Title	PREDICTIVE AND IMITATIVE EXPECTATIONS IN MACRO-DYNAMIC THEORIES
Sub Title	
Author	大山, 道廣(OHYAMA, Michihiro)
Publisher	Keio Economic Society, Keio University
Publication year	1988
Jtitle	Keio economic studies Vol.25, No.2 (1988.) ,p.1- 17
JaLC DOI	
Abstract	This paper introduces two distinct hypotheses of price expectations within the framework of deterministic macro-dynamic models with continuous time. One is the hypothesis of predictive expectations conceivable for the community where individuals behave as identical leaders in respect of expectations. The other is the hypothesis of imitative expectations applicable to the community where individuals behave as identical followers in respect of expec tations. These hypotheses are shown to generate potentially stable adjustment processes toward the steady state equilibrium in the well known models of monetary dynamics which exhibits saddle-point instability under the conventional hypothesis of myopic perfect foresight.
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19880002-0001

PREDICTIVE AND IMITATIVE EXPECTATIONS IN MACRO-DYNAMIC THEORIES*

Michihiro OHYAMA

Abstract. This paper introduces two distinct hypotheses of price expectations within the framework of deterministic macro-dynamic models with continuous time. One is the hypothesis of predictive expectations conceivable for the community where individuals behave as identical leaders in respect of expectations. The other is the hypothesis of imitative expectations applicable to the community where individuals behave as identical followers in respect of expectations. These hypotheses are shown to generate potentially stable adjustment processes toward the steady state equilibrium in the well known models of monetary dynamics which exhibits saddle-point instability under the conventional hypothesis of myopic perfect foresight.

1. INTRODUCTION

It is well recognized that the dynamic behavior of macro-economic models depends crucially on the way the public is assumed to form expectations of future economic variables. In the deterministic models of macro-dynamics, a number of hypotheses, ranging form static expectations to myopic perfect foresight, have been employed to yield drastically different conclusions concerning the stability of steady-state equilibria. Many of these expectations hypotheses are, however, vulnerable to criticism because of the basically *ad hoc* nature of their formulation. In this respect, the theory of rational expectations, attributable to Muth (1961), stands out by emphasizing the relevance of the public's knowledge of the structure of the economy, in particular, the structure of stochastic processes generating the economy, to the formation of its expectations.

The growing body of literature in the theory of rational expectations is, however, saddled with restrictive assumptions of its own. In particular, it is largely concerned with the limiting case in which the public knows perfectly the true structure of the economy. This bias against ignorance and misinformation on the part of the public is not generally acceptable. Furthermore, individual citizens are usually assumed not only to be identical in every respect but also to act in complete unison to make their expectations self-fulfilling, which is certainly far more restrictive than the standard assumption of representative citizen.

* I am greatful to Professors Yoichi Shinkai and Hirofumi Uzawa for very helpful comments on the earlier draft of this paper. The research reported here is accomplished partially with financial support from the Seimeikai Foundation.

In this paper, we put forward two alternative hypotheses of price expectations, each of which reflects the representative citizens' (possibly inaccurate) knowledge of the underlying economic structure but depart from the basic assumption of rational expectations that individuals act in collusion to realize self-fulfilling expectations. The first hypothesis, to be labelled the predictive expectations hypothesis, arises in the case where individuals behave identically as leaders with regard to price expectations. The second hypothesis, to be called the imitative expectations hypothesis, is conceivable in the case where individuals behave identically as followers with regard to price expectations. While the former is compatible (in fact, most agreeable) with perfect information, the latter breaks down when the public knows the true structure of the economy.

The plan of the paper is as follows. In Sections 2 and 3 we formulate in general terms the predictive and imitative expectations hypothesis confining ourselves to the deterministic models of macro-dynamics with continuously flowing time. We then proceed to illustrate their implications for some well-known macroeconomic models. In Section 4, we consider a model of monetary economy à la Tobin (1965, 1969), the dynamic behavior of which is unstable under the hypothesis of myopic perfect foresight[1] but potentially stable under the hypothesis of adaptive expectations [Cagan (1956)],[2] or that of long run perfect foresight [Sargent and Wallis (1973)]. These traditional hypotheses of price expectations are, however, more or less extemporaneous in nature and difficult to swallow either logically or economically. Thus, it may be worthwhile to show that the steady state equilibrium of the model is also potentially stable under our hypothesis of predictive or imitative expectations. In Section 5, we turn to a slightly more complex model of exchange rate dynamics developed by Kouri (1976) and investigate the stability of its steady state equilibrium in the same spirit. As is shown by Kouri, the system exhibits saddle-point property with myopic perfect foresight. In contrast, predictive or imitative expectations mechanism is here again seen to yield potentially stable adjustment processes toward the equilibrium.

