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URBAN UNEMPLOYMENT, ECONOMIES OF SCALE AND THE THEORY OF CUSTOMS UNIONS

Hamid BELADI*

Abstract. The purpose of this paper is to re-examine the welfare implications of customs unions in a mobile capital version of the H-T model in the presence of variable returns to scale. In this context, among other things, we show that trade creation II is welfare-improving if the manufacturing sector is characterized by decreasing or constant returns to scale. However, the welfare effect of trade creation I is ambiguous if the urban sector is characterized by increasing returns to scale.

I. INTRODUCTION

In recent years there has been a growing literature on the welfare implications of customs unions. Batra (1973), using the standard two-commodity, two-factor model of trade, showed that while welfare effect of trade diversion is ambiguous, trade creation is normally welfare improving.¹ Using a trade model incorporating diminishing returns to scale and unemployment caused by wage rigidity, Yu (1982) has shown that while trade creation may lower welfare, trade diversion could lead to an improvement in welfare. In a recent study, Choi and Yu (1984) obtained similar results for an economy in the presence of variable returns to scale.² The purpose of this paper is to examine the welfare implications of customs unions in a Harris-Todaro type of economy characterized by variable returns to scale. It has been correctly observed that the Harris-Todaro model (1970) is superior to the orthodox model of intersectoral wage differentials in describing reality in many developing countries which are suffering from inter-industry wage differentials as well as from large scale urban unemployment.³

The layout of the paper is as follows: In section II we present the model and its assumptions. In section III we analyze the traditional theory of customs unions in the presence of economies of scale for the Harris-Todaro type of economy with capital mobility. Finally, the conclusions are set out in section IV.

* I would like to thank an anonymous referee for helpful comments to a previous draft of this paper.

¹ Batra's results confirm the propositions obtained by others, such as Viner (1950), Lipsey (1957) and Gehrels (1956).

² For an illuminating discussion of variable returns to scale, see Panagariya (1980, 1981).

³ On the point, see Beladi and Naqvi (1988).

II. ASSUMPTIONS AND THE MODEL

The main features of the model may be described as follows: there are two products, agricultural output (X_a) and manufacturing output (X_m), produced respectively in rural and urban sectors of the economy, each utilizing two factors of production, capital (K) and labor (L). Capital is fully utilized, but labor is fully employed only in the rural sector, where the real wage rate (W_a) is flexible. In the urban sector, where the real wage rate (W_m) is rigid, there is unemployment. Perfect product markets and fixed factor supplies are also assumed.

Following Panagariya and Succi (1986), the production function of a typical single firm in manufacturing sector is given by,

$$\begin{aligned} X_m^i &= g(X_m)F_m(K_m^i, L_m^i) \\ &= g(X_m)L_m f_m(k_m^i) \end{aligned} \quad (1)$$

where $k_m^i = (K_m^i/L_m^i)$ is the capital/labor ratio for the i -th firm. The function, F_m , is assumed to be linearly homogeneous, $g(X_m)$ defined on $[0, \infty]$ assumed to be an increasing function of industry output, representing scale economies. The output elasticity of returns to scale (ε), defined on $(-\infty, 1)$ may be expressed as

$$\varepsilon = (dg/dX_m)(X_m/g) . \quad (2)$$

Where $\varepsilon > 0$ represents increasing returns to scale (IRS), $\varepsilon = 0$ indicates constant returns to scale (CRS) and $\varepsilon < 0$, stands for decreasing returns to scale (DRS).

Aggregating over all identical firms, the total output of the manufacturing sector may be written as,

$$\begin{aligned} X_m &= g(X_m)F_m(K_m, L_m) \\ &= g(X_m)L_m f_m(k_m) \end{aligned} \quad (3)$$

where K_m and L_m denote total quantity of capital and labor employed in the manufacturing sector respectively and k_m denotes the aggregate capital/labor ratio in the urban sector.

Since firms are assumed to be price takers, they equate the private value of marginal product of each factor to its price. Since all firms are identical, we obtain,

$$P_m g(X_m) (\partial F_m / \partial L_m^i) = P_m g(X_m) (\partial F_m / \partial L_m) = W_m \quad (4)$$

$$P_m g(X_m) (\partial F_m / \partial K_m^i) = P_m g(X_m) (\partial F_m / \partial K_m) = r_m \quad (5)$$

where P_m denotes the price of manufacturing product and W_m and r_m are respectively the real wage rate and rental of capital in the manufacturing sector.

