Title	DYNAMIC ADVERTISING AND LIMIT PRICING
Sub Title	
Author	DAS, Satya P. NIHO, Yoshio
Publisher	Keio Economic Society, Keio University
Publication year	1990
Jtitle	Keio economic studies Vol.27, No.1 (1990.) ,p.21- 39
JaLC DOI	
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Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19900001-0021

DYNAMIC ADVERTISING AND LIMIT PRICING

Satya P. DAS and Yoshio NIHO

Abstract: Optimal pricing and advertising strategies by the established firms are investigated under a dynamic condition of entry. It is shown that under the threat of entry the optimal advertising policy by the established firms is to lower their advertising expenses gradually rather than raise it as one may expect. Also, an easier or more liberal entry policy leads to a greater market concentration in the long run. Higher market growth is found to have the same consequence as those of easier entry.

I. DYNAMIC ADVERTISING AND LIMIT PRICING

The earlier literature on limit pricing focussed on the behavior of the existing firms in an industry trying to *prevent* entry at any instant of time by limiting the price for their product and increasing their outputs (e.g. Modigliani (1958)). It was recognized later by Osborne (1973) that entry prevention (or 'limit pricing' as it is called) may not be the rational strategy for the existing firms: it may be most profitable to allow some entry by charging a higher price than the limit price. A simple diagrammatic exposition of this point can be found in Needlam (1976). The well known paper by Gaskins (1971) on dynamic limit pricing can be regraded as a dynamic generalization of the Osborne-Needlam analysis. Not only does he show that entry prevention is profitable only in the steady state, not outside, but he also goes on to characterize the optimal path of the pricing policy. He finds that under the threat of entry, the established firms would charge price higher than the limit price initially and then gradually lower it. He also examines both the short run and the long run effects of various changes such as those in entry response coefficient and market growth on the price, market structure and market shares in an industry.

The price-output strategy is not the only strategy that can be available to the established firms in an industry in order to deter entry. In fact, as early as 1956, Bain has recognized that selling or promotional expenditures (or advertising briefly) by the established firms are one of the most important barriers to entry. However, advertising as a barrier to entry has received some criticisms in the literature. For example, Scherer (1980) has argued to the effect that if entrants to an industry advertise and their advertisement is as effective as that of the existing firms, then advertisement is no longer a barrier. Cubin (1981) has shown, however, that even if advertisement by the entrants is as effective as that of the existing firms, by virtue of simply being the incumbent the latter can use advertising to deter entry.[1] This finding does not however imply that advertising expenditures and industry concentration are positively correlated. It has in fact been argued by Schmalensee (1983, 1986), Needlam and Cubin that entry prevention through heavy advertising may not be the optimal policy for the existing firms.[2]

While these roles of price and advertising have been recognized, clarified and analyzed in the literature on entry deterrence, the literature lacks an analysis of pricing and advertising strategies under *dynamic conditions of entry*. This is what we intend to explore in this paper. Our pursuit hardly needs any justifying in view of the fact that the phenomenon of entry is essentially dynamic in nature. The format of our analysis, as the reader will see below, resembles that of Gaskins and the significance of our results can therefore by judged by a comparative study. As one may expect, some results shown by Gaskins do generalize to the presence of advertising and some do not. The behavior of the advertising policy itself is however the new element in our analysis. Here are some of our main findings.

(a) First, we find that as in Gaskins' analysis, entry prevention by the established firms is optimal only in the steady state; and under the threat of entry, the optimal pricing policy is to set the price initially above the limit price and then lower it gradually. This monotonically decreasing pricing policy tends to make entry harder and harder, and hence is intuitive to understand. We however find, quite surprisingly, that *under the threat of entry, the optimal advertising policy by the established firms is to lower their advertising expenses gradually rather than to raise it as one may expect.*

(b) After determining the dynamics of pricing and advertising strategies, we examine the effects of an easier or more liberal entry (or licensing) policy. We find that in the long run, quite paradoxically, *this policy leads to a greater market concentration.* If the effect of the established firms' advertising on the entrants' product is sufficiently strong, we also find that in the long run, as a result of such a policy, the established firms not only increase their advertising efforts as is expected, but *they also increase the price of their product.* In other words, easier entry may actually raise the market price of the product in the long run. In the short run however, it is likely that the established firms lower the price of their product; their advertising expenses may go up, remain unchanged or may even go down.

(c) We further show that the results reported above are valid even when the

[1] For example, at the time of the breakdown, ATT, being the incubent, advertised in order to deter entry of new firms.

[2] Recently Milgrom and Roberts (1986) and Bagwell and Ramey (1988) have considered the cases in which an incumbent is allowed to signal its cost (Bagwell and Ramey) or the product quality (Milgrom and Roberts) with price and advertisements. In Bagwell and Ramey a downward distortion in price and an upward distortion in demand-enhancing advertising are expected to occur. While dissipating advertising will not be used in Bagwell and Ramey, in Milgrom and Roberts such advertising may occur.

markets for the established firms' product and the entrants' product are growing over time. Moreover, *higher market growth has the same consequences as those of easier entry.*

In what follows, we shall have the opportunity to provide explanations for all these findings. Finally (in Section VII) we demonstrate that our findings are immune to some generalizations and alternative specifications, and hence are quite general.

II. BASIC MODEL

The industry at a given point of time is assumed to consist of two types of firms: the established firms acting collusively as one group and the group of new (identical) firms (that have already entered), each producing a unique brand. The latter group of firms is called a competitive fringe. A firm in the competitive fringe acts as a price taker. The price of the product however will decrease as the number of firms in the competitive frings increases. The decision variables of the established firms are the price of their product and their advertising expenditures. The number of firms in the competitive fringe is assumed to be large so that any advertising expenditure by a single competitive firm is expected to have negligible impact on the demand for its product, and hence is not incurred at all.

