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BARGAINING WITH DIFFERENTIAL SKILLS

Michihiro OHYAMA*

Abstract. The Nash solution of two-person bargaining game is based on the axiom that all players have equal bargaining power. In this paper, it is extended to allow for possible differentials in individual bargaining skills. Assuming that the frontier of payoffs between players is convex, the solution is shown to obtain at the point where a Cobb-Douglas function defined over payoff space takes on the highest value along the frontier.

As is well known, Nash (1950) gives a definite solution for the bargaining problem which involves two rational individuals with equal bargaining skill. In actual bargaining situations of monopoly versus monopsony, of employer versus labor union, and of one state versus another, however, differentials in bargaining skill between the two parties involved often lead to results which apparently deviate from the solution given by Nash.[1] In this note, we present an extension of Nash's model to allow for differentials in bargaining skill.

For the purpose of reference, let us first describe Nash's model briefly. There are two individuals, 1 and 2, with von Neuman-Morgenstern utility function defined over their joint strategy set. Let u_i denote the payoff (i.e., utility) of individual i, U the attainable set of (u_1, u_2) and $\bar{u}_i$ the level of individual i's guaranteed payoff to be obtained without negotiation (i.e., by his threat strategy). Their negotiation set defined by

$$N=\{(u_1, u_2)\in U: u_1 \geqq \bar{u}_1, u_2 \geqq \bar{u}_2\}$$

is assumed to be compact and convex. Nash's bargaining solution is the unique pair of utilities, $u_1{}^*$, $u_2{}^*$, depending on N. It may be expressed as the functional relationship, $(u_1{}^*, u_2{}^*)=f(N)$. Nash introduces the following axioms to restrict this relation.

A1. (Individual rationality). Individual i does not agree to an outcome giving him a lower payoff than $\bar{u}_i$.

A2. (Pareto optimality). If there exists (u_1, u_2) in N such that $u_1 \geqq u_1{}^*$ and

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[1] For example, McDonald and Solow (1981) applies the Nash solution to contracts between a firm and a union. In so doing, they implicitly assume that the firm and the union have equal bargaining power across the periods of prosperity and recession. More often that not, we encounter similar neglect of bargaining power differentials in other applications of the Nash solution.

$u_2 \geqq u_2^*$, then $(u_1, u_2)=(u_1^*, u_2^*)$.

A3. (Independence from irrelevant outcomes). Let N_A and N_B be negotiation sets such that $N_A \supset N_b$ with the same threat point $(\bar{u}_1, \bar{u}_2)$. If $(u_1^*, u_2^*)=f(N_A)$ and $(u_1^*, u_2^*) \in N_B$, then $(u_1^*, u_2^*)=f(N_B)$.

A4. (Invariance with linear transformation). Let N_A and N_B be two negotiation sets such that N_A is related to N_B by a positive linear transformation of u_1 and another of u_2, then the solutions to the two games are related by the same pair of transformations.

A5. (Equal bargaining skill). If the negotiation set is symmetric around a 45° line through the threat point $(\bar{u}_1, \bar{u}_2)$, then the solution lies on that line.

Nash proves that a two-person bargaining problem with negotiation set N and threat point $(\bar{u}_1, \bar{u}_2)$ has a unique solution (u_1^*, u_2^*) satisfying A1–A5 and

$$(u_1^* - \bar{u}_2)(u_2^* - \bar{u}_2) = \max_{u \in N}(u_1 - \bar{u}_1)(u_2 - \bar{u}_2)\,. \tag{1}$$

This solution is plausible for a bargaining problem between two individuals with equal bargaining skill since it gives each one of them an identical payoff if the set of possible outcomes is completely symmetric between them. Obviously, we must modify A5 appropriately to allow for differential bargaining skills. This is our task in what follows.

Let us suppose that the outer boundary of the negotiation set N, or the payoff frontier for short, is the set of (u_1, u_2) satisfying

$$g(u_1 - \bar{u}_1, u_2 - \bar{u}_2) = 0 \tag{2}$$

where $g(x_1, x_2)$ is a convex function. We say that the payoff frontier is symmetric around a 45° line through the threat point if $g(x_1, x_2)=0$ implies $g(x_2, x_1)=0$. Now we replace A5 by

A5′. (Bargaining skill differential). If the payoff frontier is symmetric around a 45° line through the threat point, then the elasticities of individual 1's payoff with respect to individual 2's payoff along the frontier evaluated at the solution point satisfy

$$\frac{u_1^* - \bar{u}_1}{u_2^* - \bar{u}_2} \cdot \frac{g_1{}^L}{g_2{}^L} \leqq \alpha \leqq \frac{u_1^* - \bar{u}_1}{u_2^* - \bar{u}_2} \cdot \frac{g_1{}^R}{g_2{}^R} \tag{3}$$

where α is a given number and $g_i{}^L$ (resp. $g_i{}^R$) denotes the left-hand (resp. right-hand) derivative of $g(x_1, x_2)$ with respect to x_i.

