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MULTIPERIOD INSURANCE CONTRACTS UNDER ASYMMETRIC INFORMATION

Toru HANEDA

Abstracts: This paper examines the role of multiperiod insurance arrangements in alleviating the inefficiency associated with the adverse selection problem, in the case of both private insurance and public insurance. We argue that the inefficiency can reduced by making the insurance payment to an individual, in any given period, depend on the realized states of nature in preceding periods, and thus there is reason enough to use multiperiod insurance contracts in the face of asymmetric information.

1. INTRODUCTION

Participants in insurance markets must deal with several types of information problems. One important type is the 'Adverse Selection Problem', which arises when potential customers belong to different classes but, because these risk differences are unobservable, the insurer cannot offer insurance contracts that discriminate between customers on the basis of their risk class. The negative externality that results from the existence of high risk customers, damages the low risk customers and generally impedes the performance of the market. Much of the work that has been done, on markets that must cope with the adverse selection problem, has been done in the context of single-period models (see Akerlof (1970), Rothschild and Stiglitz (1976), Wilson (1977), Miyazaki (1977), Spence (1978), and Riley (1979).

The purpose of this paper is to explore the role that multiperiod insurance contracts might play in alleviating the inefficiency associated with the adverse selection problem. When multiperiod contracts are used, information about customers' risk types can be accumulated over time and the insurance payment to an individual in any given period can be made to depend on this accumulated information. Thus the inefficiency associated with the adverse selection problem may be reduced as compared with a sequence of one-period relationships. This argument applies to public as well as private insurance.

Townsend (1982) showed that there are gains form forming multiperiod contractual relationships when an insurer is not able to observe the state of nature. In the context of moral hazard, Radner (1981), and Rubinstein and Yaari (1983), considered the moral hazard problem in an infinite-horizon model. In this paper, we consider a two-period model and argue that, even if the number of periods covered is fairly small, multiperiod insurance contracts are an effective mechanism for alleviating the inefficiency that arises from informational asymmetries.

The paper is organized as follows. Section 2 describes the basic model. In section 3, we consider the competitive equilibrium of a market for multiperiod insurance contracts and in section 4 we summarize the properties of equilibrium multiperiod insurance contracts under asymmetric information. In section 5, we examine the welfare implications of using multiperiod contracts while in section 6 we consider the optimal design of a multiperiod social insurance system under asymmetric information. Some concluding remarks make up section 7.

2. THE MODEL

For simplicity, we confine our anlysis to a two-period model. There are a finite number of insureres who are all identical and risk neutral. There is a continuum of agents represented by the unit interval [0, 1]. Each agent receives a random endowment of single nondurable consumption good $w^t \in \{w_1, \cdots, w_m\}$ in each period, with $0<w_1<\cdots<w_m$. It is assumed that w^t is an identically and independently distributed random variable, the agents' risks are statistically independent, and the realization of w^t is public information. We assume that each agent has a Von Neumann-Morgernstern utility function, $u(c^t)$, in each period, which depends on his consumption, c^t. u is defined on E_+, the set of all non-negative real numbers. We assume:

Assumption 1: u is twice continuously differentiable on E_+. In addition $u'>0$, $u''<0$ on E_+ and $u'(0)=\infty$.

There are only two types of agents, indexed by $s=H, L$, who differ in the probability distribution of their endowments. Each agent of type s receives w_i with probability q_i^s, where $q_i^s>0$ for all i. An element of informational asymmetry is introduced by assuming that although each agent knows his own risk type, the insurer is not able to verify the risk type of any agent. The fraction of type s in the population is denoted by $\theta_s(>0)$, $\theta_H+\theta_L=1$, and is public information. We assume:

Assumption 2: q_i^L/q_i^H is increasing in i.

This property is known as the strict monotone likelihood ratio proeprty (strict MLRP). The strict MLRP implies that $\sum_{j=1}^{i} q_i^H > \sum_{j=1}^{i} q_i^L$ for $i=1, \cdots, m-1$.[1] (See Milgrom (1981)). That is, the probability distribution of type L dominates that of type H in the sense of first-order stochastic dominance. Note that $\sum_{i=1}^{m} q_i^L w_i > \sum_{i=1}^{m} q_i^H w_i$ by first-order stochastic dominance. Hereafter, H-type agents and L-type agents are referred as high risk people and low risk people, respectively.

