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Notes

SUFFICIENT CONDITIONS FOR THE NON-OPTIMALITY OF MONOPOLISTIC EQUILIBRIUM IN A PURE EXCHANGE ECONOMY*

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Abstract: Consider a two commodity and two agent economy. Agent 1 is a price taker and agent 2 is a price setter. Barring the satisfaction of stringent conditions, it is possible for a monopolistic equilibrium allocation to be Pareto optimal.

I. INTRODUCTION

It is commonly held that in a classical environment a monopolistic equilibrium allocation cannot be Pareto optimal. We will show that such a conclusion requires that a stringent set of sufficient conditions be satisfied; by constructing counter examples, we will also show that the set of conditions cannot be dispensed with.

II. ECONOMY, EQUILIBRIUM AND ASSUMPTIONS

Consider the following economy: Two commodities (commodity x and commodity y); and Two agents (agent 1 and agent 2);

Consumption set: $X(1), X(2) \subset R^2_+$, where R^2_+ is the nonnegative quadrant of R^2.

Preferences: $u^i \in U^i$, where U^i is the class of all utility functions u^i: $X(i) \to R$ of agent i for $i=1$ and 2.

Initial endowments: $e(i)=(\hat{x}_i, \hat{y}_i) \in R^2$.

DEFINITION 1.[1] A *pure exchange private ownership economy* $\mathscr{E}$ is a map from the set of agents $N=\{1, 2\}$ to the space of agents' characteristics $U^1 \times R^2 \times U^2 \times R^2$, $i \mapsto [u^i, e(i)]$.

DEFINITION 2. An *allocation* for an economy $\mathscr{E}$ is a map from N into R^2. A *feasible allocation* is an allocation f for $\mathscr{E}$ with $f(1)+f(2) \leq e(1)+e(2)$ and $f(1) \in X(1)$ and $f(2) \in X(2)$.

DEFINITION 3. A feasible allocation f for an economy $\mathscr{E}$ is *Pareto optimal* if and only if: there is no feasible allocation f' for $\mathscr{E}$ such that

* This paper is based upon the Master thesis by Ndosi on August, 1970 and a note by Saijo on August, 1983.

[1] In what follows we shall abbreviate Definition 1 as (D-1) and Property 1 as (P-1) etc.

$$u^i(f'(i)) > u^i(f(i)) \qquad \text{for all } i\,.$$[2]

DEFINITION 4. An allocation f is *fair* for an economy $\mathscr{E}$ if and only if: it is Pareto optimal and $u^i(f(i)) > u^i(f(j))$ for all i and j with $i \neq j$.[3]

Let Δ be the set of prices with $\Delta = \{q \in R^2_{++} : q = (p, 1)\}$.

We shall introduce the following behavioral assumption: (B) Agent 1 behaves as a price taker (or treats prices parametrically) and we shall call him a competitor; and agent 2 behaves as a price setter and we shall call him a monopolist.

We shall assume (B) throughout this paper.

DEFINITION 5. An *offer* (or *demand*) *curve* for the competitor is a correspondence $C: R^1_{++} \twoheadrightarrow X(1)$ defined by

$$C(p) = \{z = (x, y) \in B(p) : u^1(z) \geq u^1(z') \text{ for all } z' \in B(p)\}\,,$$

where $B(p) = \{(x, y) \in X(1) : px + y \leq p\hat{x}_1 + \hat{y}_1\}$

DEFINITION 6. A *demand set* D of the monopolist is defined by:

$$D = \{z = (x, y) \in F : u^2(z) \geq u^2(z') \text{ for all } z' \in F\}\,,$$

where $F = \{(x_2, y_2) \in X(2) : (x_1, y_1) + (x_2, y_2) \leq e(1) + e(2) \text{ with } (x_1, y_1) \in C(p) \text{ for some } p \in R_{++}\}$.

DEFINITION 7. A price and allocation $((p, 1), f(1), f(2)) \in \Delta \times X(1) \times X(2)$ is a *monopolistic equilibrium* for $\mathscr{E}$ if and only if:

(i) $f(1) = (x_1, y_1) \in C(p)$; and

(ii) $f(2) = (x_2, y_2) \in D$.

Remark. Note that the monopolistic equilibrium concept itself presupposes (B).

