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LONG-RUN EQUILIBRIA FOR PERFECTLY COMPETITIVE MARKETS

Hiroaki OSANA*

Abstract. A concept of long-run equilibrium is considered for the Arrow-Debreu model of a perfectly competitive market. Some properties of long-run equilibrium are investigated and a unified view is presented concerning the classical problem of the exhaustion of products.

The postulate of free entry and free exit is one of the defining characteristics of perfect competition. The market structure having all properties of perfect competition other than free entry and free exit is referred to as *pure competition* by Samuelson (1947) following Chamberlin (1938). When one speaks of a competitive market or a competitive economy, it is not always evident whether the adjective "competitive" means perfectly competitive or purely competitive. In formulating their models of competitive markets, Arrow and Debreu (1954), Debreu (1959), and Arrow and Hahn (1971) assume that the number of producers is a fixed natural number. As the possibility of free entry or free exit is not explicitly allowed for, their models may be interpreted as formalizing pure competition.

In the present paper, a period will be said to be *short-run* (resp. *long-run*) if there is at least one (resp. there is no) factor of production whose input level is fixed within the period. We assume that every factor of production is variable in the period during which entrepreneurship is a variable factor of production. Then a period is short-run or long-run according as entrepreneurship is fixed or variable. For the sake of simplicity, one unit of entrepreneurship will be assumed to be in a one-to-one correspondence to a producer. Under this assumption, the entry or exit of a producer belongs to problems in the long-run. Therefore, the Arrow-Debreu formulation of competitive equilibrium may be looked upon as concerning short-run equilibrium, though we note here at the same time that there is a viewpoint which regards the formulation as taking into account the problem of entry and exit by interpreting the model as including all the potentially conceivable producers (cf. Negishi (1965, p. 74)).

Assuming that constant returns to scale prevail in the long-run as a result of entry and exit of producers, McKenzie (1959) adopts a formulation in which the

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number of producers does not appear explicitly. It is adequate and in accordance with his intention to interpret his model as formalizing long-run equilibrium. In the latter part of the same paper, he also presents another interpretation of his model that seems to have a short-run flavor. The possibility of decreasing returns to scale in the Arrow-Debreu model is due to the presence of a factor of production, i.e., entrepreneurship, which does not appear in the commodity space of which the production sets are supposed to be subsets (cf. Hicks (1939)). McKenzie asserts that his model is general enough to contain the Arrow-Debreu model as a special case, since the assumption of non-increasing returns to scale can be replaced by the seemingly stronger assumption of constant returns to scale by taking into explicit account entrepreneurship as a factor of production. Various kinds of entrepreneurship are assumed to be possessed by economic agents as resource endowments, the quantity of which is fixed in the short-run. In the present paper, we shall not go further into this second interpretation. Whenever we refer to McKenzie, we shall always bear in mind his first long-run interpretation of his model.

The purpose of the present paper is to introduce a concept of long-run equilibrium for a perfectly competitive market in which the number of producers is taken into account, and then to investigate its properties.

1. NOTATION

Denote the *set of commodities* by a nonempty finite set H and the *set of consumers* by a nonempty finite set I. These two sets are assumed to be fixed in the long-run. The set $\boldsymbol{R}^H$ of real-valued functions on H is regarded as the *commodity space* and is identified with the $\sharp H$-dimensional Euclidean space, where $\sharp H$ stands for the cardinality of H. Each consumer $i \in I$ can consume an element of his consumption set X_i, which is a nonempty subset of $\boldsymbol{R}^H$. Each consumer i has a preference relation Q_i, which is a total, reflexive, transitive binary relation on X_i. Each consumer i has an endowment ω_i, which is a point of $\boldsymbol{R}^H$. The set of technology which can be used by each potential producer is represented by a correspondence Y whose domain, dom Y, is a nonempty finite set and whose range, range Y, is a nonempty subset of $\boldsymbol{R}^H$. Put $J=\operatorname{dom} Y$. The finiteness of J, meaning that there is a finite number of technologies available, is assumed for simplifying analysis. For each $j \in J$, $Y(j)$ is called a *production set of type j*. The correspondence Y is assumed to be fixed in the long-run. This means that the possibility of technical progress is not taken into account. A concept of long-run equilibrium is not simple to formalize in the presence of technical progress and is beyond the scope of the present paper. Denote by $\boldsymbol{N}$ the set of natural numbers and put $K=J\times \boldsymbol{N}$, which represents the set of conceivable potential producers. An element (j, n) of K stands for the n-th producer who adopts the technology of type j. To each producer $k \in K$ there is assumed to correspond an element θ^k of $(\boldsymbol{R}_+)^I$ such that $\sum_{i\in I}\theta_i^k=1$, where θ_i^k stands for consumer i's share of the profits of producer k. In the short-run, a nonempty finite subset L of K is fixed and only the producers in this subset are engaged in production activities. For each nonempty finite subset L of K define

