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MULTILATERAL TFP INDEX AND MULTILATERAL SIMILARITY OF TECHNOLOGY*

Shinichiro NAKAMURA

Abstract. The Toernqvist spatial TFP index presupposes spatial equality of the second order parameters of translog cost/production functions. In this paper I tested equality between Germany, Japan, and the U.S. of the second order parameters of a KLM translog cost function for disaggregated industry sectors. The equality hypothesis was rejected, implying that the Toernqvist TFP index would be biased due to its neglect of the term involving the second order parameters. Still, the extent of biases was found to be quantitatively small. In particular, the biases did not cause any change in the trend pattern of TFP development of the three countries.

1. INTRODUCTION

In recent years, international comparison of the productivity level has been the focus of a growing number of researchers (see Caves, Christensen and Diewert (1982), Conrad and Jorgenson (1985), Jorgenson, Nishimizu and Kuroda (1985), and Nakamura (1989), among others). The current standard is to use Total Factor Productivity (TFP) as a measure of the productivity level instead of a partial productivity measure such as labour productivity. The use of the latter is considered to be inappropriate, because it incorporates both TFP and the effect of factor substitution (see Norsworthy and Malmquist (1983) for the decomposition of labour productivity into these two factors). Because TFP takes into account all factors of production, and thus gives an overall measure of efficiency which is free of substitution effects, it is superior to a partial productivity measure.

While TFP is thus theoretically superior to a partial productivity measure, its empirical implementation is more demanding. In particular, it requires information on the underlying technology in the form of production, cost, or profit functions. Although this kind of information could be obtained by a structural estimation under fairly general conditions based on the flexible functional specification, it is a tedious procedure, especially if one is only interested in the measurement of TFP.

Fortunately, Diewert (1976) has shown that under certain conditions (such as

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cost minimization with given factor prices and linear homogeneous technology) TFP index over time can be estimated without any structural estimation. In particular, he showed that the translog unit cost function is exact to the Toernqvist index, which is a widely used discrete version of the famous Divisia index. This is an important result, because the translog function is the most widely used flexible functional form and the Divisia index has several desirable properties as an index number (see the reference cited in Diewert (1976)). In terms of the TFP measurement this implies that TFP can be measured simply by computing the Toernqvist index.

Caves, Christensen, and Diewert (1982) subsequently extended this useful result of Diewert for the TFP measurement over space, and the resulting spatial Toernqvist TFP index has since been widely used for international productivity comparison. (The empirical papers cited above are all based on this index.) We could thus save the tedious process of structural estimation of translog functions by simply computing the Toernqvist index.

Denny and Fuss (1983) showed, however, that this useful feature is not without any cost. In particular, it requires that the spatial difference in the structure of technology be represented by the zero and first order parameters only of the translog function. The second order parameters are required to be identical over spatial units, implying that the Slutsky matrix is the same up to the terms involving cost shares. If this condition is not satisfied, the Toernqvist index is no longer exact to the translog function and its use would yield biased results.

In spite of the wide use of the Toernqvist TFP index in applied work, however, empirical validity of this critical condition has received surprisingly little attention (Denny and Fuss (1983) is an exception). On the other hand, estimation results of separate translog functions for Japan and the U.S. by Kuroda, Yosioka and Jorgenson (1984) appear to indicate the existence of substantial difference in first as well as second order translog parameters among the three countries. Since they were not concerned with the measurement of the spatial TFP index, they did no testing of spatial equality of the translog parameters.

The purpose of this paper is to test the equality of second order KLM translog parameters among Germany, Japan and the U.S. for nine industry sectors for the period 1960–1979, and to analyze its implications on the Toernqvist TFP index. Although a similar test was done by Denny and Fuss within a bilateral context, this paper is characterized by two novelties. First, it is concerned with spatial equality of technology parameters within a multilateral context involving three countries. Second, I estimate the cost function together with the system of share functions, and thus will be able to get richer implications than the model based on share functions only. (The model of Denny and Fuss is of the latter type.) Because the data set used in this paper has been the subject of several empirical studies dealing with TFP measurement (Jorgenson, Kuroda, and Nishimizu (1985), Nakamura (1989)), our concern below is mainly limited to the testing of spatial equality of technology parameters and its implications on the TFP measurement.

