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UNCERTAINTY, LIQUIDITY AND THE DEMAND FOR MOMEY*

Anjan MUKHERJI and Amal SANYAL

Abstract. An agent holding liquid assets may believe that by doing so he would buy time to ascertain more definitely the parameters relating to investment in plant, equipment or real estate i.e., investments which are either irreversible or can be reversed only at substantial cost. We show that this consideration is sufficient to generate a positive demand for liquid assets, although such assets provide a lower (often zero) yield. The individual is assumed to be completely ignorant about the furture and accordingly uses the maximin criterion to choose among various alternatives. We also examine the implications for aggregate demand for money, bonds and 'real' investment and compare them with the Keynesian formulation which models the demand for liquidity (money) as arising out of fear of capital loss in the bond portfolio.

I. INTRODUCTION

In Keynes, liquidity preference is synonymous with the asset demand for money. But if by liquidity we mean the property of convertibility into money at short notice without significant loss, then this is a property of many assets, including bonds, against which Keynes posited the agent's demand for liquidity! Since Keynes (4, Chapters 13 & 17) derived the demand for liquidity by invoking the fear of capital loss from bond holding, it is difficult to extend his argument to rationalise the demand for other liquid assets, particularly so because they often compete with assets which have both a promise of higher yield and carry no substantial fear of capital loss.

The present paper tries to derive the demand for liquid assets as originating in the advantage associated with the property of liquidity itself, i.e. from the contrast that these assets make with another group of high yielding assets which are inconvertible or illiquid, in the sense that they cannot be sold off without significant loss in future. Classification of assets on the basis of this property has been done by Marschak (6, pp. 182–183) and has been used in a somewhat different context by Hirshleifer (3). Hicks (2, Chapter 2, pages 38–43 in particular) suggests that if there is uncertainty relating to the future yield of illiquid assets,

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then the cost of making a wrong choice of these assets may prove large over its life time, since it cannot be sold off when the mistake is realised. If the agent feels that more certain knowledge about the yield on these assets can be obtained by waiting for a while, and the cost of making a mistake is larger than that of carrying a low yielding but liquid portfolio, then there is a *prima facie* reason for demanding liquidity. Note that even if the yield on the various illiquid assets under different states of nature were all larger than that on a liquid asset, the argument applies as long as there is large enough variation among these yield rates, making the cost of a mistake large. Below we attempt to develop these arguments more formally.

A related enquiry also developed in the paper is to examine the implications of this argument on the demand for money. We derive a positive demand for money at low interest rates. Regarding the slope of the demand for money with respect to interest rates, we may note that the usual negative slope in Keynesian theory follows from the continuous shift of households from money to bonds as the interest rate rises. In our formulation, the demand for money changes through substitution not only by bonds but also by other illiquid assets. The present value of the cost of making a wrong choice in the illiquid assets declines as the rate of interest rises, since it reduces the time rate of discount. Thus a rise in the interest rate encourages, other things remaining equal, a substitution of money by illiquid assets. The negative slope of the demand for money as the interest rises, thus materialises only if the switch from money to bonds by households occurs continuously so as to neutralise the substitution by money of illiquid assets within the individual portfolios.

The model used below considers a choice problem involving money, bonds and two illiquid assets. The return on the last two are larger than that on bonds under all eventualities. The rates of return on the illiquid assets are given a configuration such that neither is dominating under all eventualities. For simplification, it is assumed that initially there is complete ignorance about the future state of the world; but complete knowledge prevails after one period. Finally for modelling choice under complete ignorance, we use the maxmin criterion, the rationale for which will be explained below

II. THE MODEL

The agent is assumed to possess a nominal quantity of money M at the beginning of period 1; writing the nominal holdings of money, bonds and assets A_i $(i=1, 2)$ as x_0, x_B, x_i respectively, the budget constraint is defined by the set

$$B=\{x=(x_0, x_B, x_1, x_2): x_0, x_B, x_1, x_2 \geqq 0 ; \quad x_0+x_B+x_1+x_2 \leqq M\} . \tag{1}$$

Money carries no return. Bonds yield r per period per unit of nominal holding. They can be converted into money after the first period and the expected rate of conversion is 1 nominal unit of bond $\equiv c$ units of money, $c \geqq 0$. Clearly $c \gtrless 1$

implies the expectation regarding capital gains or loss on the bonds. Assets A_i ($i=1, 2$) are illiquid; if acquired, *they last infinitely*. Two events can materialise at the end of period 1; these events E_j ($j=1, 2$) are known today but the agent has no assessment of the probability of their occurrence. The uncertainty is expected to decline over time and this is captured by the assumption that the agent has no assessment about the outcomes at the beginning of period 1, but at the beginning of period 2, he would know with certainty which event has occurred. The return per unit of nominal holding of the asset A_i in the eventuality that E_j occurs is known to be r_{ij}.

Since we wish to explore the demand for assets arising from their property of liquidity or convertibility, we assume that in terms of yield, bonds are dominated by illiquid assets under all eventualities, i.e.,

$$r < r_{ij} \quad \text{all} \quad i \quad \text{and} \quad j\,.$$

Further, since the net return on bonds (i.e. net of capital gains or losses) is $r+c-1$ per unit of nominal holding, we sjhall assume that

$$r+c-1 < r_{ij} \quad \text{all} \quad i \quad \text{and} \quad j\,.$$

Without any loss of generality of the future argument, we assume further that

$$r_{22} > r_{11} > r_{12} > r_{21}\,. \tag{R}$$

Additionally, we need to assume that r_{12} and r_{21} are small relative to r_{11} and r_{22}. In otherwords, that by making a wrong decision, one may incur a substantial loss. The exact relationship between the r_{ij}'s will be spelt out in sections below and some examples would be provided to show that the configuration required for money holding or bond holding is plausible.

The elements x_1, x_2 chosen at the beginning of period 1 cannot, by assumption, be ever *decreased*. However, since the state of the world would be completely revealed at the beginning of period 2, if $x_0 > 0$ in period 1, it will be converted at the beginning of period 2 into the A_i for which the return is now known to be higher. Similarly, for bonds. As $r < r_{ij}$ for all i, j, bonds cannot be purchased at the beginning of period 1 with a view to hold them for ever. Thus if $x_B > 0$ in period 1, in period 2 it will be converted into the A_i with the higher yield (see Claim 1, Appendix). Finally, the nominal earnings in period 1 (from bonds and assets) would all be converted into the higher yielding asset in period 2. After period 2, there would be no further change in the portfolio.