$$J(L)=\{j\in J: (j, n)\in L \text{ for some } n\in N\}\,,$$

which represents the set of technologies used by the producers in L. For each nonempty finite subset L of K and each element j of J define

$$F(L, j)=\{n\in N: (j, n)\in L\}\,,$$

which represents the set of indices of producers using the technology of type j.

2. SHORT-RUN EQUILIBRIA AND LONG-RUN EQUILIBRIA

In what follows, different producers may participate in production activities in different short-run periods. To simplify notation, each producer not engaged in production activities will be viewed as selecting the special activity of inaction $0\in \boldsymbol{R}^H$. For each type $j\in J$, inaction is assumed to be a possible option, i.e., $0\in Y(j)$. Then every producer can entry or exit without cost. For each $j\in J$ put $\Upsilon_j=(Y_j)^N$. For each nonempty finite subset L of K define

$$\begin{aligned}A(L)=\{(x, y)\in \Pi X\times \Pi\Upsilon :\ &\text{(a) } y_j=0 \text{ for every } j\in J\backslash J(L),\\ &\text{(b) } y_{jn}=0 \text{ for every } j\in J(L) \text{ and every } n\in N\backslash F(L, j),\\ &\text{and (c) } \textstyle\sum_{i\in I}x_i-\sum_{i\in I}\omega_i\leq\sum_{j\in J(L)}\sum_{n\in F(L,j)}y_{jn}\}\,,\end{aligned}$$

which represents the set of feasible allocations for the set L of producers. In the presence of conditions (a) and (b), condition (c) can be written as $\sum_{i\in I}x_i-\sum_{i\in I}\omega_i\leq\sum_{k\in K}y_k$.

DEFINITION. For each nonempty finite subset L of K, an element (x, y, p) of $A(L)\times(\boldsymbol{R}_+)^H$ is called a *short-run competitive equilibrium for* L if

(1) for every $i\in I$, $p\cdot x_i\leq p\cdot\omega_i+\sum_{k\in K}\theta_i^k p\cdot y_k$ & $(x_i, z)\in Q_i$ for every $z\in X_i$ such that $p\cdot z\leq p\cdot\omega_i+\sum_{k\in K}\theta_i^k p\cdot y_k$,

(2) $p\cdot y_{jn}=\max p\cdot Y_j$ for every $j\in J(L)$ and every $n\in F(L, j)$,

(3) $p\cdot(\sum_{i\in I}x_i-\sum_{i\in I}\omega_i-\sum_{k\in K}y_k)=0$.

This definition coincides with the Arrow-Debreu definition of competitive equilibrium. The set of feasible allocations in the long-run is defined by

$$A=\{(x, y)\in \Pi X\times\Pi\Upsilon: (x, y)\in A(L) \quad \text{for some nonempty finite subset } L \text{ of } K\}\,.$$

DEFINITION. An element (x, y, p) of $A\times(\boldsymbol{R}_+)^H$ is called a *long-run competitive equilibrium* if

(4) there is a nonempty finite subset L of K for which (x, y, p) is a short-run competitive equilibrium,

(5) $\sup p\cdot Y_j\leq 0$ for every $j\in J$.