2. PREDICTIVE EXPECTATIONS—THE CASE OF IDENTICAL LEADERS—

We confine our attention throughout this paper to the deterministic stock-flow macro-equilibrium model of an economy with continuous time. The individual agents of the economy, whether firms or households, are assumed to be identical with respect to their knowledge of economic structure and with respect to their expectations of future rate of inflation. More specifically, let us assume that the value of the community's per capita real cash balances is determined at each point of time in terms of the community's expected rate of inflation and the objective state variables of the economy. The reduced form of the value of per capita real

[1] See Nagatani (1979) and Burmeister and Dobell (1970), Chapter 6.
[2] See Sidrasuki (1967).

balances may be written generally as

$$X_t = X(Z_{1t}, \cdots, Z_{nt}, \pi_t) \tag{1}$$

where X_t denotes the logarithm of the community's per capita real cash balance, Z_{it} $(i=1, \cdots, n)$ the ith objective state variable and π_t the community's expected rate of inflation, each at time t.

In this section, we assume that each individual forecasts future price movements independently in the belief that others follow his initiative promptly but adhere to the forecasted rate of inflation for some time to come. This assumption reflects the view that ordinary people are not only influenced by their leader's opinion but also governed by inertia. It is internally consistent in the steady state of the economy where the forecasted rate of inflation is continuously realized though time but could be self-contradictory in general. Under such circumstances, it is natural to suppose that each individual seeks to predict the future rate of inflation as connectly as possible in the light of his knowledge of the structure of the economy.

Let us first consider the limiting case where individuals are endowed with perfect information in the sense that they know the true structure of the economy, or the reduced form of the value of per capita real balances, (1). In this case, each individual will expect and forecast the rate of inflation which would be realized if all others held on to it for an infinitesimal span of time. To find such a rate of inflation, differentiable (1) with respect to time regarding π_t as an unknown constant. We obtain

$$\dot{p}_t/p_t = m_t - \sum X_{it}(Z_{1t}, \cdots, Z_{nt}, \pi_t)\dot{Z}_{it}$$

where p_t and m_t denote respectively the price level and the rate of per capita monetary expansion at time t and

$$X_{it} = \partial X/\partial Z_{it} \qquad (i=1, \cdots, n)$$

with a dot ($\dot{}$) over a variable indicating its time derivative.

Letting $\pi_t = \dot{p}_t/p_t$ in the above equation, we get

$$\pi_t = m_t - \sum X_{it}(Z_{1t}, \cdots, Z_{nt}, \pi_t)\dot{Z}_{it}\,. \tag{2}$$

It may be assumed that the time derivative of objective state variables are generally functions of objective state variables and the expected rate of inflation, i.e.,

$$\dot{Z}_{it} = \Omega(Z_{1t}, \cdots, Z_{nt}, \pi_t) \qquad (i=1, \cdots, n)\,.$$

Then, the desired expected rate of inflation can be obtained by solving (2) for π_t as

$$\pi_t^p = \pi^p(Z_{1t}, \cdots, Z_{nt}, m_t)\,. \tag{3}$$

We may label π_t^p the predictively expected rate of inflation with perfect information. Note that it differs from the expected rate of inflation under myopic perfect foresight. The latter is obtained by directly solving (1) for π_t in the form

$$\pi_t^m = \pi^m(Z_{1t}, \cdots, Z_{nt}, X_t)$$

where π_t^m is assumed equal to $\dot{p}_t/p_t$ thereby implying

$$\dot{X}_t = m_t - \dot{p}_t/p_t = m_t - \pi_t^m .$$

The obvious difficulty with this procedure is that the expected rate of inflation, rather than the price level, has to be interpreted as an equilibrating factor in the short-run markets of the economy. Furthermore, there is no plausible mechanism which ensures the coincidence of the expected rate of inflation with the actual rate of inflation even if individuals are assumed to be endowed with perfect information.[3]

Needless to say, it is generally unwarranted to presume that individuals are informed of the precise structure of the economy. Nonetheless, they may still be assumed to estimate, however imperfectly, the reduced form of the community's per capita real balances on the basis of available information at each point of time. For simplicity, let us assume that the estimated reduced form is written as

$$X_t = \tilde{X}(Z_{1t}, \cdots, Z_{nt}, \pi_t) \tag{4}$$

where function $\tilde{X}(\cdot)$ is given exactly and independently of time. Individuals must now work with (4), rather than (1), to derive the desired expected rate of expectation. If they are informed of the rate of per capita monetary expansion and the values of objective variables and are capable of foreseeing the short-run movements of state variables correctly, they should be able to find the expected rate of inflation which satisfies

$$\pi_t = m_t - \sum \tilde{X}_{it}(Z_{1t}, \cdots, Z_{nt}, \pi_t)\dot{Z}_{it} \tag{5}$$

where

$$\tilde{X}_{it} = \partial \tilde{X}/\partial Z_{it} \qquad (i = 1, \cdots, n) .$$

Thus, the pretictively expected rate of inflation with imperfect information may be generally expressed as

$$\tilde{\pi}_t^p = \tilde{\pi}_t^p(Z_{1t}, \cdots, Z_{nt}, m_t) \tag{6}$$

which coincides with π_t^p only in the limiting case of perfect information.

The steady state of the economy is defined as the state where the rate of per capita monetary expansion and all the other state variables of the system remain constant through time, i.e.,

$$m_t = m$$

[3] Sargent and Wallis (1973) proposed the hypothesis of long-run perfect foresight which dissolves these difficulties while retaining the assumption of rationality. Their view is, however, based on the extraordinarily strong assumption that the public is perfectly informed not only of the current state of economy but also of its entire future development.

and

$$\dot{Z}_{it}=0 \qquad (i=1, \cdots, n).$$

From (2) or (5), it should be clear that the expected rate of inflation falls in with the rate of per capita monetary expansion and also with the actual rate of inflation.