Given the above description, the income I_{ij} in period i if event j occurs, from a portfolio $x=(x_0, x_B, x_1, x_2)$ may be written as

$$I_{11} = x_B(r+c-1) + x_1 r_{11} + x_2 r_{21}\,,$$

$$\begin{aligned} I_{21} &= \{x_0 + x_B(r+c) + x_1 r_{11} + x_2 r_{21}\} r_{11} + x_1 r_{11} + x_2 r_{21} \\ &= \{x_0 + x_B(r+c) + x_1(1+r_{11}) + x_2 r_{21}\} r_{11} + x_2 r_{21}\,, \end{aligned}$$

$$I_{m1}=I_{21} \quad \text{for all} \quad m \geqq 2\,;$$

$$I_{12}=x_B(r+c-1)+x_1r_{12}+x_2r_{22}\,,$$

$$\begin{aligned} I_{22} &= \{x_0+x_B(r+c)+x_1r_{12}+x_2r_{22}\}r_{22}+x_1r_{12}+x_2r_{22} \\ &= \{x_0+x_B(r+c)+x_1r_{12}+x_2(1+r_{22})\}r_{22}+x_1r_{12}\,, \end{aligned}$$

and

$$I_{m2}=I_{22} \quad \text{for all} \quad m \geqq 2\,.$$

Now writing the sum of discounted stream of incomes, taking a discount rate δ, we have

$$I_1=I_{11}+\delta I_{21}+\delta^2 I_{31}+\cdots$$

and

$$I_2=I_{12}+\delta I_{22}+\delta^2 I_{32}+\cdots\,.$$

so that

$$I_1=I_{11}+\mu I_{21} \qquad \text{where} \quad \mu=\delta/1-\delta$$

and

$$I_2=I_{12}+\mu I_{22}.$$

Thus

$$\begin{aligned} I_1(x) &= \mu r_{11}x_0+\{(r+c)(1+\mu r_{11})-1\}x_B \\ &\quad +\{1+\mu(1+r_{11})\}\{r_{11}x_1+r_{21}x_2\} \end{aligned}$$

and

$$\begin{aligned} I_2(x) &= \mu r_{22}x_0+\{(r+c)(1+\mu r_{22})-1\}x_B \\ &\quad +\{1+\mu(1+r_{22})\}\{r_{12}x_1+r_{22}x_2\}. \end{aligned}$$

In the above formulation of discounted income, we can analyse the individual household's behaviour with any subjective rate of discount. However, since we will later look at the outcome for the entire market for various assets, it will be more appropriate for our purpose to take this rate as $1/(1+r)$ for all individuals. Admittedly this implies a belief in an equilibrating market process which establishes a market rate of discount to which as price takers individual households equate their subjective rates through equilibrating adjustments. No attempt has been made to analyse such a process in the current paper. When necessary, then, we shall use the above rate of discount which in turn would imply

$$\mu=\frac{\delta}{1-\delta}=\frac{1}{r}\,.$$

We shall assume that the individual chooses x from B so as to solve

$$\operatorname*{Max}_{x \in B} \operatorname*{Min}_{1,2} \{I_1(x),\ I_2(x)\}. \tag{P}$$

Before proceeding to an analysis of the above optimisation problem, we should explain why such a problem may be of interest for our decision maker. Recall that in our introduction, we mentioned that the investor is completely ignorant about what may occur. In such situations, if x is an act (a choice of a portfolio from B, in the present situation) and $I_j(x)$ is the outcome if E_j occurs, then under a rather straightforward characterisation due to Arrow and Hurwicz (1), any decision criterion satisfying some appealing axioms, takes into account only the maximum and minimum outcome associated with any act. In otherwords, if $m(x) = \text{Min}_j\, I_j(x)$ and $M(x) = \text{Max}_j\, I_j(x)$, then all x's are ordered according to some ordering of the ordered pairs $(m(x), M(x))$. We choose, in our context, an ordering of $(m(x), M(x))$ which is lexicographic in nature. Amongst all $x \in B$, choose the one with the largest $m(x)$.

Thus on a formal basis appealing to the Arrow-Hurwicz axioms, we may come up with (P) as a possible decision rule: the *maximin criterion.* For a discussion of this rule and the contribution of Arrow and Hurwicz, see Luce and Raiffa (5, pages 278–306). In particular, however, recall the behaviour of agents we are trying to model: these agents wish to buy time and find out more about the events and so they want to make a correct decision. In otherwords, our typical investor is conservative. For such decision makers, the maxmin criterion makes good sense (see, for example, Luce and Raiffa (5, page 279, third para)).

It is shown in the appendix, that given (R) if r_{12} is small enough and if there are no substantial capital gains, solving the problem (P) reduces to solving the following linear programming problem (see Claims 2 –5 and subsequent discussion in the Appendix):

$$\begin{aligned} &\text{Max } \gamma_0 x_0 + \gamma_B x_B + \gamma, \\ &\text{s.t. } x_0(1+\theta_B) + x_B\{1+(r+c)\theta_B\} \leqq M\,, \\ &\quad x_0,\ \ x_B \geqq 0, \end{aligned}$$

where

$$\gamma_0 = \left[\mu r_{11} - \frac{1+\mu(1+r_{11})}{1+\theta_2}\{r_{11}(\theta_2-\theta_B) + r_{21}(1+\theta_B)\}\right], \tag{2}$$

$$\gamma_B = \left[(r+c)(1+\mu r_{11}) - 1 - \frac{1+\mu(1+r_{11})}{1+\theta_2}\{r_{11}(\theta_2-(r+c)\theta_B) + r_{21}(1+(r+c)\theta_B)\}\right], \tag{3}$$

$$\gamma = \frac{1+\mu(1+r_{11})}{1+\theta_2}[\theta_2 M r_{11} + r_{21} M]\,, \tag{4}$$

$$\theta_B = \frac{\mu(r_{22}-r_{11})}{(r_{11}-r_{12})(1+\mu)+\mu(r_{11}^2-r_{22}r_{12})}, \tag{5}$$

and

$$\theta_2 = \frac{(r_{22}-r_{21})(1+\mu)+\mu(r_{22}^2-r_{11}r_{21})}{(r_{11}-r_{12})(1+\mu)+\mu(r_{11}^2-r_{22}r_{12})}. \tag{6}$$

The above problem determines x_0^*, x_B^* and thereafter, x_1^* and x_2^* are determined by

$$x_2^* = \frac{M-(1+\theta_B)x_0^*-(1+(r+c)\theta_B)x_B^*}{1+\theta_2} \tag{7}$$

and

$$x_1^* = \theta_B\{x_0^*+(r+c)x_B^*\}+\theta_2 x_2^*. \tag{8}$$

A complete listing of all possibilities is given in Table 1 in the appendix.

