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ENTRY IN BERTRAND AND COURNOT OLIGOPOLIES WITH PRODUCT DIFFERENTIATION

Koji OKUGUCHI and Ferenc SZIDAROVSZKY

Abstract. The effects of entry in Bertrand and Cournot oligopolies with product differentiation and single product firms are analyzed, assuming that firms producing differentiated goods are uniformly increased similarly as consumers in a replicated economy. It is found that under general conditions, the limit points of the equilibrium Bertrand prices do not coincide with the perfect competitive equilibrium. We prove that the limit points always exceed the marginal costs. Two cases are considered for Cournot oligopoly, in one of which firms producing the same goods collude, and in the other collusion is ruled out. Under some simplifying assumptions, the limit points of the equilibrium Cournot prices always exceed the competitive prices in the former case, but in the latter one the equilibrium Cournot prices converge to the competitive prices.

I. INTRODUCTION

The effects of entry in a Cournot oligopoly without product differentiation have been first analyzed systematically by Ruffin (1971) (see Frank (1965) and Horowitz (1970) for earlier contributions), whose results for identical firms have been generalized by Okuguchi (1973, 1976) to the case where firms' cost functions differ similarly to consumers' differences in endowments in a replicated economy. Szidarovszky and Okuguchi (1989) have extended Okuguchi's result allowing for the existence of multi-product firms. In all these works firms were assumed to adjust their outputs to maximize their expected profits. Under mild conditions the total outputs in the Cournot oligopoly equilibrium have been shown to increase with an increase in the number of firms and the Cournot equilibrium market prices have been shown to converge to the perfect competitive equilibrium prices. The effects of entry in a Bertrand oligopoly with product differentiation have been analyzed by Krelle (1976), Levitan and Shubik (1971), and Hurwicz (1989) for a simple symmetric case for which explicit derivations of the equilibrium prices were possible. Their models, however, were not formulated in a framework of replicated economy.

In this paper we will systematically analyze the effects of entry in Bertrand and Cournot oligopolies with product differentiation and single product firms. Following Okuguchi (1973, 1976), we consider a replicated industry in the sense that the number of firms is uniformly increased for all differentiated products.

In Section 2 Bertrand oligopoly with product differentiation and price strategies is analyzed, assuming that only one firms exists for producing one product. A set of general conditions which ensures the existence of a unique interior Bertrand equilibrium prices is derived. In Section 3 the effects of entry in Bertrand oligopoly is examined. It will be found that under very general conditions, any limit point of the equilibrium does not coincide with the perfect competitive equilibrium prices as the number of firms becomes infinite. This result is in sharp contrast with the ones obtained by Krelle. In Section 4 Cournot oligopoly with product differentiation and output strategies is analyzed for two cases, in one of which all firms producing identical product are assumed to collude among themselves, and in the other collusion is ruled out. Under general condition any limit point of the equilibrium prices is shown to differ from the competitive equilibrium prices without collusion, but they coincide in the cases with collusion. Section 5 concludes the paper.

II. BERTRAND OLIGOPOLY WITH PRODUCT DIFFERENTIATION

In this section which serves as a preliminary to our investigation into the effects of entry in Bertrand oligopoly with price strategies, we formulate Bertrand oligopoly with fixed n firms producing differentiated goods for one another, and derive a set of general sufficient conditions for the existence of a unique interior Bertrand equilibrium.

Let therefore

$$x_i = g^i(p_1, p_2, \cdots, p_n), \qquad i=1, 2, \cdots, n, \tag{1}$$

and $C_i(x_i)$ be the demand and cost functions for the i-th firm, respectively, where x_i and p_i are the demand for and the price of the i-th firm, respectively.

ASSUMPTION 1. g^i is twice continuously differentiable, bounded and

$$g^i_i \equiv \partial g^i/\partial p_i < 0, \quad g^i_j \equiv \partial g^i/\partial p_j \geq 0, \qquad i \neq j, \quad i, j = 1, 2, \cdots, n.$$

ASSUMPTION 2. C_i is convex, twice continuously differentiable.

