* = p_3 , \tag{18}$$

$$(r_1^* + r_2)X_3^* = p_3^* X_3^* \quad \text{or} \quad r_1^* + r_2 = p_3^* , \tag{19}$$

where each government collects taxes to disburse to competitive public good producers in each country. Now, maximize the conjoined welfare function with respect to r_1 in order to find the domestic optimum tax system in a market economy:

$$\partial W/\partial r_1 = W_1 U_1 \sum (U_k/U_1)\partial D_k/\partial r_1 + W_2 U_1^* \sum (U_k^*/U_1^*)\partial D_k^*/\partial r_1 = 0 , \quad k = 1, 2, 3, 4 . \tag{20}$$

Differentiate the national budget constraints, (16) and (17) with respect to r_1 and divide them by p_1:

$$\sum (p_k/p_1)\partial D_k/\partial r_1 + (r_1/p_1)\partial X_3/\partial r_1 + (r_2/p_1)\partial X_3^*/\partial r_1 + X_3/p_1 = 0 , \quad k = 1, 2 , \tag{21}$$

$$\sum (p_k/p_1)\partial D_k^*/\partial r_1 + (r_1^*/p_1)\partial X_3^*/\partial r_1 + (r_2^*/p_1)\partial X_3/\partial r_1 - X_3/p_1 = 0 , \quad k = 1, 2 , \tag{22}$$

where we make use of the following relationships that the factor incomes in (16) and (17), and unit cost of the domestic public goods in (18) depend only upon the given international prices:

$$\partial y/\partial r_1 = \partial y^*/\partial r_1 = 0 , \quad \partial r_2^*/\partial r_1 = -1 . \tag{23}$$

Then, plug (21) and (22) into (20) to obtain

$$\begin{aligned} \partial W/\partial r_1 = {} & W_1 U_1 [(U_2/U_1 - p_2/p_1)\partial D_2/\partial r_1 + (U_3/U_1 - r_1/p_1)\partial X_3/\partial r_1 \\ & + (U_4/U_1 - r_2/p_1)\partial X_3^*/\partial r_1] + W_2 U_1^* [(U_2^*/U_1^* - p_2/p_1)\partial D_2^*/\partial r_1 \\ & + (U_3^*/U_1^* - r_1^*/p_1)\partial X_3^*/\partial r_1 + (U_4^*/U_1^* - r_2^*/p_1)\partial X_3/\partial r_1] \\ & + (W_2 U_1^* - W_1 U_1)X_3/p_1 = 0 . \end{aligned} \tag{24}$$

Consequently, the sufficient conditions for maximization of the conjoined welfare function are as follows:

$$U_2/U_1 = U_2^*/U_1^* = p_2/p_1 , \tag{25}$$

$$U_3/U_1 = r_1/p_1 , \tag{26}$$

$$U_4/U_1 = r_2/p_1 , \tag{27}$$

$$U_3^*/U_1^* = r_1^*/p_1 , \tag{28}$$

$$U_4^*/U_1^* = r_2^*/p_1 , \tag{29}$$

$$W_1 U_1 = W_2 U_1^* . \tag{30}$$

Now, we have ten equations, (16)–(19) and (25)–(29) in all, which we assume will generate a unique solution to ten variables of the cooperative model in market economies, D_k, D_k^*, r_h, r_h^*, X_3 and X_3^*, $h, k=1, 2$. Thus, for optimality the two countries have to solve the system with cooperation and make an agreement that international distributional weight times marginal utility of the first good in their home country equals that of the foreign country as shown in (30). From (18), (26) and (29) we obtain

$$p_3 = r_1 + r_2^* = p_1(U_3/U_1 + U_4^*/U_1^*) . \tag{31}$$

Then, we plug the conditions of profit maximization, $q_1 = p_1 X_{i1} = p_3 X_{i3}$, into (31) to obtain

$$\mathrm{U}_3/U_1 + U_4^*/U_1^* = X_{i1}/X_{i3}, \qquad i=1,2 . \tag{32}$$

Similarly, from (19), (27) and (28), we obtain

$$U_4/U_1 + U_3^*/U_1^* = X_{i1}^*/X_{i3}^*, \qquad i=1,2 , \tag{33}$$

which satisfies the Samuelson (1954) conditions, (14), with (32). In addition, since we assume that each individual has the same homothetic preference with respect to two private goods, utility maximization as consumer behavior assures (25), with which the agreement satisfies (10). Thus, the optimum international tax system in a market economy does exist.