Regarding γ_0, it may be shown that under plausible situations there is a $\hat{r}>0$ such that

$$\gamma_0>0 \quad \text{whenever} \quad 0<r<\hat{r}$$

and

$$\gamma_0<0 \quad \text{whenever} \quad r>\hat{r}.$$

If no such $\hat{r}$ exists, then $\gamma_0<0$ for all possible r.

For γ_B, it is shown that

whenever $r=1-c$, $\gamma_B=\gamma_0$;

for $r<\hat{r}$, $\dfrac{\gamma_B}{1+(r+c)\theta_B}-\dfrac{\gamma_0}{1+\theta_B}\gtrless 0$ according as $r\gtrless 1-c$;

but for a single configuration of r_{ij}'s and c, if $\gamma_B>0$ for some r, it remains positive for larger values of r. For these and other conclusions, the reader is referred to the appendix.

III. DEMAND FOR MONEY, BONDS AND ASSETS

We shall consider the interval $(0,r_{21})$ for the variation of r, since by assumption $r<r_{ij}$ for all i,j. For an individual agent, r_{ij} and c are given and we shall study how the optimal portfolio changes when r varies. In particular, we shall assume

$$r_{11}^2 \geq r_{22}r_{12}$$

and

$$\beta_1 = r_{11}\{r_{22}(1-r_{11}r_{22}-r_{12}-r_{21})-r_{12}\} + r_{21}\{r_{12}(1+r_{11}+r_{22}+r_{11}r_{22})-r_{22}\}>0.$$

(These are conditions (ii) and (viii) in the appendix and it is discussed there how these may follow if r_{12} and r_{21} are small relative to r_{11} and r_{22}). Also, we shall assume that r^* (defined in Claim 1) $> r_{21}$, to ease the discussion. Adjustments to the arguments can be made easily to indicate what happens when these restrictions are violated and we shall indicate them. We shall consider three representative cases which broadly cover all situations:

Case 1. $0 < 1-c < \hat{r} < r_{21}$.

Recall that $r \lesseqgtr \hat{r} \Rightarrow \gamma_0 \gtreqless 0$ (refer to the discussion on the nature of γ_0 in the appendix). Thus we shall obtain: in $(0, 1-c)$, $\gamma_0 > 0$ and since $r < 1-c$, $x_0^* = M/(1+\theta_B)$ and $x_1^* = \theta_B M/(1+\theta_B)$ constitute the optimal portfolio; At $r = 1-c < \hat{r}$, $\gamma_0 > 0$ and $\gamma_B > 0$; so in the range $(1-c, r_{21})$ we have

$$x_B^* = \frac{M}{1+(r+c)\theta_B}, \quad x_1^* = \frac{(r+c)\theta_B M}{1+(r+c)\theta_B}.$$

There is a possibility, also, of a range $(\tilde{r}_1, \tilde{r}_2)$ in $(1-c, r_{21})$ where the optimal portfolio is

$$x_1^* = \frac{\theta_2 M}{1+\theta_2}, \quad x_2^* = \frac{M}{1+\theta_2};$$

refer to the discussion of the nature of γ_B in the appendix for such a possibility.

Case 2. $0 < \hat{r} < 1-c < r_{21}$.

As before in $(0, \hat{r})$, $\gamma_0 > 0$ and $r < 1-c \Rightarrow$ the optimal portfolio is

$$x_0^* = \frac{M}{1+\theta_B}, \quad x_1^* = \frac{\theta_B M}{1+\theta_B}.$$

At $r = \hat{r} (< 1-c)$, $\gamma_0 = 0$, and $\gamma_B < 0 \Rightarrow$ that at $\hat{r}$, the optimal portfolio changes to

$$x_1^* = \frac{\theta_2 M}{1+\theta_2}, \quad x_2^* = \frac{M}{1+\theta_2},$$

and this portfolio is maintained beyond $r = 1-c$, since γ_B at $r = 1-c$, must be negative as γ_0 is negative there. So there is some $\tilde{r}_1 > 1-c$ such that the above portfolio is held over the range $(\hat{r}, \tilde{r}_1)$. From $\tilde{r}_1$, where γ_B becomes positive, the portfolio changes to

$$x_B^* = \frac{M}{1+(r+c)\theta_B}, \quad x_1^* = \frac{(r+c)\theta_B M}{1+(r+c)\theta_B}.$$

And there may or may not be a subsequent switch from the (x_B^*, x_1^*) portfolio to the (x_1^*, x_2^*) portfolio as mentioned in Case 1.

Case 3. $1-c \leqq 0 < \hat{r} < r_{21}$.

Now notice that in $(0, \hat{r})$, although $\gamma_0 > 0$, since $r > 1-c$, $\gamma_B > 0$ too and moreover

$$\frac{\gamma_B}{1+(r+c)\theta_B}-\frac{\gamma_0}{1+\theta_B}>0$$

(see the appendix: The Nature of γ_B). Thus in the range $(0, r_{21})$, the portfolio would be

$$x_B^*=\frac{M}{1+(r+c)\theta_B}, \quad x_1^*=\frac{(r+c)\theta_B M}{1+(r+c)\theta_B}.$$

There may or may not be a switch to an (x_1^*, x_2^*) portfolio, as remarked in Case 1.

We devote the remaining part of this section to conditions (ii) and (viii) noted at the beginning. What happens, when these are violated?