The i-th firm's profit π_i is given as a function of all prices;

$$\pi_i = p_i g^i(p_1, \cdots, p_n) - C_i(g^i(p_1, \cdots, p_n)), \qquad i=1, 2, \cdots, n. \tag{2}$$

Let

$$p=(p_1, \cdots, p_n), \quad p_{-i}=(p_1, \cdots, p_{i-1}, p_{i+1}, \cdots, p_n), \quad p=(p_i, p_{-i}).$$

The equilibrium Bertrand prices $p^*=(p^*_1, \cdots, p^*_n)$ are defined to satisfy

$$\pi_i(p^*_i, p^*_{-i}) \geq \pi_i(p_i, p^*_{-i}), \qquad \text{for any} \quad p_i \in P_i, \quad i=1, 2, \cdots, n, \tag{3}$$

where

$$P_i = \{p_i \mid 0 \leq p_i \leq \bar{p}_i\}$$

and

$$g^i(p_i, p_{-i}) = 0$$

for $p_i \geq \bar{p}_i$ irrespective of the value of p_{-i}.

ASSUMPTION 3.

$$\partial^2 \pi_i / \partial p_i^2 < 0 , \qquad i = 1, 2, \cdots, n .$$

ASSUMPTION 4.

$$\begin{array}{ll} \partial \pi_i / \partial p_i \,|_{p_i = 0} , \quad \text{any} \quad p_{-i} > 0 , & \\ & i = 1, 2, \cdots, n , \\ \partial \pi_i / \partial p_i \,|_{p_i = \bar{p}_i} , \quad \text{any} \quad p_{-i} < 0 . & \end{array}$$

Let the maximizer of π_i for any given p_{-i} be denoted as

$$p_i = \varphi^i(p_{-i}) , \qquad i = 1, 2, \cdots, n . \tag{4}$$

If we introduce the notation

$$\varphi(p) = (\varphi^1(p_{-1}), \varphi^2(p_{-2}), \cdots, \varphi^n(p_{-n})) ,$$

then p^* is the equilibrium Bertrand price vector if and only if

$$p^* = \varphi(p^*) . \tag{5}$$

Under Assumptions 1–4 the equilibrium Bertrand prices exist and are interior. The uniqueness of the equilibrium is guaranteed if φ is a contraction, or one of functions $p - \varphi(p)$ and $\varphi(p) - p$ is strictly monotone, since they imply the uniqueness of the solution of the fixed point problem.

III. THE EFFECTS OF ENTRY: BERTRAND OLIGOPOLY

We are now in a position to analyze the effects of entry in a Bertrand oligopoly where firms are replicated. Let n denote the number of differentiated products, and let each product be produced by N firms with identical cost functions and without capacity constraints. The price of any product must be the same for all firms producing it. Otherwise, any firms selling at the lowest price captures the whole market as it can produce any amount to satisfy the demand for the whole market, and no other firm which produces the same product can survive. We assume that the demand for any firm is equal to the $1/N$ multiple of the market demand for the product it produces. Hence the profit of j-th firm producing the i-th good, π_{ij}, is given by

$$\pi_{ij}^{(N)} \equiv \pi_i^{(N)} \equiv p_i g^i(p)/N - C_i(g^i(p)/N) , \qquad \begin{array}{l} i = 1, 2, \cdots, n \\ j = 1, 2, \cdots, N . \end{array} \tag{6}$$

Note that π_{ij} does not depend on j. We assume that $\pi_i^{(N)}$ satisfies Assumptions 2–4 for any N. Hence, any equilibrium is interior, and the equilibrium Bertrand prices $p^*(N) = (p_1^*(N), \cdots, p_n^*(N))$ for arbitrary given N are the simultaneous solutions of the first order condition for profit maximization,

$$\partial\pi_i^{(N)}/\partial p_i=(g^i(p)+p_ig_i^i(p)-C_i'(g^i(p)/N)g_i^i(p))/N=0 \qquad i=1,2,\cdots,n. \tag{7}$$

Since sequence $\{p^*(N)\}$ is bounded, it has at least one limit point p^*. By letting $N\to\infty$ in (7) we conclude that

$$g^i(p^*)+p_i^*g_i^i(p^*)-C_i'(0)g_i^i(p^*)=0,$$

that is,

$$p_i^*-C_i'(0)=-\frac{g^i(p^*)}{g_i^i(p^*)}. \qquad i=1,2,\cdots,n \tag{8}$$

Introduce the additional notation

$$P^\circ=\{p=(p_i)\,|\,g^i(p)=0, \quad i=1,2,\cdots,n\},$$

and assume that

Assumption 5. For all $p\in P^\circ$, $C_i'(0)\neq p_i$ for at least one i.