We shall introduce the following assumptions:

ASSUMPTION 1. U^i is a class of utility functions satisfying

(i) u^1 and u^2 are continuous on convex sets $X(1)$ and $X(2)$ respectively, and $u^1 \in C^2$ (i.e. twice differentiable) on the interior of $X(2)$.

(ii) The first-order partial derivatives $u^i_j(z) > 0$ for each commodity $j = 1$ and 2.

(iii) u^i is strictly quasiconcave for each i.

(iv) The bordered Hessian determinant

$$H(z) = \begin{vmatrix} 0 & u^1_1(z) & u^1_2(z) \\ u^1_1(z) & u^1_{11}(z) & u^1_{12}(z) \\ u^1_2(z) & u^1_{21}(z) & u^1_{22}(z) \end{vmatrix}$$

[2] Since we shall introduce monotonicity and continuity of utility functions, this definition is equivalent to the usual one (see Hildenbrand and Kirman (1976), page 49).

[3] See H. Varian (1974).

does not vanish on the interior of $X(1)$

ASSUMPTION 2. $e(i)=(\hat{x}_i, \hat{y}_i)\in$ interior of $X(i)$ for each i.

ASSUMPTION 3. If $f=(f(1), f(2))$ is a monopolistic equilibrium allocation, then $f(1)\in$ interior of $X(1)$ and $f(2)\in$ interior of $X(2)$.

ASSUMPTION 4. Any monopolistic equilibrium allocation f is not the same as the initial allocation. That is, $(f(1), f(2)) \neq (e(1), e(2))$, where $(f(1), f(2))$ is any monopolistic equilibrium allocation.
(A-1-iii) implies:

PROPERTY 1. $C(p)$ is a function, not a correspondence.

Let $C(p)=(g(p), h(p))$. (A-1) and (A-2) imply:

PROPERTY 2. g and h are differentiable and g' and h' are finite.[4]

III. AN IMPOSSIBILITY RESULT FOR PARETO OPTIMALITY

Our problem is to check whether or not an monopolistic equilibrium allocation under (A-1), (A-2), (A-3) and (A-4) is Pareto optimal. The following lemma characterizes a condition when there exists an monopolistic equilibrium being Pareto optimal;

LEMMA. *Suppose (A-1), (A-2), (A-3) and (A-4). Let $((p, 1), f(1), f(2))$ be any monopolistic equilibrium. If f is Pareto optimal, then we have*

(T) $$pg'(p)+h'(p)=0\,.$$

Proof. Since f is Pareto optimal, and we have (P-1), (P-2) and (A-3), we can use the first order necessary condition for interior Pareto optimality. By (A-1-ii), we have $u_j^i(\cdot)>0$ for each i and j. Therefore,

(1) $$\frac{u_2^1(f(1))}{u_1^1(f(1))}=\frac{u_2^2(f(2))}{u_1^2(f(2))}\,.$$

On the other hand, since $((p, 1), f(1), f(2))$ is a monopolistic equilibrium,

(2) $$p=\frac{u_2^1(f(1))}{u_1^1(f(1))} \quad \text{and}$$

(3) $$0<\frac{u_2^2(f(2))}{u_1^2(f(2))}=-\frac{g'(p)}{h'(p)}=<+\infty\,.^5$$

Hence (1), (2) and (3) give the result. ■

See figure 1. $c(1)$–$c(1)$ is the offer curve of the competitor. Let f be a monopolistic

[4] See D. Katzner (1968).

[5] The nonzero condition for $h'(p)$ is guaranteed by Arrow-Hurwicz (1958), see section 12.