For comparison, it may be worth dwelling for a moment upon the hypothesis of adaptive expectations which is often considered to be plasusible in the context of imperfect information. With time continuously flowing, it is formulated as

$$\dot{\pi}_t=\beta(\dot{p}_t/p_t-\pi_t) \qquad \beta>0$$

meaning that individuals partially revise their expected rate of inflation when it did not materialize.[4] In view of (1), however, the actual rate of inflation $(\dot{p}_t/p_t)$ cannot be determined prior to the revision of expected rate of inflation. Thus, it should be generally impossible for individuals with imperfect information to revise their expectations in the postulated adaptive manner.[5] On the other hand, there seems to be no reason why individuals behave in conformity with the hypothesis if they know the precise structure of the economy.

3. IMITATIVE EXPECTATIONS—THE CASE OF IDENTICAL FOLLOWERS—

Discussing possible rational behavior under uncertainty, Keynes (1937) observed:

"Knowing that our individual judgement is worthless, we endeavour to fall back on the judgement of the rest of the world which is perhaps better informed. That is, we endeavour to conform with the behavior of the majority or the average. The psychology of a society of individuals each of which is endeavouring to copy the others leads to what we may strictly term a *conventional* judgement."[6]

Thus, individuals may alternatively be assumed to assess and copy in an identical manner what they believe to be the community's dominant expectations with regard to the future rate of inflation. They may start by adopting the leading opinion of experts which appears in newspapers or journals. But, as rational, economic men, they will soon learn to modify their assessment as they observe discrepancies between *actual* price levels and those which are consistent with their assessment of the community's expected rate of inflation in the light of their best knowledge of underlying economic structure.

To be more specific, let us consider the price level equation which is consistent with (1):

$$\ln p_t=\ln M_t-X(X_{1t}, \cdots, X_{nt}, \pi_t)$$

where M_t denotes the community's per capita nominal cash balances. In the

[4] This formulation of expectations mechanism was originally introduced by Cagan (1956).

[5] We owe this point to professor Hirofumi Uzawa.

[6] Keynes (1937), p. 214.

neighborhood of steady state, this can be approximated as

$$\ln p_t = \ln M_t - \bar{X}_t - \sum X_{it}(Z_{it} - \bar{Z}_i) - X_{\pi t}(\pi_t - \bar{\pi}) \tag{7}$$

where a bar ($\bar{}$) on the top of a variable indicates its steady state value and

$$X_{it} = \partial X / \partial Z_{it} \qquad (i = 1, \cdots, n)$$

and

$$X_{\pi t} = \partial X / \partial \pi_t$$

with derivatives being evaluated as $Z_{it} = \bar{Z}$ and $\pi_t = \bar{\pi}$.

Individuals are assumed to envision their own model of economy which generally differs from its true structure and behave identically on the assumption that the economy is in the vicinity of its steady state. They may then be taken to entertain their own price level equation

$$\ln p_t = \ln M_t - \bar{X}_t - \sum a_i(Z_{it} - \bar{Z}_{it}) + b(\pi_t - \bar{\pi}_t) \tag{8}$$

where $\bar{X}_t$, $\bar{Z}_{it}$ $(i = 1, \cdots, n)$ and $\bar{\pi}_t$ denote respectively the value of the community's per capita real balances, that of the ith objective variables and the community's expected rate of inflation which they anticipate at time t to prevail in the steady state. For simplicity, we assume that coefficients a_i $(i = 1, \cdots, n)$ and b are constant over time.

At each point of time, individuals endeavour to assess what they believe to be the community's expected rate of inflation and adopt it as their own. Given this assessed rate of inflation (π_t^a), they are able to compute the price level (p_t^a) which they expect to materialize in the light of (8) if their assessment of the community's expected rate of inflation is correct. The formula is given by

$$\ln p_t^a = \ln M_t - \bar{X}_t - \sum a_i(Z_{it} - \bar{Z}_{it}) + b(\pi_t^a - \bar{\pi}_t)\,. \tag{9}$$

On the other hand, individuals are also able to ascertain the expected rate of inflation (π_t^c) which is in consonance with the actually observed price level (p_t) and other relevant variables at time t. It is implicitly defined by

$$\ln p_t = \ln M_t - \bar{X}_t - \sum a_i(Z_{it} - \bar{Z}_{it}) + b(\pi_t^c - \bar{\pi}_t)\,. \tag{10}$$

From (9) and (10), we have

$$\begin{aligned} \pi_t^c - \pi_t^a &= (1/b)(\ln p_t - \ln p_t^a) \\ &= (1/b)\{(\bar{X}_t - X_t) + \sum a_i(Z_{it} - \bar{Z}_{it}) + b(\bar{\pi}_t - \pi_t^a)\} \end{aligned}$$

for $b \neq 0$. Clearly, the observed price level (p_t) deviates from the assessed price level (p_t^a) unless individuals are possessed of perfect information. They are likely to attribute the discrepancy to their misassessment of the community's expected rate of inflation to the extent that they believe in their comprehension of economic structure. In this light, they may be assumed to modify their assessment in the following adaptive manner:

$$\dot{\pi}_t^a = \beta(\pi_t^c - \pi_t^a)$$
$$= (\beta/b)\{(\bar{X}_t - X_t) + \sum a_i(Z_{it} - \bar{Z}_{it}) + b(\bar{\pi}_t - \pi_t^a)\} \tag{11}$$

where β is a positive constant. This means that individuals revise their assessment of the community's expected rate of inflation in an upward (resp. downward) direction when the actual price level is higher (resp. lower) than the price level consistent with the assessment.