Since, the other sector's (Agricultural sector) production function exhibits constant returns to scale, we have

$$X_a = F_a(K_a, L_a) \quad (6)$$

where X_a , K_a , L_a are output, capital and labor respectively in the agricultural

sector. Cost minimization by firms, again, implies

$$P_a(\partial F_a/\partial L_a) = W_a \quad (7)$$

$$P_a(\partial F_a/\partial K_a) = r_a \quad (8)$$

where P_a , W_a and r_a denote commodity price, wage rate and rental of capital in the agricultural sector.

Since capital is completely mobile between these two sectors, the consequent equilibrium is guaranteed, hence we have,

$$r_m = r_a \quad (9)$$

With regards to the labor market, following Harris and Todaro, in the labor market equilibrium, W_a equals the expected wage rate in manufacturing, which equals W_m times the probability of employment in manufacturing sector. Let us denote λ as the ratio of unemployed to the employed labor in the urban sector. Then, the labor force in the urban sector equals $L_m(1 + \lambda)$ and the probability of employment is $[1/(1 + \lambda)]$. So that, the expected wage rate is $W_m/(1 + \lambda)$. Hence, for the labor market equilibrium, we have

$$(1 + \lambda)W_a = W_m \quad (10)$$

Finally, denoting the total endowments of capital and labor by \bar{K} and \bar{L} , respectively, we can write

$$L_a k_a + L_m k_m = \bar{K} \quad (11)$$

$$L_m(1 + \lambda) + L_a = \bar{L} \quad (12)$$

It is fairly clear that (11) implies full employment of capital, whereas (12) allows for the existence of urban unemployment.

With this last equation the production side of our model is complete. Given that \bar{K} , \bar{L} , P_a , P_m and W_m are exogenous, they can be solved for ten variables X_m , X_a , L_m , L_a , K_m , K_a , λ , W_a , r_m and r_a .

The demand side of the model is represented by strictly quasi-concave social utility function (U) which is dependent upon the consumption demand for the two commodities (D_a and D_m). Hence

$$U = U(D_a, D_m) \quad (13)$$

where $U_i > 0$, and $U_{ii} < 0$, $i = a, m$ (U_i represents the first derivative of the utility function).

Economy's budget constraint stipulates that the value of production is matched by the value of consumption:

$$X_a + P X_m = D_a + P D_m \quad (14)$$

where P is the relative price of the manufacturing commodity in terms of the agricultural good (i.e. $p = P_m/P_a$). It is assumed that the home country exports the

agricultural good and imports the manufacturing good, we consequently have

$$D_a = X_a - E_a \quad (15)$$

$$D_m = X_m + E_m \quad (16)$$

where E_a and E_m represent the exports of an agricultural good, and imports of a manufacturing good, respectively. It should be noted that in this model we assume that the system is stable and with given terms of trade, a rise in the production of any commodity causes a rise in demand for any factors of production.

Before analyzing the welfare implications of custom unions in our model, we first derive the transformation curve, exhibiting the relationship between X_a and X_m . The relationship can be written as,

$$X_m = X_m(X_a). \quad (17)$$

From the budget constraint relation (14) we get ($Y = X_a + PX_m$). Note that P is exogenously determined, hence,

$$dY = dX_a + PdX_m$$

which can be written as

$$F_{K_a}dK_a + F_{L_a}dL_a + [Pg/(1-\varepsilon)](F_{K_m}dK_m + F_{L_m}dL_m). \quad (18)$$

Using the factor supply and factor market equilibrium conditions (9)–(12) we obtain,

$$dY = [1 - 1/(1-\varepsilon)]dX_a - [Pg/(1+\lambda)(1-\varepsilon)]F_{L_m}L_m d\lambda. \quad (19)$$

This implies that,

$$\begin{aligned} dX_a/dX_m &= -P[(1-\varepsilon) + \{g/(1+\lambda)\}F_{L_m}L_m d\lambda/dX_m] \\ &= -Pb \end{aligned} \quad (20)$$

where $b = [(1-\varepsilon) + (g/(1+\lambda))F_{L_m}L_m d\lambda/dX_m]$. Thus, the slope of the transformation curve depends on the sign of $(d\lambda/dX_m)$. In the appendix we show that $(d\lambda/dP) > 0$ and given that the price-output response is positive, $(dX_m/dP) > 0$.⁴ Therefore,

$$(d\lambda/dX_m) = [(d\lambda/dP)/(dX_m/dP)].$$

Following Panagariya (1980), $(1-\varepsilon)$ is always positive so that $b > 0$. Hence it implies that the slope of transformation curve is negative. It is also clear that in the absence of urban unemployment and variable returns to scale (i.e., $\varepsilon = 0$, $g = 1$, $\lambda = 0$), we obtain the traditional result, $(dX_a/dX_m) = -P$.