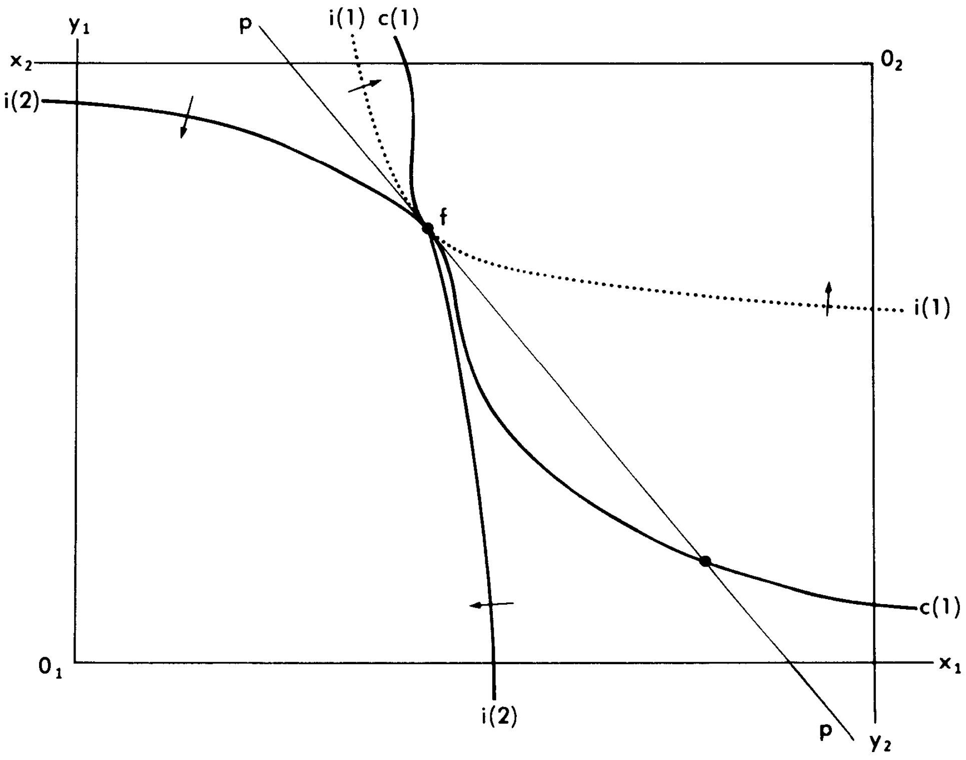

Figure 1

equilibrium allocation. Suppose f is Pareto optimal. Then the competitor's indifference curve ($i(1)$–$i(1)$) and the monopolist indifference curve ($i(2)$–$i(2)$) going through f have to have a common tangent. This is exactly (1) in the lemma. Since f is a monopolistic equilibrium allocation, condition (i) of (D-7) requires that $i(1)$–$i(1)$ be tangent to the price line p–p ((2) in the lemma). Condition (ii) of (D-7) says that the competitor's offer curve should be tangent to the indifference curve $i(2)$–$i(2)$ at f ((3) in the lemma).

The lemma does not claim the possibility of Pareto optimal monopolistic equilibrium allocation.

THEOREM. *Suppose* (A-1), (A-2), (A-3) and (A-4). *Let f be any monopolistic equilibrium allocation. Then f is not Pareto optimal.*

Proof. Suppose by way of contradiction that f is Pareto optimal. Hence it satisfies (T). Since we assumed monotonicity of preferences, i.e., (A-1-ii), the budget inequality should be satisfied with equality:

$$pg(p)+h(p)=p\hat{x}_1+\hat{y}_1 .$$

By differentiation, we have

$$pg'(p)+g(p)+h'(p)=\hat{x}_1 . \tag{4}$$

(T) and (4) show $g(p)=\hat{x}_1$, which contradicts $f \neq e$. ∎

IV. COUNTEREXAMPLES

In this section we shall see that the violation of any assumption in the theorem lead us to the possibility result. That is, the monopolistic equilibrium allocation can be Pareto optimal. Furthermore, our examples will demonstrate that it is fair. In the following, agent 1 is the competitor (i.e., price taker) and agent 2 is the monopolist (i.e., price setter). Table 1 shows the violation of an assumption or a property in examples.

TABLE 1. VIOLATION OF AN ASSUMPTION OR A PROPERTY IN EXAMPLES.

	Example 1 (Fig. 2)	Example 2 (Fig. 3)	Example 3 (Fig. 4)	Example 4 (Fig. 5)
Property 1	×	O	O	O
Property 2	(×)	×	O	O
Assumption 3	O	O	×	O
Assumption 4	O	O	O	×

O indicates the corresponding assumption or property is satisfied.
× indicates the corresponding assumption or property is violated.
(×) indicates the corresponding property is not applicable.

