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A MODEL OF BARGAINING OVER WAGES AND EMPLOYMENT*

Masao TAKESHIMA

Abstract. In this paper, we consider the problem of bargaining over wages and employment between a trade union and a firm. We extend the model developed by McDonald and Solow (1981) by introducing the element of incomplete information into their model. By means of comparative static analysis, it is shown that the way the agents form expectations about general price level affect importantly the fluctuations of wages and employment determined as the bargaining solution.

1. INTRODUCTION

The purpose of this paper is to formulate the problem of bargaining over wages and employment between a trade union and a firm in a partial equilibrium model and derive some macroeconomic implications from the property of its solution. The basic framework of the model is the extended version of the bargaining model developed by McDonald and Solow (1981).

We investigate, by means of comparative static analysis, how the money wage and the level of employment determined as the bargaining solution vary with the product price of the firm. The paper is stimulated by the work of McDonald and Solow (1981). We extend their model by introducing the element of incomplete information about general economic condition on the part of individual agents.[1]

To be specific, we assume that in each period, the firm and the trade union start the bargaining after observing the product price of the firm in that period. When they start the bargaining, however, we suppose that they do not know the general price level in that period. Therefore, both parties have to figure out the general price level in some way or another. We adopt the special form of expectation function which depends on a parameter concerning the observed product price of the firm and investigate how the comparative static results depend on the value of this parameter. When the firm and the trade union make static expectations about

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[1] Since both parties start the bargaining after observing the product price of the firm in real terms in McDonald-Solow (1981), the element of incomplete information as considered here is absent in their model.

the general price level, the comparative statics results depend crucially on the technological condition of the firm. The necessary and sufficient condition for money wage rigidity will be derived in this case. We also show that if both parties identify the expected rate of change in the general price level with the rate of change in the product price of the firm, the level of employment does not fluctuate with the product price of the firm.

The paper is organized as follows. We present the basic framework of the model and define the expectation function of general price level in Section 2. In Section 3, the comparative static analysis of the model will be carried out for alternative values of the parameter in the expectation function. We conclude the paper by noting some qualifications of the model in Section 4.

2. BASIC MODEL

Consider a firm taking price in its product market and assume that its objective is to maximize real profit $(Pf(L) - WL)/q$ where P is the price of its product, f the production function relating employment to output, L the employed member of the union, W the wage income per employee, and q the expected general price level. In this paper, we assume that in each period, the firm and the trade union start the bargaining after observing P. But at the beginning of the bargaining, both parties are assumed to be ignorant of q in that period. So, to begin the bargaining, both parties have to forecast q somehow. We assume that both parties expect q in the following way:[2]

$$\frac{q-\bar{q}}{\bar{q}} = m \cdot \frac{P-\bar{P}}{\bar{P}} \tag{1}$$

$\bar{P}$; product price of the firm in the last period
$\bar{q}$; general price level in the last period
m; constant real number.

In other words both parties expect that the rate of change in the general price level is in proportion to the rate of change in the product price of the firm via constant parameter m.

$f' > 0, f'' < 0$ and $f(0) = 0$ are assumed to hold. In the (L, W) plane, the slope of the iso-profit curve through (L, W) can be expressed as follows:

$$\frac{dW}{dL} = \frac{Pf'(L) - W}{L}.$$

In Fig. 1, the iso-profit curves are drawn. As is shown, they are horizontal where $W = Pf'(L)$.

The other bargaining party, the trade union, has N members all alike and its

[2] Professor S. Yabushita pointed out that in general the expectation formed by the firm does not necessary coincide with that formed by the union.

objective is to maximize the total sum of utility of its members, or equivalently the expected utility of its representative member, i.e. to maximize $L(u(W/q)-k)+(N-L)u(W_u/q)$ where u represents the worker's utility of real income, k the worker's (fixed, additive) disutility of working and W_u the obtainable income when unemployed. The working hour is assumed to be fixed (For example, working for eight hours). We also assume $u'>0$ and $u''<0$. To put it differently, workers are assumed to be risk averse. $\bar{W}$, reservation wage (in nominal term),[3] is defined by

$$u\left(\frac{\bar{W}}{q}\right)=u\left(\frac{W_u}{q}\right)+k.$$

If $\bar{W}\leq W$ is not satisfied, the worker will not work for that firm. The slope of the indifference curve of the union through (L, W) is given by

$$-\frac{u\left(\frac{W}{q}\right)-u\left(\frac{\bar{W}}{q}\right)}{\frac{L}{q}u'\left(\frac{W}{q}\right)}. \tag{3}$$