By referring to the appendix, it may be seen that we require

$$(1+\mu)(r_{11}-r_{12})+\mu(r_{11}^2-r_{22}r_{12})>0$$

(which is so if (ii) holds, given (R)). Rewriting $\mu=1/r$, we require

$$(1+r)(r_{11}-r_{12})+(r_{11}^2-r_{22}r_{12})>0,$$

so that even if (ii) is violated, we have the above if

$$1+r>\frac{r_{22}r_{12}-r_{11}^2}{r_{11}-r_{12}}$$

or

$$r>\frac{r_{12}(r_{22}+1)-r_{11}(1+r_{11})}{r_{11}-r_{12}},$$

so that so long as

$$r_{12}<\frac{r_{11}(1+r_{11})}{(1+r_{22})},$$

we can still proceed as before; notice that

$$\frac{r_{11}(1+r_{11})}{1+r_{22}}>\frac{r_{11}^2}{r_{22}},$$

so that we may replace (ii) by a somewhat larger bound, without altering anything. But if

$$r_{12}>\frac{r_{11}(1+r_{11})}{1+r_{22}},$$

then one may straightaway see that for any $x\in B$, $\text{Min}\,(I_1(x),I_2(x))=I_1(x)$. Thus solving (P) reduces to

$$\underset{x\in B}{\text{Max}}\, I_1(x).$$

Again returning to the expression for I_1, it may be seen that the coefficient of x_1 is the largest, (see proof of Claim 4), hence the optimal portfolio is $x_1^* = M$. This portfolio does not change when r varies.

Finally when (viii) is violated i.e., $\gamma_0 < 0$ for all $r \geqq 0$, it may be noted that if $c < 1$, there is some $\tilde{r}_1 > 1 - c$ such that in $(0, \tilde{r}_1)$, the optimal portfolio is

$$x_1^* = \frac{\theta_2 M}{1+\theta_2}, \quad x_2^* = \frac{M}{1+\theta_2}.$$

Beyond $\tilde{r}_1$, the optimal portfolio changes to

$$x_B^* = \frac{M}{1+(r+c)\theta_B}, \quad x_1^* = \frac{(r+c)\theta_B M}{1+(r+c)\theta_B}$$

and thereafter, as in Case 1.

Again, for the sake of convenience, we have assumed that the switch of portfolio occurs in $(0, r_{21})$; this of course, need not be the case. The portfolio in such cases consists of the portfolio valid for the first stage of the argument noted above.

To sum up, under the restrictions employed, one would expect the following general pattern:
at low levels of the interest rate, the portfolio is made up of money and A_1; if c is close to 1 (but less), the next stage is of bonds and A_1; whereas if c is smaller, the next stage is of A_1 and A_2; also notice that at any r, the optimal portfolio is made up of only two assets.

IV. MARKET DEMAND

In general, individual agents differ in their assessment of r_{ij}'s and c; thus the switch points, where one portfolio changes to another, are likely to differ. Let $[r_{ij}^H]$ denote the assessment of individual H, and define $r_{21} = \mathrm{Max}_H r_{21}^H$. For $r = r_{21}$, then, most individuals have a portfolio of bonds and A_1; some may even have a portfolio of A_1 and A_2—these are individuals with a very low assessment of c. If we consider such 'bears' to be in the minority, then most people are holding bonds and A_1. A fall in r may allow some individuals to attain their switch points where they convert their bond holdings into money; there may once again be switches into holding A_1 and A_2 but we may consider them to be of minor importance. So far as the market demand for money is concerned, we may proceed as follows:—let $I(r) = \{H : x_0^{H*} > 0\}$: the set of individuals who hold money. Thus for any r, the aggregate demand for money is

$$\sum_{H \in I(r)} \frac{M^H}{1+\theta_B^H(r)},$$

where we use the formula for x_0^* developed in Section III earlier and

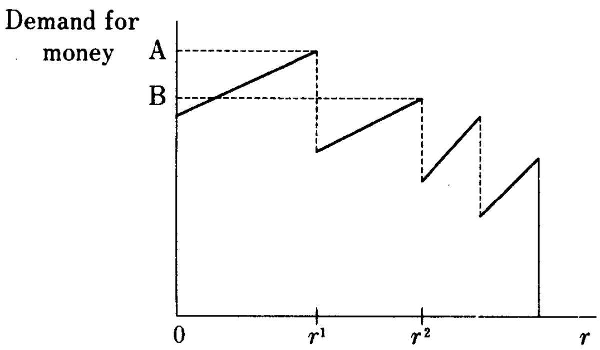

Fig. 1.

$$\theta_B^H(r) = \frac{(r_{22}^H - r_{11}^H)}{(r_{11}^H - r_{12}^H)(r+1) + \{(r_{11}^H)^2 - r_{12}^H r_{22}^H\}}\,.$$

As r changes, the above demond for money may change due to the following two factors:
(a) a change in θ_B^H
and/or
(b) a change in the set $I(r)$.
Suppose $r^1 > 0$ is such that for $r < r^1$, $I(r)$ is made up of the whole set of individuals; at r^1, one individual drops money holding and moves into a portfolio not involving money. Let the second such individual leave at r^2 and so on. Then noting that θ_B^H is a decreasing function of r and hence $1/(1+\theta_B^H)$ is an increasing function of r, we have the following demand curve for money.

In $0 < r < r^1$, all individuals belong to $I(r)$; consequently, the demand for money is

$$\sum_H \frac{M^H}{1+\theta_B^H(r)}$$

in $(0, r^1)$. At r^1, any H_1 drops out, so that $H_1 \notin I(r^1 + \varepsilon)$ for any $\varepsilon > 0$ however small. Consequently in (r^1, r^2) the demand for money is

$$\sum_{H \neq H_1} \frac{M^H}{1+\theta_B^H(r)}\,.$$

Again at r^2, say H_2 drops out so that $H_2 \notin I(r^2 + \varepsilon)$ for any $\varepsilon > 0$ however small. The difference $0A - 0B$ (see diagram) is

$$\sum_H \frac{M^H}{1+\theta_B^H(r^1)} - \sum_{H \neq H_1} \frac{M^H}{1+\theta_B^H(r^2)}$$
$$= \frac{M^{H_1}}{1+\theta_B^{H_1}(r^1)} + \sum_{H \neq H_1} M^H \cdot \left\{ \frac{\theta_B^H(r_2) - \theta_B^H(r_1)}{(1+\theta_B^H(r^1))(1+\theta_B^H(r^2))} \right\}$$
$$= \frac{M^{H_1}}{1+\theta_B^{H_1}(r^1)} + (-ve) \text{ term}\,,$$

given the property of $\theta_B^H(r)$ mentioned above. However the closer is r^2 to r^1, the smaller is the absolute value of the $-ve$ term and $0A-0B$ will be positive as drawn. Thus when a large number of agents have their switch points uniformly distributed over an interval (r^1, r^1+h), the step function nature would be smoothened out into a downward sloping demand curve for money over this interval.