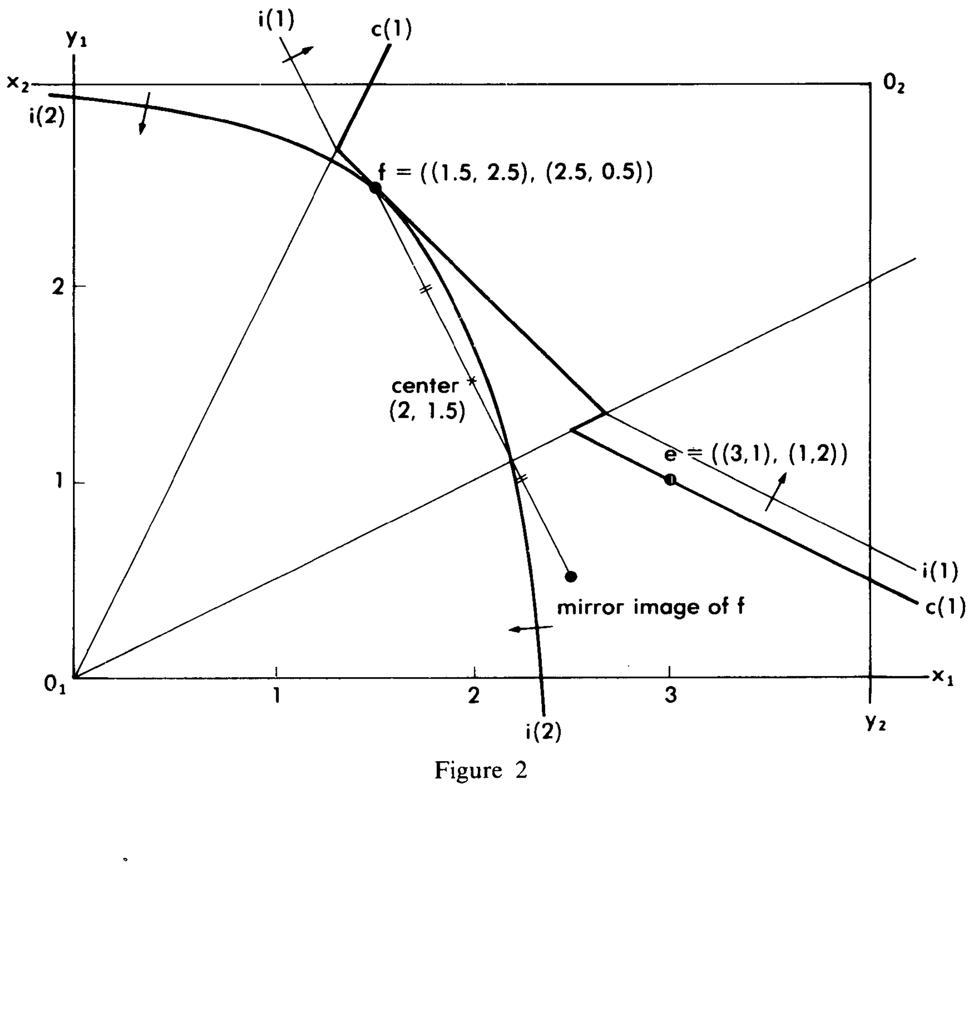

Figure 2

EXAMPLE 1 (see figure 2). Let the agents' utility functions on the nonnegative quadrant be

$$u^1(x_1, y_1) = \begin{cases} 2x_1 + y_1 & \text{if } y_1 \geq 2x_1 \\ (4/3)(x_1 + y_1) & \text{if } (1/2)x_1 < y_1 \leq 2x_1 \quad \text{and} \\ x_1 + 2y_1 & \text{if } x_1 \geq 2y_1 , \end{cases}$$

$$u^2(x_2, y_2) = x_2^{2.5} y_2^{0.5} .$$

Let $e = (e(1), e(2)) = ((3, 1), (1, 2))$. Clearly f is Pareto optimal and fair. (P-1) and (P-2) are violated. The offer curve of agent 1 is not a function, but a correspondence. Roughly speaking, note that the elasticity of demand of x and y at $p = 1$ is infinity.

We shall now construct an example which will keep the Pareto optimal property of example 1 but the offer curve will be a function.

