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Sub Title	
Author	SAKAI, Yasuhiro
Publisher	Keio Economic Society, Keio University
Publication year	1991
Jtitle	Keio economic studies Vol.28, No.1 (1991.) ,p.51- 71
JaLC DOI	
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Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19910001-0051

INFORMATION SHARING IN OLIGOPOLY: OVERVIEW AND EVALUATION PART II. PRIVATE RISKS AND OLIGOPOLY MODELS

Yasuhiro SAKAI*

Abstract. Part I of this paper has first discussed the dual relationship between the Cournot and Bertrand duopoly models in the absence of uncertainty, and has then proceeded to focus on various types of duopoly models facing a common risk of demand or cost. It has been shown that the welfare consequences of an information transmission agreement between the firms are clasified under four headings: own and cross variation effects, and own and cross efficiency effects.

Part II of the paper will now turn to the situation under which each firm is subject to its own risk. We will show that as was seen in the case of a common risk, the welfare implications of information sharing are sensitive to many factors. They are: the type of competition (Cournot or Bertrand), the nature of uncertainty (demand or cost), the number of participating firms, and the degree and direction of physical and stochastic interdependence among demand or cost parameters.

It will also be argued that our theoretical investigation of information pooling sheds new light both on the desirability of trade associations and on the merits or demerits of industrial policies.

I. THE CASE OF PRIVATE RISKS

In Part I, we have been concerned with the case of a common disturbance in the sense that the two firms face the sole common disturbance to their demand/cost functions. Such an environment is called a "common value" problem in the auction literature. However, there is another equally important environment called a "private values" problem in the same literature. In Part II, we will deal with the case of idiosyncratic disturbances: There are now two different sources of uncertainty, with each source being associated with one firm.

A. *Cournot Duopoly with Private Demand Risks*

Let us start our inquiry with a Cournot duopoly model with each firm facing its own demand uncertainty. As in the case of a common risk, there are two Cournot firms—Firm 1 and firm 2. We assume that the demand paramerters $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are random parameters whose joint distribution $\Phi(\tilde{\alpha}_1, \tilde{\alpha}_2)$ is a common

* I wish again to express my indebtedness to the persons and institutions listed in the first footnote of Part I of this paper.

knowledge to both firms.[1]

Concerning $\Phi(\tilde{\alpha}_1, \tilde{\alpha}_2)$, it is usually assumed that its regression equations are linear. The bivariate normal distribution represents a distinguished member of such a family. The linearity of regression equations makes our calculations fairly manageable, otherwise we would be entangled in a mathematical jungle, perhaps with no exit in sight.

It is convenient to express the information structure of our model in terms of a symbol η_{ik} ($i=1, 2$; $k=1, 2$) such that $\eta_{ik}=1$ if firm i knows the realized value of $\tilde{\alpha}_k$, and $\eta_{ik}=0$ otherwise. Let $\eta=[\eta_{11}\eta_{12}, \eta_{21}\eta_{22}]$. Since each η_{ik} takes on either 1 or 0, there are $2^4=16$ information structures among which the following three cases are to be discussed in this paper.[2] They are: [00, 00], [10, 01], and [11, 11].

Given one of the information structures, each firm is assumed to play Nash, so that it has no incentive to deviate from an equilibrium whenever it is reached. Formally, the pair (x_1^0, x_1^0) of output strategies is said to be an equilibrium pair under no information [00, 00] if

$$x_i^0 = \operatorname*{Arg\,Max}_{x_i \geqq 0} E[\Pi_i(x_i, x_j^0, \tilde{\alpha}_i)] . \qquad (i, j=1, 2;\ i \neq j)$$

With no information about $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ available, each firm's optimal strategy must be a routine action in the sense that it does not take account of specific values of $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$.

Now suppose that firm i knows its own demand $\tilde{\alpha}_i$. Then its optimal strategy is no longer a routine action, but an action contingent on the true value of $\tilde{\alpha}_i$. Therefore, given $\eta^N=[10, 01]$, we call the pair $(x_1^N(\tilde{\alpha}_1), x_2^N(\tilde{\alpha}_2))$ of output strategies an equilibrium pair under η^N if for each $\tilde{\alpha}_i$,

$$x_i^N(\tilde{\alpha}_i) = \operatorname*{Arg\,Max}_{x_i \geqq 0} E[\Pi_i(x_i, x_j^N(\tilde{\alpha}_j), \tilde{\alpha}_i) \,|\, \tilde{\alpha}_i] \qquad (i, j=1, 2;\ i \neq j) ,$$

where the expectation is taken over $\tilde{\alpha}_j$.

If both firms agree to exchange its private information with each other through a trade association or the like, we come to the situation of shared information $\eta^S=[11, 11]$. The pair $(x_1^S(\tilde{\alpha}_1, \tilde{\alpha}_2), x_2^S(\tilde{\alpha}_1, \tilde{\alpha}_2))$ of output strategies is called an equilibrium pair under η^S if for each $(\tilde{\alpha}_1, \tilde{\alpha}_2)$,

$$x_1^S(\tilde{\alpha}_1, \tilde{\alpha}_2) = \operatorname*{Arg\,Max}_{x_i \geqq 0} \Pi_i(x_i, x_j^S(\tilde{\alpha}_1, \tilde{\alpha}_2), \tilde{\alpha}_i) . \qquad (i, j=1, 2;\ i \neq j)$$

[1] As far as the Cournot model is concerned, whether the information in question is demand information or cost information does not matter at all. The line of research with Cournot duopoly under private uncertainty was initiated by Okada [1982] and Sakai [1985] for a homogenous case, and was extended by Gal-Or [1986], Fried [1984], Sakai [1987], and others to cover a wide range of product differentiation.

[2] All sixteen cases were extentively discussed by Sakai [1985] for the extreme case of perfect substitutes (viz., $\theta=1$). While Fried [1984] did a splended welfare analysis of information sharing within a similar framework, he restricted his attention to just nine cases out of possible sixteen, and failed to consider the welfare impact on consumers and the whole society. In his pioneering work [1982], Okada considered only four cases.

(a) $\theta > 0$ and $\rho = 0$ (b) $\theta < 0$ and $\rho = 0$

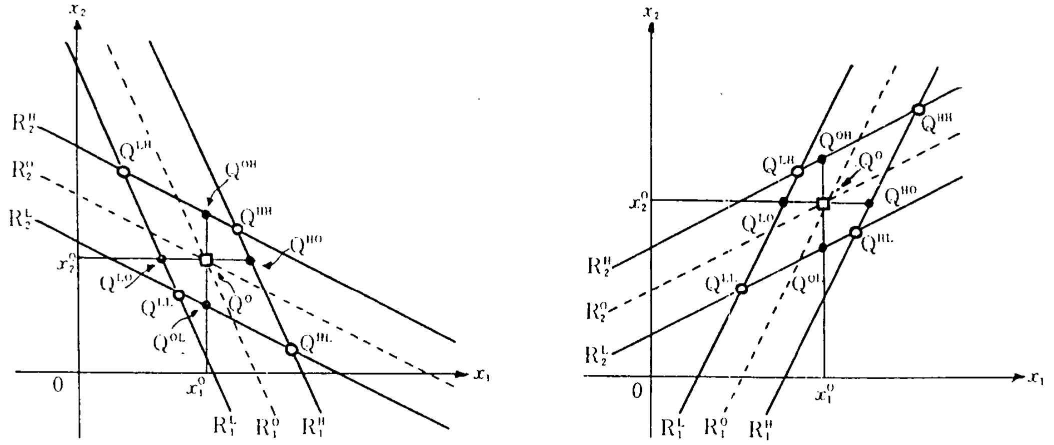

Fig. 1. Cournot Duopoly Equilibria under η^N and η^S: The Case of Private Demand Uncertainty $(\tilde{\alpha}_1, \tilde{\alpha}_2)$

In this case, each firm's optimal strategy ought to be a contingent action with its contingency depending on both $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$.