Then, at $W=\bar{W}$ the indifference curves become horizontal. And by the concavity of u, we obtain;

$$\frac{d^2W}{dL^2}=-\frac{u'\left(\frac{W}{q}\right)^2\left(\frac{L}{q^2}\right)\frac{dW}{dL}-\left(u\left(\frac{W}{q}\right)-u\left(\frac{\bar{W}}{q}\right)\right)\left(\frac{1}{q}u'\left(\frac{W}{q}\right)+\frac{L}{q^2}u''\left(\frac{W}{q}\right)\frac{dW}{dL}\right)}{\left(\frac{L}{q}u'\left(\frac{W}{q}\right)\right)^2}>0.$$

Hence, the indifference curves have usual downward sloping convex shape in the (L, W) plane as is depicted in Fig. 2. In order to exclude the possibility of corner

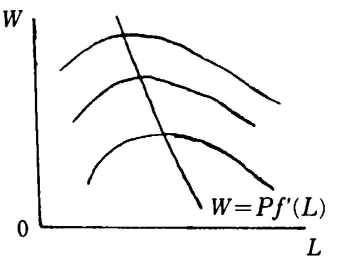

Fig. 1.

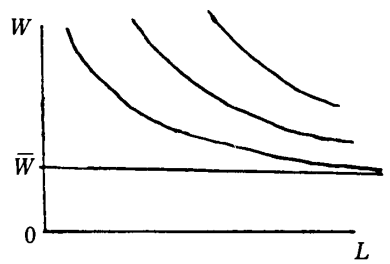

Fig. 2.

solution, we restrict our attention to the case where $Pf'(N)<\bar{W}$ holds. The equation of the contract curve is obtained by equating the slope of the iso-profit curve with that of the indifference curve. It is represented as the following;

[3] The original form of workers' utility function may be expressed as $u(W/q)-s(h)$ where h is the working hour and $s'(h)>0$ and $s''(h)>0$ are assumed to be satisfied. Here, we assume $h=\bar{h}$ and $s(\bar{h})=k$.

$$\frac{W-Pf'(L)}{q}=\frac{u\left(\frac{W}{q}\right)-u\left(\frac{\bar{W}}{q}\right)}{u'\left(\frac{W}{q}\right)}. \tag{4}$$

The slope of the contract curve is positive because the following can be obtained from (4);

$$\frac{dW}{dL}=\frac{u'\left(\frac{W}{q}\right)Pf''(L)}{\left(\frac{W}{q}-\frac{Pf'(L)}{q}\right)u''\left(\frac{W}{q}\right)}>0 \quad (\text{for } \bar{W}<W).$$

The contract curve intersects the curve $W=Pf'(L)$ at $W=\bar{W}$ as is shown in Fig. 3.

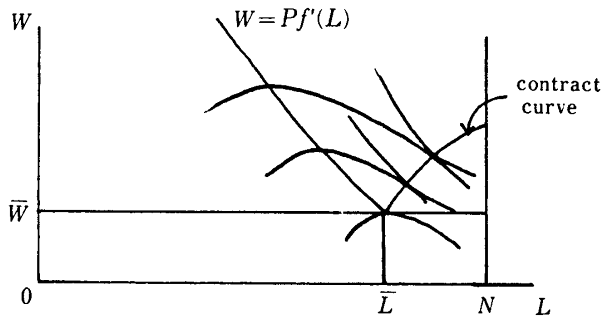

Fig. 3.

To select a point on the contract curve, we have recourse to the Nash bargaining solution. The convexity of the bargaining possibility set which is taken for granted in McDonald-Solow (1981) is explicitly proved in Appendix. For the employer, the payoff is $V(W, L)=(Pf(L)-WL)/q$ and for the union, the payoff is $U(W, L)=L(u(W/q)-k)+(N-L)u(W_u/q)$. If no bargain is struck, the payoffs will be $V(0, 0)=0$ and $U(0, 0)=Nu(W_u/q)$. The Nash bargaining solution is obtained by solving the following problem;

$$\underset{W,L}{\text{MAX.}}\,(V(W, L)-V(0, 0))(U(W, L)-U(0, 0)).$$

This is equivalent to the following;

$$\underset{W,L}{\text{MAX.}}\left(\frac{Pf(L)-WL}{q}\right)L\left(u\left(\frac{W}{q}\right)-u\left(\frac{\bar{W}}{q}\right)\right).$$

As a first order condition, we obtain (4) and the following;[4]

[4] Since, as we show in Appendix, the bargaining possibility set is convex, (4) and (5) are necessary and sufficient condition for the maximization problem.

$$W=\frac{1}{2}\left(\frac{Pf(L)}{L}+Pf'(L)\right) \tag{5}$$

Then, at the bargaining solution, the money wage is expressed as the average of average value product of labor and marginal value product of labor. The Nash bargaining solution is the pair (W, L) satisfying (4) and (5).

