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# MULTIPLIER PROCESS AND PRICE FLUCTUATIONS* 

Takashi Nishi

Abstract. In this note we consider a multiplier process with price fluctuations. By so doing we formulate a process of true inflation and semi-inflation in the sense of Keynes [4]. In particular, we take account of investment in working capital, which is an important link between output and the price level as Keynes [3] suggests. We derive the condition for fluctuation and convergence, and show the nature of movements of the variables by numerical examples.

In explaining multiplier process, Keynes [4] concerns himself with the relation between real income and real investment (more exactly income and investment in terms of wage units). Although he does not consider the movement of the price level explicitly, he argues that when full employment is attained, any attempt to increase investment will be associated with rising prices. He calls it "true inflation". On the other hand, he also admits that the price level might increase before full employment is attained. He calls it "semi-inflation". Certainly, the price level has some relations to multiplier process.

In this note we formulate a multiplier process with price fluctuations to consider some (not all) of the facts mentioned above. There are many factors which cause semi-inflation. We consider a time structure of production, which makes supply of output inelastic. In particular, we take account of investment in working capital, which is an important link between output and the price level as Keynes [3] suggests. In fact, it is important if we recall that production needs time. Introducing such lag structures, we have a dynamic model of price movement in stead of a static model of the price level determination. In our model, output shows rather complicated movement than monotonous convergence process of usual type because of the assumed lag structures. Also the price level fluctuates according to it. Here, the nature of the lags are decisive of the movement of the price level.
In Section II, we present the general framework of our model. In Section III, we examine the case where output is constant over time at full employment level. We show a simple multiplier process of the price level with monotonous convergence,

[^0]which can be called true inflation process. In Section IV, We analyse a semiinflation process, where condition of fluctuation and convergence is derived by specifying the forms of the various functions. Section $V$ is devoted to concluding remarks.

## II

In this section we present the general framework of our model.
We denote as $Y$ total money income which consists of wage income, $W$ and profit income, $\Pi$ :

$$
\begin{equation*}
Y_{t}=W_{t}+\Pi_{t} . \tag{1}
\end{equation*}
$$

We use capital letters for representing the nominal variables. The subscript $t$ indicates the time period of the variables.
We introduce Robertson's lag: consumption in period $t, C_{t}$ comes from income in period $t-1, Y_{t-1}$. We also assume that savings from wage income are zero and that consumption from prifit income is a constant fraction $\mu(0 \leqq \mu<1)$ of profit income. Thus, consumption in period $t$ is represented as

$$
\begin{equation*}
C_{t}=W_{t-1}+\mu \Pi_{t-1}, \tag{2}
\end{equation*}
$$

and planned savings in period $t$ as

$$
\begin{equation*}
S_{t}=(1-\mu) \Pi_{t-1} . \tag{3}
\end{equation*}
$$

We denote as $I$ gross invesiment, which is assumed to be exogenously constant throughout this paper. I includes maintenance or replacement of the productive equipment. Notice that in real terms gross investment varies with price fluctuations. The productive capacity may or may not grow in our economy. We consider that production in every period is always carried out under the capacity limit.

Introducing Robertson's lag, planned savings, $S_{\mathrm{t}}$ and gross investment, $I$ are not equal in general.

Effective demand in period $t$ is

$$
\begin{equation*}
E_{t}=C_{t}+I \tag{4}
\end{equation*}
$$

Let $q_{t}$ and $P_{t}$ be physical output and the price level in period $t$, respectively. We assume that the price level is determined so as to equate effective demand and the money value of output in the same period: $P_{t}$ satisfies,

$$
\begin{equation*}
P_{t} q_{t}=E_{t}^{1} \tag{5}
\end{equation*}
$$

In period $t$, effective demand, independent of the volume of $q_{t}$, determines total revenue, which is equal to $Y_{t}$. We have,

[^1]\[

$$
\begin{equation*}
E_{t}=Y_{t} \tag{6}
\end{equation*}
$$

\]

We denote as $X_{t}$ the expenditure for production planned in period $t$, which is assumed to consist exclusively of the wage payment. We also assume that the money wage is constant. So we have $X_{t}=\Omega n_{t}$, where $\Omega$ and $n_{t}$ indicate the money wage rate and the planned employment in period $t$, respectively.