⁴ It is fairly easy to differentiate (3), (11) and (12) totally and using (A.6)–(A.8) in the appendix to show that $(dX_m/dP) > 0$. In the interest of brevity, however, we leave this task to the reader.

III. THE ANALYSIS

Following Batra (1973), Yu (1981), and Choi and Yu (1984), we assume that the world consists of three countries, the home country H and two potential trading partners, C and F. All three countries produce two commodities, X_a (agricultural product) and X_m (manufacturing product). Countries C and F are similar but different from H and do not trade with each other whereas H is a small country, i.e., a price taker and when it engages in trade, it will export X_a to C and F but will only import X_m from C or F but not both. Initially, H is under autarky due to a prohibitive tariff levied against both C and F. Furthermore, country F is the least cost producer of X_m .

To examine the welfare implications of a customs union in the presence of urban unemployment and economies of scale, we assume that the home country, H exports the agricultural good (X_a) and imports the manufacturing product (X_m). Differentiating the social utility function (13), we get

$$dU = U_a dD_a + U_m dD_m. \quad (21)$$

This can be rewritten as

$$(dU/U_a) = dD_a + (U_m/U_a) dD_m. \quad (22)$$

To maximize social utility, consumers equate the marginal rate of substitution to the relative prices of the two commodities (i.e. $U_m/U_a = P_m/P_a$). Therefore, we obtain

$$(dU/U_a) = dD_a + P dD_m. \quad (23)$$

From the budget constraint (14) (with exogenously determined world price P^*) we get

$$dX_a + P^* dX_m = dD_a + P^* dD_m. \quad (24)$$

Finally, a tariff in the case of a small country changes the domestic price ratio facing both producers and consumers. Hence we have a link between the domestic price ratio and the foreign price ratio as

$$P = P^*(1 + t). \quad (25)$$

Using (15), (16), (20) and (25), we obtain

$$dU/U_a = \varepsilon P dX_m + [Pg/(1 + \lambda)] W_a L_m d\lambda + P^* t dE_m - E_m dP^*. \quad (26)$$

Given the fact that import is a function of tariff and the terms of trade, $E_m = E_m(t, P^*)$ and $dE_m = (\partial E_m / \partial P^*) dP^* + (\partial E_m / \partial t) dt$. Substituting for dE_m in (26), we have

$$\begin{aligned} dU/U_a = & \varepsilon P dX_m + [Pg/(1 + \lambda)] W_a L_m d\lambda \\ & + [P^* t (\partial E_m / \partial P^*) - E_m] [dP^* + P^* t (\partial E_m / \partial t) dt]. \end{aligned} \quad (27)$$

In (27) the first term indicates the welfare effect of economies of scale and the second term shows the effect on the urban unemployment rate,⁵ where as the third (fourth) term indicate respectively the effect of an exogenously changed tariff (terms of trade) on welfare.

Partially differentiating $P = P^*(1+t)$, we obtain $(\partial P/\partial t) = P^*$ and $\partial P/\partial P^* = (1+t)$ and since X_m depends on P^* and t , we have $dX_m = (\partial X_m/\partial t)dt + (\partial X_m/\partial P^*)dP^*$. Substituting these into (27) we obtain,

$$\begin{aligned} dU/U_a = & [\varepsilon P P^* (\partial X_m/\partial P) + P^{*2} t (\partial E_m/\partial P) - (Pg/1 + \lambda) W_a L_m (d\lambda/dP) P^*] dt \\ & + (1+t) [\varepsilon P (\partial X_m/\partial P) + P^* t (\partial E_m/\partial P) - (E_m/1 + t) \\ & - (Pg/1 + \lambda) W_a L_m (d\lambda/dP)] dP^* . \end{aligned} \quad (28)$$