In the range $r<r^1$, again, note that the demand for A_1 must be a decreasing function of r. After a switch, A_1 holding for an individual changes

$$\text{from} \quad \frac{\theta_B^H M^H}{1+\theta_B^H} \quad \text{with money}$$

$$\text{to either} \quad \frac{(r+c^H)\theta_B^H M^H}{1+(r+c^H)\theta_B^H} \quad \text{if held width bonds}$$

$$\text{or} \quad \frac{\theta_2^H M^H}{1+\theta_2^H} \quad \text{if held with } A_2 .$$

It therefore follows that when there are no switches, the demand for A_1 may either increase or decrease with r; although when only A_1 and money is held, it is definitely a decreasing function of r.

Imagine an open market operation by the Central Bank aiming to buy bonds from the market. As the Bank buys bonds, it reduces the long rate, persuading some agents to give up their bonds for money. However at the lower end of the market, since the substitution of bonds by money has already taken place, such operations would become impossible. We are now, in some sense, in a situation similar to the Liquidity Trap of Keynes. But the so-called liquidity trap in our model has a rather interesting aspect. Suppose $r<r^1$ (see diagram) and no bonds are held. Suppose for some reason located outside the asset markets, M^H, the quantity of money with H, is increased to $M^H+\Delta M^H$. In the Keynesian trap situation, this additional amount will be absorbed as additional money holding without a change in r being required. In our model, this increase in M^H would not cause any change in r; a part of the surplus will be held as money while the remaining will be used to increase the holding of A_1. Thus in the trap region, even though the rate of interest may not change, we may have an expansion in the investment on the real asset with the smaller spread of returns induced by monetary policy!

If the economic situation is characterised by overwhelming bullishness ($c>1$ for most), then money becomes inferior to bonds and as we have noted above, no one wants to hold money. A more interesting exceptional case occurs when the market is overwhelmingly bearish with c being close to zero. In this situation characterised by

$$1-c^H>r_{21}^H$$

for most H, the demand for money continues to exhibit features mentioned above; but in the range $(0, r_{21}^H)$ individual H refuses to hold bonds. We do not enter into a discussion of the implications of this aspect here.

APPENDIX

Consider the problem (P)

$$\underset{x\in B}{\mathrm{Max}}\,\underset{1,2}{\mathrm{Min}}\,(I_1(x),I_2(x))$$

mentioned in Section II of the paper. In particular, it may be recalled that $I_i(x)$ are written with the presumption that if $x_B>0$, then the total bond holding is converted at the end of period 1 into the higher yielding asset. We first put this presumption on firmer ground by means of

Claim 1. .There is $r^*>0$ such that for all $r<r^*$, bonds held at the beginning of period 1 would be converted at the end of period 1.

Proof. Consider one nominal unit of bond held in period 1; it promises a nominal return, r, per period; moreover, r units of money earned in period 1 can be invested into the highest yielding asset at the beginning of period 2.

By not converting the bond at the end of period 1, the investor has a stream of returns whose present value is *at most* $[1/(1-\delta)]r+[\delta/(1-\delta)]r\cdot r_{22}$.

While by converting the bond at the end of period 1, the investor has a stream of returns whose present value is *at least* $(r+c-1)+(r+c)\cdot r_{11}[\delta/(1-\delta)]$.

The difference between the latter and the former is $(c-2)+(r_{11}-r_{22})+c\,r_{11}/r$ when we recall that $\delta=1/(1+r)$; further, let $r^*=c\,r_{11}/[|c-2|+r_{22}-r_{11}]$; for $r<r^*$, the above difference is positive, so that conversion is the better alternative. □

In the text, we have mentioned that $r<r_{21}+1-c$; we extend this restriction to

$$0<r<\mathrm{Min}\,(r^*,r_{21}+1-c) \qquad \text{(i)}$$

here and shall operate within these limits. We may add, that in the light of the other limits to be imposed, (i) may not be more restrictive than the original limits imposed on r so long as c is not too small, as the examples at the end of the appendix indicate.

Returning to the problem (P), it should be noted that B is a non empty and compact subset of R^4_+ (the 4-dimensional Euclidean space) and that $\mathrm{Min}_{1,2}\{I_1(x);I_2(x)\}$is a continuous function on B. Hence the problem (P) is solvable and there is $x^*\in B$ which solves (P). We provide below some properties of x^* which help us to characterise the solution.

Consider the two related problems

$$\begin{aligned}&\mathrm{Max}\ I_1(x)\\ &\text{s.t.}\ \ I_1(x)-I_2(x)\leqq 0\\ &\qquad x\in B\end{aligned} \qquad \text{(P}'\text{)}$$

and

$$\begin{aligned} &\text{Max } I_2(x) \\ &\text{s.t. } \ I_2(x) - I_1(x) \leqq 0. \\ &\qquad x \in B \end{aligned} \tag{P''}$$

We note

Claim 2. x^* solves (P)$\Rightarrow$either x^* solves (P′) or (P″) or both.

