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HUMAN CAPITAL AND ENDOGENOUS ECONOMIC GROWTH

Michihiro OHYAMA*

Abstract. This paper provides an extension of the aggregative neoclassical model of growth *à la* Solow (1956) with a view to elucidating important implications of investment in human capital. It is shown that the long-run (steady-state) growth rate of the economy under consideration is positively affected by a rise in saving rate or by an improvement in production technology. Furthermore, the trade-off between current consumption and its growth rate is considered and the optimal choice between them is characterized to generalize the golden rule of economic growth within the framework of the present model.

I. INTRODUCTION

The neoclassical theory of economic growth focuses upon three basic factors of growth, i.e., population growth, technical progress and accumulation of physical capital. As Schultz (1962, 1971) and Becker (1964) forcefully argued, however, there is one more, and perhaps most important factor of growth, namely investment in *human* capital. Apparently, it is prerequisite to the vast capacity of production and high standard of living achieved in many advanced countries. As a matter of fact, the growth of labor measured in efficiency units has been paid a considerable amount of attention even within the framework of neoclassical growth models, but it is usually introduced not as an endogenous variable to be determined by investment in human capital, but as an exogenous factor brought about by "labor-augmenting" technical progress occuring outside the models.

One of the salient features of neoclassical growth theory is that the long-run steady state rate of growth is determined by the exogenously given rate of labor growth independently of the savings ratio of the economy. As Solow (1988) put it, "a developing economy that succeeds permanently increasing its saving (investment) rate will have a higher level of output than if it had not done so, and must therefore grow faster for a while. But it will not achieve a permanently higher rate of growth of output." This remarkable implication of his theory is partly attributable to the diminishing returns of *physical* capital, but it also comes from the neglect of investment in *human* capital which may serve to alleviate them. The complete independence of long-run growth rate from saving (investment) rate may be unrealistic in the presence of investment in human capital.

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In contrast to the bulky literature on economic growth and human capital, we still lack studies which connect them, or incorporate accumulation of human capital into modern growth models. There are some exceptions such as Razin (1972a, b), Aarrestad (1975, 1978), Manning (1975, 1985), Hu (1976), Romer (1986) and Lucas (1988). They considered investment in human capital or in the stock of knowledge using various models of optimal growth. This paper is yet another attempt to fill in the gap in the literature. To facilitate comparison with the original aggregative model of growth *à la* Solow (1956), we shall extend it straightforwardly and elucidate the more important implications of investment in human capital for economic development. In particular, we intend to illustrate most clearly the possibility that the rate of growth is endogenously determined within the model depending on the saving rate and state of technology in the broad sense.

The plan of the paper is as follows. In Section 2, we explain the model of an economy relying on physical and human capital for the production of a single commodity, i.e., its national product. Section 3 considers the existence and stability of the steady state where all components of the economy grows at a uniform rate. In Section 4, we investigate the effects of exogenous changes in the saving rate and technology of the economy on its steady state growth rate and capital intensity. Section 5 analyzes the optimal choice between current consumption and growth rate in the steady state and modify the well-known golden rule of economic growth. Section 6 concludes the paper with a summary and qualifications.

II. SHORT-RUN EQUILIBRIUM AND COMPARATIVE STATICS

Let there be one national product produced by two factors of production, i.e., human capital and physical capital. Human capital is the stock of labor force measured in efficiency units, whereas physical capital is the stock of produced means of production. We assume that there exists a production function relating human capital, K_1, and physical capital, K_2, to national output X:

$$X = F(K_1, K_2)\,. \tag{1}$$

Function F is assumed to be linear homogeneous and twice-continuously differentiable. Thus it may also be written,

$$x = f(k)\,, \tag{2}$$

where $x = X/K_1$ and $k = K_2/K_1$ and we assume $f(0)=0$, $f'(k)>0$ and $f''(k)<0$.[1] Furthermore, let us assume that competitive conditions prevail in all markets. We then have

[1] Thus the marginal productivity of each type of capital is positive and decreasing to proportion.

$$r_1 = f(k) - kf'(k)\,, \tag{3}$$

$$r_2 = f'(k)\,, \tag{4}$$

where r_1 denotes the rental on human capital (or the wage rate per efficiency unit of labor) and r_2 the rental on physical capital.