In general, however, the agreement on the optimum tax rates as well as the conjoined welfare function is seriously difficult to set in addition to the calculation of them, so a competitive circumstance is supposed to be usual and natural in producing international public final goods as we will discuss in the following sections. In fact, two countries can raise their respective utilities by decreasing the international tax transfer and by increasing the international distributional weight. Furthermore, each country has to collect income or lump-sum taxes from factor owners such that

$$r_1 X_3 + r_2 X_3^* = \sum t_i q_i V_i = \sum T_i V_i , \qquad 0 < t_i < 1 , \quad 0 < T_i < q_i , \quad i=1,2 ,$$

$$r_1^* X_3^* + r_2^* X_3 = \sum t_i^* q_i^* V_i^* = \sum T_i^* V_i^* , \quad 0 < t_i^* < 1 , \quad 0 < T_i^* < q_i^* , \quad i=1,2 ,$$

where t_i and t_i^* denote income tax rate on i-th factor and T_i and T_i^* stand for the lump-sum tax on i-th factor. Each income tax rate as well as each lump-sum tax is not uniquely determined although the total tax is uniquely determined. Tax burdens between the two factor owners are left to the government's discretion, giving rise to no deadweight loss because of an inelastic supply of each factor as assumed in (2).

Anyway, if the calculation cost of the first-best solution, the negotiation cost for the agreement and the maintenance cost of the system are huge, or if international mutual distrust previals, or if each country does not have reciprocally complete information about the foreign economic structure, a competitive supply of international public good is more reasonable than a cooperative supply.

5. A COMPETITIVE SUPPLY OF INTERNATIONAL PUBLIC GOOD

Suppose there is no international tax transfer so that $r_1 = p_3$, $r_1^* = p_3^*$ and $r_2 = r_2^* = 0$. The problem of the home country is to maximize the domestic welfare function with respect to the supply of domestic public goods under a given supply of foreign public goods or no conjectural variations. Thus, the problem becomes:

$$\max H = U(D_1, D_2, X_3, X_3^*) + \phi(y - p_1 D_1 - p_2 D_2 - p_3 X_3)$$

with respect to D_1, D_2 and X_3 where ϕ stands for a Lagrangian multiplier for the income constraint. The first-order conditions for this problem are

$$\partial H/\partial D_k = U_k - \phi p_k = 0\,, \qquad k = 1, 2\,, \tag{34}$$

$$\partial H/\partial X_3 = U_3 - \phi p_3 = 0\,, \tag{35}$$

and the income constraint. There are four equations in all, which we assume will generate a unique solution to four variables, D_k, X_3 and ϕ.

A shape of domestic reaction curve is obtained by differentiating (34), (35) and the income constraint with respect to X_3^*:

$$\textstyle\sum_k U_{jk}\partial D_k/\partial X_3^* + U_{j3}\partial X_3/\partial X_3^* - p_j \partial\phi/\partial X_3^* = -U_{j4}\,, \qquad k = 1, 2\,,\ j = 1, 2, 3\,, \tag{36}$$

$$\textstyle\sum_k p_k \partial D_k/\partial X_3^* + p_3 \partial X_3/\partial X_3^* = 0\,, \qquad k = 1, 2\,. \tag{37}$$

Then, we have the following solution as a slope of the domestic reaction curve, $-R$:

$$\partial X_3/\partial X_3^* = -\textstyle\sum U_{j4} S_{j3}/\phi \equiv -R\,, \qquad j = 1, 2, 3\,, \tag{38}$$

where

$$S_{j3} \equiv \phi \Delta_{j3}/\Delta\,, \qquad j = 1, 2, 3\,,$$

and Δ_{j3} is co-factor of U_{j3} in the determinant of coefficient, Δ, given by (36) and (37), so that S_{j3} denotes Slutsky's substitution term between the j-th good and a domestic public good. Noting that $\sum p_j S_{j3} = 0$ and $S_{33} < 0$, we assume that S_{13} and S_{23} are positive, *i.e.* private goods are substitutes for a public good in a Hicksian sense. Besides that, we assume that U_{14} and U_{24} are positive, but U_{34} is non-positive, *i.e.* an increase in a foreign public good raises the marginal utility of a private good while it does not increase the marginal utility of a domestic public good. With these assumptions, we have a negative slope of the domestic reaction curve, *i.e.* $R > 0$.