Condition (5) states that no producer either already participating in production activities or seeking for the opportunity to enter cannot expect positive profits at the current prices so that there is no incentive for potential producers to enter. This is a weak definition in that the possibility for positive profits to emerge as a result of changes in prices caused by the entry of some producers is not taken into account. Under perfect competition, no cooperative actions are assumed to be taken by either incumbent or potential producers so that the entry whose possibility matters here will be sought for by a single producer. As the entry of a single producer. As the entry of a single producer will not affect the current prices under perfect competition, the above definition seems to be an adequate description of long-run competitive equilibrium. The simultaneous entry of many producers, if any, would have some influences on the current prices. A stronger definition of long-run competitive equilibrium is given by requiring that there be no incentives for producers to enter even if prices change as a result of the entry of producers.

DEFINITION. An element (x, y, p) of $A\times(\boldsymbol{R}_+)^H$ is called a *strong long-run competitive equilibrium* if

(4a) there is a minimal nonempty finite subset L of K for which (x, y, p) is a short-run competitive equilibrium,

(5) $\sup p\cdot Y_j\leq 0$ for every $j\in J$,

(6) there is no nonempty finite subset M of K such that (6.1) $L\subseteq M$ and (6.2) there is a short-run competitive equilibrium (a, b, q) for M such that $q\cdot b_k>0$ for every $k\in M\backslash L$.

Condition (6) means that no new producer can enter industries, while (6.2) expecting positive profits and (6.1) not forcing any incumbent producers of the same type to exit. This does not necessarily rule out the possibility that an incumbent producer is, as a result of the entry of a producer of another type, virtually forced to exit as inaction $0\in\boldsymbol{R}^H$ is the only profit-maximiz ingbehavior. Without this requirement, no strong long-run competitive equilibrium exists, as long as there is a short-run competitive equilibrium in which some producer makes a profit, which is usually the case in the Arrow-Debreu model. Thus the requirement is necessary for the definition of strong long-run competitive equilibrium to be actually meaningful. By this requirement, condition (6) is likely to be fulfilled whenever L is large enough; then there is no reason to have a separate definition of strong long-run competitive equilibrium. So we require the minimality of L in condition (4a).

Clearly, every strong long-run competitive equilibrium is a long-run competitive equilibrium and every long-run competitive equilibrium is a short-run competitive equilibrium. Several sets of sufficient conditions are known for the existence of a short-run competitive equilibrium (cf. Arrow-Debreu (1954)). A long-run competitive equilibrium, as will be shown later, is a (short-run, as it were) competitive equilibrium of a special economy, called a long-run economy and the existence of the latter equilibrium is that of a usual short-run competitive equilibrium. As the following example shows, however, strong long-run competitive equilibria seem not to exist in general.

Example 1. Put $Y=\{(a_1, a_2)\in \boldsymbol{R}_-\times \boldsymbol{R}_+: a_2\leq -\sqrt{2a_1}\ \&\ a_2\leq \sqrt{2}-1-a_1\}$ and assume that Y is the only technology available. Assume that there are two consumers who have the same consumption set $X=\{(a_1, a_2)\in \boldsymbol{R}\times \boldsymbol{R}_+: a_1\geq -4\}$. Let their endowments be given by $\omega_1=(1+\sqrt{2}/4, 0)$ and $\omega_2=-\omega_1$ and their utility functions defined by $u(a_1, a_2)=2\sqrt{a_1+4}+a_2$ and $v(a_1, a_2)=4+a_1+2\sqrt{a_2}$. As there is only one type of technology, we can assume that $K=\boldsymbol{N}$. Put $\theta_1=(9\sqrt{2}-12)\times 8(\sqrt{2}-1)$ and $\theta_2=(4-\sqrt{2})/8(\sqrt{2}-1)$. Each consumer i is assumed to have a share θ_i of the profits of each producer.

Put $L=\{1\}$ & $p=(\sqrt{2}, 1)$ & $y_1=(-1, \sqrt{2})$ & $y_j=(0, 0)$ for each $j\in \boldsymbol{N}\backslash L$. Then $p\cdot y_j=0=\max p\cdot Y$ for every $j\in \boldsymbol{N}$. Put $x_1=(-2, \sqrt{2}-1/2)$ & $x_2=(1, 1/2)$. Then $p\cdot x_1=p\cdot \omega_1$ & $p\cdot x_2=p\cdot \omega_2$ & $u(x_1)\geq u(a)$ for every $a\in X$ such that $p\cdot a\leq p\cdot \omega_1$ & $v(x_2)\geq v(a)$ for every $a\in X$ such that $p\cdot a\leq p\cdot \omega_2$; therefore, (x, y, p) is a long-run competitive equilibrium. The property of technology entails that p is the unique price for which the maximum profit vanishes, so that (x, y, p) is the unique long-run competitive equilibrium. The minimality of L is obvious.