The national product is assumed to be used for consumption and for production of investment goods as well. For simplicity, let us introduce a function relating the input of national product, Z, to the output of human capital, Y_1, and that of physical capital, Y_2:

$$G(Y_1, K_1) + Y_2 = Z\,, \tag{5}$$

where function G is linear homogeneous and twice-continuously differentiable. This relationship may be taken as a generalization of the neoclassical model of capital accumulation *à la* Solow (1956).[2] In fact, the production of one unit of physical capital requires the input of one unit of national product as in the Solow model, while the production of one unit of human capital requires the input of $G(1, K_1/Y_1)$ units of national product.

The linear homogeneity of function G is admittedly a heroic assumption. It presumes the endowment of unlimited human capacity to improve labor productivity, or the availability of unlimited supply of primitive labor that can be transfered at costs into matured labor. In any case, it is a theoretical assumption designed to bring into sharp relief certain features of the economy in which we are interested. In this respect, it is comparable to the linear homogeneity of function F already introduced without appology for the reason that it is commonly adopted in the standard literature on economic growth.

Following Solow (1956), let us further assume that a constant fraction, s, of national income is saved and used for investment. This assumption, together with the linear homogeneity of G, enables us to rewrite (5) as

$$g(y_1) + y_2 = sf(k)\,, \tag{6}$$

where $y_i = Y_i/K_1$ $(i = 1, 2)$. We assume that $g(0) = 0$, $g'(y_1) > 0$ and $g''(y_1) > 0$.[3]

Under competitive conditions, the marginal cost of a product is equalized to its price. Therefore, we have

$$p_1 = g'(y_1)\,, \tag{7}$$

$$p_2 = 1\,, \tag{8}$$

where p_1 and p_2 denote the prices of human and physical capital (in units of

[2] Given Z, equation (5) may be called the production possibility function of human and physical capital goods. It is highly simplified here for expository purposes. More generally, it should be written as

$$H(Y_1, Y_2; K_1, K_2) = Z\,.$$

[3] In words, the marginal cost of human capital is positive and increasing to scale.

national product) respectively.[4] Competitive arbitrage in capital markets leads to the equalization of expected rates of return on human and physical capital, i.e.,

$$\frac{r_1}{p_1}+\pi=\frac{r_2}{p_2}, \tag{9}$$

where π is the expected rate of rise in the price of human capital relative to physical capital, p_1/p_2. Substituting (3), (4), (7) and (8) into (9), we obtain

$$g'(y_1)=\frac{f(k)-kf'(k)}{f'(k)-\pi}\quad\left(=\frac{r_1}{r_2-\pi}\right). \tag{10}$$

The short-run equilibrium of the economy is defined as of given k, s and π, by the saving-investment equation (6) and the capital-market arbitrage condition (10). They are assumed to determine uniquely the equilibrium values of y_1 and y_2.

At this point, let us consider the effects of a change in k on the short-run equilibrium. They are interesting in their own right, and the knowledge of them is necessary for the analysis of the steady state (or the long-run equilibrium) that will be undertaken in the next section. Defferentiating (6) and (9) with respect to k, we get

$$\frac{dy_1}{dk}=-\frac{(g'+k)f''}{r_1g''}>0\,, \tag{11}$$

$$\frac{dy_2}{dk}=sr_1+\frac{(g'+k)f''}{r_1g''}\gtrless 0\,. \tag{12}$$

In words, an increase in the proportion of physical capital to human capital raises the rate of investment in human capital, but its effect on investment in physical capital is ambiguous. There are two distinct forces at work to bring about these outcomes. On one hand, saving per unit of human capital increases to enhance investment in general. On the other hand, the rental on human capital, r_1, increases and the rental on physical capital, r_2, decreases thereby encouraging investment in human capital but discouraging investment in physical capital.