We consider that production requires $\tau$ periods, i.e., $q_{t}$ is a function of $n_{t-\tau}$ :

$$
\begin{align*}
q_{t} & =\tilde{f}\left(n_{t-\tau}\right) \\
& =f\left(X_{t-\tau}\right), \quad f^{\prime}>0 . \tag{7}
\end{align*}
$$

For the convenience of the following analysis, we use the latter expression.
We suppose that the planned expenditure for production is carried out evenly within $\tau$ periods: $1 / \tau$ of $X_{t}$ is executed in every period from $t$ to $t+\tau-1$. Therefore, we have,

$$
\begin{equation*}
W_{t}=(1 / \tau) \sum_{i=0}^{i=\tau-1} X_{t-i}, \tag{8}
\end{equation*}
$$

as wage income in period $t$.
Frisch [1] uses a similar formulation in continuous form in his analysis, ${ }^{2}$ and calls it "carry-on-activity". We represent it in value units, while Frisch represents it in physical units. According to Tinbergen [6], $W_{t}$ corresponds to investment in working capital in the sense of Keynes [3], though our model has some simplifying assumptions.

Finally, we assume the following.

$$
\begin{equation*}
X_{t}=\operatorname{Min} .\left\{\Omega n^{f}, g\left(\Pi_{t-1}\right)\right\}, \quad g^{\prime}>0 \tag{9}
\end{equation*}
$$

where $n^{f}$ indicates full employment level of $n_{t}$. Equation (9) states that until full employment is attained, the planned expenditure for production, $X_{t}$ depends positively on profit income with one period lag, and that once full employment is attained, it is constant at the level of $\Omega n^{f} .^{3}$

We have nine equations (1)-(9) determining nine endogeneous variables: $Y, C$, $S, W, \Pi, E, X, q$, and $P .^{4}$

Attention should be directed not only to the lag structure, but also to the way in which prices are determined, i.e., to equation (5). The expenditure is always decided in money terms, and the price level is determined so as to equate it with the money value of physical output. Demand and supply are equal, in physical terms as well, whenever the price level is so determined. Equation (5), therefore, assures the attainability of our economy in each period without bothering us with the

[^2]question: "What happens in our economy when demand and supply of the physical products are not equal?"

## III

In this section we consider the case where $X_{t}$ is constant over time at full employment level, that is, $\Omega n^{f}$ is binding in equation (9):

$$
X_{t}=\Omega n^{f}=\text { constant } .
$$

We have the following proposition.
[P1] Suppose $I>S_{0}$ in the initial condition, and $X_{t}$ is constant over time. Then profit income, $\Pi_{t}$ is adjusted so that savings, $S_{t}$ is equal to investment, $I$. The pice level, $P_{t}$ continues to rise so long as $I>S_{t}$.

This is shown as follows:
From (1), (2), (4), and (6), we have

$$
\Pi_{t}=W_{t-1}-W_{t}+\mu \Pi_{t-1}+I
$$

Since $X_{t}$ is constant, it follows from (8) that

$$
W_{t-1}-W_{t}=0
$$

and we have

$$
\Pi_{t}=\mu \Pi_{t-1}+I
$$

This is solved for $\Pi_{t}$ with the initial profit income $\Pi_{0}$ as

$$
\Pi_{t}=\mu^{t} \Pi_{0}+\frac{1-\mu^{t}}{1-\mu} I
$$

Since $o \leqq \mu<1$, this converges to

$$
\Pi \infty=\frac{1}{1-\mu} I,
$$

where $(1-\mu) \Pi \infty$ is $S \infty$. Profit income continues to rise from the initial situation: $(1-\mu) \Pi_{0}<I$. So is $E_{t}$ by (4). Since $q_{t}$ is constant over time by (7) and constancy of $X_{t}$, it follows from (5) that the price level $P_{t}$ continues to rise. This establishes the proposition.

This "true inflation process" is also a kind of multiplier process of investment generating its necessary savings (in this case by price rises), reminding us of Keynes' famous allusion to "widow's cruse", or Kalecki's dictum, "capitalists earn what they spent": the greater is $\mu$, the greater is $\Pi \infty$.

By using (1) and (5) $P \infty$ is calculated from $\Pi \infty$ as

$$
\boldsymbol{P}_{\infty}=\frac{W}{q}+\frac{I}{(1-\mu) q} .
$$

Thus, the greater is $\mu$ or $I$, the greater is $P \infty$. Also the higher money wage rate corresponds to the higher $W$, so it raises $P \infty$.

## IV

In this section, we examine the case where economy is always below the full empolyment, i.e., in equation (9) $\Omega n^{f}$ is not binding.