3. COMPARATIVE STATIC ANALYSIS

Using (4) and (5), we are able to conduct comparative static analysis. According to the value of m, we distinguish four cases.

[1] $m=0$

From (1), if $m=0$, we have $q=\bar{q}$. In this case, both parties are supposed to form static expectations about the general price level. What is most essential in this case is that $dq/dP=0$ always holds.

Differentiating (4) and (5) with respect to W, L, and P, we can obtain the following equation;

$$\begin{pmatrix} \dfrac{\left(u\left(\dfrac{W}{q}\right)-u\left(\dfrac{\bar{W}}{q}\right)\right)u''\left(\dfrac{W}{q}\right)\dfrac{1}{q}}{\left(u'\left(\dfrac{W}{q}\right)\right)^2} & -\dfrac{Pf''(L)}{q} \\ 1 & -\dfrac{1}{2}\left(\dfrac{Pf'(L)L-Pf(L)}{L^2}+Pf''(L)\right) \end{pmatrix}\begin{pmatrix} \dfrac{dW}{dP} \\ \dfrac{dL}{dP} \end{pmatrix}$$

$$=\begin{pmatrix} \dfrac{f'(L)}{q} \\ \dfrac{1}{2}\left(\dfrac{f(L)}{L}+f'(L)\right) \end{pmatrix}$$

The coefficient matrix on the left hand side has sign pattern $\begin{pmatrix} - & + \\ + & + \end{pmatrix}$ and its determinant is therefore negative. From this, we obtain

$$\text{sign}\left(\frac{dW}{dP}\right)=\text{sign}\left(\frac{f'(L)(f'(L)L-f(L))}{L}-f(L)f''(L)\right) \tag{6}$$

$$\frac{dL}{dP}=\frac{1}{|D|}\left(\frac{\left(u\left(\frac{W}{q}\right)-u\left(\frac{\bar{W}}{q}\right)\right)u''\left(\frac{W}{q}\right)}{\left(u'\left(\frac{W}{q}\right)\right)^2}\frac{W}{P}-\frac{Pf'(L)}{q}\right) \tag{7}$$

where D is the coefficient matrix and $|D|$ is the value of its determinant.

From (7), $dL/dP>0$ is straightforward. About the sign of dW/dP, however, there is some ambiguity. Differentiating (5) and using (7),

$$\begin{aligned}\frac{dW}{dP}&=\frac{1}{2}\left(\frac{f(L)}{L}+f'(L)\right)+\frac{P}{2}\left(\frac{f'(L)L-f(L)}{L^2}+f''(L)\right)\frac{dL}{dP}\\&=\frac{W}{P}+\frac{P}{2}\left(\frac{f'(L)L-f(L)}{L^2}+f''(L)\right)\frac{dL}{dP}\\&<\frac{W}{P}\end{aligned}$$

whence, $(dW/dP)\cdot(P/W)<1$. If the product price of the firm rises, the money wage rises less than proportionately. We might say that under $m=0$, the money wage becomes somewhat rigid. Moreover from (6), we can establish the following Proposition.

Proposition 1. *Under $m=0$, the following holds;*

$$\frac{dW}{dP}\gtreqless 0 \Leftrightarrow \varepsilon'(L)\lesseqgtr 0 \qquad \text{where} \quad \varepsilon(L)=\frac{f'(L)L}{f(L)}.$$

Proof. From (6), $\operatorname{sign} dW/dP=-\operatorname{sign}\varepsilon'(L)$. Q.E.D.

Let us present three examples with different signs of $\varepsilon'(L)$.

Example 1. $f(L)=KL^c$ ($K>0$ and $0<c<1$). Since $\varepsilon'(L)=0$ in this case $dW/dP=0$ holds.

Example 2. $f(L)=1-e^{-L}$. $\varepsilon'(L)<0$ so, $dW/dP>0$.

Example 3. $f(L)=L+L^c$ $(0<c<1)$. $\varepsilon'(L)>0$ so, $dW/dP<0$.

The interesting case is $\varepsilon'(L)=0$. The result can be restated as follows; under $m=0$, the money wage determined as the bargaining solution does not vary with the product price of the firm if and only if the firm's technology exhibits constant elasticity. From (7), on the other hand, the level of employment is positively related to the product price of the firm. Thus, our model may be taken to provide a microeconomic foundation of quantity adjustment mechanism in the labor market.