Proof. If x^* solves (P), either $I_1(x^*) \leqq I_2(x^*)$ or $I_2(x^*) \leqq I_1(x^*)$. In the former case, x^* is feasible for (P′) while in the latter case, x^* is feasible for (P″). Then in the former case, if x^* does not solve (P′), there is $x' \in B$, $I_1(x') = \text{Min}\,(I_1(x'), I_2(x')) > I_1(x^*) = \text{Min}\,(I_1(x^*), I_2(x^*))$: a contradiction. Hence x^* solves (P′). In the latter case, x^* solves (P″), exactly as above. Finally, $I_1(x^*) = I_2(x^*) \Rightarrow x^*$ solves both (P′) and (P″). □

Chaim 3. Let x' solve (P′) and x'' solve (P″); let $I_1(x') - I_2(x'') = \theta$. Then

$$\begin{aligned} \theta > 0 &\Rightarrow x' \text{ solves (P)} \\ < 0 &\Rightarrow x'' \text{ solves (P)} \\ = 0 &\Rightarrow \text{both } x' \text{ and } x'' \text{ solve (P).} \end{aligned}$$

Proof. Suppose $\theta > 0$; note that x' is feasible for (P). If x' does not solve (P), then the solution x^* to (P) must satisfy

$$\text{Min}\,(I_1(x^*), I_2(x^*)) > \text{Min}\,(I_1(x'), I_2(x')) = I_1(x').$$

Hence x^* cannot be feasible for (P′) i.e., $I_2(x^*) < I_1(x^*)$ so that by Claim 2, x^* solves (P″) i.e.,

$$I_2(x^*) = \text{Min}\,(I_1(x^*), I_2(x^*)) = I_2(x'') < I_1(x'):$$

a contradiction. Hence x' solves (P).

The conclusion in the cases $\theta \leqq 0$ follow as above. □

Thus, to solve (P), we need to study the solutions of (P′) and (P″). We have first of all

Claim 4. Under (R), if

$$r_{11}^2 \geqq r_{12} r_{22}, \tag{ii}$$

then x' solves (P)$\Rightarrow I_1(x') = I_2(x')$ and $x_0' + x_B' + x_1' + x_2' = M$.

Proof. It would be more convenient to rewrite (P′) as the following standard linear program:

$$\begin{aligned} &\text{Max } \mu r_{11} x_0 + \{(r+c)(1+\mu r_{11}) - 1\} x_B + (1 + \mu(1 + r_{11}))(r_{11} x_1 + r_{21} x_2) \\ &\text{s.t. } \ \mu(r_{11} - r_{22})\{x_0 + (r+c) x_B\} + \{(1 + \mu(1 + r_{11})) r_{11} - (1 + \mu(1 + r_{22})) r_{12}\} x_1 \\ &\qquad + \{(1 + \mu(1 + r_{11})) r_{21} - (1 + \mu(1 + r_{22})) r_{22}\} x_2 \leqq 0, \end{aligned}$$

$$x_0+x_B+x_1+x_2 \leqq M,$$
$$x_0,x_B,x_1,x_2 \geqq 0.$$

The dual to the above problem is

$$\text{Min } y_2 M$$

s.t.
$$\mu(r_{11}-r_{22})y_1+y_2 \geqq \mu r_{11},$$
$$\mu(r_{11}-r_{22})(r+c)y_1+y_2 \geqq \{(r+c)(1+\mu r_{11})-1\},$$
$$\{(1+\mu(1+r_{11}))r_{11}-(1+\mu(1+r_{22}))r_{12}\}y_1+y_2 \geqq (1+\mu(1+r_{11}))r_{11},$$
$$\{(1+\mu(1+r_{11}))r_{21}-(1+\mu(1+r_{22}))r_{22}\}y_1+y_2 \geqq (1+\mu(1+r_{11}))r_{21},$$
$$y_1,y_2 \geqq 0.$$

Let x' solve the problem (P′); then there is $y'=(y_1',y_2')$ solving the minimum problem.

From the constraints of the minimum problem, it follows that $y_2' \neq 0$; since otherwise

$$y_1' \cdot \mu(r_{11}-r_{22}) \geqq \mu r_{11} > 0:$$

a contradiction to (R). Hence $y_2'>0$.

$$\therefore x_0'+x_B'+x_1'+x_2'=M.$$

Next, if possible let $I_1(x')<I_2(x')$ i.e.,

$$y_1'=0.$$

Then one may note from the constraints of the minimum problem that

$$(1+\mu(1+r_{11}))r_{11} > \mu r_{11}, \quad (\text{since } r_{11}>0)$$
$$(1+\mu(1+r_{11}))r_{11} > (1+\mu(1+r_{11}))r_{21} \qquad \text{by} \quad (\text{R}),$$

and

$$(1+\mu(1+r_{11}))r_{11} > (r+c)(1+\mu r_{11})-1,$$
$$\text{since} \quad r+c-1<r_{ij} \quad \text{all} \quad i \quad \text{and} \quad j.$$

Hence $y_2'=(1+(1+\mu r_{11}))r_{11}$; thus all but one of the inequalities in the minimum problem are strict inequalities so that

$$x_0'=0, \quad x_B'=0, \quad x_2'=0$$
$$\therefore x_1'=M.$$

But now by virtue of (ii),

$$\{(1+\mu(1+r_{11}))r_{11}-(1+\mu(1+r_{22}))r_{12}\}M>0:$$

a contradiction to the constraints of the maximum problem. Consequently, $y_1'>0$ and $I_1(x')=I_2(x')$, and the claim is established. □

Claim 5. Given (R) x'' solves $(\mathrm{P}'') \Rightarrow I_1(x'') = I_2(x'')$ and $x_0'' + x_B'' + x_1'' + x_2'' = .$

Proof. Follows exactly as the proof of Claim 4; except that the counterpart of (ii)

$$r_{22}^2 > r_{11} r_{21}$$

is true given (R) and hence no further restriction is necessary. □

Combining the above claims, it follows that given (R), (i) and (ii), x^* solving (P) is characterised by the following conditions:

$$I_1(x^*) = I_2(x^*) \tag{iii}$$

and

$$x_0^* + x_B^* + x_1^* + x_2 = M. \tag{iv}$$

From (iii), it follows that

$$x_1^* = \theta_B(x_0^* + (r+c)x_B^*) + \theta_2 x_2^*, \tag{v}$$

where

$$\theta_B = \frac{\mu(r_{22} - r_{11})}{(r_{11} - r_{12})(1+\mu) + \mu(r_{11}^2 - r_{22} r_{12})}$$

and

$$\theta_2 = \frac{(r_{22} - r_{21})(1+\mu) + \mu(r_{22}^2 - r_{11} r_{21})}{(r_{11} - r_{12})(1+\mu) + \mu(r_{11}^2 - r_{22} r_{12})},$$

substituting (vi) in (v), we have

$$x_2^* = \frac{M - (1+\theta_B)x_0^* - (1 + (r+c)\theta_B)x_B^*}{1+\theta_2}. \tag{vi}$$