Figure 1 depicts Cournot-Nash equilibria with private demand risks and under various information situations. It is assumed that each demand intercept $(\tilde{\alpha}_i)$ must be one of two equally likely values—high (H) or Low (L). In order to make a diagrammatic illustration quite easy, we suppose that the stochastic variables, $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$, are uncorrelated (namely, $\rho = 0$).[3] As in Figure 2 of Part I, two parallel lines R_i^H and R_i^L respectively represent firm i's reaction function when its private demand is high and low $(i = 1, 2)$; and a dotted middle line R_i^0 stands for the average of these two reaction functions.

When neither $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ is known to both firms, Q^0 represents an equilibrium and (x_1^0, x_2^0) the pair of equilibrium output strategies. In case each firm becomes informed of its own demand but not its opponent's, an equilibrium is shown by a set of the four solid points, Q^{HO}, Q^{LO}, Q^{OH}, and Q^{OL}, with $(x_1^{HO}, x_1^{LO}; x_2^{OH}, x_2^{OL})$ being the vector of equilibrium output strategies. On the other hand, an equilibrium when both firms exchange its private information with each other is represented by a set of the four hollow point, Q^{HH}, Q^{HL}, Q^{LH} and Q^{LL}, and the vector of equilibrium output strategies by $(x_1^{HH}, x_1^{HL}, x_1^{LH}, x_1^{LL}; x_2^{HH}, x_2^{HL}, x_2^{LH}, x_2^{LL})$. Diagrams (a) and (b) show the cases of substitutes and of complements, respectively. It would be quite instructive to graphically see the way in which a set of four equilibrium points spreads out through information pooling. However, a graph is no more than a graph and cannot totally replace real computation.

[3] For a graphical presentation only, we make the assumption of no correlation at this point. The welfare analysis of this paper can cover the whole range of correlation from minus unity to plus unity. Since Gal-Or [1986] assumes that goods are substitutes and that stochastic parameters are noncorrelated, her analysis corresponds well to Figure 4(a).

Provided one of the information structures, we are able to first find the equilibrium pair of output strategies, and to proceed to compute each firm's expected profits, expected producer surplus, expected consumer surplus, and expected total surplus. Since the computation is analogous to the one we did for the case of a common risk, it may be omitted here. We are content to record the following useful set of welfare equations:

$$\Delta E\Pi_i = -\beta\Delta\,\mathrm{Var}(x_i) - \beta\theta\Delta\,\mathrm{Cov}(x_1, x_2) + \Delta\,\mathrm{Cov}(\tilde{\alpha}_i, x_i)\,, \quad (i=1,2) \tag{5.1}$$

$$\Delta EPS = -\beta\textstyle\sum_i\Delta\,\mathrm{Var}(x_i) - 2\beta\theta\Delta\,\mathrm{Cov}(x_1, x_2) + \sum_i\Delta\,\mathrm{Cov}(\tilde{\alpha}_i, x_i)\,, \tag{5.2}$$

$$\Delta ECS = (\beta/2)\textstyle\sum_i\Delta\,\mathrm{Var}(x_i) + \beta\theta\Delta\,\mathrm{Cov}(x_1, x_2)\,, \tag{5.3}$$

$$\Delta ETS = -(\beta/2)\textstyle\sum_i\Delta\,\mathrm{Var}(x_i) - \beta\theta\Delta\,\mathrm{Cov}(x_1, x_2) + \sum_i\Delta\,\mathrm{Cov}(\tilde{\alpha}_i, x_i)\,. \tag{5.4}$$

The above system (5.1)–(5.4) for the case of private demand risks is seen to be similar to the system (3.8)–(3.11) for the case of a common demand risk, only the difference being that there is now a firm-specific parameter $\tilde{\alpha}_i$ instead of a industry-wide parameter $\tilde{\alpha}$. As in the previous situations, there exist two channels through which information sharing between the two firms affects the equilibrium values of welfare quantities. They are: variation and efficiency channels.

A good summary of the welfare effects of information pooling for Cournot duopoly with private demand risks is provided by Table 1. Let us compare this table with Table 4 of Part I made for a common demand risk. Then we immediately see that a mosaic-type diagram enchased with plus, minus and zero signs becomes much simpler in the sense that only one sign is attached to each block regardless of the value of θ. The reason for it is that the transmission of information between the two firms is now "two-way" and thus both firms may be treated symmetrically.

By taking a close look at Table 1, we are led to the following welfare results:

(i) The exchange of demand information between two Cournot firms makes

TABLE 1. COURNOT DUOPOLY WITH PRIVATE DEMAND RISKS $(\tilde{\alpha}_1, \tilde{\alpha}_2)$: VARIATION AND EFFICIENCY CHANNELS

The Impact of Information Sharing	OWN VARIATION		CROSS VARIATION	OWN EFFICIENCY		CROSS EFFICIENCY		TOTAL
	ΔVar (x_1)	ΔVar (x_2)	$\theta\Delta$Cov (x_1, x_2)	ΔCov $(\tilde{\alpha}_1, x_1)$	ΔCov $(\tilde{\alpha}_2, x_2)$	ΔCov $(\tilde{\alpha}_1, x_2)$	ΔCov $(\tilde{\alpha}_2, x_1)$	
	+	+	−	+	+	0	0	
$\Delta E\Pi_1$	−	0	+	+	0	0	0	+
$\Delta E\Pi_2$	0	−	+	0	+	0	0	+
ΔEPS	−	−	+	+	+	0	0	+
ΔECS	+	+	−	0	0	0	0	−
ΔETS	−	−	+	+	+	0	0	+

each firm's production activity more responsive to a change in demand, so that it increases the variance of each output (the own variation effect). This results in a fall in expected producer surplus and to a rise in expected consumer surplus.

(ii) Information sharing has a tendency to reinforce the degree of interaction between the strategies of the two firms (the cross variation effect). Since the reaction curves of firms are negatively (or positively) sloped whenever goods are substitutes (or complements), information pooling increases the product of θ and $-\mathrm{Cov}(x_1, x_2)$ (see Figure 1). The greater the strategic interaction between both firms, the more advantageous the position of "producers as insiders" and the more disadvantages the position of "consumers as outsiders."