Needless to say, A5′ coincides with A5 if $\alpha=1$. We may interpret α as a measure of the bargaining skill of individual 1 relative to individual 2. To see this point, it is useful to consider the special case where $g(x_1, x_2)$ is differentiable. In this special case, (3) reduces to

$$\frac{u_1{}^* - \bar{u}_1}{u_2{}^* - \bar{u}_2} \cdot \frac{g_1}{g_2} = \alpha \tag{3'}$$

with the following implications. First, (3′) means that a 1% decrease in individual 1's payoff would bring about an α% increase in individual 2's payoff along the frontier at the solution point. Thus, the greater the value of α is, the greater is the marginal gain which individual 1 secures for himself at the sacrifice of individual 2. Second, if the solution is of the form (x, x) or $u_1{}^* - \bar{u}_1 = u_2{}^* - \bar{u}_2$, we have $\alpha = 1$ by the symmetry of the payoff frontier. Third, an increase in α leads to an increase in $u_1{}^*$ and a decrease in $u_2{}^*$ in view of the convexity of the function $g(u_1 - \bar{u}_1, u_2 - \bar{u}_2)$ (implied by the convexity of the negotiation set, N). When $g(x_1, x_2)$ is not differentiable, we must replace (3′) by (3) with at least one strict inequality. The modification would affect each of the foregoing implications of (3′) in an obvious way, but it would not nullify the interpretation of α as a measure of bargaining skill.

THEOREM. *A two-person bargaining problem with negotiation set N and threat point $(\bar{u}_1, \bar{u}_2)$ has a unique solution $(u_1{}^*, u_2{}^*)$ satisfying* (A1)–(A4) *and* (A5′) *and*

$$(u_1{}^* - \bar{u}_1)^{\theta}(u_2{}^* - \bar{u}_2)^{1-\theta} = \max_{u \in N}(u_1 - \bar{u}_1)^{\theta}(u_2 - \bar{u}_2)^{1-\theta} \tag{4}$$

where $\theta = \alpha/(1+\alpha)$.[2]

Proof. Let us first show that the point $(u_1{}^*, u_2{}^*)$ which satisfies (4) is unique and that it satisfies (A1)–(A4) and (A5′). The uniqueness of $(u_1{}^*, u_2{}^*)$ satisfying (4) is immediate from the convexity of N. It is also straightforward that it satisfies (A1)–(A4). In view of the first-order conditions for the maximization of $(u_1 - \bar{u}_1)^{\theta}(u_2 - \bar{u}_2)^{1-\theta}$ subject to $g(u_1, u_2) = 0$, we have

$$\frac{u_1{}^* - \bar{u}_1}{u_2{}^* - \bar{u}_2} \cdot \frac{g_1{}^L}{g_2{}^L} \leqq \frac{\theta}{1-\theta} = \alpha \leqq \frac{u_1{}^* - \bar{u}_1}{u_2{}^* - \bar{u}_2} \cdot \frac{g_1{}^R}{g_2{}^R}$$

Thus (A5′) is also satisfied.

It remains to prove that no other point in N satisfy (4). Define

$$h(u_1, u_2) = \theta\left(\frac{u_2{}^* - \bar{u}_2}{u_1{}^* - \bar{u}_1}\right)^{1-\theta}(u_1 - \bar{u}_1) + (1-\theta)\left(\frac{u_1{}^* - \bar{u}_1}{u_2{}^* - \bar{u}_2}\right)^{\theta}(u_2 - \bar{u}_2)$$

By construction, $h(u_1, u_2) \leqq h(u_1{}^*, u_2{}^*)$ for all (u_1, u_2) such that $(u_1, u_2) \in N$. Define

$$H = \{(u_1, u_2) : h(u_1, u_2) \leqq h(u_1{}^*, u_2{}^*)\}$$

[2] Professor Kunio Kawamata brought to my notice the paper by Harsanyi and Selten (1972) which extended Nash's theory of two-person bargaining to situations with each player having incomplete information of the other. Their result may also be interpreted as deriving the solution of bargaing games with differential bargaining powers. Their solution maximizes the generalized Nash product of the payoffs of all possible types of players each raised to the power of its subjective marginal probability.

Let H' be the set of points (u_1', u_2') obtained by the following linear transformation of points in H.

$$u_1' = 2\theta\left(\frac{u_1 - \bar{u}_1}{u_1^* - \bar{u}_1}\right), \qquad u_2' = 2(1-\theta)\left(\frac{u_2 - \bar{u}_2}{u_2^* - \bar{u}_2}\right)$$

Then, $h(u_1, u_2) \leqq h(u_1^*, u_2^*)$ implies $u_1' + u_2' \leqq 2$, $u_i = \bar{u}_i$ implies $u_i' = 0$. The set H' is symmetric around a 45° line through the origin. Thus the solution of the bargaining problem with negotiation set H' which satisfies (A1)–(A4) and (A5′) is clearly $(2\theta, 2(1-\theta))$ or $(2\alpha/(1+\alpha), 2/(1+\alpha))$. By A4 (invariance with linear transformation), (u_1^*, u_2^*) is the solution of the bargaining problem with negotiation set H. By A3 (independence from irrelevant outcomes), (u_1^*, u_2^*) is also the solution of the original problem with negotiation set N, which proves the theorem.

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