Put $M=\{1, 2\}$ & $q=(1, 1)$ & $w_1=w_2=(-2, 1+\sqrt{2})$ & $w_j=(0, 0)$ for each $j\in \boldsymbol{N}\backslash M$. Then $q\cdot w_j=\max q\cdot Y=\sqrt{2}-1>0$ for every $j\in M$. Put $z_1=(-3, 2\sqrt{2}-1)$ & $z_2=(1, 1)$. Then $q\cdot z_1=2\sqrt{2}-4=q\cdot \omega_1+\theta_1(q\cdot w_1+q\cdot w_2)$ & $q\cdot z_2=2=q\cdot \omega_2+\theta_2(q\cdot w_1+q\cdot w_2)$ & $u(z_1)\geq u(a)$ for every $a\in X$ such that $q\cdot a\leq q\cdot \omega_1+\theta_1(q\cdot w_1+q\cdot w_2)$ & $v(z_2)\geq v(a)$ for every $a\in X$ such that $q\cdot a\leq q\cdot \omega_2+\theta_2(q\cdot w_1+q\cdot w_2)$; therefore, (z, w, q) is a short-run competitive equilibrium for M. Consequently, there is a room for producer 2 to make a profit after entry, which is an incentive for entry. Thus there is no strong long-run competitive equilibrium in this example (cf. Fig. 1).

There is nothing pathological in this example. This fact indicates that the strong definition is unduly strong as a concept of long-run competitive equilibrium. In fact, even if potential producers seeking for the opportunity of entry happen to know that equilibrium prices will change as a result of their entry, they do not have enough information needed to exactly calculate new equilibrium prices and hence can at most anticipate them. Under perfect competition, it is natural to suppose that they would anticipate the prices to remain unchanged after entry. So a long-run competitive equilibrium but not a strong long-run competitive equilibrium seems to be a suitable concept for our purpose. In what follows, we shall not deal with the strong definition.

3. LONG-RUN TECHNOLOGY AND CONSTANT RETURNS TO SCALE

Every producer makes zero profits in a long-run competitive equilibrium so that the value of products equals the income of the factors of production which have

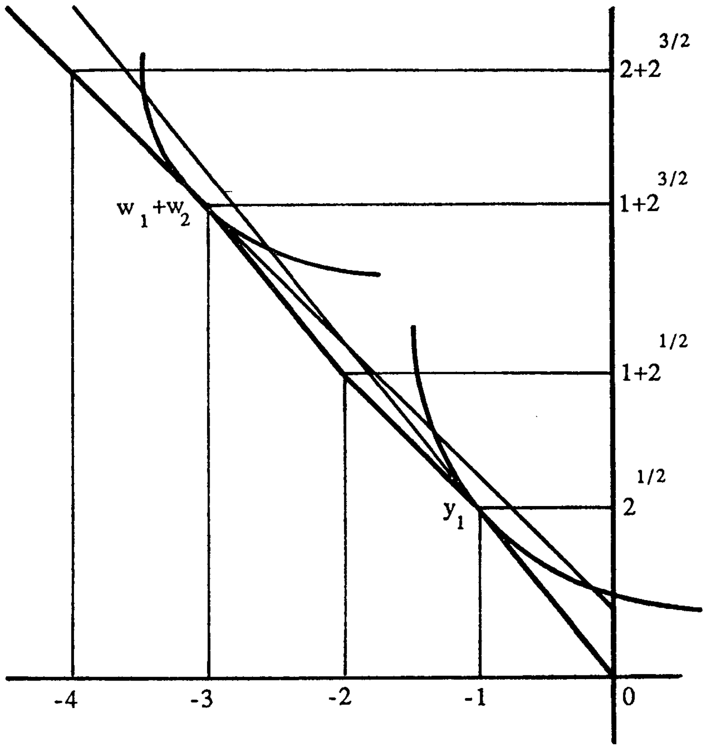

Fig. 1.

contributed to production activities. This fact certainly does not depend on the assumption that each producer's technology is subject to constant returns to scale. As will be shown below, however, the economy-wide technology in the long-run is subject to constant returns to scale. Hence the proposition of exhaustion may better be taken as virtually depending on the assumption of constant returns to scale.