Let the demand function for the product by the established firms be $Q(\underset{-}{p}, \underset{+}{p_r}, \underset{+}{s})$, where p is the price of the product by the established firms, p_r is the price of the product by the competitive fringe and s is the sales expenditure by the established firms. The signs of the partial derivatives shown above are very much what one would expect. The price of the product by the competitive fringe is assumed to increase with an increase in the price charged by the established firms, decrease with an increase in advertising by the established firms and decrease with an increase in the size of the competitive fringe. That is, $p_r(\underset{+}{p}, \underset{-}{s}, \underset{-}{n})$ where n is the number of firms in the competitive fringe. Then, the demand function facing the established firms can be written as

$$Q = Q(p, s, n) \equiv Q[p, p_r(p, s, n), s] \, . \tag{1}$$

The impact of change in n on Q is evident; as the number of firms in the competitive fringe increases their product price goes down and thus the established firms notice a decline in their sales. To determine the effects of changes in p and s on Q it is assumed that the direct effects of p and s on Q outweigh their indirect effects through changes in p_r, so the net effects are that an increase in own price reduces the sales and an increase in advertising increases the sales by the established firms. The demand functions specified in (1) will not be able to take us far in our analysis. In order to derive meaningful conclusions we require a little stronger structure, that is, Q is separable and linear in n. (We do not consider this to be restrictive at all vis-a-vis the existing literature on market structure which abounds with examples of specific demand functions.) Then Q_n is a negative constant and we set it -1 without loss of generality. In keeping with the literature on advertising, we impose one further restriction on the demand function Q, that is, an increase in advertising reduces the price elasticity of Q.

Denoting the (given) unit cost of production by c, the profits of the established firms in period t is given by

$$\pi(t) = (p(t) - c)Q(n(t), p(t), s(t)) - s(t). \tag{2}$$

We assume that this profit function is strictly concave in p and s for each t and $n(t)$, so that

$$\pi_{pp} < 0, \quad \pi_{ss} < 0 \quad \text{and} \quad \pi_{pp}\pi_{ss} - \pi_{ps}^2 > 0$$

for each t. The objective of the established firms is to choose the paths of $p(t)$ and $s(t)$ such that the discounted sum of profits, $\int_0^\infty e^{-rt}\pi(t)dt$, is maximized.

We assume that the established firms are constrained by the following entry condition:

$$\dot{n}(t) = k[\pi_r(t) - \pi^0], \qquad n(0) = n^0, \quad \pi^0 > 0, \tag{3}$$

where $k \equiv$ entry response coefficient, $n^0 \equiv$ given initial size of the competitive fringe, $\pi_r \equiv$ the profit level of a competitive firm and $\pi^0 \equiv$ some (given) minimum profit level for the prospective entrants. Eq. (3) says that some firms enter or leave the industry according as the current profits earned by a competitive firm are above or below a minimum threshold level. Note that the entry resonse coefficient (k) can be interpreted as a policy parameter such as the licensing policy.

The profit function of a firm in the competitive fringe can be written as

$$\pi_r = [p_r(\underset{-}{X_r}, \underset{+}{p}, \underset{-}{s}) - c_r]x_r$$

where X_r is the aggregate output of the competitive fringe, x_r the output of a firm in the competitive fringe, $p_r(X_r, p, s)$ is the inverse demand function, assumed to be linear and separable in X_r, and c_r is the unit cost of production, assumed to be given. Supposing Cournot expectations a firm maximizes its profit with respect to x_r, assuming that the other firms in the competitive fringe do not change their output. The first order condition is

$$p_r(X_r, p, s) - c_r + x_r\partial p_r/\partial X_r = 0$$

Nothing $X_r = nx_r$, the above condition yields a firm's output as a function of n, p and s:

$$x_r = x_r(\underset{-}{n}, \underset{+}{p}, \underset{-}{s})$$

$$\frac{\partial x_r}{\partial n} = \frac{-x_r}{n+1} < 0; \quad \frac{\partial x_r}{\partial p} = \left(\frac{-1}{n+1}\right)\left(\frac{\partial p_r/\partial p}{\partial p_r/\partial X_r}\right) > 0; \quad \frac{\partial x_r}{\partial s} = \left(\frac{-1}{n+1}\right)\left(\frac{\partial p_r/\partial s}{\partial p_r/\partial X_r}\right) < 0.$$

Since π_r can be written as $-x_r^2(\partial p_r/\partial X_r)$ and p_r is linear and separable in X_r, $\pi_r=\pi_0$ defines a certain value of x_r, say x_r^0. Thus, $\pi_r=\pi_0$ is equivalent to

$$x_r(\underset{-}{n}, \underset{+}{p}, \underset{-}{s})=x_r^0$$

The above defines the following 'limit price' function:

$$p(t)=\bar{p}(\underset{+}{n(t)}, \underset{+}{s(t)}) \tag{4}$$

(4) defines the configuration of $p(t)$, $n(t)$ and $s(t)$ which implies no entry or exit. One may note that our limit price function is more general than the ones that have appeared in the literature. For example, without product differentiation and advertising, and with a perfectly competitive fringe, (4) reduces to $p(t)=$ a constant, which is used by Gaskins. Also, the limit price function used by Williamson (1962): $p(t)=\bar{p}(s(t))$ is a special case of (4) where the number of entrants is implicitly given. The sign pattern in (4) implies that at any given level of advertising expenditures by the established firms, if the size of the competitive fringe is greater, the profits of a firm in the competitive fringe are less and therefore a higher $n(t)$ can be coupled with a higher $p(t)$ so that entry is blockaded. Similarly, given the size of the competitive fringe, since a firm's profits are respectively positively and negatively related to the price charged and the selling expenses incurred by the established firms, a higher $p(t)$ can be coupled with a higher $s(t)$ so as to blockade entry.

We assume that for the relevant range of operation, p is greater than the unit cost of output (c) by the established firms. This means that the price that forces the competitive profits down to zero earns a positive profit (gross of advertising expenses) for the established firms—in other words, the latter possess a cost advantage over the competitive fringe.

Finally we assume that the effects of changes in n and s on the limit price are constant, i.e., $\partial\bar{p}/\partial n\equiv\alpha_n$ and $\partial\bar{p}/\partial s=\alpha_s$ where α_n and α_s are positive constants. One may note that α_n is likely to be small in magnitude, because the impact of an increase in n on p_r is likely to be small when the size of the competitive fringe is large. Hence, only a small increase in p is likely to restore p_r to the level which permits a competitive firm to earn the minimum threshold level of profit, since the products produced by the established firms and by the competitive fringe must be very close substitutes.

Now, the profits in the competitive fringe are positive or negative according as $p(t)$ is greater or less than p. Thus, the entry equation (3) can be equivalently stated as

$$\dot{n}(t)=k[p(t)-\bar{p}(n(t), s(t))], \qquad n(0)=n^0. \tag{5}$$

To summarize, the established firms

$$\underset{p(t),\, s(t)}{\text{maximize}} \int_0^\infty e^{-rt}\pi(t)dt \equiv \int_0^\infty e^{-rt}[(p(t)-c)Q(n(t), p(t), s(t))-s(t)]dt$$

subject to (5).