(iii) Information pooling contributes to the efficiency allocation of resources across firms (the efficiency effect). In fact, it increases the value of $\mathrm{Cov}(\tilde{\alpha}_i, x_i)$, meaning that the firm facing greater (or lesser) demand is likely to have a larger (or smaller) market share. A better correspondence between demands and outputs means an additional gain in the welfare of producers. It is noted that consumers are not *directly* affected by such reallocation, even if it could be *indirectly* influenced through corresponding changes in outputs.

(iv) The last column indicates the total welfare impact combining (own and cross) variation and efficiency effects. Information sharing increases producer welfare as well as overall welfare, but decreases consumer welfare; which clearly would agree with common sense.

B. *Other Duopoly Models with Private Risks*

Let us continue to assume that firms act as Cournot competitors and thus employ quantities as their strategic variables. Then as we noted in 4.A, whether private risks are about demands or costs does not matter at all. If we discuss the situation under which the vector $(\tilde{k}_1, \tilde{k}_2)$ of cost parameters rather than the vector $(\tilde{\alpha}_1, \tilde{\alpha}_2)$ of demand parameters is a stochastic vector, we are able to draw a table analogous to Table 10, only the difference being that we must now compute the value of $(-\Delta\,\mathrm{Cov}(\tilde{k}_i, x_i))$ instead of that of $\Delta\,\mathrm{Cov}(\tilde{\alpha}_i, x_i)$. Therefore, the welfare results obtained for the case of private demands can be applied to the present case of private costs with appropriate modifications.

Now, let us turn our attention to the situation under which firms play as Bertrand competitors and thus use prices as their strategic variables. In such a case of Bertrand duopoly, the question of whether private risks are about demand or costs becomes important, and may significantly affect the concluding part of welfare implications of information sharing in oligopoly.

Let us assume that each of Bertrand competitors faces its own demand risk. Specifically, we assume that the demand parameters $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are random variables whose joint distribution is a bivariate normal distribution. We are concerned with comparing nonsharing information and sharing information equilibria on an *ex* *ante* basis.[4]

By drawing a table similar to Table 1 above, we may obtain the following welfare results:[5]

(i) If two Bertrand firms agree to exchange private demand information with each other, then each firm's price level becomes more responsive to a change in its private demand (the own variation effect). As a result, expected producer surplus and expected total surplus fall while expected consumer surplus rises.

(ii) Information pooling has an effect of reinforcing the strategic interaction between the two firms (the cross variation effect). The greater such an interaction, the more beneficial the position of producers and the less beneficial the position of consumers.

(iii) A better correpondence between demands and prices is now possible by information exchange between two Betrand firms (the (own) efficiency effect). This is not only beneficial to producers, but is now *harmful* to consumers; which is a new feature of Bertrand model with demand uncertainty.

(iv) In a sharp contrast to the Cournot case, there emerges a new sign pattern for the welfare impact on *ECS* and *ETS* through the own efficiency channel. Note that a gain in *EPS* and a loss in *ECS* via this route are just counterbalanced so that *ETS* remains unaffected.

(v) In spite of the appearance of the own efficiency effect on the part of consumers, it is remarkable to see that the total welfare impact of demand information sharing between Bertrand firms is the same as the one between Cournot firms. As in the Cournot case, information pooling increases the welfare of producers and the total welfare, but it decreases the welfare of consumers.

Finally, let us discuss the situation that Bertrand firms are subject to private cost uncertainty. Among the four cases of private risks, this constitutes the most delicate case to obtain the welfare results.[6]

(i) As in the previous cases, the pooling of cost information between Bertrand firms tends to increase the variance of each firm's price (the own variation effect) and to strengthen the degree of interaction between the two prices (the cross variation effect). The own variation effect contributes negatively to the welfare of producers and the whole society, and positively to the welfare of consumers. The cross variation effect has exactly opposite welfare implications from the own

[4] Bertrand duopoly model with private demand risks was analyzed by Sakai [1987]. However, the welfare analysis of information sharing there was not complete. This earlier work not only failed to investigate the welfare impact on consumers and the whole society, but also neglected the decomposition into variation and efficiency channels.

[5] To save the space, we omit detailed tables indicating the welfare impact through variation and efficiency channels for the present and following cases. For a fuller explanation, see Sakai [1989].

[6] Bertrand duopoly under private cost uncertainty was studied by Gal-Or [1986] for the special case where goods are substitutes and costs are not correlated (i.e., $\theta > 0$ and $\theta = 0$). However, the welfare impact on consumers and the whole society was not discussed in her otherwise excellent work. A more complete welfare analysis, which allows for complementary goods and for positively or negatively correlated costs, was independently done by Sakai & Yamato [1989].

variation effect.

(ii) An information exchange yields an improved correspondence between the cost and price of each firm (the own efficiency effect). Therefore, just as in the case of private demand risks, this has a beneficial effect of the welfare of producers and whole society. However, contrary to the situation of private demand uncertainty, it has no effect whatever on the welfare of consumers.

(iii) Remarkably, there is another kind of allocation repercussion across firms (the cross efficiency effect). It can be shown that if goods are substitutes (or complements) then information pooling increases (or decreases) the covariance between the cost of one firm and the price of the other. Such a repercussion has a disturbing impact on resource allocation across firms, regardless of technical substitution between goods. Presence of this cross efficiency effect distinguishes the welfare analysis of Bertrand dupoly with private cost risks from all other duopoly cases with private risks.

(iv) Information sharing may benefit firms in some situations but it hurts them in other situations, depending on the degree of substitutability, θ, and the degree of correlation, ρ. The product $(\theta\rho)$ of these two parameters measures the degree of combined interaction between the two firms, considering both physical and stochastic factors. In general, the total variation and efficiency effects operate in mutually opposing directions.[7] If, and only if, combined interaction is large enough (more exactly, $\theta\rho > (4-3\theta^2)/[2(2-\theta^2)]$, the total variation effect would dominate the total efficiency effect, so that the exchange of cost information would benefit participating firms. Concerning the consumer side, only the total variation effect is operating against consumer surplus because there is no (own or cross) efficiency effect present. The result is that information sharing is detrimental to consumers.

(v) In sharp contrast to all previous cases with private risks, information pooling is not socially desirable. More significantly, except when combined interaction is positive and strong, the pooling case must be Pareto inferior to the non-pooling case. This is presumably the worst possible situation we face among all types of duopoly under private uncertainty.

II. OLIGOPOLY MODELS

In the above, we have found that the welfare implications of information transmission between firms are sensitive to strategic variables (output versus prices), the source of risk (demand versus cost), and the type of uncertainty (a common value versus private values). What we are going to do in this section is to show that those implications are also very dependent on the number of firms in an

[7] We can show that concerning the variation aspect, the own effect is overpowered by the cross effect, meaning that the total variation works for the welfare of producers. On the other hand, since the own effect is dominated by the cross effect on the efficiency side, the total efficiency effect operates against producers. See Sakai & Yamato [1989].

industry. As will be shown, the possibility that information sharing among producers benefits consumers as "outsiders" would arise and grow as the number of "insiders" increases.

While we aim to extend our welfare analysis to the general case of oligopoly, we limit our attention to the situation under which each Cournot or Bertrand firm faces private cost uncertainty. We believe that this constitutes the most interesting case we can think of, and that we may handle other cases in a similar fashion.