For each subset S of $\boldsymbol{R}^H$ define

$$T(S)=\{v\in\boldsymbol{R}^H: \text{There is a function } f \text{ such that } \mathrm{dom}\, f \text{ is a nonempty finite set, range } f \text{ is a subset of } S \text{ and } v=\textstyle\sum_{n\in\mathrm{dom}\, f} f(n)\}.$$

For each $j\in J$ put $V_j=T(Y_j)$. For every $j\in J$, each element of V_j is an input-output vector obtained by some activities of a finite number (which depends on the particular choice of the element of V_j) of producers of type j and hence V_j is the long-run production set of type j. We shall show that the long-run production set V_j is subject to constant returns to scale if the short-run production set Y_j is subject to non-increasing returns to scale.

THEOREM 1. *$T(S)$ is a convex cone containing S for every convex subset S of $\boldsymbol{R}^H$ owning* 0.

Proof. Clearly $S\subseteq T(S)$. Let $(v, t)\in T(S)\times\boldsymbol{R}_+$. If $t=0$ then $tv=0\in S\subseteq T(S)$. Suppose $t\neq 0$. Then there is a natural number m such that $t\leq m$. By the definition of $T(S)$, there is a function f such that $\mathrm{dom}\, f$ is a nonempty finite set, range f is a subset of S, and $v=\sum_{n\in\mathrm{dom}\, f} f(n)$. For each $n\in\mathrm{dom}\, f$ put $g(n)=(t/m)f(n)$. Since S is a convex set owning 0, it follows that $g(n)\in S$ for every $n\in\mathrm{dom}\, f$ and hence that $tv=\sum_{n\in\mathrm{dom}\, f} tf(n)$ & $\sum_{n\in\mathrm{dom}\, f} mg(n)\in T(S)$. Thus $T(S)$ is a cone containing S.

Let $(x, y, t)\in T(S)\times T(S)\times]0, 1[$. Then there are functions a and b such that dom a and dom b are nonempty finite sets, range $a\cup$ range b is a subset of S, and $x=\sum_{n\in\mathrm{dom}\, a} a(n)$ & $y=\sum_{n\in\mathrm{dom}\, b} b(n)$. Define $c(n)=(1-t)a(n)+tb(n)$ for each $n\in\mathrm{dom}\, a\cap\mathrm{dom}\, b$, $c(n)=(1-t)a(n)$ for each $n\in\mathrm{dom}\, a\setminus\mathrm{dom}\, b$, and $c(n)=tb(n)$ for each $n\in\mathrm{dom}\, b\setminus\mathrm{dom}\, a$. Since S is a convex set owning 0, it follows that $c(n)\in S$ for every $n\in\mathrm{dom}\, a\cup\mathrm{dom}\, b$. Therefore, $(1-t)x+ty=(1-t)\sum_{n\in\mathrm{dom}\, a} a(n)+t\sum_{n\in\mathrm{dom}\, b} b(n)=\sum_{n\in\mathrm{dom}\, c} c(n)\in T(S)$ so that $T(S)$ is convex.

The following example shows that the long-run production set is not necessarily closed even if the short-run production set is.

Example 2. Put $S=\{(x, y)\in \boldsymbol{R}_-\times\boldsymbol{R}_+ :\ y\le\sqrt{-x}\}$. Then S is closed but $T(S)=\{(x, y)\in\boldsymbol{R}_-\times\boldsymbol{R}_+ : y=0 \text{ or } x<0\}$, which is not closed.