Similarly, we obtain the following solution as a slope of the foreign reaction curve, $-R^*$:

$$\partial X_3^*/\partial X_3 = -\textstyle\sum U_{j4}^* S_{j3}^*/\phi^* \equiv -R^*\,, \qquad j = 1, 2, 3\,. \tag{39}$$

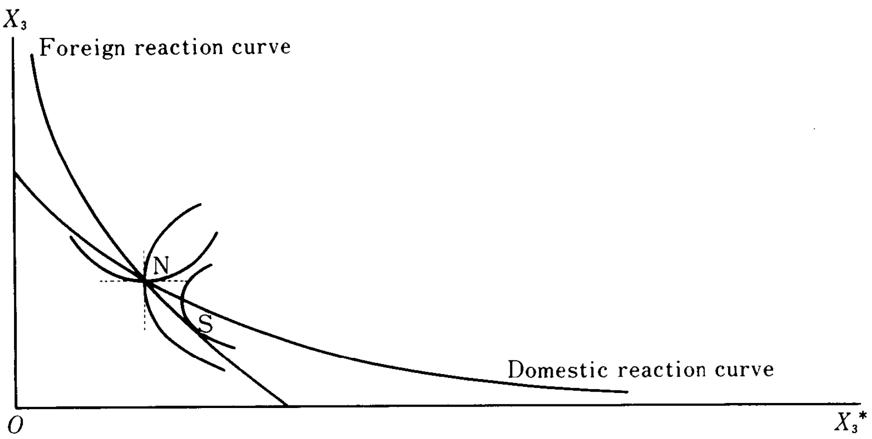

Figure 1.

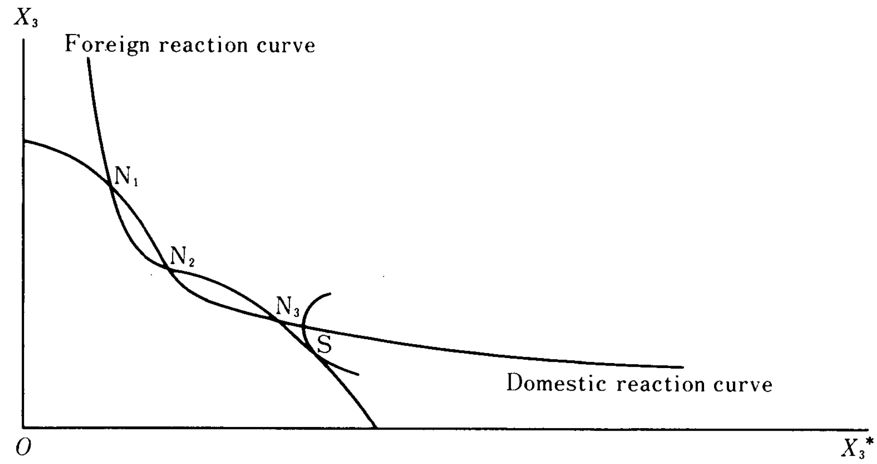

Figure 2.

Define $\partial R/\partial X_3^* < 0$ as a monotone decreasing reaction. Then, there exists a unique Nash equilibrium which is stable since $R < 1/R^*$ at the equilibrium if both reaction curves are of a monotone decreasing reaction as shown in Figure 1.

PROPOSITION 1. *If both reaction curves are of a monotone decreasing reaction, then there exists a unique and stable Nash equilibrium.*

Unless both reaction curves are of a monotone decreasing reaction or monotonously convex to the origin, we might have multiple Nash equilibria such as N_1, N_2 and N_3 shown in Figure 2. Since the stability condition of a Nash equilibrium is that the domestic slope of a reaction curve is steeper than the foreign one, the two Nash equilibria, N_1 and N_3, are stable but N_2 is not. Between the two stable Nash equilibria, the home country prefers N_3 to N_1 while the foreign country prefers N_1 to N_3 so that these Nash equilibrium are locally stable at most. In this sense, there is no globally stable equilibrium under the multiple equilibria case.