We suppose that the periods of production $\tau$ to be two, and specify the functional forms of (7) and (9) as follows:

$$
\begin{align*}
q_{t} & =\alpha X_{t-2}, \quad \alpha>0,  \tag{7'}\\
W_{t} & =(1 / 2)\left(X_{t}+X_{t-1}\right),
\end{align*}
$$

and

$$
\begin{equation*}
X_{t}=\beta_{1} \Pi_{t-1}+\beta_{0}, \quad \beta_{1}>0 . \tag{9'}
\end{equation*}
$$

The coefficient $\alpha$ is the labour productivity multiplied by reciprocal of the money wage. Equation ( $7^{\prime}$ ) means that with no labour input no output can be obtained. The coefficient $\beta_{1}$ represents the capitalists' marginal propensity for the productive expenditure.

In the first place, we examine the stationary solution of the system described by (1), (2), (3), (4), (5), (6), (7'), (8'), and ( $9^{\prime}$ ). From (1), (2), (4), and (6), we have

$$
\Pi_{t}=W_{t-1}-W_{t}+\mu \Pi_{t-1}+I
$$

Using ( $8^{\prime}$ ) and ( $9^{\prime}$ ) we have

$$
\begin{equation*}
\Pi_{t}=\frac{1}{2} \beta_{1} \Pi_{t-3}-\left(\frac{1}{2} \beta_{1}-\mu\right) \Pi_{t-1}+I \tag{10}
\end{equation*}
$$

This is a difference equation of third order, and if we put $\Pi_{t}=\Pi^{*}$ for all $t$, we have

$$
\begin{equation*}
(1-\mu) \Pi^{*}=I \tag{11}
\end{equation*}
$$

(11) implies that $I=S$ in the stationary state.

Substituting (2), (4), $\left(7^{\prime}\right),\left(8^{\prime}\right)$, and ( $9^{\prime}$ ) into (5), and solving for $P_{t}$, we have

$$
\begin{equation*}
P_{t}=\frac{(1 / 2) \beta_{1} \Pi_{t-2}+(1 / 2) \beta_{1} \Pi_{t-3}+\mu \Pi_{t-1}+\beta_{0}+I}{\alpha \beta_{1} \Pi_{t-3}+\alpha \beta_{0}} \tag{12}
\end{equation*}
$$

Substituting (11) into (12), we have the stationary solution for $P_{t}$ :

$$
\begin{equation*}
P^{*}=\frac{\left(1+\beta_{1}\right) I+(1-\mu) \beta_{0}}{\alpha \beta_{1} I+(1-\mu) \alpha \beta_{0}} . \tag{13}
\end{equation*}
$$

Differentiating (13) with respect to $I$ yields,

$$
\frac{\partial P^{*}}{\partial I}=\frac{(1-\mu) \alpha \beta_{0}}{\left\{\alpha \beta_{1} I+(1-\mu) \alpha \beta_{0}\right\}^{2}} .
$$

If $\beta_{0}>0$, an increase in investment $I$ increases the stationary price level. However, if $\beta_{0}=0$, that is, $X_{t}$ is proportional to $\Pi_{t-1}, P^{*}$ is independent of the level of $I$, and is solely determined by the parameters of the model.

In the same way, from (13) we can derive,

$$
\begin{equation*}
\frac{\partial P^{*}}{\partial \alpha}=\frac{-\left\{\left(1+\beta_{1}\right) I+(1-\mu) \beta_{0}\right\} \beta_{1} I}{\left\{\alpha \beta_{1} I+(1-\mu) \alpha \beta_{0}\right\}^{2}} . \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial P^{*}}{\partial \beta_{1}}=\frac{-\alpha I^{2}}{\left\{\alpha \beta_{1} I+(1-\mu) \alpha \beta_{0}\right\}^{2}}<0 . \tag{15}
\end{equation*}
$$

In (14) $\partial P^{*} / \partial \alpha$ is negative if $\beta_{0}$ is nonnegative. Thus (14) is interpreted as follows. If the labour productivity is increased by technical progress, then the stationary price level of the economy decreases when $\beta_{0}$ is nonnegative. The coefficient $\alpha$ also represents, as mentioned above, reciprocal of the money wage. Therefore, for the case $\beta_{0} \geqq 0$, the higher is the money wage, the higher is the stationary price level.
$\partial P^{*} / \partial \beta_{1}$ is always negative. An increase in the coefficient $\beta_{1}$ affects the price level $P$ in two ways. On the one hand, it increases production in physical terms, tends to lower $P$. On the other hand, it increases effective demand through the wage payment, and tends to raise $P$. (15) states that when the stationary state is restored, the former dominates the latter.