Using (v) and (vi) in the expression for $I_1(x^*)$ we obtain

$$I_1(x^*) = \gamma_0 x_0^* + \gamma_B x_B^* + \gamma, \tag{vii}$$

where

$$\gamma_0 = \left[\mu r_{11} + \frac{1+\mu(1+r_{11})}{1+\theta_2}\{r_{11}(\theta_B - \theta_2) - r_{21}(1+\theta_B)\}\right],$$

$$\gamma_B = \left[(r+c)(1+\mu r_{11}) - 1 + \frac{1+\mu(1+r_{11})}{1+\theta_2}\{r_{11}((r+c)\theta_B - \theta_2) - r_{21}(1+(r+c)\theta_B)\}\right],$$

and

$$\gamma = \frac{1+\mu(1+r_{11})}{1+\theta_2}[\theta_2 M r_{11} + r_{21} M].$$

TABLE 1.

$\gamma_0<0$	$\to \gamma_B<0 \Rightarrow x_0^*=0,\quad x_B^*=0;\quad x_2^*=M/(1+\theta_2),\quad x_1^*=\theta_2 M/(1+\theta_2)$		(1.1)
	$\to \gamma_B=0 \Rightarrow x_0^*=0$, x_B^* may take any value in $[0, M/1+(r+c)\theta_B]$ according to each such value x_1^*, x_2^* from (vi) and (vii).		(1.2)
	$\to \gamma_B>0 \Rightarrow x_0^*=0,\quad x_B^*=M/\{1+(r+c)\theta_B\},\quad x_2^*=0,\quad x_1^*=\{(r+c)\theta_B M\}/\{1+(r+c)\theta_B\}$		(1.3)
$\gamma_0=0$	$\to \gamma_B<0 \Rightarrow x_B^*=0$; $x_0^*\in[0, M/(1+\theta_B)]$; x_1^*, x_2^* defined for each value of x_0^* via (vi) and (vii)		(2.1)
	$\to \gamma_B=0 \Rightarrow x_0^*$, x_B^* take on any value satisfying $x_0^*(1+\theta_B)+x_B^*\{1+(r+c)\theta_B\}=M$; x_1^*, x_2^* defined accordingly		(2.2)
	$\to \gamma_B>0 \Rightarrow x_0^*=0,\quad x_B^*=M/\{1+(r+c)\theta_B\},\quad x_2^*=0,\quad x_1^*=\{(r+c)\theta_B M\}/\{(1+(r+c)\theta_B\}$		(2.3)
$\gamma_0>0$	$\to \gamma_B<0 \Rightarrow x_0^*=M/1+\theta_B,\quad x_B^*=0,\quad x_2^*=0,\quad x_1^*=\theta_B M/(1+\theta_B)$		(3.1)
	$\to \gamma_B=0 \Rightarrow$ as above		(3.2)
	$\to \gamma_B>0$	$\to \gamma_0/(1+\theta_B)>\gamma_B/\{1+(r+c)\theta_B\} \Rightarrow x_0^*=M/(1+\theta_B)$, $x_1^*=\theta_B M/(1+\theta_B),\quad x_B^*=0,\quad x_2^*=0$	(3.3.1)
		$\to \gamma_0/(1+\theta_B)=\gamma_B/\{1+(r+c)\theta_B\} \Rightarrow x_0^*$, x_B^* may take any value subject to $x_0^*(1+\theta_B)+x_B^*\{1+(r+c)\theta_B\}=M$;	(3.3.2)
		$\to \gamma_0/1+\theta_B<\gamma_B/\{1+(r+c)\theta_B\} \Rightarrow x_0^*=0$, $x_B^*=M/\{1+(r+c)\theta_B\},\quad x_2^*=0,\quad x_1^*=\{(r+c)\theta_B M\}/\{1+(r+c)\theta_B\}$	{3.3.3}

Thus, solution to our problem (P) may be replaced by the solution to

$$\begin{aligned} &\text{Max } \gamma_0 x_0^* + \gamma_B x_B^* + \gamma \\ &\text{s.t.} \quad (1+\theta_B)x_0^* + (1+(r+c)\theta_B)x_B^* \leqq M, \\ &\qquad\qquad x_0^*,\quad x_B^* \geqq 0, \end{aligned} \tag{Q}$$

The constraint in (Q) guarantees that $x_2^* \geqq 0$ (see (vi)) and the corresponding value for x_1^* may be computed from (v).

Thus solutions to (Q) depend on signs (and magnitudes, if both positive) of γ_0 and γ_B. A complete listing of all possibilities are contained in Table 1. Before analysing γ_0 and γ_B, it should be mentioned that the above conclusions are based on (R) (i) and (ii). It turns out that (i) (as we mentioned before) is not necessarily more restrictive than $r<r_{21}$; (ii) essentially holds whenever r_{12} is sufficiently small when compared to r_{11} and r_{22}.

The Nature of γ_0.

It may be shown that

$$\gamma_0 = (\beta_1\mu^2 + \beta_2\mu + \beta_3)/\{(1+\mu)(a+b) + \mu(r_{11}a + r_{22}b)\},$$

where

$$a = r_{11} - r_{21} > 0,$$
$$b = r_{22} - r_{12} > 0,$$
$$\beta_1 = \{(a + r_{11}(r_{22} - r_{21})\}b - r_{22}(1 + r_{11})(r_{11}r_{22} - r_{12}r_{21}),$$
$$\beta_2 = ab - (1 + r_{11} + r_{22})(r_{11}r_{22} - r_{12}r_{21}) < 0,$$
$$\beta_3 = -(r_{11}r_{22} - r_{12}r_{21}) < 0.$$

Thus if $\beta_1 \leq 0$, then $\gamma_0 < 0$ for all values of $\mu > 0$; the implications for this can be seen from Table 1 that $x_0^* = 0$.