By the way, the conjoined income constraint (4) is reduced to

$$y+y^*-p_3X_3-p_3^*X_3^*=\sum p_k(D_k+D_k^*)\,,\qquad k=1,2\,.$$

At the Nash equilibrium, differentiate the conjoined welfare function with respect to X_3 subject to the constraint above to obtain

$$\partial W/\partial X_3=W_2U_4^*+(W_2U_1^*-W_1U_1)[\partial D_1^*/\partial X_3+(p_2/p_1)\partial D_2^*/\partial X_3]\,. \tag{40}$$

Thus, under the agreement, $W_2U_1^*=W_1U_1$, we have $\partial W/\partial X_3=W_2U_4^*>0$ since the foreign country is a free rider on the domestic public goods and the domestic welfare is maximized with respect to X_3 at the Nash equilibrium. By the same token, we obtain $\partial W/\partial X_3^*=W_1U_4>0$. Therefore, the conjoined welfare goes up by increasing the output of either public good.

Define an indifference curve on the reaction plane as a curve on which a country's national welfare is constant with the income constraint and an efficient combination of private good consumption, *i.e.* $U_1/U_2=p_1/p_2$. Thus, a move on the domestic indifference curve on the reaction plane is given by the following equation:

$$(\sum U_k\partial D_k/\partial X_3+U_3)dX_3+U_4dX_3^*=0\,,\qquad k=1,2\,. \tag{41}$$

Making use of the income constraint and the consumption efficiency of private goods, we have the following marginal rate of substitution on the reaction plane:

$$MRS\equiv dX_3^*/dX_3=(p_3\phi-U_3)/U_4\,. \tag{42}$$

Thus, the marginal rate of substitution on the reaction plane takes the value of zero on the domestic reaction curve as shown in Figure 1.

Now, we consider an asymmetric information case where the home country has a thorough knowledge of the foreign reaction curve while the foreign country is not be posted up with the domestic reaction curve since, for instance, the foreign country is a new producer of international public goods by means of international technology transfer from the home country, or the home country has an absolute advantage by knowing information from foreign sources. Suppose the home country is a leader who decides the supply of public goods on the foreign reaction curve. Then, the home country can maximize the national welfare such that

$$0=\partial U/\partial X_3=(\sum U_k\partial D_k/\partial X_3+U_3)+U_4\partial X_3^*/\partial X_3\,,\qquad k=1,2\,, \tag{43}$$

where the last term on the right-hand side of the equation above equals the slope of the foreign reaction curve such that

$$\partial X_3^*/\partial X_3=-R^*\,, \tag{44}$$

so that the condition for the Stackelberg equilibrium with a positive supply of domestic public goods is given by

$$MRS=-R^*\,, \tag{45}$$

i.e. at the Stackelberg equilibrium the marginal rate of substitution on the domestic indifference curve equals the slope of the foreign reaction curve at a point such as S in Figure 1 and 2.

Since in the Nash equilibrium, in (43) $\sum U_k \partial D_k/\partial X_3 + U_3 = 0$, we have the following change in the national welfare of the home country when the country changes the output of the domestic public goods:

$$\partial U/\partial X_3 = U_4 \partial X_3^*/\partial X_3 < 0 , \tag{46}$$

i.e. the home country can raise her utility by decreasing the domestically produced public goods of the Nash equilibrium along the foreign reaction curve. By the same token, as for a change in the foreign national welfare at the Nash equilibrium along the foreign reaction curve when the home country changes domestic output of a public good, we obtain

$$\partial U^*/\partial X_3 = U_4^* > 0 . \tag{47}$$

Thus, a leader becomes better off and a follower becomes worse off by moving from the Nash equilibrium to the Stackelberg equilibrium.

If the size of a country or national preferences are so different that a smaller country in produces no public good in terms of reaction function in a Nash equilibrium, the country is a free rider and the equilibrium is the same as a Stackelberg equilibrium when the country is a leader.