In this example, S, viewed as a short-run production set, is subject to decreasing returns to scale in a neighborhood of the origin (cf. Fig. 2). It will be shown

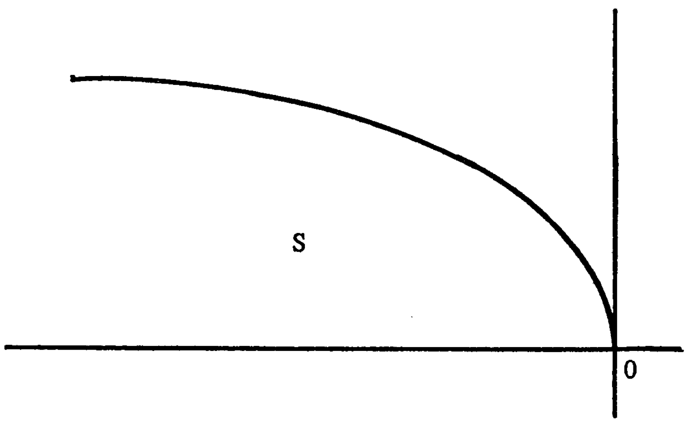

Fig. 2.

below that the closedness of the short-run production set implies that of the long-run production set, provided that the short-run production set is subject to constant returns to scale in a neighborhood of the origin.

THEOREM 2. *Let S be a closed convex subset of $\boldsymbol{R}^H$ owing* 0. *If there is a positive real number δ such that* $(\delta/|y|)y\in S$ for every $y\in S$ *such that* $0<|y|<\delta$, *then* $T(S)$ *is closed.*

Proof. Put $Z=\{y\in S: |y|\le\delta/2\}$. Clearly $Z\subseteq S$ so that $T(Z)\subseteq T(S)$. Let $y\in T(S)$. Then there is a function x such that dom x is a nonempty finite set, range x is a subset of $S\setminus\{0\}$, and $y=\sum_{i\in\mathrm{dom}\, x} x(i)$. Let $i\in\mathrm{dom}\, x$. Then there is a natural number $n(i)$ such that $n(i) \geq 2|x(i)|/\delta$. Put $a(i)=x(i)/n(i)$. Then $a(i) \in S$ & $|a(i)|=|x(i)|/n(i) \leq \delta/2$ so that $a(i) \in Z$. Put $D=\{(i,j) \in \operatorname{dom} x \times \boldsymbol{N}$: $j \in \{1, \cdots, n(i)\}\}$ & $f(i,j)=a(i)$ for each $(i,j) \in D$. Then $\operatorname{dom} f=D$ is a nonempty finite set, range f is a subset of Z, and $y=\sum_{(i,j) \in \operatorname{dom} f} f(i,j)$; therefore, $y \in T(Z)$. Thus $T(S) \subseteq T(Z)$ so that $T(S)=T(Z)$.

It remains to show that $T(Z)$ is closed. Let $\{z^\nu\}$ be a sequence in $T(Z)$ converging to some point z of $\boldsymbol{R}^H$. There is a natural number n such that $n \geq 4|z|/\delta$. Since $\{z^\nu\}$ converges to z, we can assume without loss of generality that $|z^\nu| \leq n\delta/2$ for every $\nu \in \boldsymbol{N}$. For each $\nu \in \boldsymbol{N}$ put $a^\nu=z^\nu/n$. Let $\nu \in \boldsymbol{N}$. Then there is a function y such that dom y is a nonempty finite set, range y is a subset of Z, and $z^\nu= \sum_{i \in \operatorname{dom} y} y(i)$. Put $b=\sum_{i \in \operatorname{dom} y} y(i)/\#\operatorname{dom} y$. Then $b \in Z$ since Z is convex. Since $a^\nu=z^\nu/n=\sum_{i \in \operatorname{dom} y} y(i)/n=(\#\operatorname{dom} y/n)b$, i.e., $(n/\#\operatorname{dom} y)a^\nu=b \in Z \subseteq S$ & $|a^\nu|=(\#\operatorname{dom} y/n)|b|=(\#\operatorname{dom} y/n)|z^\nu/\#\operatorname{dom} y|=|z^\nu|/n \leq \delta/2<\delta$, it follows by hypothesis that $a^\nu \in S$ so that $a^\nu \in Z$. Put $a=z/n$. Since Z is closed and $\{a^\nu\}$ converges to a, it follows that $a \in Z$. Hence $z=na \in T(Z)$ so that $T(Z)$ is closed.