To consider the application of expectations adjustment mechanism (11), it is necessary to specify the time paths of $\bar{X}_t$, Z_{it} $(i=1, \cdots, n)$ and $\bar{\pi}_t$. To simplify matters, let us consider only three special cases. The first special case postulates that individuals identify the current state with the steady state and set $\bar{X}_t = X_t$, $\bar{Z}_{it} = Z_{it}$ $(i=1, \cdots, n)$ and $\pi_t^a = \bar{\pi}_t$. Clearly, this amounts to static expectation. In the second special case, individuals are assumed to set $\bar{X}_t = X_t$ and $\bar{Z}_{it} = Z_{it}$ $(i=1, \cdots, n)$ while adhering to the unique expected rate of inflation $(\bar{\pi}_0)$ which they anticipate to prevail in the steady state. In this case, (11) simplifies to

$$\dot{\pi}_t^a = \beta(\bar{\pi}_0 - \pi_t^a) \tag{12}$$

that is, the simple hypothesis of regressive-imitative expectation.[7] The third special case arises when individuals persevere with their initial anticipation of the steady state values of all relevant variables. (11) then becomes

$$\dot{\pi}_t^a = (\beta/b)\{(\bar{X}_0 - X_t) + \sum a_i(Z_{i0} - \bar{Z}_{it}) + b(\bar{\pi}_0 - \pi_t^a)\}$$

where X_0 and Z_{i0} $(i=1, \cdots, n)$ denote respectively the value of the community's per capita real balances and the ith objective state variable initially anticipated to materialize in the steady state. This formula may be called the general hypothesis of regressive-imitative expectation.

The actual rate of inflation equals the rate of per capita monetary expansion (m) in the steady state of the economy. If individuals correctly anticipate this rate from the beginning, the simple hypothesis of regressive-imitative expectation implies that the community's expected rate of inflation coincides with the actual rate of inflation in the steady state. Similarly, if individuals anticipate correctly the steady state values of all relevant variables, the community's expected rate of inflation will be self-fulfilling in the steady state under the general hypothesis of regressive-imitative expectation.

[7] One is tempted to formulate the hypothesis of regressive expectations in the form such as

$$\dot{\pi}_t^a = \beta(\bar{\pi}_0 - \dot{p}_t/p_t)$$

in the present context. It should be noted, however, that the rate of inflation $(\dot{p}_t/p_t)$ is not determined prior to the revision of the expected rate of inflation. Clearly, the formulation of regressive-imitative expectations in the text is exempt from this logical difficulty.

4. A SIMPLE MODEL OF MONETARY DYNAMICS

To illustrate the implications of alternative mechanisms of price expectation, let us first consider the stability of a simple model of monetary dynamics. It has been argued that if individuals correctly but myopically perceive the rate of inflation, the steady state equilibrium in the models of money and growth becomes a "saddlepoint" in the sense that the system diverges more and more from the equilibrium as time goes on if it is not initially placed on the right path. To focus upon the crucial property of such instability, however, it would suffice to investigate a model of monetary expansion with static population and constant capital stock. In what follows, we omit the subscript t to variables to ease our notation.

Let us assume that the economy produces a constant output (Y) by fully employing the given labor force and capital stock (K). There are two assets, i.e., money (M) and equity capital (K), or the entitlements to the existing capital stock. The stock of real wealth (W) is then defined by

$$W=M/p+qK=N+qK \tag{13}$$

where p denotes the price level, N the community's real cash balances and q the "real" price of equity capital. The demand for real cash balances is a function of the expected rate of inflation (π), the real price of equity capital, the level of output and the stock of wealth. Therefore, in the monetary equilibrium, we have

$$L(\pi, q, Y, W)=N \tag{14}$$

which, in view of the wealth constraint of the economy, also ensures that the demand for equity capital equals its supply.[8]

On the other hand, the aggregate demand for commodities consists of private consumption (C) and government expenditure (G). For the sake of simplicity, the government is assumed to finance its expenditure by increasing the stock of money by a constant rate (m), or

$$G=mN \qquad (m\geqq 0)\,.$$

Private consumption is a function of real income (Y) and the stock of real wealth. The flow equilibrium of the economy is then described by

$$Y=C(Y, W)+mN\,. \tag{15}$$

We define the short-run equilibrium of the economy as the state where (14) and (15) hold simultaneously.

Focusing on the dynamics of monetary expansion, we shall first examine the implications of myopic perfect foresight and then those of predictive and imitative expectation formations developed in the preceding sections of this paper.

[8] The present model is adapted from Tobin's formulation of a "money-capital economy." See Tobin (1969), pp. 19–23.