[1] First-order stochastic dominance does not always imply the strict MLRP. The strict MLRP is required to get concrete results.

Since the agents are risk averse, they have incentive to exchange their random endowments for some other consumption pattern. It is assumed that the agents can make such an exchange only by purchasing a single insturace contract from some insurer. A multiperiod insurance contract is defined as a $(m+m^2)$-dimentional vector, $\delta=(x_i^1, x_{ij}^2)$. x_i^1 denotes the number of units of consumption good transferred from the insurer to the agent in the first period given that $w^1=w_i$. x_{ij}^2 denotes the number of units of consumption good transferred from the insurer to the agent in the second period given that $w^1=w_i$ and $w^2=w_j$.[2] Negative components of this vector represent payments from the agent to the insurer. Multiperiod insurance contracts are agreed upon at the beginning of the first period, and we will confine our consideration to those in the set:

$$\Delta=\{\delta: w_i+x_i^1\geqq 0 \quad \text{for all } i \quad \text{and} \quad w_j+x_{ij}^2 \geqq 0 \quad \text{for all } i,j\}.$$

This restriction seems reasonable, because the agent cannot pay the insurer more than his endowment in each period. (We are assuming that the agent can neither borrow nor save).

The expected utility, over his two period horizon, of an agent who has pucrhased the multiperiod insurance contract, δ, is defined by:

$$V_s(\delta)=\sum_{i=1}^m q_i^s u(w_1+x_i^1)+\sum_{i=1}^m \sum_{j=1}^m q_i^s q_j^s u(w_j+x_{ij}^2)\,,$$

while that of an agent who has not purchased insurance is defined by:

$$\bar{V}_s=\sum_{i=1}^m q_i^s u(w_i)+\sum_{j=1}^m q_j^s u(w_j)\,.$$

An insurer's expected two-period profit, when the multiperiod insurance contract, δ, is purchased by the s-type agent, is defined by:

$$R_s(\delta)=\sum_{i=1}^m q_i^s(-x_i^1)+\sum_{i=1}^m \sum_{j=1}^m q_i^s q_j^s(-x_{ij}^2)\,.$$

3. THE COMPETITIVE EQUILIBRIUM OF THE INSURANCE MARKET

We will consider the situation where multiperiod insurance contracts are traded on an insurance market at the beggining of the first period, and there is no market in the second period. The insurance market is competitive in the sense that there is no collusion among participants in the market and there are only two kinds of participants, the agents who purchase insurance contracts, and the insurers who offer them.

[2] This does not quite correspond to conventional definition of an insurance contract. Let p^1 be the first period insurance premium which the agent pays to the insurer regardless of w^1, and let p_i^2 be the second period conditional insurance premium, given that $w^1=w_i$, which the agent pays to the insurer regardless of w^2. Then, in the conventional terminology, x_i+p^1 and $x_{ij}+p_i^2$ are the first period insurance coverage in the event that $w^1=w_i$, and the second period insurance coverage in the event that $w^1=w_i$ and $w^2=w_j$, respectively. For expositional convenience, we adopt this definition.

The Insurers

Since there are only two types of agents, the insurer offeres at most a pair of contracts. (Any insurer wishing to offer just one contract can simply equate its two contracts). We will frequently use the term 'contract structure' to denote a pair of contracts offered by a single insurer. Thus an insurer is identified by its contract structure. Let $\delta_H=(x_i^{1H}, x_{ij}^{2H})$ and $\delta_L=(x_i^{1L}, x_{ij}^{2L})$ denote the multiperiod insurance contracts intended for the H-type agents and the L-type agents, respectively. The per capita expected two-period profit of an insurer when the contract structure (δ_H, δ_L) attracts both H-type agents, (N_H), and L-type agents, (N_L), is given by:

$$(N_H/N_H+N_L)\cdot R_H(\delta_H)+(N_L/N_H+N_L)\cdot R_L(\delta_L)\,.$$

The contract structure is said to be profitable (resp. unprofitable) if the per capita expected profit is non-negative (resp. negative). We assume that insurer is only concerned about per capita profit and he offers a given contract structure if and only if it is profitable.