As is observed in many existence theorems, the closedness of production sets is one of the crucial conditions assuring the existence of a competitive equilibrium. The sufficient condition given in Theorem 2 for the closedness of the long-run production set can be directly seen to be indispensable for the existence of a long-run competitive equilibrium. In fact, if the short-run production set does not satisfy the hypothesis of Theorem 2, being strongly convex in a neighborhood of the origin, as in Example 2, then the producer's maximum profit vanishes only at the origin. That is, for the producer to have a profit-maximizing action other than the origin in the long-run competitive equilibrium, it is necessary that his technology be subject to constant returns to scale in a neighborhood of the origin. Thus, the condition given in Theorem 2 is crucial for the existence of a non-trivial long-run competitive equilibrium.

4. COMPETITIVE EQUILIBRIA FOR A LONG-RUN ECONOMY

We now define, following McKenzie, a competitive equilibrium in terms of the long-run technology.

DEFINITION. An element (x, v, p) of $\Pi X \times \sum V \times (\boldsymbol{R}_+)^H$ is called a *competitive equilibrium for the long-run economy* if

(7) for every $i \in I$, $p \cdot x_i \leq p \cdot \omega_i$ & $(x_i, z) \in Q_i$ for every $z \in X_i$ such that $p \cdot z \leq p \cdot \omega_i$,

(8) $p \cdot v=0=\max p \cdot \sum V$,

(9) $\sum_{i \in I} x_i \leq \sum_{i \in I} \omega_i + v$ & $p \cdot (\sum_{i \in I} x_i - \sum_{i \in I} \omega_i - v)=0$.

The number of producers does not appear in this definition, so that the phenomenon of entry and exit is hidden behind the model. Furthermore, the maximum profit earned by the grand producer with the production set $\sum V$ vanishes and hence the fact that each consumer's income does not contain dividends of profits is explicitly stated in condition (7). The following evident relationship holds between the two concepts of long-run competitive equilibrium.

THEOREM 3. (1) *$(x, \sum_{k\in K} y_k, p)$ is a competitive equilibrium for the long-run economy for every long-run competitive equilibrium (x, y, p), and*

(2) *for every competitive equilibrium (x, v, p) for the long-run economy there is y such that $v=\sum_{k\in K} y_k$ and (x, y, p) is a long-run competitive equilibrium.*

Proof. Obvious.

5. PARETO OPTIMA

We shall now observe a relationship between short-run Pareto optima and long-run Pareto optima. For each nonempty finite subset L of K, the set of consumption allocations feasible in the short-run is defined by

$$C(L)=\{x\in\Pi\, X\colon (x, y)\in A(L) \text{ for some } y\in \Pi Y\}\ .$$

DEFINITION. Let L be a nonempty finite subset of K. An element x of $C(L)$ is called a *short-run Pareto optimum for* L if there is no element z of $C(L)$ such that $(z_i, x_i)\in Q_i$ for every $i\in I$ and $(x_i, z_i)\notin Q_i$ for some $i\in I$.

The set of consumption allocations feasible in the long-run is defined by

$$C=\{x\in\Pi\, X\colon \text{There is a nonempty finite subset } L \text{ of } K \text{ and an element } y \text{ of } \Pi Y \text{ such that } (x, y)\in A(L)\}\ .$$

DEFINITION. An element x of C is called a *long-run Pareto optimum* if there is no element z of C such that $(z_i, x_i)\in Q_i$ for every $i\in I$ and $(x, z_i)\notin Q_i$ for some $i\in I$.

THEOREM 4. *For every long-run Pareto optimum x there is a nonempty finite subset L of K such that x is a short-run Pareto optimum for L.*

Proof. Obvious.

As the set $C(L)$ of consumption allocations feasible in the short-run is generally a proper subset of the set C of consumption allocations feasible in the long-run, we cannot always expect that a short-run Pareto optimum is a long-run Pareto optimum.