PROPOSITION 2. *The Stackelberg equilibrium is not inferior for a leader but not superior for a follower in the Nash equilibrium.*

If each domestic good is indispensable for positive utility in each country, there is no case for a free rider. So, a free-rider condition is tedious. As we have mentioned right above, if the size of a country or the social preferences are so different that there is no intersection between reaction curves, then in terms of a reaction function the smaller country will be a free rider.

PROPOSITION 3. *If the sizes of respective countries are so different that there is no intersection between reaction curves, then a smaller country free-rides.*

6. IMMISERIZING GROWTH

Suppose the home country experiences economic growth in terms of i-th factor endowment, V_i, so that the domestic reaction curve will shift so as to satisfy the following equations:

$$\textstyle\sum_k U_{jk} \partial D_k/\partial V_i + U_{j3} \partial X_3/\partial V_i - p_j \partial \phi/\partial V_i = 0 , \qquad i, k = 1, 2 , \ j = 1, 2, 3 , \tag{48}$$

$$\textstyle\sum_k p_k \partial D_k/\partial V_i + p_3 \partial X_3/\partial V_i = q_i , \qquad i, k = 1, 2 . \tag{49}$$

Then, the degree of shift-up of the domestic reaction curve is shown as

$$\partial X_3/\partial V_i = q_i \Delta_{43}/\Delta = q_i \partial X_3/\partial y > 0\,, \qquad i = 1, 2\,, \tag{50}$$

where the cofactor of p_3 in the determinant of the coefficients in (48) and (49), Δ_{43}, is positive, and the determinant, Δ, is also positive because of the second order condition for utility maximization with income constraint or because of the assumption of quasi-concavity of the utility function, so that the domestic public good is superior and $\partial X_3/\partial V_i$ is positive.

Now, as a result of economic growth in the domestic factor endowment, the Nash equilibrium will shift along the foreign reaction curve as long as the home country is not a free rider. The change in the competitive supply of both public goods is calculated with (49) and the following equations:

$$\begin{aligned}&\textstyle\sum_k U_{jk}\partial D_k/\partial V_i + (U_{j3} - U_{j4}R^*)\partial X_3/\partial V_i - p_j\partial\phi/\partial V_i = 0\,,\\ &\qquad\qquad i, k = 1, 2\,,\ j = 1, 2, 3\,.\end{aligned} \tag{51}$$

Then, the determinant of the coefficients in (49) and (51) is as follows:

$$\Delta' \equiv (1 - RR^*)\Delta > 0\,, \tag{52}$$

where $1 - RR^* > 0$ in the Nash equilibrium since we assume that each reaction curve is of a monotone decreasing reaction so that $R < 1/R^*$,which is also a stability condition for the Nash equilibrium. Thus, the change in value of public goods at the Nash equilibrium is shown as follows:

$$\partial X_3/\partial V_i = q_i\Delta_{43}/\Delta' > 0\,, \qquad i = 1, 2\,, \tag{53}$$

$$\partial X_3^*/\partial V_i = -R^* q_i \Delta_{43}/\Delta' < 0\,, \qquad i = 1, 2\,. \tag{54}$$

Then, the output of domestic public goods increases whereas that of foreign public goods decreases. Now, we want to point out a possibility of immiserizing growth when the domestic factor endowment, V_i, increases. First of all, the change in the domestic social welfare is shown as

$$\partial U/\partial V_i = \textstyle\sum_k U_k\partial D_k/\partial V_i + U_3\partial X_3/\partial V_i + U_4\partial X_3^*/\partial V_i\,, \qquad i, k = 1, 2\,, \tag{55}$$

so that the change in the domestic social welfare is reduced to

$$\partial U/\partial V_i = \phi q_i + U_4\partial X_3^*/\partial V_i\,, \qquad i = 1, 2\,. \tag{56}$$

Therefore, if the decrease in the supply of foreign public goods is enough to eliminate an increase in the welfare caused by an increment in income so that $\partial U/\partial V_i < 0$, then an immiserizing growth could occur in the new Nash equilibrium.