A. *The Basic Model*

The generalization of a duopoly model to an oligopoly model is rather straightforward if each firm is treated symmetrically. On the production side, we have an oligopolistic sector with n firms, with firm i producing a differentiated output x_i $(i=1, \cdots, n)$, and a competitive sector producing a numéraire good x_0. Let p_i be the price of x_i $(i=1, \cdots, n)$.

On the consumption side, we have a continuum of consumers of the same type such that the utility function of the representative consumer is of the following form:

$$U = x_0 + \alpha \textstyle\sum_i x_i - (1/2)\beta(\sum_i x_i^2 + \theta \sum_i \sum_{j \neq i} x_i x_j), \tag{6.1}$$

where both α and β are positive. Without loss of generality, assume that β is unity.

If U is to be concave, the following matrix must be positive definite:

$$\theta = \begin{pmatrix} 1 & \theta & \theta & \cdots & \theta \\ \theta & 1 & \theta & \cdots & \theta \\ \vdots & & & \ddots & \vdots \\ \theta & \theta & \theta & \cdots & 1 \end{pmatrix}.$$

This implies that the value of θ must lie between $(-1)/(n-1)$ and 1.[8]

We assume that the consumer maximizes U subject to his budget constraint. Inverse demand functions are then given by the set of linear equations:

$$p_i = \alpha - x_i - \theta \textstyle\sum_{j \neq i} x_j \qquad (i=1, \cdots, n), \tag{6.2}$$

provided that prices are positive. Note that any two goods are substitutes, independent, or complements according to whether θ is greater, equal to, or less than zero.

If we solve for x_i in (34), we may obtain direct demand functions as

$$x_i = a - b[1+(n-2)\theta]p_i + b\theta \textstyle\sum_{j \neq i} p_j, \tag{6.3}$$

provided that outputs are positive. It is easy to see that $a = \alpha/[1+(n-1)\theta]$ and $b = 1/(1-\theta)[1+(n-1)\theta]$.

As in the case of duopoly, we assume that the technology exhibits constant

[8] Such a symmetric case was investigated by Dixit & Stern [1982] and Friedman [1983] for oligopoly models under no uncertainty.

returns to scale, whence firm i has constant unit cost k_i ($i=1, \cdots, n$). In order to make our computation manageable, let us assume that $(\tilde{k}_1, \cdots, \tilde{k}_n)$ is a stochastic vector, the joint distribution of which is normal with mean $(\mu, \cdots, \mu)$ and covariance matrix Σ, where

$$\Sigma=\sigma^2\begin{pmatrix} 1 & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \vdots & & & \ddots & \vdots \\ \rho & \rho & \rho & \cdots & 1 \end{pmatrix}.$$

Because the matrix Σ is positive definite, the value of ρ must lie between $(-1)/(n-1)$ and 1. Note that the taste matrix θ and the covariance matrix Σ are both symmetric and of the same form.

It should be noticed that the specific form of a normal distribution is not essential for our analysis. What we need to have for computational convenience is the property of linearity of regression equations. Indeed, the normal case meets such requirement and the regression equations are written as

$$E(\tilde{k}_j \mid \tilde{k}_i)=\rho(k_i-\mu)+\mu \qquad (i, j=1, \cdots, n;\ i\neq j). \tag{6.4}$$

Profits of firm i are given by $\Pi_i=(p_i-\tilde{k}_i)x_i$ ($i=1, \cdots, n$). While producer surplus is the sum of those profits over i, consumer surplus is simply measured by $CS=U-x^0-\sum_i p_i x_i=(1/2)\sum_i(\alpha_i-p_i)x_i$ if the utility function is provided by (6.1).

Concerning the information structure of our oligopoly model, we focus our attention to the two cases as we did for the case of duopoly with private risks: (i) the case of private information in which each firms acquires information about its own cost, but not its rival's; and (ii) the case of shared information in which each firm gets information about both costs.

B. *Cournot Oligopoly*

To begin with, let us suppose that firms are Cournot-type competitors, with outputs being their strategic variables.[9] Following the same method of computation as we did for the case of duopoly, we may find various equilibrium values for the cases of private and shared information.

For the sake of convenience, let us introduce the following notations:

$$\text{OWN VARI}=-\beta\textstyle\sum_i \text{Var}(x_i), \tag{6.5}$$

$$\text{CROSS VARI}=-\beta\theta\textstyle\sum_i\sum_{i\neq j}\text{Cov}(x_i, x_j), \tag{6.6}$$

[9] The problem of information sharing in Cournot oligopoly has been much concern in the modern theory of oligopoly and industrial organization. Gal-Or [1985], Li [1985], and Shapiro [1986] attacked the problem for a simple case of homogenous goods while Sakai [1988] worked with a more general case of product differentiation. There exist another group of papers such as Ponssard [1979], Clarke [1983], and Nalebuff & Zeckhauser [1986] which limit attention on the presence of only one risk, maintaining the assumption of homogeneous goods. Note that if all private risks are perfectly and positively correlated, then the case of private risks may boil down to the one of a common risk.

TABLE 2. COURNOT OLIGOPOLY WITH PRIVATE COST RISKS ($\tilde{k}_1, \cdots, \tilde{k}_n$)

The Impact of Information Sharing	VARIATION		EFFICIENCY		TOTAL
	OWN	CROSS	OWN	CROSS	
ΔEPS	−	+	+	0	+
ΔECS	+	−	0	0	? $\begin{cases} -(n<9) \\ \pm(n \geqq 10) \end{cases}$
ΔETS	−	+	+	0	+

$$\text{OWN EFFI} = -\textstyle\sum_i \text{Cov}(\tilde{k}_i, x_i)\,. \tag{6.7}$$

Then by making use of (6.5)–(6.7), it is a cubersome yet straightforward task to obtain the following set of welfare equations:[10]

$$\Delta EPS = \Delta(\text{OWN VARI}) + \Delta(\text{CROSS VARI}) + \Delta(\text{OWN EFFI})\,, \tag{6.8}$$

$$\Delta ECS = -(1/2)\Delta(\text{OWN VARI}) - (1/2)\Delta(\text{CROSS VARI})\,, \tag{6.9}$$

$$\Delta ETS = (1/2)\Delta(\text{OWN VARI}) + (1/2)\Delta(\text{CROSS VARI}) + \Delta(\text{OWN EFFI})\,. \tag{6.10}$$

It would be natural to refer to the terms OWN VARI, CROSS VARI and OWN EFFI as the *own variation* term, the *cross variation* term and the *own efficiency* term, respectively. The first term (OWN VARI) consists of the variances of x_i while the second term (CROSS VARI) comprises the covariances of x_i and x_j ($i \neq j$) and the degree of technical substitution between them. These two are related to the variation side of firms' strategic variables. In contrast, the third term (OWN EFFI) is associated with the covariance between the cost and output of each firm, shedding light on the efficiency side of firms in an industry.

Table 2 summarizes the welfare impact of information exchange via variation and efficiency channels. This table can be regarded as a generalization of Table 1 to the case of oligopoly since the latter table is also applicable to the Cournot duopoly case with private *cost* risks.