2. MODEL

Our maintained hypothesis is that the structure of sectoral technology for Germany, Japan and the U.S. is given by the following KLM translog (TL) unit cost function:

$$\begin{aligned} \ln c^r &= f^r(z^r, t) \\ &= a^r + B_p^{r\prime} z^r + B_t^r t + 1/2 z^{r\prime} B_{pp}^r z^r + 1/2 z^{r\prime} B_{pt}^r t + B_{tt}^r t \end{aligned} \qquad r = A, B, J \quad (1)$$

where z is the vector of logarithms of prices of capital services, labour services, and material, $'$ refers to the transpose, t is the time trend used as a proxy for the state of technology, and r is the spatial index with A referring to the U.S., J to Japan, and D to Germany. I thus allow the parameters a, B_p, B_t, B_{pp}, B_{pt}, and B_{tt} to be spatially different. The parameters are subject to the well known symmetry and adding up conditions. Note that (1) does not include the output variable implying linear homogeneity of the underlying technology.

The spatial TFP index corresponding to (1), μ_{sr}, can be obtained in correspondence with the method of Caves, Christensen and Diewert (1982) as follows (for more details of the derivation see Nakamura (1989)):

$$\begin{aligned} \ln \mu_{sr} = {} & \ln(c^r/c^s) + 1/2[(w^\circ + w^s)'(z^s - z^\circ) - (w^\circ + w^r)'(z^r - z^\circ)] \\ & + 1/4[(z^s - z^\circ)'(B_{pp}^\circ - B_{pp}^s)(z^s - z^\circ) - (z^r - z^\circ)'(B_{pp}^\circ - B_{pp}^r)(z^r - z^\circ)] , \end{aligned} \qquad s, r = A, D, J \quad (2)$$

where the superscript $^\circ$ denotes that the variable/parameter attached to it refers to the mean over the three countries (for example, $z^\circ = (z^A + z^D + z^J)/3$, and $B_{pp}^\circ = 1/3(B_{pp}^A + B_{pp}^D + B_{pp}^J)$). w^r refers to the cost minimizing cost shares of the three inputs:

$$w^r = B_p^r + B_{pp}^r z^r + B_{pt}^r t , \qquad r = A, D, J \quad (3)$$

Note that μ_{sr} is a multilateral extension of the bilateral index of Denny and Fuss, and satisfies the transitivity condition.

If the third term of the r.h.s. of (2) (the expression in 1/4[ ]) vanishes, μ_{sr} reduces to the usual Toernqvist TFP index which does not include the second order parameters. This third term consists of spatial differences in both price and technology, while the second term is the spatial Toernqvist price index. Thus, when B_{pp}^r are different for $r = A, D, J$, the spatial cost difference can no longer be exclusively decomposed into the price and TFP factors.

Since the r.h.s. of (2) does not explicitly involve second order technical change parameters B_{pt}^r and B_{tt}^r, it follows that spatial equality of these parameters will not be necessary for the Toernqvist TFP index to be exact. (Note that this is due to the use of a simple time trend as a proxy for the state of technology. This feature will no longer hold, if we use an economic measure such as R&D capital instead of a

TABLE 1. TESTED MODELS (MODEL 1 IS THE MAINTAINED HYPOTESIS)*

Models	Covariance matrix Σ^r	Second order price parameters B^r_{PP}	Second order technical change parameters B^r_{Pt}, B^r_{tt}
Model 1 (48)	different	different	different
Model 2 (36)	same	different	different
Model 3 (30)	same	same	different
Model 4 (24)	same	same	same

* The term "different" indicates that the corresponding parameter is different among countries (the space index r is retained). The term "same" refers to its spatial equality, that is the space index disappears. The number in the parentheses refers to the number of unrestricted parameters.

time trend.) For econometric implementation of the model I assume that a disturbance term could be added to (1) and (3), and that the resulting disturbance vector would be independently and identically multivariate normally distributed with mean zero and constant covariance matrix Σ^r of rank 3, the singularity of which follows from the adding up condition of (3). I further assume that the disturbance vectors of different countries are uncorrelated.

Table 1 shows four models that are nested to the above model together with corresponding parameter restrictions. Model 1 represents the maintained hypothesis, while Models 2 to 4 represent hypotheses to be tested. Model 1 is estimated separately for each country data. Model 2 assumes cross country equality of the covariance matrix. Its estimation method is to pool the German, Japanese, and U.S. data and use dummy variables to specify different coefficients for the cost functions. If this hypothesis is correct, efficiency gains in estimators can occur.