The reader may now notice that even though our formulation has a close resemblance with that of Gaskins, ours is a much more general one. Not only have we allowed for product differentiation and advertising that are absent in Gaskins' analysis, the competitive price (and hence the competitive output) is also sensitive to changes in price (as well as in advertising)—which was ruled out in Gaskins' analysis.

III. SOLUTION TO THE OPTIMAL CONTROL PROBLEM AND COMPARISON WITH NO ENTRY SITUATION

The current value Hamiltonian of this control problem is

$$H \equiv \pi(t) + \mu(t)k[p(t) - \bar{p}(n(t), s(t))] \equiv \pi(t) + \lambda(t)[p(t) - \bar{p}(n(t), s(t))]$$

where $\lambda(t) = k\mu(t)$. The necessary conditions for the optimal time path, besides the entry-exist equation (5) are

(a) H is maximized with respect to p and s at each t.[3] We assume a regular interior solution for p and s. Thus

$$H_p \equiv \pi_p + \lambda = (p-c)Q_p + Q + \lambda = 0 \tag{6}$$

$$H_s \equiv \pi_s - \lambda\alpha_s = (p-c)Q_s - \lambda\alpha_s - 1 = 0 \tag{7}$$

and strict concavity of π assures

$$H_{pp} = \pi_{pp} < 0\,, \quad H_{ss} = \pi_{ss} < 0\,, \quad H_{pp}H_{ss} - H_{ps}^2 = D \equiv \pi_{pp}\pi_{ss} - \pi_{ps}^2 > 0 \tag{8}$$

(b)
$$\dot{\lambda} = (r + k\alpha_n)\lambda + k(p-c)\,. \tag{9}$$

One may note that λ is the shadow price of entry (normalized on the speed of adjustment k) and is negative. The optimal pricing and advertising strategies are governed by the solution of eqs. (5), (6), (7) and (9).

One of the main concerns in the theory of entry is to see how behavior of the established firms under the threat of entry differs from that without any threat of entry. This can be illustrated graphically. First note that if the price elasticity of Q goes down as advertising expenses increase, the cross partial $H_{ps} = \pi_{ps}$ is positive, implying that the $H_p = 0$ curve and the $H_s = 0$ curve are both positively sloped. Also condition (8) implies that $H_p = 0$ is steeper than $H_s = 0$. In Figure 1 the optimal solutions are p^* and s^*. However, without any threat of entry profits are maximized with $\pi_p = 0$ and $\pi_s = 0$, which imply $H_p = \lambda < 0$ and $H_s = -\lambda\alpha_s > 0$. Thus the optimal price and advertising without any threat of entry are at a point in the shaded area.

[3] From here onwards, the time notation will be suppressed unless it is essential.

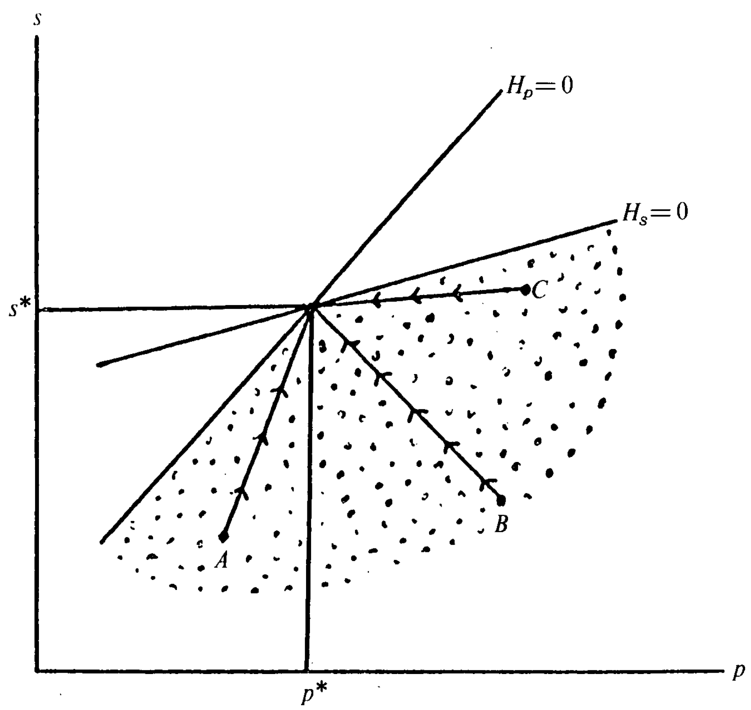

Fig. 1

There are therefore more than one possibility, say points A, B or C. Hence with an initial situation of no entry, *the threat of entry does not necessarily motivate the established firms in an industry to charge a lower price and to engage in more advertising effects* (which is consistent with a point such as B only); *the established firms may increase or decrease both price and advertisement* as illustrated by points A and C. Considering the advertising policy alone, it follows that advertising expenditures are not necessarily positively correlated with entry barriers or (as a proxy) industry concentration. This has incidentally been shown empirically by Telser (1964, 1969) and is consistent with the views expressed by Schmalensee, Needlam and Gubin.

IV. OPTIMAL TIME PATH

We examine how the optimal price and advertising expenditures change over time as the size of the competitive fringe or the shadow price of entry changes. This is obtained by solving (6) and (7) for p and s.

$$p=p(n,\lambda)\,; \qquad s=s(n,\lambda) \tag{10}$$

Differentiating (6) and (7) we also have

$$\begin{aligned} p_n&=\pi_{ss}/D<0\,; \qquad s_n=-\pi_{ps}/D<0 \\ p_\lambda&=-(\pi_{ss}+\alpha_s\pi_{ps})/D\,; \qquad s_\lambda=(\alpha_s\pi_{pp}+\pi_{ps})/D \end{aligned} \tag{11}$$

where $D=\pi_{pp}\pi_{ss}-\pi_{ps}^2>0$. We now substitute (10) in (5) and (9) to obtain:

$$\dot{n}=k[p(n,\lambda)-\bar{p}(n,s(n,\lambda))]\equiv kg(n,\lambda) \tag{12}$$

$$\dot{\lambda}=(r+k\alpha_n)\lambda+k[p(n,\lambda)-c]\equiv h(n,\lambda;k) \tag{13}$$

Equations (12) and (13) are two dynamic equations in two variables, n and λ. We assume that there exists an interior steady state $(\hat{n},\hat{\lambda})$ which is a saddle point of the system. Then the Jacobian of (12) and (13) must be negative, implying that the following expression is positive:

$$Z\equiv\alpha_n-(\pi_{ss}+\alpha_s\pi_{ps})/D>0\,. \tag{14}$$

Also note that since α_n is likely to be small in magnitude (see the last section), for Z to be positive, it is likely that $\pi_{ss}+\alpha_s\pi_{ps}$ is negative.