Note that the right hand sides of equation (8) are nonnegative under Assumption 1, and if Assumption 5 holds, then it is positive for at least one i. Hence

$$p_i^*\geq C_i'(0), \qquad i=1,2,\cdots,n$$

and strict inequality holds for at least one i. In summary, at any limit point, the Bertrand and perfect competitive equilibrium prices do not coincide. This result implies much more than showing that in the case of convergent Bertrand equilibrium prices the limit point differs from the perfect competitive equilibrium. It also implies that the entire sequence of the Bertrand equilibrium prices is isolated from the perfect competitive equilibrium. Note that this result is in sharp contrast with the one earlier obtained by Krelle (1976) and others for simple symmetric Bertrand oligopoly as well as with the one for Cournot oligopoly without product differentiation.

Finally we note that in Okuguchi and Szidarovszky (1989) special cases were investigated which implied the monotonicity and hence the convergence of the equilibrium sequence $p^*(N)$. These earlier results are all special cases of the above findings.

IV. COURNOT OLIGOPOLY WITH PRODUCT DIFFERENTIATION

It is now well known that if the number of firms is fixed, the equilibrium prices for Bertrand oligopoly with price strategies and the ones for Cournot oligopoly with output strategies differ generally. See Levitan and Shubik (1971), Hathaway and Rickard (1979), Cheng (1985a), Vives (1985), Okuguchi (1987) and Friedman (1988) for the detail. In this section we analyze the effects of entry on the equilibrium prices for Cournot oligopoly with product differentiation and output strategies.

First introduce

ASSUMPTION 6.

$$|g_i^i| > \sum_{j \neq i} |g_j^i|, \qquad i=1, 2, \cdots, n.$$

It is known that under Assumptions 1 and 6, the market demand functions are globally invertible. See, for example, Cheng (1985b) and Okuguchi (1987) for the detail. Hence we can write

$$p_i = f^i(x_1, x_2, \cdots, x_n), \qquad i=1, 2, \cdots, n, \tag{9}$$

where f^i's are differentiable and

$$f_i^i \equiv \partial f^i/\partial x_i < 0, \quad f_j^i \equiv \partial f^i/\partial x_j \leq 0, \qquad i \neq j, \quad i, j=1, 2, \cdots, n. \tag{10}$$

If there are N identical firms for each product and if firms producing the same product collude among themselves, and if, in addition, side payment is impossible, the profit function of any firm producing the i-th product is given by

$$v_i^{(N)} = p_i x_i - C_i(x_i) = x_i f^i(Nx_1, \cdots, Nx_n) - C_i(x_i), \qquad i=1, 2, \cdots, n.$$

Similarly to price adjusting oligopolies denote

$$F = \{x=(x_i) \,|\, f^i(x)=0, \quad i=1, 2, \cdots, n\}.$$

Instead of going into the detail of the conditions for the existence of a unique interior equilibrium Cournot outputs, we simply introduce

ASSUMPTION 7. There exists a unique interior equilibrium Cournot output vector $x^*(N) \notin F$ for all $N \geq 1$.

ASSUMPTION 8. Set $R_+^n \backslash F$ is bounded.