Figure 1 illustrates the relationship between y_1 and k more generally. In the first quadrant, the schedule *OR* depicts the price of human capital $(r_1/(r_2-\pi))$ as an increasing function of k, while in the second quadrant the schedute *MC* shows the marginal cost of human capital investment as an increasing function of y_1. From these schedules we can infer that y_1 is a non-decreasing function of k. It is shown in the fourth quadrant as the schedule *OKH*. Note that it coincides with the horizontal axis for $k\leq\underline{k}$, where $\underline{k}$ corresponds to the price of human capital just equal to the marginal cost of the production of human capital at the start

[4] It is not necessary to assume that human capital is literally marketable. The price of human capital may be interpreted as the cost of education or training required to produce an additional efficiency unit of human capital. This education or training may be provided by schools as in Aarestad (1975) or it may be realized through self-teaching as in Razin (1972).

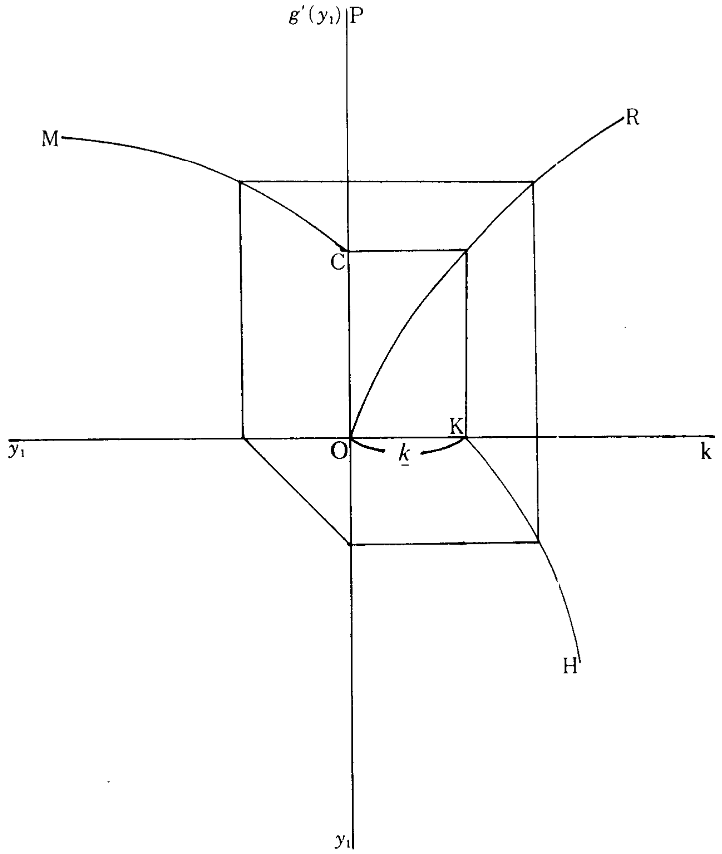

Figure 1

(i.e., at $y_1=0$). Clearly, $y_1=0$ for $k \leqq \underline{k}$ and $y_1>0$ for $k>\underline{k}$. This means that investment in human capital takes place only if the economy is endowed with enough physical capital to make it worthwhile. Strictly speaking, (10) and (11) are valid only for $k \geqq \underline{k}$.

III. CAPITAL ACCUMULATION AND THE STEADY STATE

In this section we study the process of capital accumulation toward the steady state and its stability properties. The seminal work by Hahn (1966) demonstrated that the steady state (or the long-run equilibrium) of the economy with heterogenous capital goods becomes a saddle point under perfect foresight. The present model with two distinct capital stocks is also expected to exhibit a similar property. It will indeed be shown that the steady state of the model becomes a saddle point under perfect foresight, while it is stable under static expectations.