EXAMPLE 2 (see figure 3). Let competitor's utility function be $u_1(v, w) = w - v^4$ with $-0.4 < v < 0.4$ and $w > 0$. The budget constraint is $pv + w = (p, 1) \cdot e(1)$ with $-1 < p < \infty$. Let $e(1) = (\sqrt{2}, 2\sqrt{2})$. Now rotate clockwise both the indifference curves and the budget constraint 45 degrees. The previous line with $w = v$ becomes $0 - x_1$ axis and $w = -v$ becomes $0 - y_1$ axis. We shall put one more restriction on the consumption set. $x_1 > 0$ and $y_1 > 0$. The restriction on v is to preserve monotonicity. The restriction on p should be self evident. We shall compute the original system.[6]

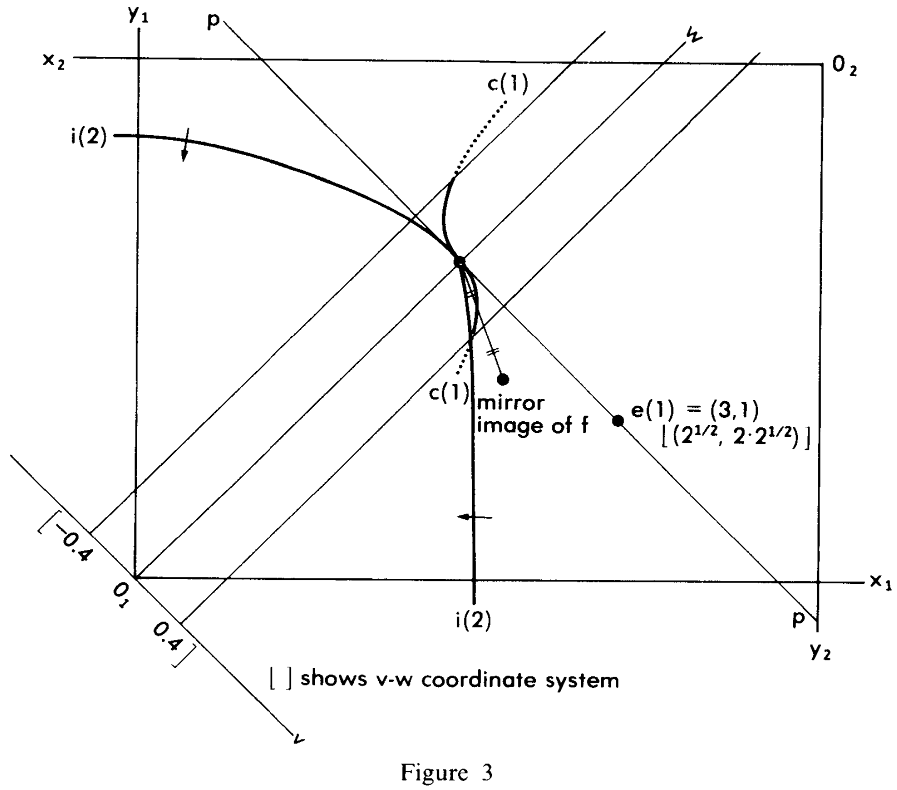

Figure 3

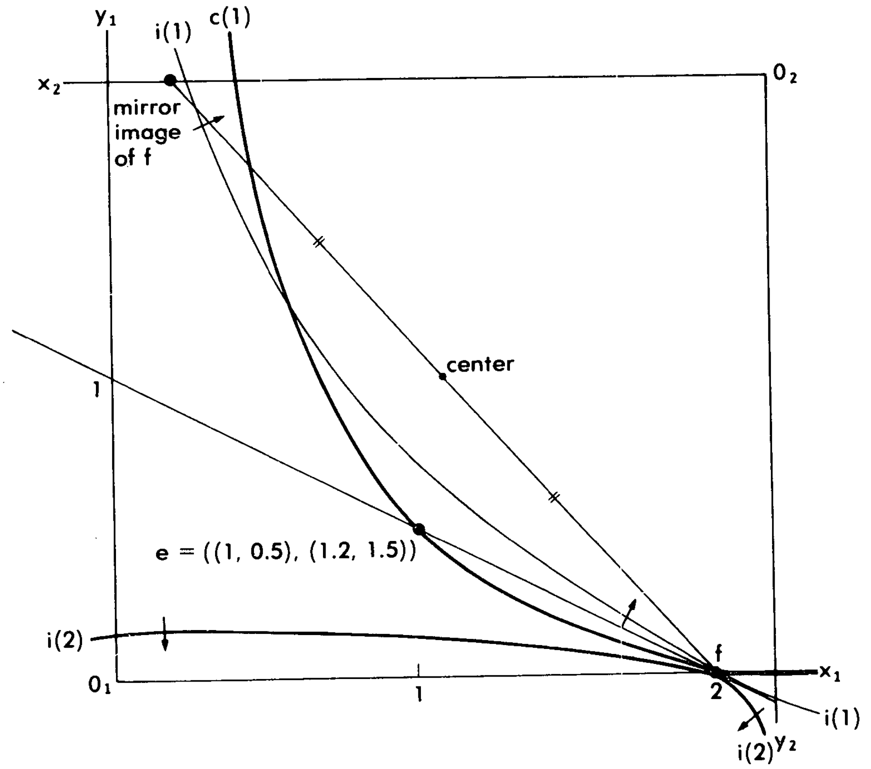

Figure 4

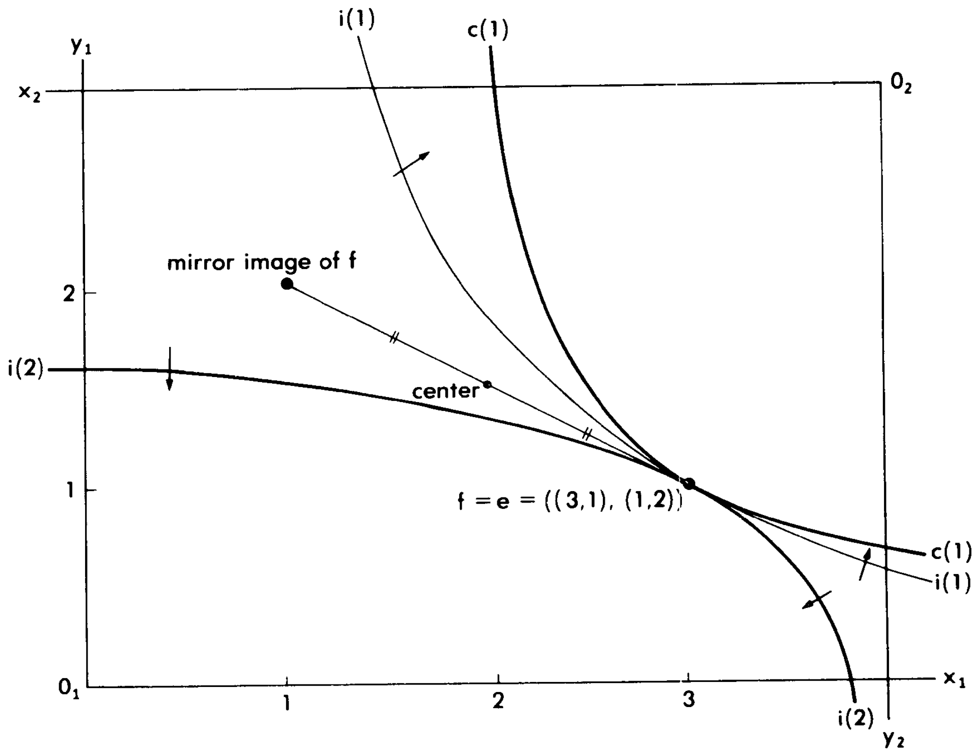

Figure 5

The first order necessary condition and the budget constraint give us:

$$p = -4v^3 \qquad \text{and} \qquad pv + w = \sqrt{2}\,(2+p)\,. \tag{5}$$