We are able to draw several welfare implications of information sharing among Cournot firms from Table 2:

(i) On the one hand, if we look at this table from top to down, we can see in what direction the welfare of producers, consumers and the whole society is influenced through each given channel. On the other hand, if we look at the table from left to right, we may understand how the welfare of producers or consumers must change through variation and efficiency channels. By taking a look at Table 2 either vertically or horizontally, we can see that there is a general tendency that a minus sign is followed by a plus sign that is in turn followed by a minus sign... Such a sequence of minus and plus signs makes our welfare analysis considerably

[10] For the detailed derivation of these formulas, see Sakai [1988].

complicated yet extremely interesting.

(ii) The last column teaches us the total welfare impact of information sharing, taking account of many opposing effects working on the variation and efficiency sides. First, information pooling tends to increase the welfare of producers, regardless of the number of firms in an industry. Second, information pooling has a tendency to improve the overall welfare as well, meaning that information is good for the society. These welfare results are the same as those obtained for duopoly.

(iii) When we turn to the welfare impact for consumers, the situation changes drastically and becomes much more intricate in the sense that the impact is quite sensitive to the number of firms. The quantity ΔEPS may move in either direction, depending on the number of producers. Since there is no efficiency effect term present on the consumer side, the direction of change is determined by the relative strength of the own and cross variation effects. If there are a few firms (more exactly, less than nine firms for our model), then information sharing increases the variance of each firm's output so much that the own variation effect overwhelms the cross variation effect. Note that this result is obtained regardless of the degree of substitutability, θ, and the direction of correlation, ρ. If, however, there are many firms (at least as many as ten firms for our model), then the power of the cross effect is weakened and thus the position of consumers as outsiders is relatively strengthened. The result is that information pooling among firms may even be beneficial to consumers as well.[11]

Result (iii) is of great importance, for it shows the possibility that the information pooling situation is Pareto superior to the non-pooling situation if the number of participating firms is large enough. Figure 1 indicates more specifically how the sign of the quantity ΔECS is sensitive to the values of θ and ρ when n takes on four values: $n=1$, 10, 20, and 50.

In the interior of the shaded area in Figure 2, the quantity ΔECS takes on positive values, meaning that information sharing among firms benefits consumers in terms of expected consumer surplus. On those solid lines in which $\theta=0$ or $\rho=1$, $(-1)/(n-1)$, and on those solid curves $H_nM_nK_n$ $(n=10, 20, 50)$ in which the pair (θ, ρ) satisfies the equation $\rho=[4+(n-1)\theta^2]/\theta^2(n-1)[(n-2)-(n-1)\theta]$, the quantity just vanishes, so that consumers' gains due to information pooling are nil. And any point in the remaining blank area represents the situation under which information exchange has a harmful effect on consumers. Besides, the coordinates of points H_n, M_n and K_n are approximately given as follows: $H_{10}=(0.334, 1)$, $M_{10}=(0.5, 0.794)$, $K_{10}=(0.666, 1)$; $H_{20}=(0.120, 1)$, $M_{20}=(0.5, 0.217)$, $K_{20}=(0.880, 1)$; and $H_{50}=(0.043, 1)$, $M_{50}=(0.5, 0.057)$, $K_{50}=(0.957, 1)$.

[11] Such a distinction between "a few" and "many" may be compared with Selten's famous result [1973] saying that four are "few" and six are "many." By using a simple model of cartel formation, Selten has shown that if there are at least as many as six firms then there emerges the situation where any one firm prefers staying out of the cartel to being an insider. In our oligopoly setting, consumers are supposed to take the position of outsiders.

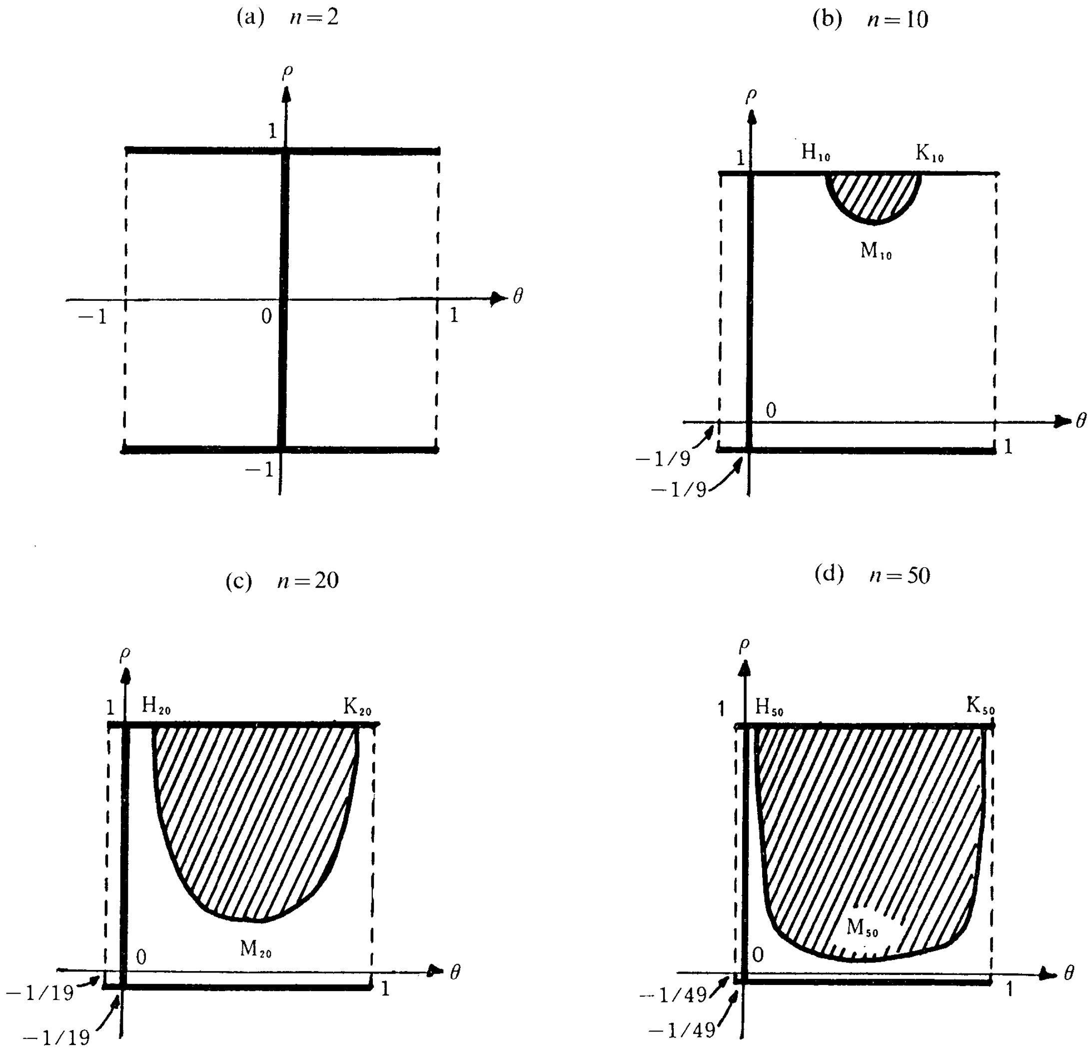

Fig. 2. The Effect of Information Sharing on Consumers: Cournot Oligopoly with Private Cost Risks