(a) *Myopic Perfect Forsight*

The assumption of myopic perfect foresight postulates

$$\pi = \dot{p}/p = m - \dot{X} \tag{16}$$

where X is, as before, the logarithm of the stock of real balances ($\ln N$). Totally differentiating (14) and (15) with respect to time, and substituting from (16), we get

$$\dot{\pi} = \frac{\{(1-L_W)C_W K + (L_q + L_W K)(m + C_W)\}N}{L_\pi C_W K}(m - \pi) \tag{17}$$

where functional symbols with subscripts signifies the partial derivatives with respect to variables indicated by subscripts, e.g., $L_\pi \equiv \partial L/\partial \pi$. It may be assumed that $L_q > 0$, $C_W > 0$, $L_\pi < 0$ and $0 < L_W < 1$. Then, we have $\partial \dot{\pi}/\partial \pi > 0$ implying the instability of the steady state equilibrium where $\pi = m$. Once the actual and expected rate of inflation diverges from the given rate of monetary expansion, a cumulative process of inflation or deflation is bound to develop.

It should be noted that the short-run equilibrium conditions (14) and (15) are interpreted to determine $\pi \equiv \dot{p}/p$ and q given N, K, Y and m. In other words, the expected rate of inflation and the price of equity capital are adjusted to bring about asset and commodity market equilibria while the price level and the nominal stock of money remain constant in the short run. This interpretation of short-run equilibrium is at variance with the normal conception of market clearance.

(b) *Predictive Expectations*

The inspection of the short-run equilibrium conditions (14) and (15) reveals that the equilibrium value of real balances is a function of the expected rate of inflation, the given stock of capital, the real output and the rate of monetary expansion, i.e.,

$$N = N(K, Y, m, \pi) . \tag{18}$$

Individuals are now supposed to adopt the expected rate of inflation which would be self-fulfilling if they sticked to it knowing that K, Y and m are constant through time. If the expected rate of inflation is not to be modified, (18) implies that the real balances of the community $N(= M/P)$ remain unchanged through time, or

$$m - \dot{p}/p = 0 . \tag{19}$$

Letting $\dot{p}/p = \pi$ in (19), we find that the desired expected rate of inflation equals the rate of monetary expansion, or,

$$\pi = m . \tag{20}$$

Note that the expected rate of inflation is self-fulfilling in this case. To obtain this result, individuals are not required to have perfect information about the basic structure of the economy. They have to know that the reduced form of real balances is a function of the expected rate of inflation, the rate of monetary expansion, the level of real output and the stock of capital, but they need not know

its precise functional form.

(c) *Regressive-Imitative Expectation*

Give K, Y and m, individuals are supposed to conform with the expected rate of inflation which they believe to be dominant in the community at each point of time. This gives rise to the modification of price expectation in the regressive-imitative manner, or

$$\dot{\pi}=(\beta/b)\{(\bar{X}_0-X)+b(\bar{\pi}_0-\pi)\} \tag{21}$$

where π_0 is the rate of inflation which individuals expect to prevail in the steady state. If individuals know that the rate of monetary expansion (m) is to be constant over time, it is reasonable to assume that $\pi_0=m$. In view of (14) and (15), the differentiation of (21) with respect to π yields

$$\frac{\partial\dot{\pi}}{\partial\pi}=-\beta(1-\eta_\pi/b)$$

where

$$\eta_\pi\equiv-\frac{\partial X}{\partial\pi}=-\frac{L_\pi C_W K}{\{(1-L_W)C_W K+(C_W+m)(L_q+L_W K)\}N}>0$$

measuring the responsiveness of the community's equilibrium real cash balances to a change in the expected rate of inflation.[9] Thus, the dynamic adjustment process described by (21) is globally stable if $b>\eta_\pi$ everywhere, or if individuals always overstimate the rate of change in the community's real cash balances resulting from a change in price expectation. Instability would prevail otherwise.

5. A SIMPLE MODEL OF EXCHANGE RATE DYNAMICS

The model of monetary dynamics which we have just discussed is useful in bringing into sharp focus the different implications of alternative expectation mechanisms. It is, however, inadequate in the sense that all the state variables of the economy are assumed to be constant over time. Therefore, let us consider in this section yet another example. It is a simple model of exchange rate dynamics, attributable to Kouri (1976), in which the economy's stock of foreign exchanges may vary through time as a result of its balance of trade disequilibrium.

The economy is supposed to produce only traded goods the relative price of which is fixed on the world market. The world price level is assumed to be constant

[9] Since $X=\ln N$, we have

$$n_\pi=\frac{\partial X}{\partial\pi}=\frac{1}{p}\frac{\partial p}{\partial\pi}.$$

Thus n_π may also be interpreted to measure the responsiveness of the current price level to a change in the expected rate of inflation.

and equal to unity so that the domestic price level (p) is equal to the exchange of rate. Labor is fully employed and domestic output (Y) is constant. The stock of financial wealth consists of domestic money (M) and foreign money (F):

$$W = M/p + F = N + F . \tag{22}$$

We assume that foreigners do not hold domestic money. Then, as in the foregoing section, we may represent the asset market equilibrium by

$$L(\pi, Y, W) = N \tag{23}$$

which, given Y, F, M and π, determines the equilibrium rate of exchange (p) instantaneously. Equations (22) and (23) may be solved for the reduced forms of real balances and wealth:

$$N = N(\underset{+}{Y}, \underset{+}{F}, \underset{-}{\pi}) \tag{24}$$

$$W = W(\underset{+}{Y}, \underset{+}{F}, \underset{-}{\pi}) \tag{25}$$

on the assumption that $L_\pi < 0$, $L_Y > 0$ and $L_W > 0$. For example, an increase in the expected rate of inflation reduces the stock of real balances and hence the real value of wealth by causing the exchange rate to depreciate.