PROPOSITION 4. *If a decrease in the foreign public good is enough to eliminate an increase in domestic national welfare caused by an increment in income in terms of the marginal utility of foreign public goods when the home country experiences economic growth, then in a new Nash equilibrium an immiserizing growth could occur.*

On the other hand, the foreign national welfare always increases when the home country experiences economic growth since

$$\partial U^*/\partial V_i = U_4^* \partial X_3/\partial V_i > 0\,, \qquad i=1,2\,. \tag{57}$$

7. COMPETITIVE SUPPLY OF INTERNATIONAL PUBLIC BADS

Now, consider a case where a domestic public good is a public bad for the foreign people while a foreign public good is a public bad for the local people. For example, the foreign people might dread domestic defense expenditures if they form no reciprocal alliance. In such a case we have the following signs for marginal disutilities:

$$U_4,\ U_4^* < 0\,. \tag{58}$$

Thus, under an optimum tax system, the home country has to transfer $-X_3 p_k U_4^*/U_k^*$ to the foreign country while the foreign country has to transfer $-X_3^* p_k U_4/U_k$ to the home country.

In addition we assume that an increase in foreign public goods or public bads for the local people decreases the marginal utility of the k-th consumption good (U_{14}, $U_{24} < 0$) but does not decrease that of the domestic public good ($U_{34} \geqq 0$) so that we obtain a positive slope of the domestic reaction curve, *i.e.* since U_{k4} is negative but U_{34} is non-negative, $\partial X_3/\partial X_3^* \equiv R' > 0$ from (38) and by the same token $\partial X_3^*/\partial X_3 \equiv R^{*\prime} > 0$.

Let us consider two cases: Case I deals with a monotone decreasing reaction shown in Figure 3, that is:

$$\partial R'/\partial X_3^* < 0\,, \quad \partial R^{*\prime}/\partial X_3 < 0\,, \tag{59}$$

even if an economy specializes in either private good and Case II deals with non-monotone reaction shown in Figure 4. As we assume that a domestic public

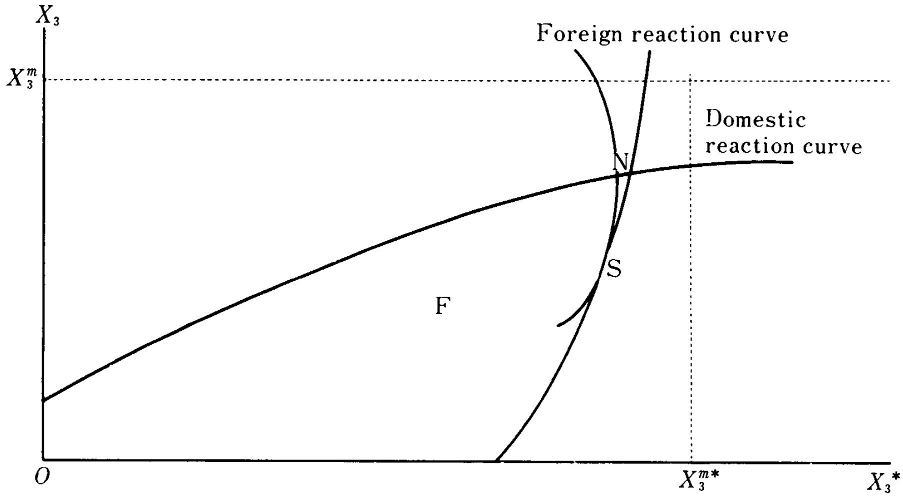

Figure 3.