In the case of duopoly (i.e., $n=2$), there is no shaded area present, whence information sharing among producers is harmful to consumers as was discussed above. The tongue-like shaded area appears in the upper middle of the (θ, ρ) square only after $n=10$, and grows very rapidly as n increases.[12] So when there are many firms in an industry, the situation under which information sharing benefits consumers takes place if goods are moderately substitutable and costs are positively correlated. Moreover, when a sufficiently large number of firms exist, a great part of the (θ, ρ) square is swallowed by the shaded tongue, meaning that consumers may almost always enjoy the benefit of a third party from information

[12] Note that any tongue-like area in Figure 1 is not exactly a symmetric figure and point M_n is near yet not equal to the minimum point of curve $H_nM_nK_n$ ($n=10$, 20, 50). In fact, this curve is not a parabola but takes a more complicate shape.

exchange among producers.[13]

C. *Bertrand Oligopoly*

Let us turn to the situation under which firms are Bertrand competitors that employ price levels as their strategic variables.[14] For the purpose of presentation, let us bring in the following notations:

$$\text{OWN VARI} = -b(1+(n-2)\theta)\textstyle\sum_i \text{Var}(p_i)\,, \tag{6.11}$$

$$\text{CROSS VARI} = b\theta\textstyle\sum_i\sum_{i\neq j}\text{Cov}(p_i, p_j)\,, \tag{6.12}$$

$$\text{OWN EFFI} = b(1+(n-2))\textstyle\sum_i \text{Cov}(\tilde{k}_i, p_i)\,, \tag{6.13}$$

$$\text{CROSS EFFI} = -b\theta\textstyle\sum_i\sum_{i\neq j}\text{Cov}(\tilde{k}_i, p_j)\,. \tag{6.14}$$

Then we are able to derive the following set of welfare equations:

$$\begin{aligned}\Delta EPS = {} & \Delta(\text{OWN VARI}) + \Delta(\text{CROSS VARI}) + \Delta(\text{OWN EFFI}) \\ & + \Delta(\text{CROSS EFFI})\,,\end{aligned} \tag{6.15}$$

$$\Delta ECS = -(1/2)\Delta(\text{OWN VARI}) - (1/2)\Delta(\text{CROSS VARI})\,, \tag{6.16}$$

$$\begin{aligned}\Delta ETS = {} & (1/2)\Delta(\text{OWN VARI}) + (1/2)\Delta(\text{CROSS VARI}) + \Delta(\text{OWN EFFI}) \\ & + \Delta(\text{CROSS EFFI})\,.\end{aligned} \tag{6.17}$$

If we compare Eqs. (6.11)–(6.14) with Eqs. (6.5)–(6.7), we can see that there is now a *cross efficiency* term (CROSS EFFI) associating $\tilde{k}_i$ with p_j $(i\neq j)$. The presence of a new cross term is expected to make our welfare analysis of Bertrand oligopoly different from the one of Cournot oligopoly. In fact, as is seen in Table 3, the own and cross efficiency effects are working in opposite directions in the determination of ΔEPS. If Bertrand firms agree to exchange their private information with each other, we can expect to have an allocation benefit arising from a better correpondence between the cost and price of each firm because the higher (or lower) cost firm is likely to have a smaller (or larger) market share. In the case of Bertrand competition, however, there is another kind of allocation repercussion across firms. If goods are substitutes (or complements) then information pooling increases (or decreases) the covariance between the cost of one firm and the price of any other firm. Such a repercussion has a disturbing impact on resource allocation across firms, regardless of technical substitution between goods.

The following welfare implications of information pooling among Betrand competitors may be drawn from Table 3:

[13] When there exist a sufficiently large number of firms, our oligopoly framework is supposedly close to the monopolistic competition framework of Chamberlin [1933]. In his recent work [1987, 88a, 88c], Vives studied incentives to share information and welfare in such large markets.

[14] For a detailed welfare analysis of Bertrand oligopoly under private cost uncertainty, see Sakai & Yamato [1988]. Vives [1987] discussed a similar problem within the framework of monopolistic competition.

TABLE 3. BERTRAND OLIGOPOLY WITH PRIVATE COST RISKS: $(\tilde{k}_1, \cdots, \tilde{k}_n)$

The Impact of Information Sharing	VARIATION		EFFICIENCY		TOTAL
	OWN	CROSS	OWN	CROSS	
ΔEPS	−	+	+	−	? possibly positive for all n
ΔECS	+	−	0	0	? $\begin{cases} -(n<9) \\ \pm(n \geqq 10) \end{cases}$
ΔETS	−	+	+	−	−

(i) We can look at Table 3 either horizontally or vertically. In either way, there are no sequences of simple sign pattern such as plus signs only or minus signs only; and indeed both plus signs and minus signs appear in any sequence. As in the case of Cournot competition, there exist both variation and efficiency channels through which information pooling affects the welfare of any member of the society. The variation channel consists of two subchannels—own and cross subchannels. Moreover, unlike the Cournot situation, the efficiency channel is now decomposed into own and cross subchannels as well. Interestingly enough, those two subchannels on the variation or the efficiency side are working in opposing directions.

(ii) The total impact taking care of all possible channels is shown in the last column. First of all, information sharing has a general tendency of decreasing expected total surplus. Therefore, in sharp contrast to the Cournot case, more information means less benefit. This is particularly so because the presence of the cross efficiency term has a strong effect of pulling down the level of welfare.

(iii) How the "economic pie" that gets smaller by information pooling is to be distributed between producers and consumers raises a more delicate question. Information sharing may make producers worse off or better off, and it may hurt or benefit consumers, depending upon the relative strength of three factors: the degree of technical substitution between any two goods, θ, the value of stochastic correlation of any two costs, ρ, and the number of Bertrand firms, n. It is noted that for *any* finite number of firms, information pooling may benefit producers. On the other hand, there exists a critical value of the number of firms beyond which information becomes beneficial to consumers.

(iv) It should be stressed that the possibility that both *EPS* and *ECS* simultaneously rise through information exchange never occurs, for *ETC*, being the sum of *EPS* and *ECS*, must decline.

Since Result (iii) above is, we believe, a remarkable one, let us do a more detailed analysis. The question at issure is how a change in *EPS* or *ECS* is related to the values of n, θ, and ρ. Figure 3 gives us an answer when $n=2$, 10, 20, 50.