The stationary output is given from $\left(7^{\prime}\right),\left(9^{\prime}\right)$, and (11) as

$$
q^{*}=\alpha \beta_{1} \frac{I}{1-\mu}+\alpha \beta_{0} .
$$

$q^{*}$ is increasing with respect to $I, \alpha, \beta_{1}$, and $\mu$, respectively, as is expected.
Next, let us examine the solution of the difference equation (10), and see how the system moves in time.

For the sake of analytical simplicity, we assume $\mu=0$ in the following. ${ }^{5}$ Then, the characteristic equation of (10) becomes

$$
\begin{equation*}
\lambda^{3}+(1 / 2) \beta_{1} \lambda^{2}-(1 / 2) \beta_{1}=0 . \tag{16}
\end{equation*}
$$

From the discriminant of (16) and applying the Schur condition, we have
$[\mathrm{P} 2]$ The time path of profit income $\Pi_{t}$ fluctuates and converges if and only if marginal propensity for productive expenditure $\beta_{1}$ is less than $\sqrt{2} .^{6}$

In the case where $\Pi_{t}$ fluctuates, we have the time path of $\Pi_{t}$ by solving (10) as

$$
\Pi_{t}=A_{1} \lambda^{t}+A_{2} r^{t}\{\cos \theta t-\varepsilon\},
$$

[^3]where $\lambda$ and $r$ are the real solution and the absolute value of the imaginary solution of (16), respectively, and $A_{1}, A_{2}$, and $\varepsilon$ are given by the initial conditions.
Substituting this solution of $\Pi_{t}$ into (12) we obtain the time path of $P_{t}$. However,


Fig. 1.


Fig. 3.


Fig. 2.


Fig. 4.
since it is too complicated to analyse qualitatively, we rely on numerical examples to see the movement of the price level. We assume that until period 0 , the economy is in the stationary state with $I=100$, and in period 1 , gross investment increases once-and-for-all to $I=120$. In Figure $1 \& 2$, we put $\beta_{1}=1 / 2, \alpha=1 / 4$, and in Figure $3 \& 4, \beta_{1}=4 / 5, \alpha=1 / 4$. We put $\beta_{0}=10$ in both figures.

From both figures, we see the cyclical movement of $P_{t}$. In both cases, the price level and physical output show movements inverse to each other. Intuitively this is due to (5) and the lag structures of our model. The effect of change in $X$ on $q$ appears later than the effect on $E$. Moreover, the effect on $E$ is dipersed over time, while the effect on $q$ is concentrated in one period of time.

## V

We have considered the relation between the IS balance and the price movement by taking into account of investment in working capital. An important meaning of this concept is not only that production needs time, but also that the wage payment, and so effective demand have a time structure according to it. Movements of the price level is affected by both. Equation (5) is representing this fact.

We have analysed the result of the initial increase in gross investment. This increase in investment may be supposed to have happened with or without an increase in money supply. From [P1], if an increase in money supply is all devoted to expenditure, then the increase in money supply brings about a rise in the stationary price level when output is constant over time. This is the weakest version of the so-called quantity theory. From [P2], however, if output is not constant over time, even this weakest version never comes to be true under the proportional specification, i.e., $\beta_{0}=0$, even if the increase in money supply is all devoted to expenditure. That is, an increase in money supply increases output and employment, but the long run price level is never affected by it.

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[^1]:    ${ }^{1}$ This relation can, as the author knows, be originally found in Lindahl [5] formulating Wicksell.

[^2]:    ${ }^{2}$ The same formulation is also used by Kalecki [2]:
    ${ }^{3}$ Interpreting equation (9) we might consider two-class economy, or three-class economy by adding entrepreneurs class. In the latter, however, entrepreneurs are assumed to behave according to interests of capitalists class. For the result of introducing one period lag, see note 6 below.
    ${ }^{4}$ In our economy we might suppose that the quantity of money flow is endogeneously determined by addapting itself to effective demand, i.e., $E$.

[^3]:    ${ }^{5}$ We can show that such a simplification never affects the following results.
    ${ }^{6}$ If we omit one period lag from ( $9^{\prime}$ ), the dynamic equation of $\Pi_{t}$ becomes second order. One can easily ascertain that in such a case the discriminant is always positive, and so $\Pi_{t}$ never fluctuates.