The Agents

We assume that each agent of type s chooses, from all offered contracts that are at least as good for him as no insurance, the contract which maximizes his expected utility $V_s(\cdot)$. If more than one insurer offers the contract that is most preferred by a type, we assume that the agents of that type distribute their purchases evenly among insurers who offer the preferred contract. On the other hand, when the agents of a type are indifferent between an insurer's two contracts, we assume that they all purchase the contract intended for them.

The Competitive Equilibrium under Full Information

As a standard for comparison, it is useful to consider the market equilibrium of the situation where the insurers know the agents' types. It may seem that a Nash type of equilibrium concept is relevant to the model described above. We give the following definition of a Nash equilibrium.

Definition. (Nash equilibrium): A set of contract structures is a Nash equilibrium if each insurer earns zero per capita expected profit, and if there is no new contract structure which would make non-negative per capita expected profit whenever the original contract structures continues to be offered.

Now consider the following maximization problem:

$$\max_{\delta_s\in\Delta} V_s(\delta_s) \qquad \text{subject to} \quad R_s(\delta_s)=0 \qquad (s=H, L) \tag{1}$$

The existence and uniqueness of the solution are assured by the aforementioned assumptions. Let $\hat{\delta}_s$ denote a solution to the problem (1) $(s=H, L)$. We will show that $(\hat{\delta}_H, \hat{\delta}_L)$ is a Nash equilibrium. Using the first-order conditions, it can be shown that $w_i+\hat{x}_i^1=\hat{c}_s$ for all i and $w_j+\hat{x}_{ij}^2=\hat{c}_s$ for all i and j, where $\hat{c}_s$ is some constant $(s=H, L)$. Thus, since $\hat{c}_s=\sum_{i=1}^m q_i^s w_i$, by Jensen's inequality we have

$V_s(\hat{\delta}_s) > \bar{V}_s$ $(s=H, L)$. Suppose that each insurer offers $(\hat{\delta}_H, \hat{\delta}_L)$. Then, under full information, it can be assured that $\hat{\delta}_H$ and $\hat{\delta}_L$ are purchased by high and low risk people, respectively. Furthermore, since the agents of each type distribute evenly among the insurers, the contracts structure $(\hat{\delta}_H, \hat{\delta}_L)$ attracts the H-type agents, (N_H), and the L-type agents, (N_L), in the ratio $N_H/N_L = \theta_H/\theta_L$. Hence, each insurer earns zero per capita profit under $(\hat{\delta}_H, \hat{\delta}_L)$. To complete the proof, it remains to check that each insurer has no incentive to offer a new contract structure. Clearly, there is no contract structure which attracts both types and makes non-negative per capita expected profit, becauses $\hat{\delta}_s$ is an nuique solution to the problem (1) $(s=H, L)$. Similarly, there is no contract structure which attracts only one type and makes non-negative per capita expected profit.

The concept of Nash eqilibrium defined above is similar to that in the usual context of a non-cooperative game. We can formulate a model of an insurance market as a game. In this game, the insurers are the players. A pure strategy for any insurer is taken to be a pair of insurance contracts (i.e. a contract structure). Each agent chooses a best insurance contract from among those offered. Then, a pure strategy combination is said to be a Nash equilibrium if each insurer's strategy maximizes his per capita expected profit when used against the strategy combination of the other insurers.