For simplicity, let us assume away the depreciation of physical capital. The process of capital accumulation may then be described by the following differential equation:

$$\dot{k}=y_2-ky_1\,, \tag{13}$$

where a dot over a variable denotes its time derivative. The steady state of the economy is defined as the state where all the components of the economy grows at a uniform rate and the price of human capital relative to physical capital is expected to be stationary, or

$$y_2=ky_1\,, \tag{14}$$

$$\pi=0\,. \tag{15}$$

These equations, together with (6) and (10), yield

$$g(y_1)+ky_1=sf(k)\,, \tag{16}$$

$$g'(y_1)=\frac{f(k)}{f'(k)}-k\,, \tag{17}$$

which contain two unknowns, k and y_1.

We wish to show that there is at least one solution for the system of equations (16) and (17) under plausible conditions. We already examined the relationship between y_1 and k implied by (17) in some details in the preceding section to establish that y_1 is a non-decreasing function of k for $k \geqq k$. Figure 2 illustrates the determination of y_1 and k by (16) on the basis of this knowledge about (17). The schedules OS and KT depict the graph of $sf(k)$ and $g(y_1)+ky_1$ respectively. The schedule OS is upward rising and strictly concave below by the properties of $f(k)$. We may further assume that the marginal productivity of physical capital tends to zero as its proportion to human capital tends to infinity, or

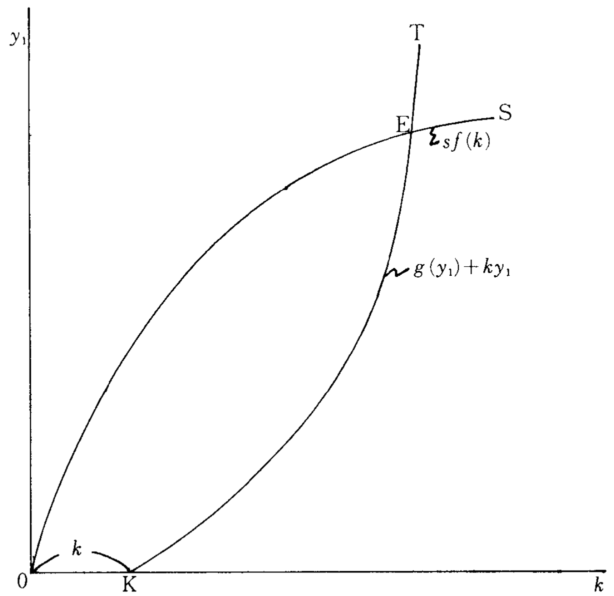

Figure 2

$$\lim_{k \to \infty} f'(k) = 0 . \tag{A}$$

This assumption seems to be plausible since human capital in the present sense of the word is indispensable to production. The schedule KT is also upward rising. Its slope is given by $y_1 + (g' + k)dy_1/dk$. Given any $\tilde{k} > \underline{k}$, it is greater than $y_1(\tilde{k}) > 0$ for $k \geqq \tilde{k}$. Thus, assumption (A) ensures that there is at least one intersection, E, of OS and KT as shown in the figure. It should be noted that assumption (A) is much weaker than the condition usually imposed on neoclassical growth model to ensure the existence of the steady state.[5] We have established

PROPOSITION 1. *If assumption (A) is satisfied, there is at least one steady state in the present model.*

The stability property of the steady state is dependent on how the public forms its expectations with regard to the price variable of the model. First, let us assume that the public entertains static expectations and let π be always equal to zero. Differentiating (13) with respect to k and making use of (11) and (12), we may then obtain.

$$\frac{d\dot{k}}{dk} = sf' - y_1 + \frac{(g' + k)^2 f''}{r_1 g''} . \tag{18}$$

In terms of Figure 2, $y_1 - (g' + k^2)f''/r_1 g''$ is the slope of KT and sf' is the slope of OS. This derivative is, therefore, negative in the neighborhood of the steady state, E, in Figure 2. We can put foward

PROPOSITION 2. *The steady state is stable under static expectations if* $sf' - y_1 + (g' + k)^2 f''/r_1 g'' < 0$.