Model 3 is conditional on Model 2, and corresponds to the case where the third term of the r.h.s. of (2) vanishes, and implies that the Slutsky matrix of the three countries is identical up to the terms involving cost shares. If this model is correct, the Toernqvist TFP index would be exact to the underlying translog technology. Finally, Model 4 corresponds to the usual index number approach where technology parameters are spatially identical up to the zero and first order ones. This model assumes that the biases as well as rate of technical change are also identical over space.

3. EMPIRICAL RESULTS

I use the annual K (capital) L (labour) M (material and energy) data for the period 1960 –79 on nine producing sectors of Germany, Japan, and the US consisting of agriculture, foods, textiles, chemicals, primary metals, machinery, electrical machinery, motor vehicles, and precision machinery. The three aggregates (KLM) are based on Divisia indices. Purchasing power parities were used to transform the original data measured in local currencies to the mutually comparable US dollar base. (See Appendix and Nakamura (1989) for further details of the data.)

Note that M includes both materials and energy inputs, the aggregation of which implies certain separability conditions. A further disaggregation of M into the two components would allow for a more general analysis. Given the quadratic nature of the specification and the limited number of observations, however, such a generalization would not be possible. Furthermore, the relatively small cost share of energy appears to make such a generalization less important within the present context.

The system of equations (1) and (3) was estimated by the method of maximum likelihood, using MLE implemented in TSP 4.1B, under different hypothesis shown in Table 1. The system was estimated subject to adding up and homogeneity conditions. The imposition of global negativity is known to imply unduly strong restrictions on flexible functions such that its very flexibility can eventually be lost (Barnett and Lee (1985)). I therefore chose to impose the negativity condition at the sample mean only. In particular, I first estimated the model with only the adding up and homogeneity restrictions imposed, and checked the non-positivity of the characteristic values of the Slutsky matrix evaluated at the sample mean. If the negativity condition was not satisfied, the model was reestimated with the condition being imposed. (The method is based on Lau (1978) and used in Nakamura (1986).)

It turned out that for Models 1 and 2 it was necessary to impose the negativity condition for four US sectors: agriculture, foods, textiles, and precision machinery. The condition was automatically satisfied for all the German and Japanese sectors and for Models 3 and 4. The number of additional parameter restrictions that resulted from the negativity condition was two for agriculture and foods, one for textile, and three for precision machinery.

Table 2 represents the test results. I use minus twice the logarithm of the likelihood ratio as the test statistic and its asymptotic property to obtain the critical level. To keep the overall level of significance for all the nested hypotheses considered simultaneously at a reasonable level, I have assigned 0.01 as the level of significance for each test.

The test results show no ambiguity. Conditional on Model 1, equality of the covariance matrix is rejected. Conditional on the equal covariance matrix, we reject equality of second order parameters. Thus, all the tested models are decisively rejected for each of the nine sectors analyzed. In view of the large value of test statistics, it seems certain that the test results would remain unchanged even under small sample corrections of the likelihood ratio as the one proposed by Italiener (1985). I conclude that the spatial equality of the second order parameters is not consistent with the present data, and that the Toernqvist index is subject to biases.

Since the equality hypotheses was rejected for all the sectors considered

TABLE 2. TEST RESULTS (CONTINUED)

	Model 2 against Model 1		
	Test statistics	d.f	$\chi^2(0.01)$
Agriculture	106.07	12	26.22
Foods	194.42	12	
Textiles	110.65	12	
Chemicals	62.27	12	
Primary metals	112.42	12	
Machinery	97.90	12	
Electrical machinery	97.18	12	
Motor vehicles	98.59	12	
Precision machinery	112.79	12	

TABLE 2. TEST RESULTS (CONCLUDED)

	Model 3 against Model 2			Model 4 against Model 3		
	Test statistics	d.f	χ^2 (0.01)	Test statistics	d.f	$\chi^2(0.01)$
Agriculture	50.02	4	13.28	111.8	6	16.81
Foods	35.58	4	13.28	57.4	6	
Textiles	41.06	5	15.09	94.48	6	
Chemicals	62.48	6	16.81	101.44	6	
Primary metals	65.62	6	16.81	62.86	6	
Machinery	29.24	6	16.81	78.72	6	
Electrical machinery	36.54	6	16.81	39.84	6	
Motor vehicles	31.32	6	16.81	54.64	6	
Precision machinery	39.64	3	11.34	30.68	6	

including the eight manufacturing sectors, it seems certain that the same would hold for the aggregate of these eight manufacturing sectors, the result of which is consistent with that of Denny and Fuss (1983) for the aggregated data on U.S. and Japanese private sector 1952 –1974 compiled by Jorgenson and Nishimizu (1978). Because output of some sectors would appear as input for some other sectors, the aggregation of eight manufacturing sectors cannot be achieved by simply adding the input and output data for individual sectors, should the resulting aggregate have a sound economic meaning. A sound aggregation would thus require exclusion of this double counting using information on intermediate input output relationships. To carry out such an aggregation is beyond the scope of this paper.