In Figure 3, the interior of the shaded area located in the upper right corner

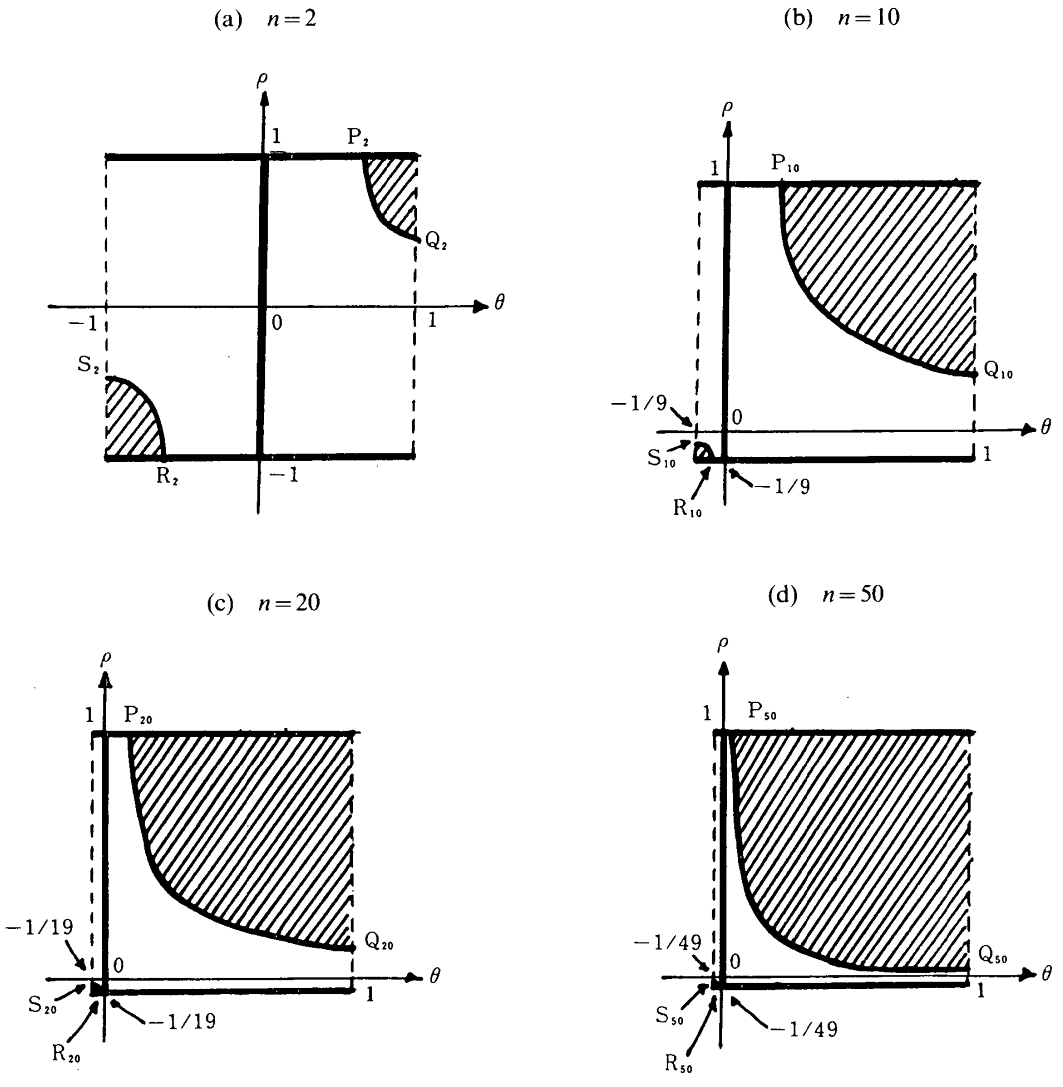

Fig. 3. The Effect of information Sharing on Producers: Bertrand Oligopoly with Private Cost Risks

and the lower left corner of the (θ, ρ) square indicates the combination of θ and ρ for which information sharing has a positive effect on producers. On those solid lines where $\theta=0$ or $\rho=1$, $(-1)/(n-1)$, and on those solid curves where the pair (θ, ρ) satisfies the equation $\rho=[1+(n-2)\theta]\{4(1-\theta)[1+(n-1)\theta]+(n-1)\theta^2\} \div (n-1)\theta\{(1-\theta)[2+(2n-3)\theta]+[1+(n-1)\theta][2+(n-3)\theta]\}$, information pooling has no influence at all on producers. And the remaining blank area shows the situation under which information exchange has a negative effect on producers. The coordinates of points P_n, Q_n, R_n and S_n are approximately given by as follows: $P_2=(0.7808, 1)$, $Q_2=(1, 0.5)$, $R_2=(-0.7808, -1)$, $S_2=(-1, -0.5)$; $P_{10}=(0.1929, 1)$, $Q_{10}=(1, 0.1)$, $R_{10}=(-0.1105, -1)$, $S_{10}=(-1, 0.1)$; $P_{20}=(0.0983, 1)$, $Q_{20}=(1, 0.05)$, $R_{20}=(-0.0525, -1)$, $S_{20}=(-1, -0.05)$; and $P_{50}=(0.0397, 1)$,

$Q_{50}=(1, 0.02)$, $R_{50}=(-0.020403, -1)$, $S_{50}=(-1, -0.02)$.

It is noted that a pair of shaded areas appear already when $n=2$, and that the shaded area in the positive quadrant gets larger as n increases. In other words, even when there are only two firms in an industry, the exchange of cost information between them may benefit firms either if goods are strong substitutes and costs are positively correlated or if goods are strong complements and costs are negatively correlated. When there are a large number of firms, a great portion of the (θ, ρ) square is swamped by the fan-like shaded area, thus showing the plausibility of the conflict of interests between producers and consumers.

In general, if there is an information exchange among Bertrand firms, it is likely to put consumers in a less advantageous position. However, the possibility that it may even be beneficial to consumers cannot be excluded. Whether consumers suffers from being outsiders or enjoys the benefit of a third party depends on three factors again. They are: θ, ρ and n. If $\theta>0$, or if $\rho\leqq 0$, then it can be shown that $\Delta ECS<0$. Besides, whenever n is at most as great as nine, ΔECS takes on a negative value. In our Bertrand model, only when n is at least as great as ten, there appears a combination of θ and ρ for which ΔECS is positive. This is due to the fact that the own variation effect gets stronger and the cross variation effect gets weaker as the number of firms gets larger.

To take an example, let us consider the case of $\theta=(-1)/n$. Then it is not hard to obtain the following relationship:

$$\Delta ECS>0 \Longleftrightarrow \rho>\frac{2(n+15)}{(n-1)(n-3)} \tag{6.18}$$

Let us denote by ρ^* the value of quantity on the right-hand side of (6.18). Clearly, the amount of this ρ^* represents a critical value on which consumers can enjoy the benefit of a third party. We can show that $\rho^*=50/63$ $(=0.7937)$ for $n=10$, $\rho^*=70/323$ $(=0.2167)$ for $n=20$, and $\rho^*=130/2303$ $(=0.05645)$ for $n=50$. This demonstrates that the possibility that information pooling benefits consumers becomes larger as there are more firms in an industry.

III. CONCLUDING REMARKS

While this paper is mainly a theory-oriented piece of work, we believe that the results obtained so far may have some policy implications regarding the effectiveness and limits of information-sharing agreements.