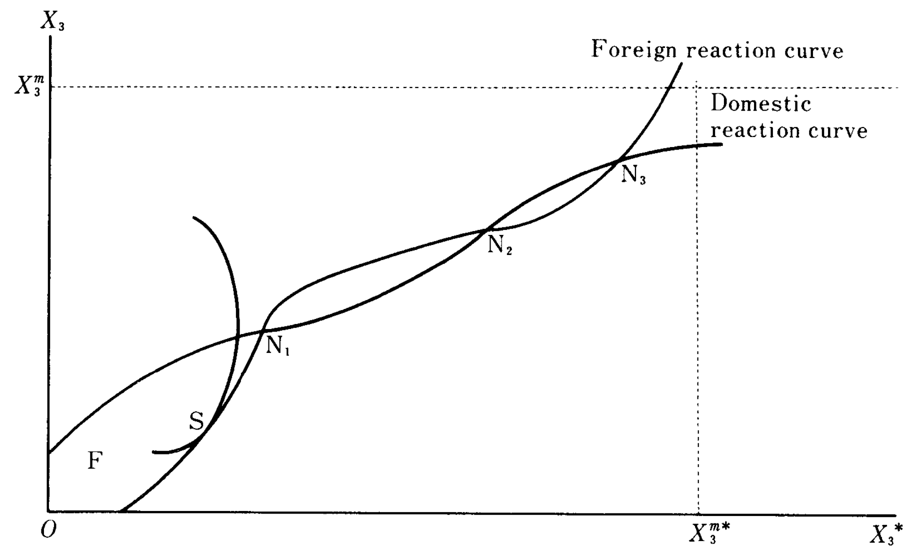

Figure 4.

good is non-traded and a private good is indispensable for positive utility, the reaction curves never reach to the maximum output volume of public good (X_3^m, X_3^{m*}) which could be turned out by all factor endowments.

In Case I there exists a unique and stable Nash equilibrium since $R' < 1/R^{*\prime}$ whereas in Case II there might be multiple Nash equilibria. Suppose there are three Nash equilibria, N_1, N_2 and N_3 as intersections between the two reaction curves. The middle equilibrium N_2 in terms of the volume of public goods is unstable while the rest of the equilibria, N_1 and N_3, are locally stable. Between the two equilibria one equilibrium near to the origin, N_1, is preferable to the other, N_3, for each country so that the former equilibrium must be chosen by each country if they have rough information that an equilibrium near to the origin is the best. In this sense the former equilibrium is not only locally but globally stable. Thus, the analytics of public bads are not of the mirror writing of public goods since there exists no globally stable equilibrium of public good production in international economy with non-monotone reactions as we have seen in Figure 2.

PROPOSITION 5. *In the case of international public bads the Nash equilibrium with monotone decreasing reaction is unique and stable. Even if there are multiple Nash equilibria, there exists a globally stable equilibrium.*

In the Nash equilibrium above, we see that $\partial W/\partial X_3 = W_2 U_4^* < 0$ and $\partial W/\partial X_3^* = W_1 U_4 < 0$ from (40) since one country maximizes her utility with respect to ones own public goods but the other is given negative utility by the goods. Therefore, the conjoined welfare goes up by decreasing output of each public good. Furthermore, as shown in Figure 3, at the first-best solution such as F the amount of each country's public good is less than that in the Nash equilibrium shown as N while in Figure 1 we could not point out that the quantities of the public good are less than at the first-best as Connolly (1970) has noticed that by (40) we mean only that welfares can be improved over the Nash position by increasing outputs of public good, that is, however, a marginal and not an absolute rule.

PROPOSITION 6. *In the case of international public bads the amount of each country's public good at the Nash equilibrium is more than that at the first-best solution.*

The Stackelberg equilibrium, where the home country is well aware of the foreign reaction curve while the foreign country is not familiar with the domestic reaction curve, will be found on the foreign reaction curve when the home country is a leader. From (46) and (47) if the home country is a leader, we obtain the following values at the Nash equilibrium:

$$\partial U/\partial X_3 = U_4 R^{*\prime} < 0\,, \qquad \partial U^*/\partial X_3 = U_4^* < 0\,. \tag{60}$$

Thus, the Stackelberg equilibrium is characterized by a lesser amount of each public good and superior to the Nash equilibrium for each country.

PROPOSITION 7. *In the case of international public bads the Stackelberg equilibrium is superior to the Nash equilibrium for each country whichever country is a leader.*

Comparing proposition 7 above with proposition 2, we again notice that the analytics of public bad are not the same as with the public good. In either the case of the public good or the public bad, a leading nation can raise her welfare by decreasing her own output of public goods. Consequently, when the public good of a leading nation decreases, this results in similarly deteriorating economic welfare for the follower. On the other hand, when the public good of a leader, which is inversely bad for the follower, decreases, this results in ameliorating economic welfare for the follower. Therefore, even if one country has information about the other's reaction curve, she can be better than at the Nash equilibrium by pretending to be a follower.