The Competitive Equilibrium under Asymmetric Information

We now consider market equilibrium under asymmetric information. Since $\hat{c}_L > \hat{c}_H$, if an insurer offers $(\hat{\delta}_H, \hat{\delta}_L)$, both types will purchase $\hat{\delta}_L$. As a result, the insurer suffers a loss because $R_H(\hat{\delta}_L) < 0$. Thus, in the presence of asymmetric information, $(\hat{\delta}_H, \hat{\delta}_L)$ is no longer a Nash equilibrium. Furthermore, as was shown by Rothschild and Stiglitz (1976), there is a robust class of examples in which a Nash equilibrium (in pure strategy) fails to exist. In order to avoid this difficaulty, we will employ the equilibrium concept, due to Wilson, that incorporates a non-myopic behavior rule. (This notion of equilibrium was first proposed by Wilson (1977), and was followed up by Miyazaki (1977) and Spence (1978)). We assume that each insurer expects the other insurers to withdraw their contract structures as soon as they become unprofitable. Thus, before an insurer actually offers a new contract structure, it must take account of the effect that this will have on the existing set of contract structures. We use the following definition of Wilson equilibrium.

Definition. (Wilson equilibrium): A set of contract structures is a Wilson eqlilibrium if each insurer earns zero per capita expected profit, and if there is no new contract structure which would make non-negative per capita expected profit after the elimination of all contract structures thereby rendered unprofitable.[3]

[3] If a set of contract structures is a Nash equillbrium, then it is also a Wilson equilibrium. For an alternative notion of non-myopic equilibrium, see Riley (1979).

In the remainder of this section, we will prove the existence of a Wilson equilibrium. Before doing so, it may be useful to make the following preliminary observations.

First, under asymmetric information, the contract structure offered by an insurer must satisfy the constratint that neither risk group prefers the contract intended for the other risk group to their own. This constraint is frequently referred to as the self-selection constraint in the literature on the economics of information.

Secondly, competition among the insurers gurantees that the high risk group obtains at least $V_H(\hat{\delta}_H)$ in equilibrium. Let (δ_H, δ_L) be an equilibrium pair of contracts. Suppose that $V_H(\delta)_H < V_H(\hat{\delta}_H)$ in equilibrium. If an insurer offers a single contract, $\hat{\delta}_H$, in place of (δ_H, δ_L), it will attract all of the H-type agents. Consider the case where $\hat{\delta}_H$ also attracts the L-type agents. Then, the deviant insurer makes strictly positive expected profits, because δ_H is a solution to the problem (1), and, by first-order stochastic dominance, $R_L(\hat{\delta}_L)>0$. In the case where $\hat{\delta}_H$ attracts only the H-type agents, the deviant insurer gets zero expected profit. Therefore, we must have that $V_H(\delta_H) \geqq V_H(\hat{\delta}_H)$ in equilibrium.

Finally, since the L-type agents have higher probabilities, than the H-type agents, of receiving large endowments, the L-type agents may be regarded as good customers by the insurer. Thus, the insurer must offer a contracts structure which can attract the low risk group.

Miyazaki (1977) and Spence (1978) established that, under the Wilson concept of competition, the market offers a pair of contracts, (δ_H, δ_L), and behaves as if it maximizes the expected utility of low risk people, subject to the well-defined constraints. In our setting, it can be shown that an equilibrium pair of contracts is a solution to the following problem:

(P.1) $$\max_{(\delta_H, \delta_L) \in \Delta \times \Delta} V_L(\delta_L)$$

subject to

$$V_H(\delta_H) \geqq V_H(\delta_L) \tag{2}$$

$$V_H(\delta_H) \geqq V_H(\hat{\delta}_H) \tag{3}$$

$$\theta_H R_H(\delta_H) + \theta_L R_L(\delta_L) \geqq 0\,. \tag{4}$$

Constraint (2) means that the H-type agents prefer δ_H to δ_L. Constraint (3) says that the H-type agents must be guranteed at least the expected utility that they would get under full information. Constraint (4) means that an insurer's per capita expected profit is non-negative when the contract structure (δ_H, δ_L) attracts the H-type agents, (N_H), and the L-type agents, (N_L), in the ratio $N_H/N_L = \theta_H/\theta_L$. As will be shown below, (4) can be replaced by the equality constraint without loss of generality.

We will now show that a solution to the problem (P.1), is a Wilson equilibrium. To do so, we require two lemmas.

Lemma 1: The problem (P.1) has a unique solution.