The balance of trade (B) is the difference between domestic output and domestic expenditure which, as in the foregoing section, is assumed to comprise private consumption (C) and government expenditure (G) being financed by monetary expansion. The rate of change in the supply of money ($m = \dot{M}/M$) is assumed to be constant over time. Note that the change in the stock of foreign money per unit of time ($\dot{F}$) equals the balance of trade. Thus,

$$\dot{F} = B = Y - C(Y, W) - mN . \tag{26}$$

In view of (24) and (25), we may rewrite (26) as

$$\dot{F} = B(\underset{+}{Y}, \underset{+}{F}, \underset{-}{m}, \underset{+}{\pi}) \tag{27}$$

on the assumption that $0 < C_Y < 1$ and $C_W > 0$.

The steady state of the system is defined by the balance of trade equilibrium:

$$B(Y, F, m, \pi) = 0 , \tag{28}$$

the monetary equilibrium:

$$N(\pi, Y, F) = N \tag{29}$$

and the expectational equilibrium:

$$\pi = m . \tag{30}$$

We wish to investigate the stability of the steady state under alternative expectations hypotheses.

(a) *Myopic Perfect Foresight*

It can be shown that the steady state is a saddle point under myopic perfect foresight. The dynamic behavior of the stock of real balance and the stock of foreign money is illustrated in Fig. 1. The *NN* curve shows the combination of the stock of real balances and that of foreign money which satisfies (29) and (30). The *FF* curve shows the combination of real balances and that of forein money which satisfies (26) for $\dot{F}=0$. The system conerges to the steady state equilibrium only if it moves along the *TT* trajectory i.e., only in the case of long run perfect foresight

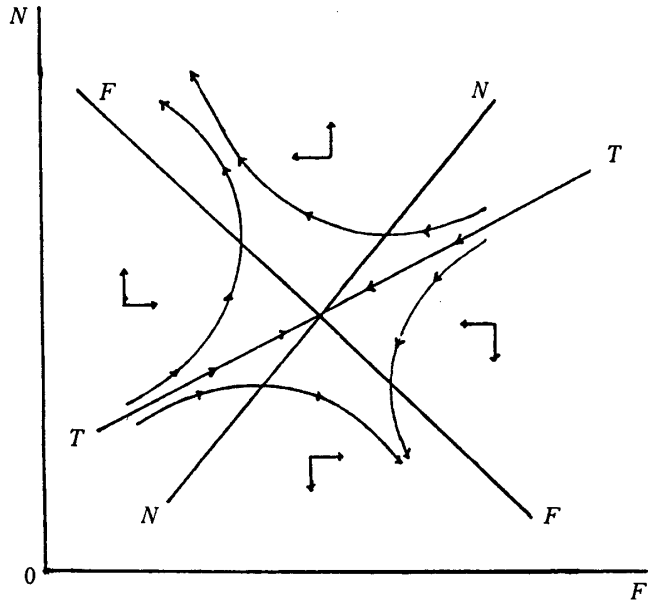

Fig. 1. The adjustment process with myopic perfect foresight.

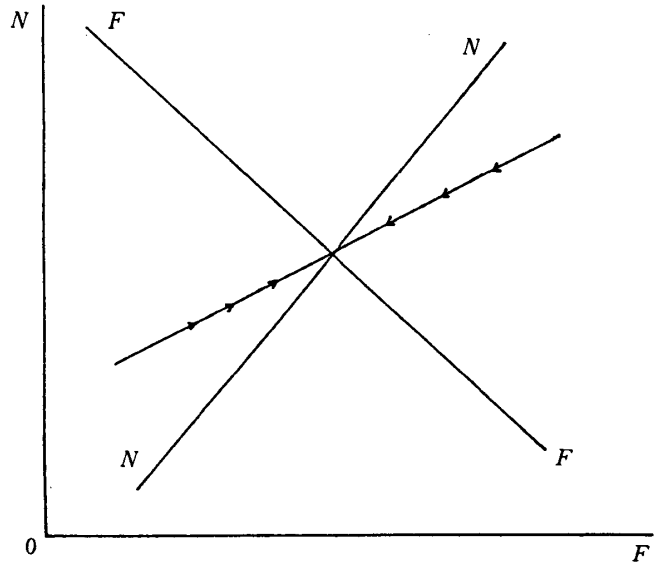

Fig. 2. The adjustment process with predictive expectations.

(Kouri (1976)) or rational expectations. But this solution has its own problem. Given the initial value of F and M, individuals must adjust p appropriately at the initial point of time to get on this trajectory. If each individual is a price taker, however, it would be impossible to achieve this adjustment were it not for complete collusion among individuals.