We draw a phase diagram in $n-\lambda$ space. From (12) and (13), the slopes of $\dot{n}=0$ and $\dot{\lambda}=0$ lines are given by

$$(d\lambda/dn)_{\dot{n}=0}=-g_n/g_\lambda \quad \text{and} \quad (d\lambda/dn)_{\dot{\lambda}=0}=-h_n/h_\lambda\,,$$

where, upon the use of (1) and (14),

$$g_n=(\pi_{ss}+\alpha_s\pi_{ps})/D-\alpha_n<0\,;\quad g_\lambda=-(\pi_{ss}+\alpha_s^2\pi_{pp}+2\alpha_s\pi_{ps})/D>0$$

$$h_n=k\pi_{ss}/D<0\,;\quad h_\lambda=r+k\alpha_n-k(\pi_{ss}+\alpha_s\pi_{ps})/D>0\,. \tag{15}$$

Thus both $\dot{n}=0$ and $\dot{\lambda}=0$ lines are positively sloped, as drawn in Figure 2. Since the Jacobian of (12) and (13) is negative, the $\dot{n}=0$ line is steeper than the $\dot{\lambda}=0$ line.

The steady state values of n and λ are indicated by $\hat{n}$ and $\hat{\lambda}$. From (12) we see that *the steady state (or long run) price charged by the established firms, $\hat{p}$, is equal to the limit price ($\bar{p}$).* Inspection of Figure 2 gives the first property of the optimal path. *If the initial size of the competitive fringe (n^0) is smaller (greater) than its steady state value, the optimal policy includes monotonic increase (decrease) in both n and λ.*

At this level of generality, it does not seem feasible to entirely characterize the time paths of optimal price and advertising expenditures. However in the neighborhood of the steady state, E, definite conclusions can be obtained. Let us now consider the dynamic path of the optimal price, $p(n,\lambda;k)$. Change in the optimal price is given by $\dot{p}=p_n\dot{n}+p_\lambda\dot{\lambda}$. In Figure 2, it is easily seen that in the neighborhood of E, the slope of the optimal path $(\dot{\lambda}/\dot{n})$ is less than that of $\dot{\lambda}=0$ line. This implies that in the neighborhood of steady state $\dot{p}/\dot{n}<0$. Hence, *the optimal pricing policy is to initially set the price higher or lower than the steady state level and then continually reduce or increase it according as the initial size of the competitive fringe is smaller or larger than its steady state size.*[4]

The optimal path of the advertising expenditures is also clear-cut. Totally differentiating (7), we obtain $\dot{s}=(\alpha_s\dot{\lambda}-\pi_{ps}\dot{p})/\pi_{ss}$. We have already shown that if the

[4] This is a generalization of Gaskins' corresponding result which ignores advertising expenses.

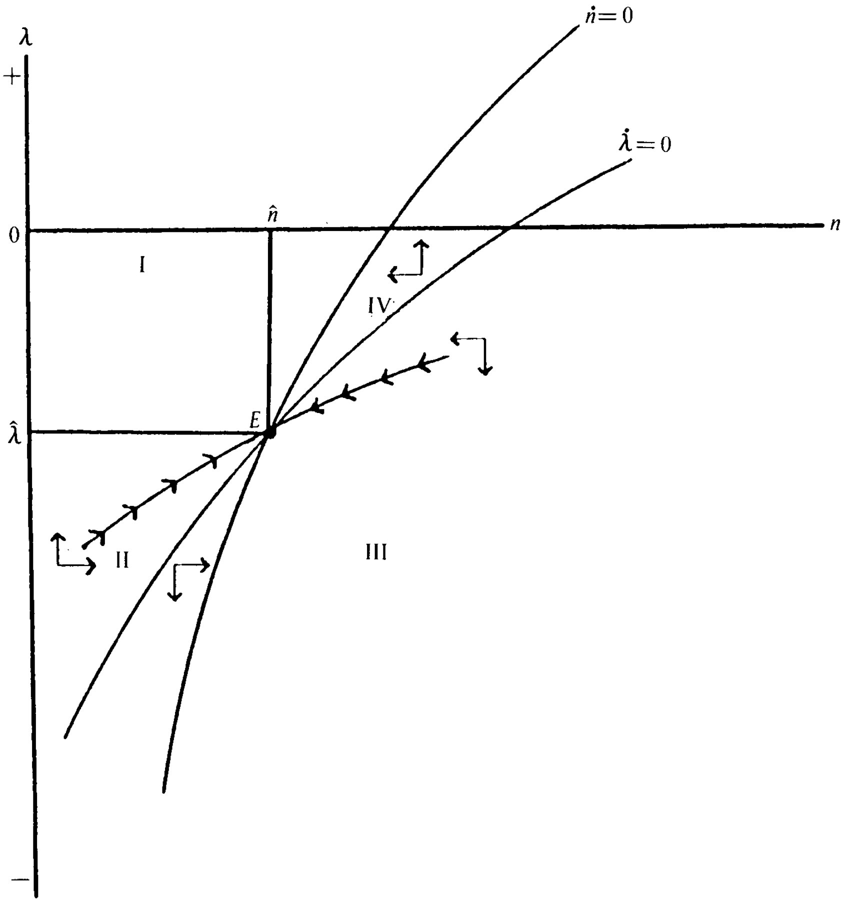

Fig. 2

price elasticity of quantity demanded (of the established firms' product) goes down when the advertisements are increased, π_{ps} is positive. Thus when $n^0 < \hat{n}$, $\dot{s} < 0$ since $\dot{\lambda} > 0$ and $\dot{p} < 0$. Similarly $\dot{s} > 0$ if $n^0 > \hat{n}$. In other words, *the optimal advertising policy is to initially engage in more or less advertising activities than their steady state level and then continually reduce or increase it according as the initial size of the competitive fringe is smaller or larger than its steady state level.*

To summarize, $p(t)$ and $s(t)$ both decline over time as n increases and they both increase as n decreases. Whereas this dynamic path of $p(t)$ is as one would expect, the dynamic path of $s(t)$ is by no means apparent. This even casts doubt on any positive correlation between observed advertising expenditures and entry barriers in actuality.