Suppose the home country experiences economic growth in terms of i-th factor endowment, V_i. Then, from (53) and (54) we directly obtain

$$\partial X_3/\partial V_i = q_i \Delta_{43}/\Delta'' > 0\,, \qquad i = 1, 2\,, \tag{61}$$

$$\partial X_3^*/\partial V_i = R^{*\prime} q_i \Delta_{43}/\Delta'' > 0\,, \qquad i = 1, 2\,. \tag{62}$$

where

$$\Delta'' \equiv (1 - R' R^{*\prime})\Delta > 0\,.$$

Thus, both public good increase when economic growth occurs. Therefore, the change in the domestic social welfare is reduced to the same expression as (56) but the sign of U_4. Hence, again an immiserizing growth could take place in the new Nash equilibrium if the increase in supply of foreign public good is enough to eliminate an increase in the welfare caused by an increment in income. Furthermore, as for a change in the foreign social welfare, we have also the same equation as (57) but the sign of U_4^*. Thus, the economic growth in one country always harms the other country. In this deleterious case no country gains from one country's economic growth even under the given international commodity prices.

PROPOSITION 8. *In the case of international public bads, if an increase in the foreign public good is enough to eliminate an increase in the domestic national welfare caused by an increment in income when the home country experiences economic growth, then in a new Nash equilibrium an immiserizing growth could occur. At the same time, the foreign national welfare also decreases regardless of the above condition.*

8. CONCLUDING SUMMARY

In this paper we have dealt with interesting topics of international spillover effects, focusing on the extreme case of purely public goods and bads of an international type. Our model is one of two small open economies in which there are two internationally immobile factors, two private internationally-traded goods where each country produces a single different public good. Each country's public good benefits the other country as well as the home country.

We derived the optimality conditions under which joint welfare of the two countries is maximized. This requires the existence of a joint welfare function. The conditions involve a Samuelson (1954) summation of marginal rates of substitution criterion for the optimal provision of a public good, and equality of opportunity costs for production efficiency. We then derived optimum tax shares and prices which would lead to satisfaction of the marginal conditions for optimality.

In addition, we have considered the case of non-cooperation, comparing the Nash and Stackelberg equilibria to the cooperative solution. There was also a discussion of the implications for factor accumulation and for the case where one country's public good is a bad for the other country.

In this paper a notion of monotone reaction is crucial for existence of a unique, stable Nash equilibrium in the case of a public good as well as in the case of public bad. Whereas in some points the analytics of public bads are the same as with those of public goods, in the case of public bad there exists a globally stable equilibrium and in the Nash equilibrium the amount of each country's public good is more than of the first-best regardless of the shape of the reaction curves.

Nishogakusha University

REFERENCES

Aliber, R. (ed.), 1977, *The political economy of monetary reform* (London, The Macmillan Press).

Buiter, W. and R. Marston (eds.), 1988, *International economic policy coordination* (Cambridge, Cambridge University Press).

Connolly, M., 1970, Public goods, externalities, and international relations, *Journal of Political Economy*, 78, 279–290.

Connolly, M., 1972, Trade in public goods: A diagrammatic analysis, *Quarterly Journal of Economics*, 86, 61–78.

Frey, B., 1984, *International political economics* (Oxford, Basil Blackwell Publisher).

Hamada, K., 1977, On the political economy of monetary integration: A public economics approach, in Aliber (1977), 13–31.

Markusen, J. R., 1975, International externalities and optimal tax structures, *Journal of International Economics*, 5, 15–29.

Ohyama, M., 1988, Economics of international coordination, in Yanaihara, Fukami and Ohyama (1988), 22–43.

Olson, M. and R. Zeckhauser, 1966, An economic theory of alliances, *Review of Economics and Statistics*, 48, 266–279.

Samuelson, P. A., 1954, The pure theory of public expenditure, *Review of Economics and Statistics*, 36, 387–389.

Yanaihara, K., H. Fukami, and M. Ohyama (eds.), 1988, *New frontier of the world economy* (Tokyo, Bunshindo).