However if $\beta_1 > 0$, then the numerator in the expression for $\gamma_0 : \beta_1\mu^2 + \beta_2\mu + \beta_3$ can be positive for some values of μ. In particular, consideration of

$$\beta_1\mu^2 + \beta_2\mu + \beta_3 = 0$$

leads us to two roots μ_1 and μ_2 such that

$$\mu_1 + \mu_2 = -\beta_2/\beta_1 > 0,$$
$$\mu_1 \cdot \mu_2 = \beta_3/\beta_1 < 0,$$

given the signs of β_1, β_2, β_3. Thus

$$\beta_1\mu^2 + \beta_2\mu + \beta_3 = (\mu - \mu_1)(\mu - \mu_2) \cdot \beta_1,$$

where $\mu_1 > 0$ and $\mu_2 < 0$; the expression is positive for all $\mu > \mu_1$ whenever $\beta_1 > 0$.

Recall that $\mu = 1/r$. It follows therefore that if $\beta_1 > 0$, $\gamma_0 > 0$ whenever $1/r > \mu_1$ or whenever $r < 1/\mu_1 = \hat{r}$, say. Thus

$$\begin{aligned} \gamma_0 &> 0 \quad \text{whenever} \quad 0 < r < \hat{r} \\ &< 0 \quad \text{whenever} \quad r > \hat{r} \end{aligned}$$

if $\beta_1 > 0$.

To examine $\beta_1 > 0$, we expand the expression in β_1 to obtain

$$\begin{aligned} \beta_1 = {} & r_{11}\{r_{22}(1 - r_{11}r_{22} - r_{12} - r_{21}) - r_{12}\} \\ & + r_{21}\{r_{12}(1 + r_{11} + r_{22} + r_{11}r_{22}) - r_{22}\}. \end{aligned}$$

The restrictions (R) and (ii) however do not imply any definite sign for β_1; however if both r_{12} and r_{21} are negligible (compared to r_{11} and r_{22}), then the above expression would have the same sign as

$$r_{11}r_{22}(1 - r_{11}r_{22}),$$

which is positive provided $r_{11}r_{22} < 1$. This, naturally is one possible configuration of r_{ij} which leads to $\beta_1 > 0$. We shall use for our analysis, the final restriction

$$\beta_1 > 0. \qquad \text{(viii)}$$

The Nature of γ_B.

Examining the expression for γ_B, one may write

$$\gamma_B=(r+c)\gamma_0+(r+c-1)\cdot\left\{1+\frac{1+\mu(1+r_{11})}{1+\theta_2}(r_{11}\theta_2+r_{21})\right\}; \tag{ix}$$

hence when $r+c=1$, $\gamma_B=\gamma_0$.

Further

$$\frac{\gamma_B}{1+(r+c)\theta_B}-\frac{\gamma_0}{1+\theta_B}=\frac{r+c-1}{1+(r+c)\theta_B}\left\{1+\frac{1+\mu(1+r_{11})}{1+\theta_2}(r_{11}\theta_2+r_{21})+\frac{\gamma_0}{1+\theta_B}\right\}.$$

Thus if $\gamma_0>0$, then

$$\frac{\gamma_B}{1+(r+c)\theta_B}-\frac{\gamma_0}{1+\theta_B}\gtrless 0 \text{ according as } r+c\gtrless 1. \tag{x}$$

Finally, we need to study whether $\gamma_B>0$ for some value of r implies γ_B remains positive for all greater values of r. For this purpose, we expand the expression for γ_B to obtain:

$$\gamma_B=\frac{1}{r\{(r+1)(a+b)+r_{11}a+r_{22}b\}}\cdot[\alpha_1 r^3+\alpha_2 r^2+\alpha_3 r+\alpha_4],$$

where a, b are as before; the substitution $\mu=1/r$ has been made, and

$$\alpha_1=(a+b)>0,$$
$$\alpha_2=ba+c(a+b)+r_{22}b+r_{11}a>0;$$

but α_3 and α_4 could be of either sign. Now the question is whether it is possible to have $\gamma_B(\bar{r})>0$ and $\gamma_B(\hat{r})<0$ for $\hat{r}>\bar{r}$. Note that then $\alpha_1 r^3+\alpha_2 r^2+\alpha_3 r+\alpha_4=0$ has one positive root r_1, $\hat{r}>r_1>\bar{r}$ and another positive root $r_2>\hat{r}$, since $\alpha_1 r^3+\alpha_2 r^2+\alpha_3 r+\alpha_4>0$ for r large enough. Given that $\alpha_1, \alpha_2>0$, there may be two positive roots *only if* $\alpha_3<0$ and $\alpha_4>0$. Apart from this sign configuration, any other sign pattern would lead to at most one positive root and hence to the conclusion that once $\gamma_B>0$, it remains so for higher values of r.

Examples.

Example a. $r_{22}=.6$, $r_{11}=.4$, $r_{12}=.15$, $r_{21}=.1$.

One may note that $r_{22}\cdot r_{12}=.09<.16=r_{11}^2$ which is (ii) and in the formula for γ_0, (see above)

$$\beta_1=.036 \text{ so that (viii) holds;}$$
$$\beta_2=-.315,$$
$$\beta_3=-.225,$$

so that $\mu_1=9.414$ and consequently for all values of $r<.106$, $\gamma_0>0$.

Specifying $c=.9$, r^* in the proof of Claim 1, may be computed to be

$$r^* = (.9 \times .4)/(1.1 + .2) = \frac{.36}{1.3} = .277\,,$$

so that $\mathrm{Min}(r_{21} + 1 - c, r^*) = r_{21} + 1 - c$ and (i) does not constitute an additional restriction.

Example b. Alter r_{12} to .27 and r_{21} to .05.

It may be noted that (ii) is violated but (viii) holds.

Example c. Set $r_{12} = .27$ and $r_{21} = .1$.

It may be noted that both (ii) and (viii) are violated.

Exampled d. Set $r_{12} = .2$ and $r_{21} = .19$.

It may be noted that both (ii) and (viii) are violated.

Thus conditions (ii) and (viii) are independent restrictions on the r_{ij}'s appearing in (R).

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