(b) *Predictive Expectations*

Suppose first that individuals are endowed with perfect information. Then, in the light of the reduced form of real balances (24) and that of the balance of trade (27), we obtain the expected rate of inflation under predictive expectations hypothesis as

$$\pi = m - X_F \dot{F} = m - X_F B(Y, F, m, \pi) \tag{31}$$

where X is the logarithm of real balance ($\ln N$) and X_F the partial derivative of X with respect to F. Since $X_F > 0$, (31) implies that the expected rate of inflation is lower (resp. higher) than the rate of monetary expansion when the balance of trade shows a surplus (resp. deficit). It should also be noted that the expected rate of inflation increases as the trade account surplus diminishes over time. This implication of (31) is contradictory to the basic assumption of the predictive expectations hypothesis, i.e., $\dot{\pi} = 0$. Thus the expected rate of inflation is not self-fulfilling here.

Taking account of (31), differentiate (27) with respect to F to obtain

$$\frac{\partial \dot{F}}{\partial F} = \frac{1}{1 + B_\pi X_F + B X_{F\pi}} \{(1 + B X_{F\pi}) B_F - B B_\pi X_{FF}\}$$

where $X_{F\pi} = \partial X_F / \partial \pi$ and $X_{FF} = \partial X_F / \partial F$. In general, we are not able to evaluate the derivative. But in the neighborhood of steady state equilibrium where $B = 0$, we have

$$\frac{\partial \dot{F}}{\partial F} = \frac{B_F}{1 + B_\pi X_F} < 0 .$$

Therefore, we may conclude that the steady state equilibrium is at least locally stable. Thus, predictive expectation, coupled with perfect information, implies that the system converges to the steady state equilibrium as time elapses if it starts from a point sufficiently near the equilibrium.

Next, consider the more realistic case in which individuals are ignorant of the precise structure of the economy. Even though they are not endowed with perfect information, they may be taken to approximate (31) as

$$\pi = m - a_F \tilde{B}(Y, F, m, \pi) \tag{32}$$

where a_F is some constant and $\tilde{B}$ the perceived reduced form of the balance of trade. From (27) and (32), we obtain

$$\frac{\partial \dot{F}}{\partial F}=\frac{(1+a_F\tilde{B}_\pi)B_F-a_F B_\pi \tilde{B}_F}{1+a_F\tilde{B}_\pi}.$$

Therefore, provided that $1+a_F\tilde{B}_\pi>0$, the necessary and sufficient condition for the global stability of the steady state equilibrium is

$$B_F<\frac{a_F B_\pi}{1+a_F\tilde{B}_\pi}\tilde{B}_F \tag{33}$$

which can be satisfied if individuals correctly perceive the reduced form of the balance of trade ($\tilde{B}_\pi=B_\pi$ and $\tilde{B}_F=B_F$) or if they incorrectly presume that the current balance of trade will persist in the immediate future ($\tilde{B}_\pi=\tilde{B}_F=0$). Under condition (33), the system will converge to the steady state equilibrium as time elapses but there is no guarantee that expectations are continuously fulfilled in the process. The time path of the stock of real balances and the stock of money is illustrated in Fig. 2.

(c) *Regressive-Imitative Expectation*

Suppose that individuals correctly perceive the steady state values of real cash balances and foreign assets and always hold on to them. Then, general regressive-imitative expectations formula reduces to

$$\dot{\pi}=\frac{\beta}{b}\{(\ln \bar{N}-\ln N)+a_F(F-\bar{F})+b(m-\pi)\} \tag{34}$$

where $\bar{N}$ and $\bar{F}$ are implicitly defined by

$$N=N(Y, \bar{F}, m);$$

$$B(Y, \bar{F}, m, m)=0.$$

Equation (34), together with

$$\dot{F}=B(Y, F, m, \pi) \tag{27}$$

constitutes the dynamic adjustment process of the system. Differentiating (27) and (34) with respect to F and π, we get

$$\frac{\partial \dot{F}}{\partial F}=B_F<0$$

$$\frac{\partial \dot{F}}{\partial \pi}=B_\pi>0$$

$$\frac{\partial \dot{\pi}}{\partial F}=(\beta/b)(a_F-\eta_F)$$

$$\frac{\partial \dot{\pi}}{\partial \pi}=-(\beta/b)(b-\eta_\pi)$$

where

$$\eta_\pi \equiv -\frac{\partial X}{\partial \pi} = -\frac{L_\pi}{(1-L_W)N} > 0$$

and

$$\eta_F \equiv \frac{\partial X}{\partial F} = \frac{L_W}{(1-L_W)N} > 0$$

measuring respectively the responsiveness of the community's demand for real cash balances to changes in the expected rate of inflation and the stock of foreign assets. Therefore, the process is locally stable if $a_F \leqq \eta_F$ and $b \geqq \eta_\pi$, which means that individuals do not underestimate (resp. overestimate) the rate of change in the community's real cash balances resulting from changes in the expected rate of inflation (resp. in the stock of foreign money). Figure 3 depicts a time path of the expected rate of inflation and the stock of foreign money. Here, the *FF* curve shows the combination of the expected rate of inflation and the stock of foreign money which satisfies (28). The $\pi\pi$ curve shows the combination of the expected rate of inflation and the stock of foreign money which satisfies (34) for $\dot{\pi} = 0$. Note that it exhibits cyclical movements around the steady state equilibrium implying overshootings in exchange rate adjustment in contrast to monotonic convergence under the hypotheses of long run perfect foresight and predictive expectations.