Next, consider the hypothesis of perfect foresight as a popular alternative to static expectations. In order to investigate its implications, let $\pi = \dot{p}/p$ and take account of (8) to rewrite (9) as

$$\dot{p}_1 = p_1 f'(k) - [f(k) - kf'(k)] . \tag{19}$$

The dynamic adjustment path of the economy is now described by a pair of differential equations (13) and (19). Differentiating (6), (7), (13) and (19) totally and rearranging terms, we get

$$d\dot{k} = -(y_1 - sf')dk - \frac{g' + k}{g''} dp_1 , \tag{20}$$

$$d\dot{p}_1 = (g' + k)f'' dk + f' dp_1 . \tag{21}$$

The steady state of the economy is known to become a saddle point if and only

[5] Usually, the existence of the steady state requires an additional restrictive condition, i.e., $\lim_{k \to 0} f'(0) = \infty$

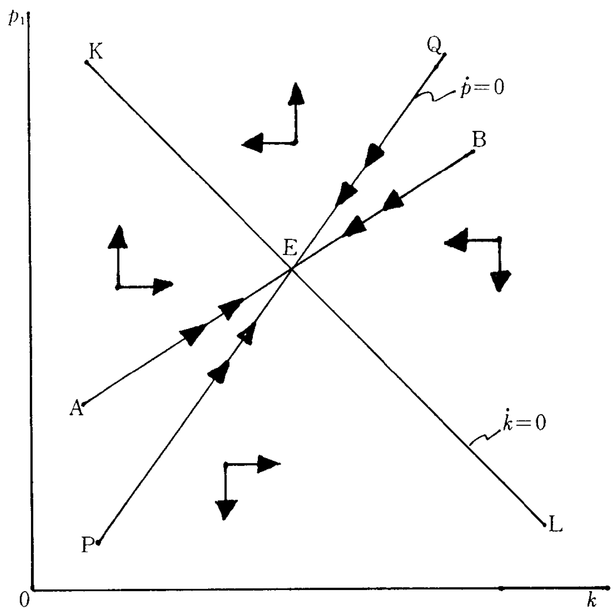

Figure 3

if the determinant of the coefficient matrix of (20) and (21) or the Jacobian of (13) and (19) is negative. This determinant condition is readily seen to be equivalent to the negativity of (18), which enables us to state

PROPOSITION 3. *The steady state is a saddle point under perfect foresight, if and only if it is stable under static expectations.*

It is customary to argue that the steady state is stable under perfect foresight since the public is able to choose the initial condition appropriately to get on the path leading to the steady state. In view of Proposition 3, however, perfect foresight is not necessary for the convergence to the steady state. The public with static expectations is also able to grope for the steady state under the same condition.[6]

Figure 3 illustrates the dynamic adjustment paths of the economy to the steady state. The schedule KL is the locus of k and p_1 that satisfy (13) with $\dot{k}=0$ and is downward sloping if $y_1 > sf'$. The schedule PQ is the locus of k and p_1 that satisfy (19) with $\dot{p}=0$. It is upward rising under the assumption of the present model. The intersection, E, of KL and PQ illustrates the steady state of the economy. In the present set-up, it is stable under static expectations and becomes a saddle point under perfect foresight. Its adjustment under static expectations proceeds along the schedule PQ while it occurs along the path AB under perfect foresight.

[6] In general, it can be shown that the steady state is stable under static expectations if and only if it is a saddle point under perfect foresight. See Ohyama (1989).

IV. SAVING, TECHNOLOGY AND GROWTH RATE

We are now in the position to investigate the role of saving and technology in the determination of long-run growth rate. Growth rate is exogenously given independently of saving rate and technology in the neoclassical model of economic growth. This convention is attributable to the neglect of human capital in those models and at least objectionable on empirical grounds. In fact, growth rate is endogenously determined in the long run in the present model with human capital.