V. EFIECTS OF A CHANGE IN THE ENTRY RESPONSE COEFICIENT

Now we examine how the dynamic paths of optimal price, advertising expenditures and of the size of the competitive fringe and their steady state values change as various parameters in the model change. From the point of view of policy making, the entry resonse coefficient (k) may be the most important parameter. A higher or lower k represents a more lenient or stringent licensing policy. We thus examine only the effects of change in k, though other parametric changes could be worked out as well.

Long Run Effects

Let us first derive the effects on steady state values. Totally differentiating (12) and (13) in steady state, we obtain

$$d\hat{n}/dk = h_k g_\lambda / J < 0$$

$$d\hat{\lambda}/dk = -h_k g_n / J < 0 \, ,$$

where $h_k = \lambda\alpha_n + (p - c) > 0$. Thus as k increases, $\hat{\lambda}$ goes down and $\hat{n}$ goes down also. The latter result is surprising, because it says that *an increase in the entry coefficient or an easier licensing policy leads to an increase in market concentraton in the long run*. This is similar to Gaskins' finding that an increase in k reduces the market share of the competitive firms.

What about the price and advertising expenses chosen by the established firms? It must be noted that in the simpler case without advertising, the limit price is a given number and hence is not affected by parametric changes (see Gaskins).[5] With advertising and imperfect competition in our analysis, the limit price ($\bar{p}$) is not a given number and there is no reason why it may not change.

Substituting $d\hat{\lambda}/dk$ and $d\hat{n}/dk$ in (10), we obtain

$$\frac{d\hat{p}}{dk} = r\hat{\lambda}[\alpha_s^2 + \alpha_n(\pi_{ss} + \alpha_s \pi_{ps})]/(DJ)$$

$$\frac{d\hat{s}}{dk} = r\hat{\lambda}[\alpha_s + \alpha_n(\alpha_s \pi_{pp} + \pi_{ps})]/(DJ)$$

The impacts of increase in k on $\hat{p}$ and $\hat{s}$ are not unambiguous. However recall that α_n is likely to be small in magnitude. In addition, if α_s is sufficiently large, i.e. *if the impact of advertising by the established firms on raising the limit price is sufficiently strong, $\hat{p}$ and $\hat{s}$ both go up*. Since advertising by the established firms tends to reduce the profits earned by the competitive firms (by lowering the competitive price), one would expect that an increase in the entry coefficient would motivate the established firms to step up their advertising activities. This explains our previous result that the size of the competitive fringe goes down in the steady state. Since an increase in advertising by the established firms reduces the profits of the competitive firm, the size of the competitive fringe must decrease in order for its price (p_r) to increase to the level which restores the profit to the threshold level. Since from the limit price function $\hat{p} = \bar{p}(\hat{n}, s)$, $d\hat{p} = \alpha_n d\hat{n} + \alpha_s d\hat{s}$, with α_n likely to be small and α_s sufficiently high, $\hat{p}$ can be raised along with s and still entry can be blockaded. Since $\hat{n}$ goes down, it also follows from $d\hat{p} = \alpha_n d\hat{n} + \alpha_s d\hat{s}$ that $\alpha_s ds > d\hat{p}$.

[5] We agree with Ireland (1972) on this point: that it is a very limiting assumption.

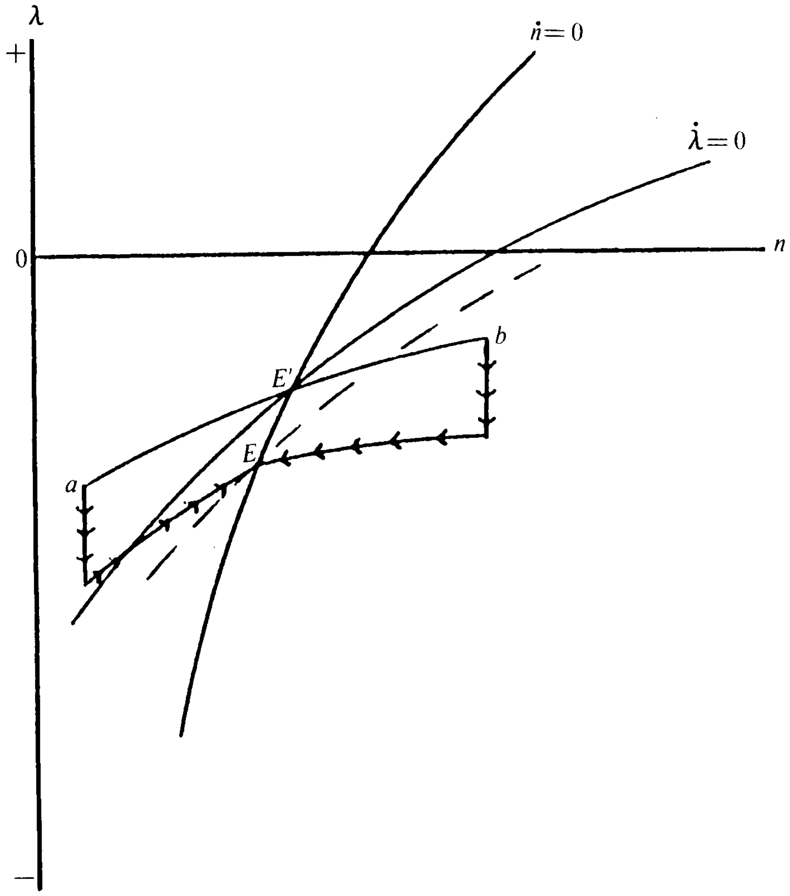

Fig. 3

This can be interpreted as saying that *in the long run advertising by the established firms is more sensitive to changes in entry conditions than their pricing policies.*

If the demand function Q (in (1)) is linear in its own price and separable from s, it follows immediately from the first order condition (6) that, *the aggregate output and sales by the established firms most likely increase as k increases.* This together with the fact that the size of the competitive fringe declines as k increases further adds to the point that easier entry conditions can actually increase the market concentrations in the long run.

Short Run Effects

Following Gaskins, the short run impacts of an increase in k can be determined by observing its impact on the slope of the optimal trajectory which equals $\dot{\lambda}/\dot{n} = [(r/k + \alpha_n)\lambda + p - c]/(p - \bar{p})$. Since λ is negative, an increase in k increases the numerator of $\dot{\lambda}/\dot{n}$. Thus the optimal trajectory becomes steeper or flatter according as $p(t)$ is greater or smaller than $\bar{p}$.