Proof: Consider $\bar{\delta}_L=(\bar{x}_{ij}^{1L}\bar{x}_{ij}^{2L})$, such that $\bar{x}_i^{1L}=0$ for all i and $\bar{x}_{ij}^{2L}=0$ for all i and j. Clearly $(\hat{\delta}_H, \bar{\delta}_L)$ satisfies the constraints (2)-(4). Thus the constraint set is nonempty. It is also closed. (δ_H, δ_L) is bounded from below because $(\delta_H, \delta_L)\in\Delta\times\Delta$. Note that $\theta_s>0$ and $q_i^s>0$ for all i $(s=H, L)$. Thus, from constraint (4), (δ_H, δ_L) is bounded from above. Hence the constraint set is compact. Since the objective function, $V_s(\cdot)$, is continuous, the maximization problem has a solution. $V_s(\cdot)$ is strictly concave by the properties of u, and $\theta_H R_H(\cdot)+\theta_L R_L(\cdot)$ is linear. Therefore, uniqueness follows directly. Q.E.D.

The first-order conditions for a solution, after some manipulation, are;

$$[\lambda_1+\lambda_2]u'(w_i+x_i^{1H})=\lambda_3\theta_H \quad \text{for all } i \tag{5}$$

$$[\lambda_1+\lambda_2]u'(w_j+x_{ij}^{2H})=\lambda_3\theta_H \quad \text{for all } i \text{ and } j \tag{6}$$

$$[1-\lambda_1(q_i^H/q_i^L)]u'(w_i+x_i^{1L})=\lambda_3\theta_L \quad \text{for all } i \tag{7}$$

$$[1-\lambda_1(q_i^H q_j^H/q_i^L q_i^L)]u'(w_j+x_{ij}^{2L})=\lambda_3\theta_L \quad \text{for all } i \text{ and } j \tag{8}$$

where λ_1, λ_2 and λ_3 are non-negative Lagrange multipliers corresponding to the constraints (2)-(4), respectively. We have also the complementary slackness conditions.

Let (δ_H^*, δ_L^*) denote a solution to the problem (P.1). Then we have the following results.

Lemma 2: A solution (δ_H^*, δ_L^*) satisfies the following properties:
(i) $w_i+x_i^{1H*}=c_H^*$, for all i, and $w_j+x_{ij}^{2H*}=c_H^*$, for all i and j; (ii) $w_i+x_i^{1L*}$ is increasing in i, and $w_i+x_i^{2L*}$ is increasing in i and j; (iii) $V_L(\delta_L^*)>V_L(\delta_H^*)$; (iv) $V_H^*(\delta_H^*)>\bar{V}_H$ and $V_L(\delta_L^*)\geqq\bar{V}_L$; (v) $\theta_H R_H(\delta_H^*)+\theta_L R_L(\delta_L^*)=0$; (vi) $R_H(\delta_H^*)\leqq 0$ and $R_L(\delta_L^*)\geqq 0$, where c_H^* is some constant.

Proof: Note that $\lambda_3>0$. If $\lambda_3=0$, we must have $u'=0$ and so Assumption 1 is violated. Thus, (i) follows from (5)-(6) and Assumption 1. (v) follows from complmentary slackness. Since $c_H^*\geqq\hat{c}_H$, by constraint (3), we get $R_H(\delta_H^*)\leqq 0$, which, together with (v), implies $R_L(\delta_L^*)\geqq 0$. Since $V_H(\hat{\delta}_H)>\bar{V}_H$ and $(\hat{\delta}_H, \bar{\delta}_L)$ belongs to the constraint set, we get (iv).

To prove the remaining results, we will show $\lambda_1>0$. Suppose that $\lambda_1=0$. Then, from (5) we have $\lambda_2>0$ and, by complementary slackness, constraint (3) is binding at the optimum. Hence, $c_H^*=\hat{c}_H$ and $R_H(\delta_H^*)=0$, which implies $R_L(\delta_L^*)=0$, because of (v). On the other hand, from (7)-(8) and Assumption 1, we have that $w_i+x_i^{1L*}=c_L^*$ for all i, and $w_j+x_{ij}^{2L*}=c_L^*$ for all i and j, where c_L^* is some constant. Thus, x_i^{1L*} is decreasing in i, and x_{ij}^{2L*} is decreasing in j for all i, and is constant over i for all j. By first-order stochastic dominance, we have $R_H(\delta_L^*)<R_L(\delta_L^*)$. Furthermore, from constraint (3), $c_H^*\geqq c_L^*$ and so $R_H(\delta_H^*)\leqq R_H(\delta_L^*)$. Therefore, we have $R_L(\delta_L^*)>0$, a contradication.