Our primary interest lies in the effects of a change in the rate of saving on growth rate. Let λ and κ denote the steady state values of y_1 and k satisfying (16) and (17) respectively. Clearly, λ represents the steady state growth rate of the economy. Differentiating (16) and (17) totally with respect to s, we get

$$\frac{d\lambda}{ds}=\frac{1}{\Delta}(g'+k)ff''>0\,, \tag{22}$$

$$\frac{d\kappa}{ds}=-\frac{1}{\Delta}ff'g''>0\,, \tag{23}$$

where $\Delta=(sf'-y_2)f'g''+(g'+k)^2f''<0$ in the neighborhood of a stable steady state. Thus we have

PROPOSITION 4. *A rise in the rate of saving raises the steady state growth rate and increases the steady state proportion of physical capital to human capital.*

An increase in the the proportion of physical to human capital brings about a fall in the rate of return on capital (the real rate of interest). This result may be taken to revive the old belief in the virtue of thrift as an important driving force of economic growth. It has been almost lost in the modern theory of economic growth which ignores investment in human capital completely.

Figure 4 illustrates the dynamic response of the economy to a rise in the average propensity to save under alternative hypotheses of expectations. It is basically a transformation of Figure 3. In view of (7), we may take y_1 instead of p_1 along the vertical axis and k_1 along the horizontal axis as in Figure 3 and reinterpret the schedules kL and PQ accordingly. The schedule KL now respresents the locus of y_1 and k that ensures $\dot{k}=0$ and the schedule PQ the locus of y_1 and k that ensures $\dot{y}_1=0$. Let the intersection, E, of the kL and PQ schedules represent the initial steady state. A rise in the rate of saving shifts the kL schedule to the right and upward to the position of $K'L'$, say, without affecting the PQ schedule. The new steady state is represented by the intersection, E', of the $K'L'$ and PQ schedules with higher y_1 and k than before.

The dynamic adjustment from E to E' differs depending on how the public expects the future price of human capital relative to physical capital. Under static expectations, the adjustment proceeds along the schedule PQ with y_1 and k rising gradually to the new steady state values. In the case of perfect foresight, there is

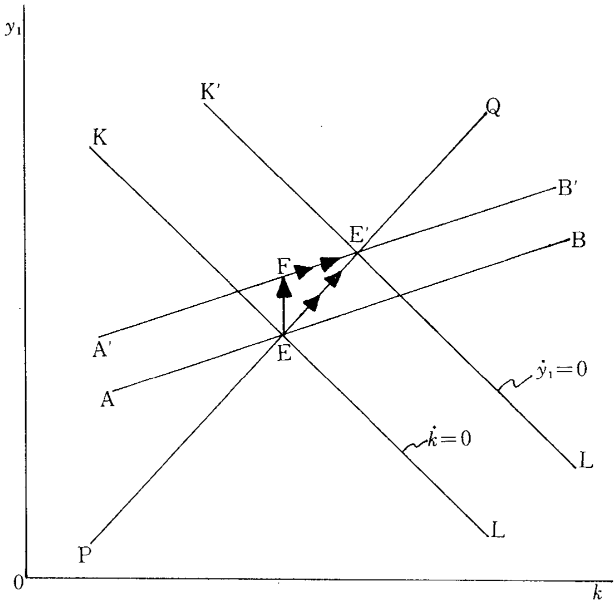

Figure 4

an immediate jump from E to F with y_1 rising discretely from the initial level, but thereafter the adjustment takes place gradually from F to E' along $A'B'$. The jump from E to F reflects the response of the public to their foresight of the new state with a higher growth rate.

Along with the propensity to save, the state of technology is also responsible for the determination of the steady state growth rate and capital formation. For simplicity, let us consider here only the effects of Hicksian neutral technological improvements. Let us introduce parameters α and β indicating the state of technology in the production of national product and human capital respectively and rewrite (16) and (17) as

$$g(y_1/\beta)+ky_1=s\alpha f(k)\,, \tag{16'}$$

$$g'(y_1/\beta)=\frac{f(k)}{f'(k)}-k\,. \tag{17'}$$

The mere inspection of this system reveals that the effect of a Hicksian neutral improvement in the production of national product (an increase in α) is identical to those of a rise in saving rate. It would, therefore, suffice to compute the effect of an improvement in the production of human capital (an increase in β). Differentiating (16′) and (17′) with respect to β, we get

$$\frac{d\lambda}{d\beta}=\frac{1}{\Delta}[g'(g'+k)f''-g''f'(y_1-sf')]y_1\gtrless 0\,, \tag{24}$$

$$\frac{d\kappa}{d\beta}=\frac{1}{\Delta}f'g''y_1<0\,, \tag{25}$$

where β is set to be unity initially. We see that $d\lambda/d\beta>0$ if $y_1>sf'$, a condition that is likely to be fulfilled under the normal circumstances.