Consider Figure 3 now. E is the initial steady state and E' is the new state after k has increased. (Note from (12) and (13) that as k increases $\dot{n} = 0$ line does not shift but $\dot{\lambda} = 0$ line shifts to the right.) Suppose initially $p(t) > \bar{p}$ and thus the system is at a point such as $\underline{a}$. Now, as k increases, if $\lambda(t)$ goes up, then the system can never reach E' since the new optimal trajectory is steeper. Thus $\lambda(t)$ must go down. The same conclusion follows when $p(t)<\bar{p}$ (see part $\underline{b}$). At the instant of time when k goes up, the size of the competitive fringe is unchanged. Therefore, the impacts on $p(t)$ and $s(t)$ are determined by how $p(t)$ and $s(t)$ change when $\lambda(t)$ changes. As expression (11) shows, p_λ and s_λ are not unambiguous in their signs and thus nothing could be said on the impacts on $p(t)$ and $s(t)$ in general. However, we have argued in Section 4 and $\pi_{ss}+\alpha_s\pi_{ps}$ is likely to be negative. Since $p_\lambda = -(\pi_{ss}+\alpha_s\pi_{ps})/D$ and $\lambda(t)$ goes down as k increases, it follows that $p(t)$ *is likely to go down as* k increases. The impact on $s(t)$ is however not clear. However, if $s(t)$ happens to go up, the aggregate sales by the established firms go up $(Q(\underset{-}{p},\underset{+}{s},\underset{-}{n}))$. This together with the size of the competitive fringe being fixed in the short run leads to a presumption that an increase in the entry coefficient also increases the market concentration in the short run. Also, it may be observed that if $s(t)$ goes up along with $p(t)$ going down, the price of the produced by the competitive fringe goes down.[6]

V. MARKET GROWTH

In this section we extend our analysis to incorporate market growth. Such an extension hardly needs any justification in view of the fact that most modern economies have been growing over time.

Let v be the growth rate of demands for the products produced by established firms as well as the competitive fringe. Then the market demand function of the established firms is specified by $e^{vt}Q(\cdot)$. In an environment of growth, it is natural that the demand for products other than the ones in question must be growing. Thus the advertising expenditures needed to exert the same influence on the demand for a particular product must be increasing in keeping with the overall growth. Let us now denote the advertising expenditures at time t as $\bar{s}(t)$ and let $s(t)\equiv \bar{s}(t)e^{-vt}$ may be interpreted as the 'effective advertising expenditures' and this, instead of $\bar{s}(t)$, should enter the demand function.

Two changes in functional forms are needed to reflect the growth of demands. First, along with p and s, the demand for the product produced by the established firms depends (linearly) on ne^{-vt} instead of n, i.e. $Q=Q(ne^{-vt},p,s)$. This implies that if the size of the competitive fringe remaining the same or does not grow as fast as the market, the demand for the product by the established firms will increase. Second, the limit price function becomes $\bar{p}(ne^{-vt},s)$, implying that with the same

[6] Effects of change in discount rate r are qualitatively exactly the oposite of change in k. Namely, in the steady state both $\hat{\lambda}$ and $\hat{n}$ increase while $\hat{p}$ and $\hat{s}$ decrease (if α_s is sufficiently large). Consequently, the size of the competitive fringe goes up while the output of the established firm goes down (if Q is linear in p and separable from s). Hence, the market concentration decreases in this case. In the short run, λ goes up, p is likely to go up, but the effect on s is ambiguous. However, effects of change in the established firm's cost (c) are not clear.

level of s, if the size of the competitive fringe does not grow as fast as the market the limit price must be lowered. This is because if the output of the competitive fringe does not increase as much as the demand, the price of the competitive product will increase, implying a higher level of profit of the competitive firm and entry of new firms. Thus, in order to blockade entry, the established firms must lower the price of its product (and/or increase the advertising expenditure more than the rate of growth of the market).

Regarding entry, we assume the specificatin used by Ireland, that is

$$\dot{n}(t) = vn(t) + ke^{vt}[p(t) - \bar{p}(n(t)e^{-vt}, s(t))] \tag{16}$$

This means that whether or not the limit price is charged, there is a growth component in the number of entrants. Second, the entry response coefficient itself grows over time in keeping with the overall growth.

The established firms now maximize

$$\int_0^\infty e^{-rt}[(p(t)-c)e^{vt}Q(p(t), s(t), n(t)e^{-vt}) - s(t)e^{vt}]dt$$

subject to (16), $n(0) = n^0$, $r > v$.[7]

Using the transformation: $m(t) = n(t)e^{-vt}$, we can write the current value Hamiltonian as

$$H' \equiv (p(t) - c)[Q(p(t), s(t), m(t))] - s(t) + \lambda(t)k[p(t) - \bar{p}(m(t), s(t))].$$

The following first order conditions are obtained:

$$H'_p = \pi_p + \lambda = 0$$

$$H'_s = \pi_s - \lambda\alpha_s = 0$$

$$\dot{m}(t) = k[p(t) - \bar{p}(m(t), s(t))] \equiv kg'(m, \lambda) \tag{17}$$

$$\dot{\lambda}(t) = (r + k\alpha_n - v)\lambda(t) + k[p(t) - c] \equiv h'(m, \lambda; k, v) \tag{18}$$

The structure of these equations is exactly the same as in the basic model. Thus all our conclusions in the previous section apply to the case of market growth. It can be observed from (18) that *the long run effects of an increase in the growth rate is qualitatively the same as those of an increase in k*, because along the steady state $h'_v = -\hat{\lambda}$, $h'_k = p - c + \hat{\lambda}\alpha_n = (-\hat{\lambda})(r-v)/k$ and thus $h'_k = h'_v(r-v)/k$. *The short run impacts of market growth are also qualitatively the same as those of the entry response coefficient.* To see this, examine the slope of the optimal path. $\dot{\lambda}/\dot{m} = [(r + k\alpha_n - v)\lambda + k(p-c)]/[k(p-\bar{p})] = [\{\alpha_n + (r-v)/k\}\lambda + p - c]/(p - \bar{p})$. Hence, an increase in v has qualitatively the same impacts on the slope of the optimal path as an increase in k. The short run effects are therefore analogous.

This is another 'surprising' result that we obtain. The explanation of this lies

[7] Note that the advertising expenditures are $\bar{s}(t) = s(t)e^{vt}$. Also note that $r > v$ is needed for the convergence of the maximand.

in our entry equation specification (16). The benefit of hgher market growth to the established firms through the shift in the demand curve for their product ($e^{vt}Q$) is neutralized by the corresponding increase in the growth rate of competitive firms (indicated in the first term in (16)). Thus the net impact of an increase in v is the same as an increase in v in $ke^{vt}(p-\bar{p})$. It is now easy to see that this increase in v is equivalent to an increase in k.