In what follows, we deal with the case where $m>0$ and q varies with P from (1).

For analytical simplicity, let us assume that W_u is fully indexed to the variation of q. Then, W_u/q and $\bar{W}/q$ become constant.[5] If $m>0$, let $\bar{w}=\bar{W}/q$.

[2] $0<m<1$

Differentiating (4) and (5) with respect to W, L and P, we obtain the following;

$$\begin{pmatrix} \dfrac{\left(u\left(\frac{W}{q}\right)-u(\bar{w})\right)u''\left(\frac{W}{q}\right)}{u'\left(\frac{W}{q}\right)} & -Pf''(L)u'\left(\frac{W}{q}\right) \\ 1 & -\dfrac{P}{2}\left(\dfrac{f'(L)L-f(L)}{L^2}+f''(L)\right) \end{pmatrix}\begin{pmatrix} dW \\ dL \end{pmatrix}$$

$$=\begin{pmatrix} u'\left(\frac{W}{q}\right)f'(L)\left(1-\dfrac{mcP}{q}\right)+\dfrac{mcW}{q}\dfrac{\left(u\left(\frac{W}{q}\right)-u(\bar{w})\right)}{u'\left(\frac{W}{q}\right)}\cdot u''\left(\frac{W}{q}\right) \\ \dfrac{W}{P} \end{pmatrix} dP \tag{8}$$

where $c=\bar{q}/\bar{P}$. In this case, the equations become somewhat complicated because of the dependence of q on P through Eq. (1). After lengthy calculations, the following Proposition can be obtained.

PROPOSITION 2.

① *If* $dW/dP\geqq 0$ *under* $m=0$, $dW/dP>0$ *for all* $m\in(0, 1)$.

② *Assume the measure of relative risk aversion* $\left(-u''\left(\frac{W}{q}\right)\frac{W}{q}\Big/u'\left(\frac{W}{q}\right)\right)$ *is constant. If* $dW/dP<0$ *under* $m=0$, *there exists* $\tilde{m}$: $\tilde{m}\in(0, 1)$ *such that for all* $m\geqq\tilde{m}$, $dW/dP>0$.

③ $dL/dP>0$ *for all* $m\in(0, 1)$.

The proof is omitted here. ③ is obvious by noting that

$$\frac{dL}{dP}=\frac{1}{|D|}\cdot\left(\frac{\left(u\left(\frac{W}{q}\right)-u(\bar{w})\right)u''\left(\frac{W}{q}\right)}{u'\left(\frac{W}{q}\right)}\cdot\frac{W}{P}-u'\left(\frac{W}{q}\right)f'(L)\right)\left(1-\frac{mcP}{q}\right) \tag{9}$$

where $|D|$ is negative. Comparing Proposition 2 with Proposition 1 we note that wage is more flexible under $m>0$ than under $m=0$.

[5] By this assumption, reservation wage in real term is constant throughout this model.

[3] $m=1$

In this case, $(P-\bar{P})/\bar{P}=(q-\bar{q})/\bar{q}$ holds from (1) and both parties expect that the rate of change in the general price level will be equal to that of the product price of the firm. $q=cP$ holds where $c=\bar{q}/\bar{P}$. We differentiate (4) and (5) with respect to W, L and P using the relation $q=cP$. Then we can obtain the following:

$$\begin{pmatrix} x & -Pf''(L)u'\left(\dfrac{W}{cP}\right) \\ 1 & -\dfrac{P}{2}\left(f''(L)+\dfrac{f'(L)L-f(L)}{L^2}\right) \end{pmatrix}\begin{pmatrix} dW \\ dL \end{pmatrix}=\begin{pmatrix} \dfrac{W}{P}x \\ \dfrac{W}{P} \end{pmatrix}dP$$

where

$$x=\frac{\left(u\left(\dfrac{W}{cP}\right)-u(\bar{w})\right)u''\left(\dfrac{W}{cP}\right)}{u'\left(\dfrac{W}{cP}\right)}$$

We can establish

PROPOSITION 3. *Under* $m=1$, $(dW/dP)\cdot(P/W)=1$ *and* $dL/dP=0$.

The proof of the proposition is straightforward from the above equation. In this case, without strong restriction on firm's technology, we may conclude that the (expected) real wage and the level of employment does not vary with P. Since W/P is constant from Proposition 3, L is constant from (5).

[4] $m>1$

Differentiating (4) and (5) with respect to W, L and P we again obtain (8) which now implies

PROPOSITION 4. *Under* $m>1$, $(dW/dP)\cdot(P/W)>1$ *and* $dL/dP<0$.