PROPOSITION 5. (1) *A Hicksian neutral technological improvement raises the steady state growth rate and proportion of physical capital to human capital.* (2) *A Hicksian neutral technological improvement in the production of human capital raises the steady state growth rate if $y_1>sf'$; It decreases the steady state proportion of physical capital to human capital.*

It should be noted here that a once-and-for all improvement in production technology leads to a permanent rise in growth rate. This strong result is attributable to the assumption that human capital is produced under constant returns to scale. As in the case of a rise in saving rate, we can easily illustrate the dynamic response of the economy to technological improvements but we leave the exercise to the interested reader.

V. CONSUMPTION NOW OR HIGHER GROWTH RATE

We have assumed so far that the average propensity to save is exogenously given. As we demonstrated in the preceding section, a rise in saving rate raises the steady state growth rate, but it may reduce the steady state consumption per unit of human capital at the same time. When there is a trade-off between current consumption and growth rate, the rate of saving may be chosen to achieve the optimal combination of them. In this section, we study this problem and relate it to the well-known golden rule to maximize the steady state consumption *per capita* discussed in the neoclassical theory of economic growth.

Suppose that the economy is in its steady state at time 0 and that population is expected to be constant over time. Denoting the stock of human capital and the aggregate consumption at time s by K_{1s} and C_s, we have

$$K_{1t}=K_{10}e^{\lambda t}\,, \tag{26}$$

$$C_t=C_0e^{\lambda t}\,. \tag{27}$$

Let γ_t be the *per capita* consumption in the steady state at time $t\geqq 0$. It is written

$$\gamma_t=\frac{C_t}{K_{10}}=\frac{C_0e^{\lambda t}}{K_{10}}=\gamma e^{\lambda t}\,, \tag{28}$$

where γ $(=C_s/K_{1s})$ is the steady state consumption per unit of human capital. The representative agent of the economy placed at an arbitrarily chosen time 0 may be assumed to face the choice of γ_0 $(=\gamma)$ and γ_t $(t>0)$ and maximize the utility function

$$u=u(\gamma,\lambda)\,. \tag{29}$$

Function u is assumed to be strictly quase-concave and twice continuously differentiable. We assume that $\partial u/\partial\gamma$, $\partial u/\partial\lambda\geqq 0$.

In view of (16), the steady state consumption *per capita* is expressed as

$$\gamma=(1-s)f(k)=f(k)-k\lambda-g(\lambda)\,. \tag{30}$$

Taking notice of (11), differentiate (30) totally with respect to λ to get

$$\frac{d\gamma}{d\lambda}=-\frac{f'g''}{(k+g')f''}(f'-\lambda)-(k+g')\,. \tag{31}$$

We assume that $d^2\gamma/dy_1^2<0$. The condition for the maximization of u subject to (30) is then given by

$$\frac{\partial u/\partial\lambda}{\partial u/\partial\gamma}=\frac{f'g''}{(k+g')f''}(f'-\lambda)+(k+g')\,. \tag{32}$$

Rearranging (32), we can state

PROPOSITION 6. *The steady state of the economy is optimal if and only if the condition*

$$f'-\lambda=\frac{f''}{f'g''}\left[\frac{\partial u/\partial\lambda}{\partial u/\partial\gamma}-(k+g')^2\right] \tag{33}$$

is satisfied at the position.