Among other things, this equivalence implies and explains the 'disturbing' result that higher market demand growth leads to greater industry concentration in the long run and in the short run there is a presumption to the same effect.

VII. EXTENSIONS AND ALTERNATIVE SPECIFITIONS

Most of the results we have derived in preceding sections are valid under more general conditions and under alternative specifications. We demonstrate this by considering two specific cases.

First, an implicit assumption that the reader may have noticed in our preceding analysis is that new firms forever remain new and followers to the already established firms. Over time however, some of the new firms may grow, get over the cost disadvantage and become established firms in the market. Two situations can arise: either they complete or they collude with the already established firms. Since the objective of our paper is to focus on competition with the entrants, we assume that the latter situation prevails: the newly growing competitive firms join the group of already established firms and exercise price-leadership. This phenomenon can be introduced in our model in a simple manner. We show that in this situation the dynamic princing and advertising strategies remain the same as in our orginal model and the impact of market growth remains unchanged. The only difference is that increase in entry coefficient is no longer equivalent to increase in market growth and its long run and short run impacts are subject to some revisions.

Second, following the well-known model of advertising by Nerlove and Arrow (1962), advertising expenses can be regarded as input to the build-up of goodwill for the product on the part of the consumers. Thus a distinction can be made between the *flow* of advertising expenses and the stock of goodwill that affects the market demand. We wish to reassess our results in such a 'goodwill' model. As will be seen below, this converts our optimal contrl problem to a two-state variable problem—a general solution of which does not seem to be possible. It is however easy to characterize the steady state and it will be shown that the steady state results in this model are the same as in the previous sections.

Growth of New Firms

The simplest way to capture the growth of new firms is to assume that at any instant of time a fixed proportion, say δ, of the existing competitive firms join the group of established firms and start producing their product. The only difference it makes is that the rate of change in the number of new firms is now given by

$$\dot{n}(t)=k[p(t)-\bar{p}(n(t), s(t))]-\delta n(t), \qquad n(0)=n^0, \tag{5'}$$

which is an extended version of (5). We also have an extended version for the change in shadow price of entry:

$$\dot{\lambda}(t)=(r+k\alpha_n+\delta)\lambda+k(p(t)-c). \tag{13'}$$

Without repeating our analysis, we may simply point out here that, as before $\dot{n}=0$ and $\dot{\lambda}=0$ line are both positive sloped. $\dot{n}=0$ line is steeper than $\dot{\lambda}=0$ line and the dynamic paths of $n(t)$, $p(t)$ and $s(t)$ are qualitatively the same.

The effects of increase in the entry coefficient are unclear in general. We can however say that as long as δ is not large, the long run impact is that $\hat{n}$ would go down and $\hat{\lambda}$ would also go down as before. The impact on $\hat{p}$ and $\hat{s}$ would also be the same qualitatively. It does not seem possible to determine the short run effects on $p(t)$ and $s(t)$ (even when δ is not very large) because $\dot{n}=0$ $\dot{\lambda}=0$ curves both shift.

The impact of market growth however is the same as in our original analysis. The dynamic equations for $m(t)$ and $\lambda(t)$ are now extended to incorporate δ:

$$\dot{m}(t)=k[p-\bar{p}(m, s)]-\delta m \tag{17'}$$

$$\dot{\lambda}(t)=(\delta+r+k\alpha_n-v)\lambda+k(p-c) \tag{18'}$$

Thus as in our original model, only the $\dot{\lambda}=0$ line shifts as v increases and consequently the long run and short run impacts are qualitatively the same.

A 'Goodwill' Model

Let $s(t)$ now stand for the stock of goodwill in period t. Following Norlove and Arrow, we assume that the stock of goodwill accumulates according to

$$\dot{s}(t)=a(t)-\rho s(t), \qquad \underline{a}\le a(t)\le \bar{a}, \quad s(0)=s^0, \tag{19}$$

where $a(t)\equiv$ advertising expenses in period t, $\rho\equiv$ rate of depreciation of goodwill, $\underline{a}$ and $\bar{a}$ are the given lower and upper limits of advertising expenses at any instant of time.

The established firm's objective is to

$$\underset{p(t),\, a(t)}{\text{maximize}} \int_0^\infty e^{-rt}[(p(t)-c)Q(p(t), s(t), n(t))-a(t)]dt$$

subject to (5) and (19)

We may write the Hamiltonian of this problem as

$$\tilde{H}=(p-c)Q(p, s, n)-a+\lambda(p-\bar{p}(n, s))+\sigma(a-\rho s)$$

The necessary conditions for this maximization are

$$\tilde{H}_p=\pi_p+\lambda=0 \tag{20}$$

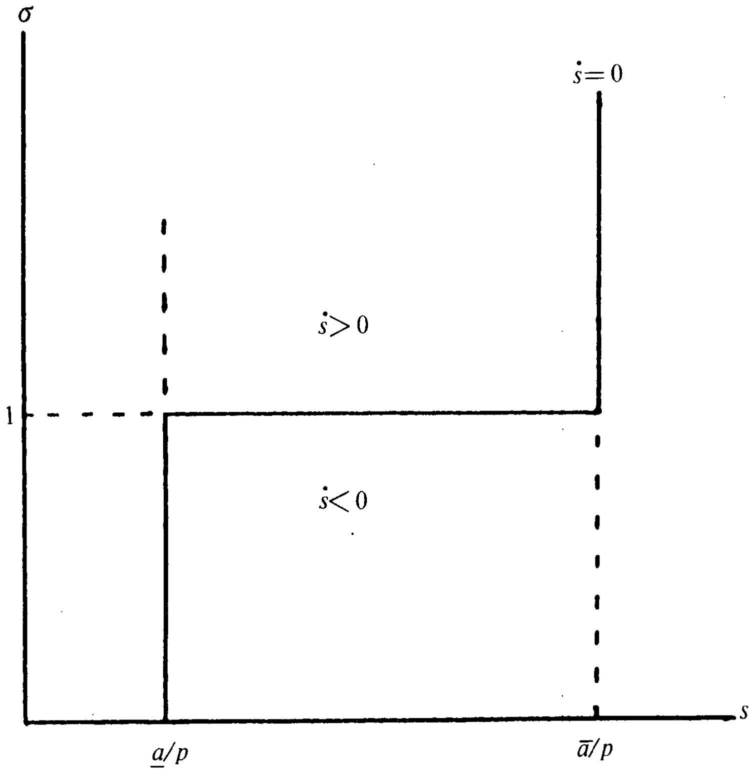

Fig. 4

$$a(t)=\begin{cases} \bar{a} \\ \underline{a}\le a(t)\le \bar{a} \\ \underline{a} \end{cases} \quad \text{according as} \quad \sigma \gtreqless 1 \tag{21}$$

$$\dot{\lambda}=(r+k\alpha_n)\lambda+k(p-c) \tag{9}$$

$$\dot{\sigma}=(r+\rho)\sigma+\alpha_s\lambda-\pi_s$$

Note that from the first order condition (20), p can be expressed as a function of s, n and λ:

$$p=\tilde{p}(s,n,\lambda)\,,\quad \tilde{p}_s=-\frac{\pi_{ps}}{\pi_{pp}}>0\,,\quad \tilde{p}_n=\frac{1}{\pi_{pp}}<0\,,\quad \tilde{p}_\lambda=\frac{-1}{\pi_{pp}}>0\,. \tag{20'}$$