Proof. $dL/dP<0$ easily follows from equation (9) since $1-mcP/q<0$ holds under $m>1$.

Differentiating (5) and using the result $dL/dP<0$ holds under $m>1$ we get

$$\begin{aligned}\frac{dW}{dP}&=\frac{W}{P}+\frac{P}{2}\left(f''(L)+\frac{f'(L)L-f(L)}{L^2}\right)\frac{dL}{dP}\\ &>\frac{W}{P}\end{aligned}$$

Q.E.D.

$m>1$ means that both the firm and the trade union expect that the rate of increase of the general price level is larger than that of the product price of the firm. Thus, the wage rate must increase more than the product price of the firm if the real wage is to be maintained. Hence, $(dW/dP)\cdot(P/W)>1$ holds at the bargaining solution. From (5) this implies that the firm has an incentive to reduce employment with higher P.

4. CONCLUSION

Extending the bargaining model developed by McDonald-Solow (1981), we contemplated the relation between the public expectation of the general price level and the comparative statics of wages and employment determined as the bargaining solution between firms and trade unions. Generally speaking, macroeconomic fluctuations are strongly affected by the expectations formed by individual agents about general economic conditions. Thus, in order to stabilize economic fluctuation, the government might have to employ those policy instruments which can affect the expectation formation of individual agents, specifically m in our model.

Our model is still too simple to analyse the dynamic fluctuations of wages and employment in the real world. It remains basically static and ignores the possible fact that the firm and the trade union may differ in their bargaining skills. We wish to generalize our model to deal with these generalizations in the future.

APPENDIX: CONCAVITY OF THE BARGAINING POSSIBILITY FRONTIER[6]

In what follows, we prove the concavity of the bargaining possibility frontier for the purpose of showing that the bargaining set is convex. P, W and W_u deflated by q are represented by lower letters p, w and w_u respectively.

Then let us consider the following problem:

$$\underset{w,L}{\text{Max.}}\ pf(L) - wL \tag{A.1}$$

s.t.

$$L(u(w)-k)+(N-L)u(w_u) \geqq U \tag{A.2}$$

$$L \leqq N \tag{A.3}$$

If we represent the solution to the above problem as $w(U, p, w_u, k, N)$ and $L(U, p, w_u, k, N)$, the resulting profit of the firm can be represented as $V(U, p, w_u, k, N)$. Our objective is to show $\partial V/\partial U < 0$ and $\partial^2 V/\partial U^2 < 0$. The necessary conditions for the maximization described by (A.1) through (A.3) are as follows:

$$pf'(L) - w + t(u(w) - u(\bar{w})) = 0 \tag{A.4}$$

$$tu'(w) = 1 \tag{A.5}$$

$$L(u(w)-k)+(N-L)u(w_u) = U \tag{A.6}$$

where t is the non-negative Lagrange multiplier corresponding to the constraint (A.2). From (A.4) and (A.5), we can represent the solution as

[6] The technique of the proof is adapted from Homma-Osano (1983).

$$w=\tilde{w}(t, p, \bar{w}) \tag{A.7}$$

$$L=\tilde{L}(t, p, \bar{w}) \tag{A.8}$$

where

$$\tilde{w}_1 \equiv \frac{\partial \tilde{w}}{\partial t} > 0 \quad \text{and} \quad \tilde{L}_1 \equiv \frac{\partial \tilde{L}}{\partial t} > 0$$

Substituting (A.7) (A.8) into $V=V(U, p, w_u, k, N)$ and (A.6), we obtain

$$V=pf(\tilde{L}(t, p, \bar{w}))-\tilde{w}(t, p, \bar{w})\tilde{L}(t, p, \bar{w}) \tag{A.9}$$

$$\tilde{L}(t, p, \bar{w})(u(\tilde{w}(t, p, \bar{w}))-u(\bar{w}))+Nu(w_u)=U \tag{A.10}$$

Differentiating (A.9) (A.10) with respect to V, t and U, we obtain

$$\begin{pmatrix} -\tilde{L}_1(\tilde{w}-pf'(\tilde{L}))-\tilde{L}\tilde{w}_1 & -1 \\ \tilde{L}_1(u(\tilde{w})-u(\bar{w}))+\tilde{L}u'(\tilde{w})\tilde{w}_1 & 0 \end{pmatrix}\begin{pmatrix} dt \\ dV \end{pmatrix}=\begin{pmatrix} 0 \\ 1 \end{pmatrix}dU \tag{A.11}$$

From (A.11) and using (A.4) and (A.5) $dV/dU=-t$ holds. So, $d^2V/dU^2=-dt/dU<0$.

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