Eliminating p in (5), we have

$$w = 4v^4 - 4\sqrt{2}\,v^3 + 2\sqrt{2}\,.$$

Note that monopolistic equilibrium allocation f is Pareto optimal but it does not satisfy (A-1-iv). It is easy to show that (0, 0) is an inflection point, and $f'(0) = \infty$.

EXAMPLE 3 (see figure 4). Let $u^1(x, y) = y + \log x$ with $x > 0$, $y \geq 0$ and $u^2(x, y) = xy^{41}$ with $x, y \geq 0$. Let $e = (e(1), e(2)) = ((1, 0.5), (1.2, 1.5))$. Then the monopolistic equilibrium allocation is $f = (f(1), f(2)) = ((2, 0), (0.2, 2))$. f is Pareto optimal and fair. This violates (A-3).

EXAMPLE 4 (see figure 5). Let $u^1(x, y) = x^3y^2$ with $x, y \geq 0$ and $u^2(x, y) = xy^4$ with $x, y \geq 0$. Let $e = (e(1), e(2)) = ((3, 1), (1, 2))$. Then the monopolistic equilibrium allocation is $f = (f(1), f(2)) = ((3, 1), (1, 2)) = e$. f is Pareto optimal and fair. This violates (A-4).

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[6] See D. Katzner (1968), page 54.