Also, note from (19) that in the steady state the advertising expenditure is at the replacement level, and we assume, quite reasonably, that $\bar{a}$ always exceeds and $\underline{a}$ (which could actually be zero) is always below the replacement level. Thus in the steady state, $\sigma=1$. The dynamics of the stock of goodwill can be shown in the $s-\sigma$ space in Figure 4. Note that the stock of goodwill is bounded between $\underline{a}/\rho$ and $\bar{a}/\rho$.

The dynamics of this system are then described by the following four differential equations in four variables: $s(t)$, $n(t)$, $\lambda(t)$ and $\sigma(t)$.

$$\dot{s}=\theta(\sigma-1)\,,\quad \theta(0)=0\,,\quad \theta(+)>0\,,\quad \theta(-)<0 \tag{21'}$$

$$\dot{n}=k[\tilde{p}(s,n,\lambda)-\bar{p}(n,s)] \tag{12'}$$

$$\dot{\lambda}=(r+k\alpha_n)\lambda+k[\tilde{p}(s,n,\lambda)-c] \tag{13'}$$

$$\dot{\sigma}=(r+\rho)\sigma+\alpha_s\lambda-\pi_s[\tilde{p}(s,n,\lambda),s]\,. \tag{22'}$$

A full dynamic characterization of this system does not seem to be feasible. The steady state implications are however straightforward to obtain. Since $\sigma=1$ in the steady state, (12′), (13′) and (22′) can be set to zero and these three equations can be regarded as determining three variables: $\hat{s}$, $\hat{n}$ and $\hat{\lambda}$. The effects of an increase in the entry coefficient are obtained by totally differentiating these three equations with respect to k. Doing so we obtain

$$\begin{vmatrix} \tilde{p}_s-\alpha_s & \tilde{p}_n-\alpha_n & -\tilde{p}_n \\ k\tilde{p}_s & k\tilde{p}_n & r+k\alpha_n-k\tilde{p}_n \\ -(\pi_{ss}+\tilde{p}_s\pi_{ps}) & -\tilde{p}_n\pi_{ps} & \alpha_s+\tilde{p}_n\pi_{ps} \end{vmatrix} \begin{vmatrix} d\hat{s}/dk \\ d\hat{n}/dk \\ d\hat{\lambda}/dk \end{vmatrix} = \begin{vmatrix} 0 \\ r\hat{\lambda}/k \\ 0 \end{vmatrix} \tag{23}$$

Solving for $d\hat{s}/dk$ and $d\hat{n}/dk$, and using (20′), we get

$$\frac{d\hat{s}}{dk}=\left(\frac{-r\hat{\lambda}}{kM\pi_{pp}}\right)[\alpha_s-\alpha_n(\alpha_s\pi_{pp}+\pi_{ps})] \tag{24}$$

$$\frac{d\hat{n}}{dk}=\left(\frac{-r\hat{\lambda}}{kM\pi_{pp}}\right)[\alpha_s^2\pi_{pp}+\pi_{ss}+2\alpha_s\pi_{ps}]\,, \tag{25}$$

where M is the determinant of (23). The saddle point property of the steady state implies that M is negative.[8] Since in the steady state $\hat{p}=\bar{p}(n,s)$,

$$\frac{d\hat{p}}{dk}=\alpha_n\frac{d\hat{n}}{dk}+\alpha_s\frac{d\hat{s}}{dk}=\left(\frac{-r\hat{\lambda}}{kM\pi_{pp}}\right)[\alpha_s^2+\alpha_n(\pi_{ss}+\alpha_s\pi_{ps})]\,. \tag{26}$$

Note that the expression in the square brackets of (25) has been proved to be negative in Section 4. Thus, as in our original analysis, an increase in the entry coefficient leads to a reduction in the size of the competitive fringe in the long run. Also comparison of (24) and (26) to the corresponding expressions for $d\hat{s}/dk$ and $d\hat{p}/dk$ in Section 4 shows that these impacts here are qualitatively exactly the same as in our original model.

This goodwill model can be extended to the case of market growth and it can be checked that our previous results of market growth are also unchanged.

VIII. CONCLUDING REMARKS

Even though our findings have been shown to be insensitive to some important

[8] The Jacobian of the system (21′), (12′), (13′) and (22′) equals $-kM$. The steady state being a saddle point implies that this Jacobian is positive, which implies that M is negative.

generalizations, it is not possible to consider in one piece all relevant generalizations one could think of. One may note the following limitations of our analysis that we are able to notice immediately.

First, one may note that our treatment of advertising as affecting market demand is essentially static. Some recent literature on advertising (e.g. Kotowitz and Mathewson (1979)) has focused on the spread of information by means of advertising about the product among the potential consumers—which is itself a dynamic henomenon. This together with the dynamics of entry as in this paper posits a worthwhile problem to be investigated.

Second, we may point out that apart from pricing and advertising strategies, the choice of capacity by the existing firms has been recognized as another variable to deter entry (see, for example, Pashingin (1968), Weders (1971), Spence (1977) and Dixit (1980)). The basic idea is that the existing firms may decide to maintain a large capacity but may not use it all, which will act as a credible threat to potential entrants that if they decide to enter, the former can expand the output to the capacity level and hence will be able to squeeze the market share of the entrants such that entry is unprofitable.[9] It will be useful to study how price/output and capacity choices can interact with the choice of advertising—which has hitherto not been attempted.

Finally, it may be realized that while in reality competition exists among the existing firms, among the entrants and between the two groups, we have emphasized the last. An integration of all the three forms of competition will be a hard but welcome task.

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[9] As Dixit has pointed out, this does not mean that it is profitable for the existing firm to *actually prevent* entry.

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