Generally speaking, this result modifies the celebrated golden rule of economic growth that the rate of return on capital be equal to growth rate, i.e., $f'=\lambda$. For instance, consider the special case where there is absolutely no preference for growth. In this case, (33) reduces to

$$f'-\lambda=-\frac{f''}{f'g''}(k+g')^2>0\,. \tag{34}$$

At the interior optimum characterized by (34), growth rate could be positive despite the assumption that growth is not desired at all. This seemingly paradoxical outcome is attributable to the fact that a rise in the rate of saving raises not only the steady state growth rate but also the steady state consumption *per capita* for some range of λ where λ is substantially smaller than f' (see (31)). Generally, λ may be greater or smaller than f' at the optimum depending on the society's preference for current consumption and growth rate.

Figure 5 illustrates the optimal position with $\lambda>f'$. The schedule CD is the graph of (30), or the frontier of the combination of possible consumption *per capita* and growth rate in the steady state. It may be upward sloping for small λ, but it is definitely downward sloping for $\lambda\geqq f'$. The schedule UV is one of the social indifference curves tangent to CD. The point of tangency, A, shows the optimal combination of γ and λ under the given preference. It should be clear that an increase in the preference for current consumption relative to growth rate increases γ and decreases λ along the schedule CD. The social optimum in the special case where there is no preference for growth is indicated by B, the

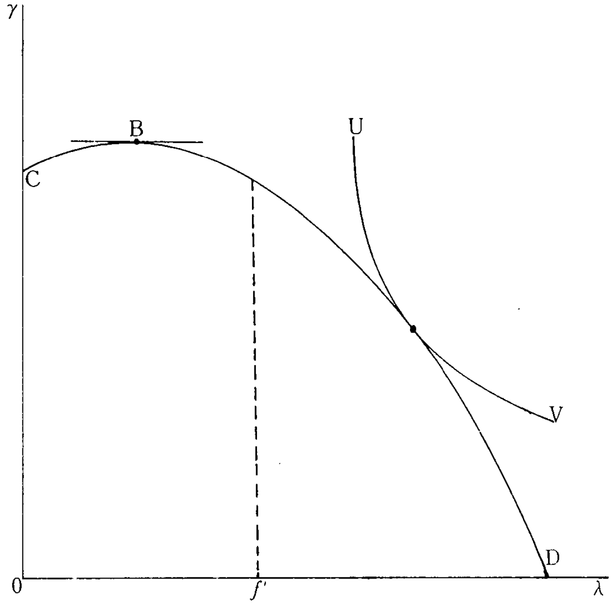

Figure 5

point at which a horizontal line is tangent to CD.

VI. CONCLUDING REMARKS

We have considered a simple model of economic growth in which investment in human capital plays an important role along with investment in physical capital in the determination of long-run growth rate and consumption per capita. It may be regarded as a straightforward generalization of neoclassical growth model of Solow (1956) in the spirit of Hahn (1966). The steady state of the model is shown to be a saddle point under perfect foresight, a property characteristic of the Hahn-type growth model with heterogenous capital goods.

One of the important implications of the model is that the long-run (steady-state) growth rate of the economy under consideration is dependent on the rate of saving and on the state of technology in production. In fact, we have shown that a rise in saving rate, as well as a Hicks-neutral improvement in the production of national output, increases the long-run growth rate of the economy. This property of our model may be considered as a most clear (although somewhat exaggerative) illustration of the relevance of thrift and industry to the speed of growth completely neglected in the neoclassical model of economic growth. It also enables us to discuss the trade-off between current consumption and growth in the future and optimal choice between them. In the present paper, it is derived as an extension of the celebrated golden rule of economic growth.

The present model may also be used to illustrate the effect of government subsidy to investment in human capital. Not surprisingly, it can be shown that a subsidy to investment in human capital leads to a rise in the long-run growth rate. Beside this point, there are a number of possible extensions of the model. For instance, the production possibility function of human and physical capital (equation (5)) adopted in this paper may be generalized in an obvious way. The saving rate of the economy, now given exogenously, may be determined endogenously from the intertemporal optimization of the representative individual.[7] These extensions are, however, more or less straightforward and not attempted here.

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[7] See Razin (1972 a, b), Manning (1975, 78), Arrestad (1975, 79) for various attempts to explain human capital investment from the viewopoint of intertemporal optimization.