The equilibrium Cournot outputs x_i^*'s form a simultaneous solution to

$$\partial v_i^{(N)}/\partial x_i = f^i(Nx_1, \cdots, Nx_n) + Nx_i f_i^i(Nx_1, \cdots, Nx_n) - C_i'(x_i) = 0,$$
$$i=1, 2, \cdots, n, \tag{11}$$

With the notation $Nx_i \equiv X_i$ for the total output of the firms having the cost function C_i, (11) is rewritten as

$$f^i(X_1, \cdots, X_n) + X_i f_i^i(X_1, \cdots, X_n) - C_i'(X_i/N) = 0, \qquad i=1, 2, \cdots, n. \tag{12}$$

Since vector $(X_1, \cdots, X_n)$ is in a bounded set, it has at least one limit point X^*. By letting $N \to \infty$ in (12), we get the identity

$$f^i(X^*) - C_i'(0) = X_i^* f_i^i(X^*). \tag{13}$$

The right hand side of (13) is nonnegative under Assumptions 1 and 6, since they imply relations (10). The right hand side is zero for all i if and only if $X^*=0$. Hence,

$$f^i(X^*) \geq C_i'(0) \qquad \text{for all} \quad i,$$

and strict inequality holds for at least one i for $X^* \neq 0$.

ASSUMPTION 9. For at least one $i, f^i(0) \neq C_i'(0)$.

Under this additional assumption $f^i(X^*) > C_i'(0)$ for at least one i even when $X^*=0$. In summary at any limit point, the Cournot and perfect competitive equilibrium prices do not coincide. Similarly to the case of Bertrand equilibrium, this result also implies that the entire sequence of the Cournot equilibrium prices is isolated from the perfect competitive equilibrium.

We also mention that in Okuguchi and Szidarovszky (1989) sufficient conditions are derived for the monotone convergence of the Cournot outputs, and the same conclusion as above is reached.

We now consider an alternative model where collusion is ruled out and all firms behave independently with Cournot expectations on all other firms' outputs. In this case the profit function of the j-th firm producing the i-th product is defined as

$$v_{ij}^{(N)} = x_{ij} f^i(X_1, \cdots, X_{i-1}, \sum_{k \neq j} x_{ik} + x_{ij}, X_{i+1}, \cdots, X_n) - C_i(x_{ij}) ,$$
$$i=1,2,\cdots,n, \quad j=1,2,\cdots,N . \tag{14}$$

The first order condition for the profit maximization for the j-th firm producing the i-th good is

$$\partial v_{ij}^{(N)} / \partial x_{ij} = f^i(X_1, \cdots, X_i, \cdots X_n) + x_{ij} f_i^i(X_1, \cdots, X_i, \cdots, X_n) - C_i'(x_{ij}) = 0 ,$$
$$i=1,2,\cdots,n, \quad j=1,2,\cdots,N , \tag{15}$$

where $X_i = \sum_j x_{ij}$ for all i.

Taking into account the fact that the outputs are the same for all firms producing the identical goods, and expressing the value at the equilibrium by asterisk, (15) is rewritten as

$$f^i(X_1^*, \cdots, X_i^*, \cdots, X_n^*) + f_i^i(X_1^*, \cdots, X_i^*, \cdots, X_n^*) X_i^*/N - C_i'(x_i^*/N) = 0 ,$$
$$i=1,2,\cdots,n . \tag{16}$$

ASSUMPTION 10. There exists a unique interior equilibrium Cournot output vector $X^*(N) = (X_{ij}^*(N)) \notin F$ for all $N \geq 1$.

Under assumptions 8 and 10, let $N \to \infty$ to get equation

$$f^i(X^*) + f_i^i(X^*) \cdot 0 - C_i'(0) = 0 ,$$

where X^* is now a limit point of the total output of the industry. This relation shows that the equilibrium Cournot prices converge to the marginal costs $C_i'(0)$ for all products.

Finally, we note that in Okuguchi and Szidarovszky (1989) sufficient conditions are derived for the monotone convergence of the outputs $X_i^*(N)$.

V. CONCLUSION

In this paper we have analyzed the effects of entry on the total outputs and market prices for Bertrand and Cournot oligopoly with product differentiation and single product firms. Under fairly general conditions any limit point of the Bertrand equilibrium prices does not coincide with perfect competitive equilibrium prices. We obtained the same result for Cournot oligopoly with product differentiation and with collusion. If, however, collusion is ruled out, the equilibrium prices do converge to the perfect equilibrium prices. These results are the generalizations of those obtained in Okuguchi and Szidarovszky (1989).

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