In (28), given the positive price-output response, $(dX_m/dP) > 0$ and $(\partial E_m/\partial P) < 0$ held in the absence of any inferior goods in social consumption. Furthermore, in (28), $\varepsilon P P^* (\partial X_m/\partial P)$, shows the production effect of a change in the tariff rate in the presence of returns to scale, $P^{*2} t (\partial E_m/\partial P)$, indicates the production and consumption effect, $(Pg/1 + \lambda) W_a L_m (d\lambda/dP)$, captures the effect on urban unemployment rate via returns to scale, $\varepsilon P (\partial X_m/\partial P)$, shows the production effect of a change in the terms of trade in the presence of returns to scale, $P^* t (\partial E_m/\partial P)$, indicates the terms of trade effect on production and consumption and finally $(E_m/1 + t)$, represents the terms of trade effect in terms of changes in the value of import.

Equation (28) is the main expression for analyzing the welfare effects of trade creation and trade diversions. Let us assume that the home country H is under autarky initially and the importable industry is protected by prohibitive tariffs at the rate t (where t is the smallest value that can prohibit trade). As in the standard customs unions theory, we define trade creation as the home country's switch of its consumption of the importable commodity from a higher cost producer to a lower cost producer and trade diversion as that from a lower cost producer to a higher cost producer. Choi and Yu (1984) has refined the traditional analysis of customs unions theory by differentiating two types of trade creation and trade diversion according to the manner in which trade has been created or diverted. Trade creation I refers to H's switch of its consumption of X_m from H's producers (higher-cost source) to F's producers (least-cost source); trade diversion I refers to H's switch of its consumption X_m from F's producers to C's producers by discriminatorily abolishing tariffs on C only. Trade Creation II is identified with H's switch of its consumption of X_m from C's producers to F's producers and finally trade diversion II refers to H's switch of its consumption of X_m from F's producers to C's producers by levying tariffs only against F.

⁵ The rate of urban unemployment (θ) may be defined as:

$$\theta = [L_u/(L_m + L_u)] = [\lambda/(1 + \lambda)]$$

where as already stated in the main text $\lambda = (L_u/L_m)$, is the ratio of unemployed to urban employed and $(d\theta/d\lambda) = [1/(1 + \lambda)^2] > 0$. Therefore, as λ falls, θ also falls.

First let us consider trade creation I where home country (H) switches its consumption of X_m from domestic producers to F's producers by reducing its tariffs against both C and F such that home country engages in trade with F only. As a result, H's domestic price ratio decreases but the foreign price ratio facing H which is given by F will remain the same, hence $dP^* = 0$. Furthermore, $dt < 0$ due to the reduction in H's tariff. Hence under trade creation I equation (28) reduces to

$$\begin{aligned} dU/U_a = & [\varepsilon P P^* (X_m/2P) + P^* t (\partial E_m / \partial P) \\ & - (Pg/(1 + \lambda) W_a L_m (d\lambda/dP) P^*] dt . \end{aligned} \quad (29)$$

In the appendix we show that $(d\lambda/dP) > 0$. It is clear that (29) reduces to $[P^* t (\partial E_m / \partial P) - (P/(1 + \lambda) W_a L_m (d\lambda/dP) P^*] dt$ if $\varepsilon = 0$ (constant returns to scale) which is necessarily positive and accords well with the standard result obtained by, for example, Batra (1973) and Choi and Yu (1984). The employment effect of trade creation I is given by, $[-Pg/(1 + \lambda) W_a L_m (d\lambda/dP) P^*] dt$, which is positive. Therefore in the HT type of economy characterized by variable returns to scale, trade creation I causes a fall in the urban unemployment rate. Note that (dU/U_a) is positive even when, $\varepsilon < 0$. The following proposition is now immediate.

PROPOSITION I. *The trade creation I is always welfare-improving in the presence of urban unemployment if the manufacturing sector is characterized by either constant returns to scale or decreasing returns to scale given the positive price-output response. However, trade creation I causes a rise in the urban unemployment rate.*

Moreover, if the manufacturing sector is characterized by increasing returns to scale, ($\varepsilon > 0$), then the welfare effect of trade creation I is ambiguous. Hence, we have,

PROPOSITION II. *In the Harris Todaro type of economy, the welfare effect of trade creation I is ambiguous if the manufacturing sector is characterized by increasing returns to scale given the positive price-output response. Furthermore the urban employment rate will decrease.*