To see the quantitative extent of biases that can result by ignoring this mixed term I computed its contribution in (2) relative to the second term (price index) by dividing the absolute value of the third term by the sum of the absolute value of the second and third terms (the exact form is shown in the top of Table 3). Table 3 shows the results in terms of the mean for the period 1960–79. The contribution of the mixed term exceeds 10 percent for one sector in the case of the U.S. -Japan comparison, and three sectors for the U.S. -German comparison. On the average, however, the contribution is smaller than 10 percent for both the U.S. -Japan and the U.S. -German comparisons. It seems safe to conclude that while the difference in the second order parameters is statistically significant, its quantitative significance for TFP measurement is minor.

TABLE 3. CONTRIBUTION OF THE MIXED TERM IN THE TFP INDEX

$$1-(|[(w^o+w^s)'(z^o-z^o)-(w^o+w^r)'(z^r-z^o)]|/$$
$$\{|[(w^o+w^s)'(z^s-z^o)-(w^o+w^r)'(z^r-z^o)]|+|1/2[(z^s-z^o)'$$
$$(B^o_{PP}-B^s_{PP})(z^s-z^o)-(z^r-z^o)'(B^o_{PP}-B^r_{PP})(z^r-z^o)]|\})$$

Sector	Mean over 1960–79	
	Japn–USA	Germany–USA
Agriculture	0.030	0.085
Foods	0.069	0.129
Textiles	0.088	0.159
Chemicals	0.045	0.133
Primary metals	0.163	0.097
Machinery	0.038	0.050
Electrical machinery	0.017	0.049
Motor vehicles	0.060	0.070
Precision machinery	0.046	0.056
Mean	0.062	0.092

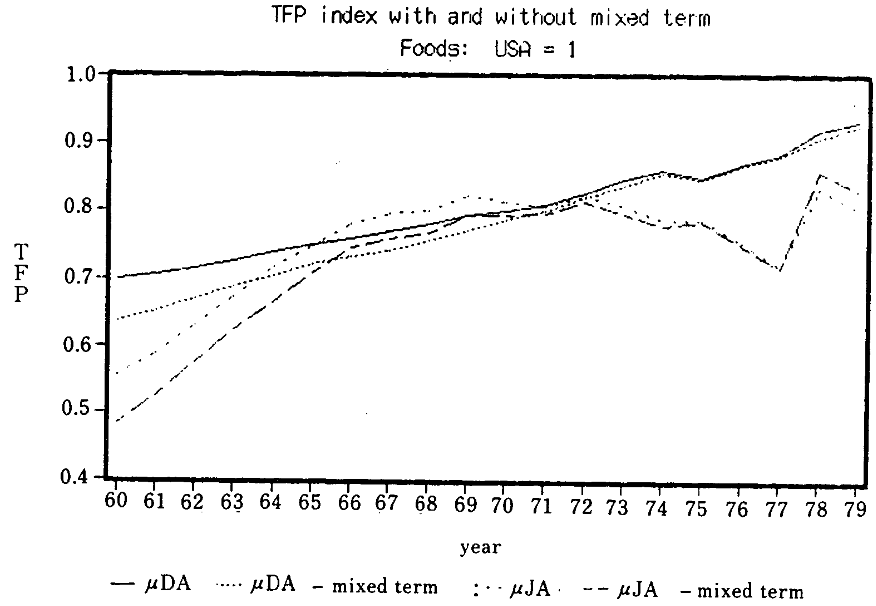

Fig. 1.