On the one hand, trade associations are one of the institutions in which information transmission takes place and is organized. Any kind of information-sharing agreement is seen to be double-edged: it may strengthen coalition among firms whereas it may enhance the efficiency of resource allocation across firms. In the light of those mutually opposing effects working behind, antitrust authorities in the U.S. have not taken a clear-cut position on agreements on information pooling.[15] On the other hand, there are many economicsts who think that, among a set of industrial policies the Japanese government undertook, those policies that explicitly or implicitly contributed to improvement of flows of industrial information were very successful measures.[16]

It is hoped that our theoretical investigation of information transmission sheds new light both on the desirability of trade associations and on the merits or demerits of industrial policies. It seems that we can derive the following set of policy implications from our theoretical analysis carried out in the previous sections:

(i) The most important thing we must bear in mind is that the welfare implications of information transmission are sensitive to many factors. They are: the type of competition (Cournot or Bertrand), the nature of uncertainty (demand or cost), the character of information (a common value or private values), and the number of participating firms. And even if every one of those factors is specified, the welfare results may as well depend on the degree of technical substitution between any two outputs and the value and direction of stochastic interdependence between any two demand or cost parameters.

(ii) It goes without saying that policy implications are closely linked to the welfare results, given a certain criterion of social welfare. Even if we regard the expected sum of the producer and consumer surpluses as a good measure of social welfare, we should be very careful of what kind of oligopoly we are discussing and of what sort of uncertainty and information we are talking about. Different assumptions on oligopoly and uncertainty are likely to lead to different policy implications.

(iii) In order to have a clear-cut conclusion on the merits or demerits of information transmission agreements, it is first necessary to determine whether the uncertainty each firm is confronted with is of a common type or of a firm-specific type. Suppose that every Cournot or Bertrand firm belonging to the same industry is subject to the same demand or cost risk. Then, as our welfare analysis can show, information flow from one firm to others results in an increase in expected social surplus, with the exception of the case that firms are Bertrand competitors facing common demand uncertainty and goods are not strong substitutes. Besides, in all those favorable cases, if side payments are permitted between firms and goods are moderately substitutable or complementary, such information transmission is most likely to represent a Pareto improvement in the sense that it makes both producers and consumers better off.

Therefore, except the situation of Bertrand oligopoly with common demand uncertainty, the government authority should pursue a policy which encourages the spreading of information among firms. If such a policy happens to harm

[15] For trade association laws and antitrust laws in the U.S., see Lambo & Shield [1971] and Areeda [1981]. Also see Scherer [1980] and Vives [1987].

[16] For evaluation of industrial policies in Japan, see Komiya [1975] and Suzumura & Okuno-Fujiwara [1987]. Also see Vives [1990].

consumers although it does increase total surplus, it appears that we are in a sort of dilemma, since consumer protection is often regarded by antitrust policy makers as their main objective. It follows that public policies for information transmission should be supplemented with income distribution policies so that some of the increased social surplus may be shifted to consumers, for instance, through taxes and subsidies.

(iv) The most troublesome case rests with the situation under which firms are Bertrand competitors facing a common demand risk. Unless goods are strong substitutes, information transmission has a rather negative effect on social welfare. In such a case, the authority should be discouraged from engaging in information transfer.

(v) Let us turn to the more interesting case where each firm faces its own demand or cost risk. In the case of such private uncertainty, the number of participating firms plays an important role in deciding the effect of information sharing on the welfare of consumers.

Apart from the case of Bertrand oligopoly with cost uncertainty, any information pooling agreement yields an increase in producer surplus and in total surplus, whatever the degree of technical substitution and the value of stochastic correlation. Regarding the effect on consumers, there appears a dividing line between "a few firms" and "many firms." When the number of firms is "small," information pooling is always harmful to consumers, showing the need of introduction of supplementary income redistribution policies. If, however, the number of firms is "large," then the situation changes completely. Then unless goods are homogenuous (which is unlikely in today's business circle) the shared information case is most likely to be Pareto superior to the non-shared information case. This is no doubt the most fortunate case we could have when we ask the authority to interfere information flows in private sectors.

(vi) If firms are Bertrand competitors facing private cost uncertainty, then more information means less social benefit in the sense that information pooling makes the "economic pie" smaller. This is presumably the most unfortunate situation among possible combinations of oligopoly and uncertainty. Although the authority is not recommended to help diffuse private cost uncertainty across firms, it may do so under the pressure of business circle because information sharing is likely to increase the share of producers in social surplus if the number of producers is sufficiently large. To make the problem even more complicated, there are some other circumstances in which information pooling may increase the welfare of consumers if the number of firms is "large."

(vii) To sum up, policy implications of an information transmission agreement depends on whether uncertainty is of an industry-wide type or of a firm-specific type, whether information is about demand or cost, and on whether interfirm competition is of the Cournot quantity type or of the Bertrand price type. Besides, those implications are also sensitive to the degree of technical substitution among goods, the value and direction of stochastic correlation among demand or cost parameters, and the number of participating firms.

The above considerations seem to lead to making a case-by-case analysis quite effective if we have to take much care of adopting a Pareto-improving policy. If, however, we allow for a certain kind of side payment among firms, the scheme of welfare-enhancing policy becomes much simpler. This is due to the fact that unless the oligopoly in question is Bertrand oligopoly with common demand uncertainty or private cost uncertainty, any government policy of promoting information flows among firms has an effect of increasing total welfare although it might decrease the welfare of certain members of the society. Since the economic pie *per se* gets larger by information transmission, it is possible to make every member better off if an information-flow-promoting policy is supplemented by a series of income redistribution policies.

On the other hand, there are a limited number of cases in which information transmission or information sharing does indeed hurt total welfare. These unfortunate cases are only two: Bertrand oligopoly with common demand uncertainty and the same oligopoly with private cost uncertainty. Besides, there are more possible cases where information pooling is harmful to consumers as outsiders if the number of producers is rather small. What we have learned from our analysis is that these "bad" cases may clearly be identified and should be distinguished from many other "good" cases. The government agencies should have sharp eyes to select "good" cases only and, if necessary, should supplement policies for information transfer with policies for income redistribution.

It should be noted that there remain some limitations in our welfare analysis and many other directions in which the analysis may be further extended. First of all, we have been working with a simple oligopoly model with explicit functional forms assumed for the utility functions of consumers, the cost functions of producers, and the density functions of stochastic variables. It is our strong belief that simplification is the essence of science and is justified if it gets straightforward to the heart of the matter.

Second, we have ignored the problem of risk aversion on the part of producers and/or consumers along with the problem of information cost.[17] Third, the question of partial information sharing and garbling has not been discussed.[18] These problems remain unsolved and will be the target of future research.

And finally, we have had no attention on the leader-follower model of Stackelberg. Stackelberg competitors could employ either quantities or prices as their strategic variables. Besides, the uncertainty in question may be of an industry-wide type or of a firm-specific type, and the information in question may be about demand or cost parameters. Taking these factors into account, we would have so many Stackelberg models to work with. Then there would be a certain

[17] For the effect of risk aversion on information sharing in oligopoly, see Sakai & Yoshizumi [1989].

[18] The problem of strategic information revelation is currently the topic of much concern. See Crawford & Sobel [1982], Dubey, Geanakoplos & Shubik [1987], Okuno-Fujiwara, Postlewaite & Suzumura [1986], and others.

class of circumstances under which a less informed firm is willing to act as a follower, with a more informed firm playing the role of a leader. Such an analysis will threw new light on the problem of the first mover advantage versus the second mover advantage.[19]

In conclusion, we believe that economists should share any kind of information with each other through oral discussions or written papers, with the strong faith that information is power in the academic circle. Laboremus!

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