THEOREM 5. *Suppose that, for every $i\in I$, Q_i is locally nonsatiated, i.e., for every $(a, \varepsilon)\in X_i\times \boldsymbol{R}_{++}$ there is $b\in X_i$ such that $(a, b)\notin Q_i$ & $|a-b|<\varepsilon$. Then*

(1) *every element x of C is a long-run Pareto optimum if there is an element (y, p)* of $\Pi Y\times(\boldsymbol{R}_+)^H$ *such that (x, y, p) is a long-run competitive equilibrium,*

(2) *for every nonempty finite subset L of K, every element x of $C(L)$ is a short-run Pareto optimum for L if there is an element (y, p) of $\Pi Y\times(\boldsymbol{R}_+)^H$ such that (x, y, p) is a short-run competitive equilibrium for L.*

Proof. The proof of (2) is exactly the same as that of the first basic theorem of welfare economics.

(1) Suppose that x is not a long-run Pareto optimum. Then there is an element a of C which is Pareto-superior to x. Hence there is a nonempty finite subset M of K and an element b of $\Pi\Upsilon$ such that $(a, b) \in A(M)$. Since (x, y, p) is a long-run competitive equilibrium, there is a nonempty finite subset L of K such that (x, y, p) is a short-run competitive equilibrium for L and $\sup p \cdot Y_j \leq 0$ for every $j \in J$. Hence

(i) for every $i \in I$, $p \cdot x_i \leq p \cdot \omega_i + \sum_{k \in K} \theta_i^k p \cdot y_k$ & $(x_i, z) \in Q_i$ for every $z \in X_i$ such that $p \cdot z \leq p \cdot \omega_i + \sum_{k \in K} \theta_i^k p \cdot y_k$,

(ii) $p \cdot y_{j_n} = \max p \cdot Y_j$ for every $j \in J(L)$ and every $n \in F(L, j)$,

(iii) $p \cdot (\sum_{i \in I} x_i - \sum_{i \in I} \omega_i - \sum_{k \in K} y_k) = 0$.

By local nonsatiation, $p \cdot x_i = p \cdot \omega_i + \sum_{k \in K} \theta_i^k p \cdot y_k$ for every $i \in I$. Since a is Pareto-superior to x, it follows that $p \cdot a_i \geq p \cdot x_i$ for every $i \in I$ and $p \cdot a_k > p \cdot x_k$ for some $k \in I$. Hence $0 \geq p \cdot \sum_{K \in K} b_k \geq p \cdot (\sum_{i \in I} a_i - \sum_{i \in I} \omega_i) > p \cdot (\sum_{i \in I} x_i - \sum_{i \in k} \omega_i) = p \cdot \sum_{k \in K} y_k = 0$, a contradiction.

6. CONCLUDING REMARKS

In the present paper, we have introduced a concept of long-run competitive equilibrium for the Arrow-Debreu model in which the number of producers is taken into explicit account, and investigated its relationship to a competitive equilibrium for the McKenzie model of the long-run economy. Our assertion that the long-run technology is subject to constant returns to scale seems to give us a unified view on the classical problem of the exhaustion of products (cf. Samuelson (1947, pp. 81–87). Each of the positive linear homogeneity of production functions and the long-run competitive equilibrium of perfect competition has been recognized as a sufficient condition for the value of each product to be exhausted when the factors of production are paid according to their value of marginal products. It has been emphasized that the positive linear homogeneity is not necessary for exhaustion in long-run competitive equilibrium. Theorem 1 in the present paper, however, asserts that the economy-wide production functions in the long-run are positively linear homogeneous.

A long-run production technology which is considered in the present paper is the set of input-output vectors obtained by actions of potential producers using the same type of short-run production technology. This has been shown to be a convex cone so that the corresponding long-run average cost curve is horizontal. Besides this, there is another concept of long-run production technology which a producer obtains by selecting, for each level of output, an optimal short-run technology among different types of short-run technologies. The long-run average cost curve corresponding to this long-run technology is not necessarily horizontal. In the present paper, we have completely neglected the latter sort of long-run technology.

Throughout the paper, we have assumed the convexity of production technology. Nonconvexities raise an important issue as technical barriers to entry or exit. But nonconvexities are in general incompatible with perfect competition. We have neglected them as we are mainly concerned with long-run equilibrium for a perfectly competitive market. They cannot be neglected when we deal with long-run equilibrium for an imperfectly competitive market.

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