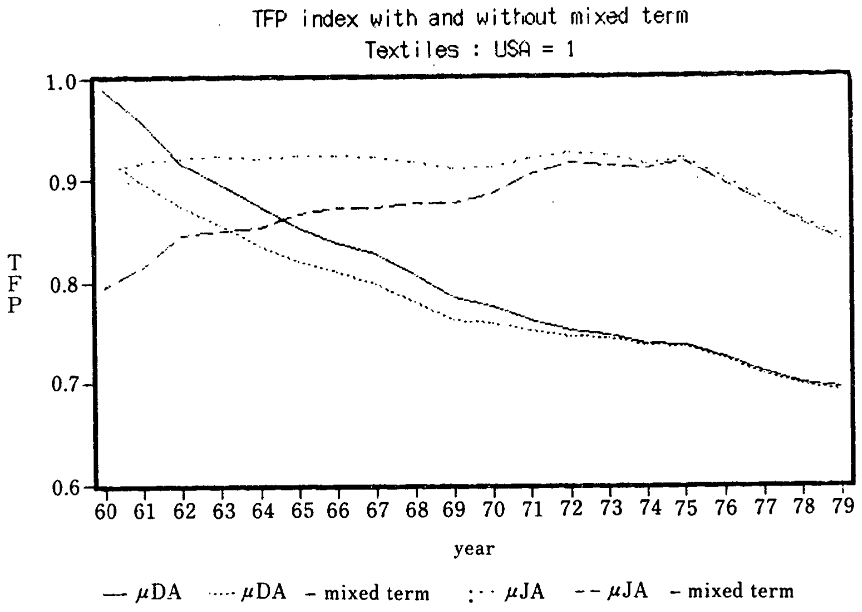

Fig. 2.

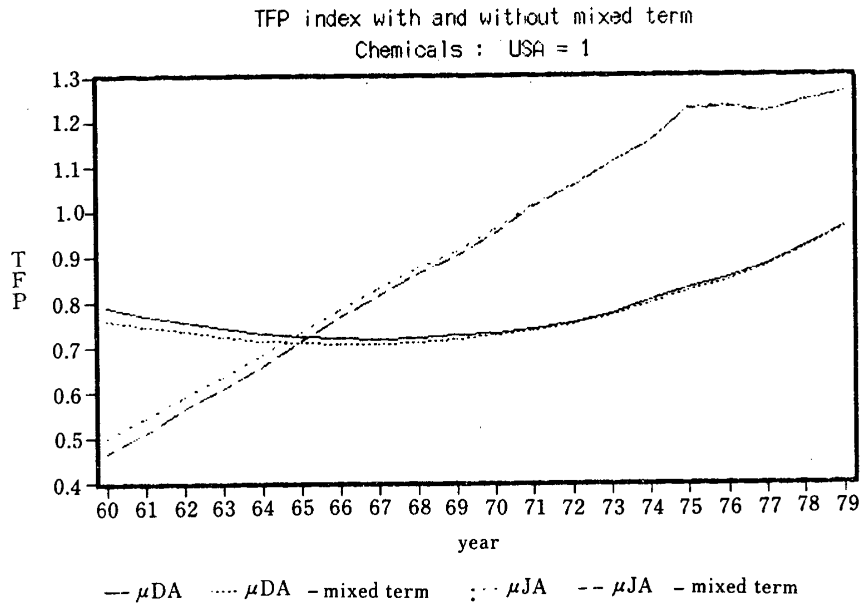

Fig. 3.

Figures 1 to 4 compare the TFP indexes with and without the mixed term for the above four sectors with relatively large contributions of the mixed term. We find that biases that result from ignoring the mixed term tend to cause a one sided shift of the TFP index, especially in the first half of the sample period, and tend to decrease with time. This pattern results from a decreasing difference over the observation period in factor price levels among the three countries. (See Nakamura (1989). Note that provided the difference in the second order para-

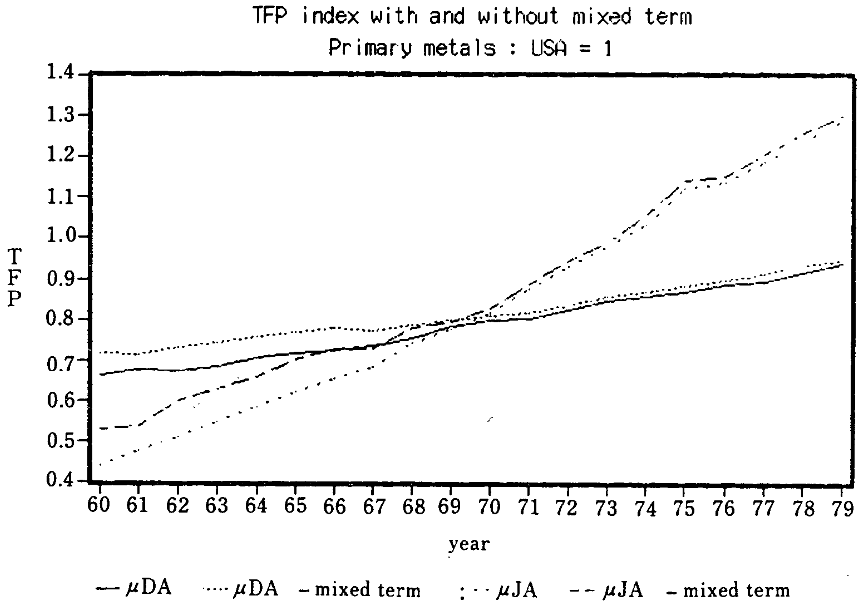

Fig. 4.