Under trade diversion I, home country remove its tariff against C, hence $dt < 0$ and H trades with C in C's terms of trade and therefore $dP^* > 0$. The welfare effect of trade diversion I is given by (28). Let us assume that $\varepsilon = 0$, then we have,

$$\begin{aligned} d/U_a = & [P^{*2} t (\partial E_m / \partial P) - (P/(1 + \lambda) W_a L_m (d\lambda/dP) P^*] dt \\ & + (1 + t) [P^* t (\partial E_m / \partial P) - (E_m / (1 + t) - (P/(1 + \lambda) W_a L_m (d\lambda/dP))] dp^* \\ = & (dU/dt) |_{dP^*=0} dt + (dU/dP^*) |_{dt=0} dP^* . \end{aligned} \quad (30)$$

Hence, in this case, $(dU/dt) |_{dP^*=0} dt \geq 0$ and $(dU/dP^*) |_{dt=0} dP^* \geq 0$, depending on the relative strength of opposite forces. However, (28) shows that the traditional results of trade diversion I (namely the welfare-improving effect of a reduction in tariff and the welfare-reducing effect of terms-of-trade deterioration) as pinpointed

by Lipsey (1957) also hold in our model *if the manufacturing sector is characterized by decreasing returns to scale* ($\varepsilon < 0$). It should be emphasized here than in the HT model, a tariff reduction, in addition to the usual production and consumption gain, causes a further welfare gain by decreasing the rate of urban unemployment. On the other hand, if the elasticity of returns to scale of the manufacturing sector is greater than unity (increasing returns to scale), the welfare effect of trade diversion I is ambiguous. It is noteworthy that under trade diversion I, the urban unemployment effect of a change in terms of trade (i.e., $[-Pg/(1+\lambda)W_aL_m(d\lambda/dP)]dP^*$) is negative and the unemployment effect of a change in the tariff rate (i.e., $[-Pg/(1+\lambda)W_aL_m(d\lambda/dP)P^*]dt$) is positive. Thus we have,

PROPOSITION III. *In the Harris-Todaro type of economy, the welfare as well as urban employment effect of trade diversion I is ambiguous if the manufacturing sector is characterized by increasing returns to scale given the positive price-output response.*

Let us now consider the trade creation II, where H removes its tariffs against F such that H engages in trade with F only. As a result H's domestic price ratio, P decreases to F's terms of trade. So that, H experiences an exogenous improvement in its terms of trade. Hence, $dP^* < 0$ and $dt = 0$, because home country has already engaged in free trade with C under trade diversion I. Therefore, the welfare effect of trade creation II is given by,

$$\begin{aligned} dU/U_a = & (1+t)[\varepsilon P(\partial X_m/\partial P) + P^*t(\partial E_m/\partial P) \\ & - (E_m/1+t) - (Pg/1+\lambda)W_aL_m(d\lambda/dP)]dP^* . \end{aligned} \quad (31)$$

Which is positive if, $\varepsilon \leq 0$. Hence trade creation II improves the social welfare and this happens even when the manufacturing sector is exhibiting decreasing returns to scale. The unemployment effect of trade creation II via a change in the terms of trade is given by $(-Pg/1+\lambda)W_aL_m(d\lambda/dP)dP^*$, which is unambiguously positive. Thus, we have,

PROPOSITION IV. *In the HT type of economy, trade creation II is welfare-improving if the manufacturing sector is characterized by decreasing or constant returns to scale given the positive price-output response. Moreover, trade creation II causes an unambiguous fall in urban employment rate.*

However, if elasticity of returns to scale in manufacturing is positive ($\varepsilon > 0$), then the welfare effect of trade creation II is ambiguous. The following proposition is now in order,

PROPOSITION V. *In the presence of urban unemployment and positive price-output response, the welfare effect of trade creation II is ambiguous if the manufacturing sector is characterized by increasing return to scale. Furthermore, urban unemployment rate will rise.*

Finally, consider the welfare effect of trade diversion II under which H imposes a discriminatory tariff against imports from F. As a result, H will trade only with C at C's terms of trade and since there is no change in the tariff rate imposed against imports from C, $dt=0$, but $dP^* > 0$ because of H's switch of its consumption of the importable good from F to C. The welfare effect of trade diversion is then given by,

$$dU/U_a = (1+t)[\varepsilon P(\partial X_m/\partial P) + P^*t(\partial E_m/\partial P) - (E_m/1+t) - (Pg/1+\lambda)W_a L_m(d\lambda/dP)]dP^* . \quad (32)$$