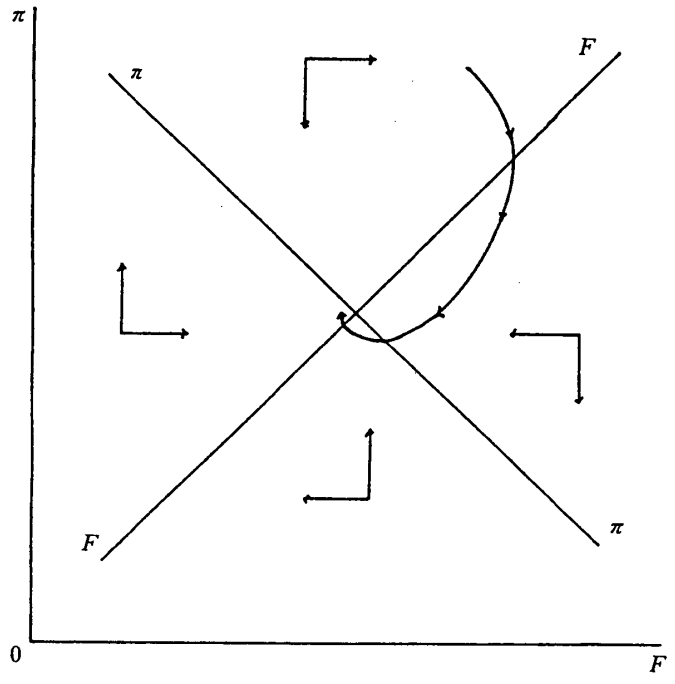

Fig. 3. The adjustment process with regressive-imitative expectation.

6. CONCLUDING REMAKRS

In this paper, we have introduced two distinct hypotheses of price expectations within the framework of deterministic macro-dynamic models with continous time. One is the hypothesis of predictive expectations conceivable for the community where individuals behave as identical leaders in respect of expectations. The other is the hypothesis of imitative expectations pertaining to the community where individuals behave as identical followers in respect of expectations. These hypotheses are based on the recognition that individuals make best use of their (possibly erroneous) knowledge of underlying economic structure in forming expectations.

Our illustrative analysis suggests that the expectations hypotheses of this paper are capable of generating stable adjustment processes in many dynamic systems. In fact, the hypothesis of predictive expectations, together with the assumption of perfect information, ensures that the steady state of the economy is at least locally stable. This conclusion forms a sharp contrast with the implication of myopic perfect foresight that self-fulfilling expectations are generally incompatible with economic stability. The stability property of predictive expectations carries over to the cases where available information is less than perfect.

The hypothesis of imitative expectations also allows the process of adjustment to be potentially stable. The stability result under this hypothesis depends, however, on how individuals estimate the community's responsiveness to changes in its state variables. For instance, if individuals underestimate the responsiveness of the current price level to a change in the expected rate of inflation which they believe to prevail in the community, they are supposed to modify their assessment of the community's expected rate of inflation in a destabilizing direction. The system may exhibit instability under such circumstances.

Needless to say, there are a host of economic variables other than the rate of inflation the future movements of which arouse the public's concern. It is merely for the sake of expository simplicity that we have confined ourselves to the expected rate of inflation. It should be clear that the basic concepts and methods employed in this paper are also applicable to the formulation of expectations mechanism regarding other key variables such as the growth rate of income and the rates of variation in the prices of securities.

Keio University

REFERENCES

Burmeister, E. and A. R. Dobell, *Mathematical Theories of Economic Growth*, London: Macmillan, 1970.

Cagan, P., "The Monetary Dynamics of Hyperinflation." in M. Friedman, ed., *Studies in the Quantity Theory of Money*, Chicago: University of Chicago Press, 1956.

Keynes, J. M., "The General Theory of Employment," *Quarterly Journal of Economics*, **51** (937), 209–223.

Kouri, P. J. K., "The Exchange Rate and The Balance of Payments in the Short Run and in the Long Run: A Monetary Approach," *The Scandinavian Journal of Economics*, **78** (1976), 280–304.

Muth, J. F., "Rational Expectations and The Theory of Price Movements," *Econometrica*, **29** (1961), 315–335.

Nagatani, K., "A Note on Professor Tobin's Money and Economic Growth," *Econometrica*, **38** (1970), 171–175.

Sargent, T. J. and N. Wallace, "The Stability of Models of Money and Growth with Perfect Foresight," *Econometrica*, **41** (1973), 1043–1048.

Sidrauski, M., "Rational Choice and Pattern of Growth in a Monetary Economy," *American Economic Review*, **57** (1967), 534–544.

Tobin, J., "Money and Economic Growth," *Econometrica*, **33** (1965), 671–684.

———, "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit and Banking*, **1** (1969), 15–29.