meters is smaller in absolute value than the difference in the value shares, w, the contribution of the mixed effect represented by the third term in (2) would decrease.) Furthermore, the trend pattern of TFP development appears to be unaffected by biases. These findings agree with those of Denny and Fuss (1983).

4. CONCLUDING REMARKS

Within the maintained hypothesis of the KLM translog unit cost function, I tested the equality of the second order parameters among Japan, Germany and the U.S. for nine industry sectors for the period 1960 –1979. The results indicate a decisive rejection of intercountry equality of the covariance matrix, the second order price parameters as well as the second order technical change parameters. There exist substantial differences in the structure of KLM technology among the three countries such that these differences cannot be fully represented by the zero and first order terms of the translog function alone. Hence, the conventional Toernqvist TFP index based on spatially equal second order parameters is biased.

Although the equality hypothesis of the second order parameters was decisively rejected, the resulting bias was found to be quantitatively less significant. Furthermore, the trend pattern of TFP turned out to be neutral to the bias. Recall that the TFP measure is by nature a residual variable and would certainly be subject to errors of various sorts including measurement errors. Therefore, the empirical result of this paper appears to support the use of the conventional Toernqvist TFP index as a practical measure of TFP. Of course one should be aware of its potential biases.

In view of the above test results, one may ask why technology should be so different between countries that appear to technologically similar such as Germany, Japan and the U.S.? Product mix certainly plays an important role. Apart from all this and similar issues dealing with data problems, however, there is a fundamental reason inherent to the present framework of TFP measurement. Recall that the present model does not include country specific variables referring to the level of efficiency or technology such as R&D stock. Hence, the difference in the cost (production) function parameters is the only source from which spatial TFP difference can emerge. In other words, within the present framework, existence of spatial TFP difference presupposes spatially different parameters. This implies that while the current framework is useful for measuring TFP, it cannot explain the difference or change in TFP. For the latter purpose it seems vital to replace the simple trend variable by an economic variable such as R&D stock. Nakamura (1989) finds that by the end of 1970s the once predominant difference in factor prices among Germany, Japan, and U.S. almost disappeared, and hence the unit cost level has become increasingly sensitive to the TFP level. A model allowing for endogenous explanation of the TFP level is thus very much in need. (See Mohnen, Nadiri, and Prucha (1986) as an example of study incorporating R&D capital. Nakamura (1988) finds that R&D capital plays a significant role in explaining TFP growth rates of Germany and Japan.)

APPENDIX: DATA

I used yearly data (1960 –79) on nine producing sectors (agriculture, foods, textiles, chemicals, primary metals, machinery, electrical machinery, motor vehicle, and precision machinery) of the U.S., Japan and Germany. The data consist of prices and quantities of output and three types of inputs (material, capital services, and labour services). The U.S. and Japanese data were taken from Jorgenson, Kuroda and Yosioka (1984), and the German data from Conrad and Jorgenson (1985). These data sets were compiled based on the accounting framework and methodology of Gollop and Jorgenson (1980). The data on labour and capital services are based on detailed information on labour input classified by education levels and other quality factors and on capital stock classified by asset types. Aggregation of individual components to an aggregate input is done by the Toernqvist index. The Japanese and German data were converted into the U.S. dollar unit using time series data on purchasing power parities for each input and output (PPP) estimated by Jorgenson, Kuroda and Nishimizu (1985) for Japan – U.S. and by Conrad and Jorgenson (1985) for Germany –U.S..

Leaving the details of the estimation of the PPP data to the cited studies, I point out here that the PPP data are based on PPP of final products obtained by Kravis and associates; for references see the cited papers. It was thus necessary to approximate the PPP of intermediate inputs, which is needed to make the price and quantity of M comparable, by that of similar final products. This already difficult issue is further complicated by the inclusion of intermediate imports. In spite of these difficulties and the resulting measurement errors, the use of PPP is more desirable than using the exchange rate, which completely neglects the difference in relative prices.

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