Which is negative if $\varepsilon < 0$. However, the unemployment effect is negative, implying a decrease in urban unemployment rate. The following proposition is now immediate,

PROPOSITION VI. *In the HT type of economy, trade diversion II is welfare-reducing if the manufacturing sector is characterized by constant or decreasing returns to scale given the positive price-output response. However, urban unemployment rate falls.*

On the other hand, if $\varepsilon > 0$, the welfare effect of trade diversion II is ambiguous. Hence, we have,

PROPOSITION VII. *In the HT type of economy, the welfare effect of trade diversion II is ambiguous if the manufacturing sector is characterized by increasing returns to scale given the positive price-output response. However, the urban unemployment rate unambiguously falls.*

V. CONCLUDING REMARKS

In this paper we have explored the welfare implications of the theory of customs unions for the Harris-Todaro type of economy in the presence of variable returns to scale. In this context, we have derived, among other things, the following results.

1. In the Harris-Todaro type of economy, the welfare effect of trade creation I is ambiguous if the manufacturing sector is characterized by increasing returns to scale. However, if the manufacturing sector is exhibiting constant or decreasing returns to scale, trade creation I is welfare-improving. But the urban unemployment rate will increase.
2. The welfare as well as urban unemployment effect of trade diversion I is ambiguous if the manufacturing sector is characterized by increasing returns to scale. However, the traditional results hold if elasticity of returns to scale is negative. Furthermore, under trade diversion I the urban unemployment effect of a change in terms of trade is negative while that of a change in the tariff rate is positive.
3. In the HT type of economy, trade creation II is welfare-improving if the

manufacturing sector is characterized by decreasing or constant returns to scale and is ambiguous if the elasticity of returns to scale is positive. However, trade creation II causes an unambiguous rise in the urban rate of unemployment.

4. In the HT type of economy, the welfare effect of trade diversion II is ambiguous if the manufacturing sector is characterized by increasing returns to scale and is welfare-reducing if the elasticity of returns to scale is zero or negative. But trade diversion II causes an unambiguous fall in the urban unemployment rate.

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APPENDIX

We will now derive the mathematical expressions supporting the arguments presented in the main text. The basic three equations used here are,

$$f'_a = Pgf'_m \quad (\text{A.1})$$

$$W_m = Pg(f_m - k_m f'_m) \quad (\text{A.2})$$

$$(1 + \lambda)(f_a - k_a f'_a) = Pg(f_m - k_m f'_m) \quad (\text{A.3})$$

These are obtained from (4), (5), (9) and (10). This system contains three equations in three variables, k_a , k_m and λ . Total differentiation of (A.1)–(A.3) yields the following matrix system,

$$\begin{bmatrix} f''_a & -Pgf''_m & 0 \\ 0 & Pfk_m f''_m & 0 \\ -(1 + \lambda)k_a f''_a & Pfk_m f''_m & F_{L_a} \end{bmatrix} \begin{bmatrix} dk_a \\ dk_m \\ d\lambda \end{bmatrix} = \begin{bmatrix} g(f'_m dP + Pf'_m dg/g) \\ M_L dP + PM_L dg/g \\ M_L dP + PM_L dg/g \end{bmatrix} \quad (\text{A.4})$$

The determinant of this system, D , is given by:

$$D = Pfk_m f''_m A_L f''_a = Pfk_m W_a f''_m f''_a > 0 \quad (\text{A.5})$$

Solving the system in (A.4) yields the following equations:

$$dk_a = (1/D)W_a [Pfk_m f''_m (gf'_m dP + Pf'_m dg) + Pgf''_m (M_L dP + PM_L dg/g)] \quad (\text{A.6})$$

$$dk_m = (1/D)W_a f''_a [M_L dP + PM_L dg/g] \quad (\text{A.7})$$

$$d\lambda = (1/D)Pgf''_a f''_m [-(1 + \lambda)k_a M_L dP - k_m gf'_m dP - (1 + \lambda)k_a PM_L (dg/g) - (1 + \lambda)k Pf'_m dg] \quad (\text{A.8})$$

from (A.8) we obtain,

$$d\lambda dP = (1/D)f''_a f''_m P [k_a (1 + \lambda)M_L + k_m gf'_m] \quad (\text{A.9})$$

Since $D > 0$, $f''_a < 0$ and $f''_m < 0$. Therefore $d\lambda/dP > 0$.

Q.E.D.

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