If $\lambda_1>0$, then (ii) follows from (7)-(8) and Assumptions 1-2. Note that constraint (2) is binding at the optimum by the complementary slackness. Thus, by (i)-(ii) and first-order stachastic dominance, we get (iii). Q.E.D.

We are now in a position to show fthat.

Proposition 1: There is an aunique Wilson equilibrium.

Proof: Suppose that each insurer offers (δ_H^*, δ_L^*). Then, by constraint (2) and Lemma 2 (iii)-(iv) high and low risk people purchase δ_H^* and δ_L^*, respectively. Since the agents of each type distribute evenly among the insurers, the contract structure (δ_H^*, δ_L^*) attracts the H-type agents, (N_H), and the L-type agents, (N_L), in the ratio $N_H/N_L=\theta_H/\theta_L$. Thus, by Lemma 2 (v), each insurer earns zero per capita expected profit. Furthermore, (δ_H^*, δ_L^*) is a unique solution to the problem (P.1). Therefore, it only remains to check that each insurer has no incentive to offer a new contract structure.

Clearly, there is no contract structure which attracts both types and makes non-negative per capita expected profit. Moreover, since $R_H^*(\delta_H^*)\leqq 0$, by Lemma 2 (vi), there is no contract structure which attracts only the high risk group and makes non-negative per capita expected profit. Suppose that there is a contract δ' such that $V_L(\delta')>V_L(\delta_L^*)$, $V_H(\delta_H^*)>V_H(\delta')$ and $R_L(\delta')\geqq 0$. Then, it must be hold that (a) $R_L(\delta_L^*)>0$, (b) $V_H(\delta')\geqq V_H(\hat{\delta}_H)$ and (c) $\theta_H R_H(\delta')+\theta_L R_L(\delta')<0$. For otherwise, (δ_L^*, δ_L^*) is no longer a solution to the problem (P.1). If an insurer offers a new single contract δ' in place of (δ_H^*, δ_L^*), it will attracts only the low risk group and it will deprive other insurers of their L-type agents. Since $R_H(\delta_H^*)<0$ by Lemma 2 (v) and (a), (δ_H^*, δ_L^*) becomes unprofitable. As it will be withdrawn by all of the other insurers, by (b), the H-type agents will all purchse δ'. Consequently, the deviating insurer will surffer a loss, because of (c). Thus, the insurer endowed with Wilson anticipations will not offer a contract, δ'. Q.E.D.

It should be noted that if constraint (3) is binding at the optimum, (δ_H^*, δ_L^*) is also a Nash equilibrium. In this case, we have $R_H(\delta^*)=0$ and thus there is no contract structure which attrcts only the low risk group and makes onn-negative per capita expected profit. But, in all other cases, there is no Nash equilibrium.

4. THE PROPERTIES OF EQUIOLIBRIUM MULTIPERIOD INSURANCE CONTRACTS

In the previous section, we have seen that (δ_H^*, δ_L^*) is s Wilson equilibrium and high and low risk people purchase δ_H^* and δ_L^*, respectively. Therefore, the properties of equilibrium multiperiod insurance contracts under asymmetric information follow immediately from Lemma 2 (i)-(ii). We say that a multipeiod insurance contract is full insurance if an agent can enjoy constant consumption, both over time and across endowment realizations, by purchasing it.

Proposition 2: Equilibrium multiperiod insurance contracts under asymmeric information satisfy the following properties: (1) high risk people are offered full insurance and the second period payment to them is independent of the realization of w^1; (2) low risk people are not offered full insurance and the second period payment to them depends on the relaization of w^1 as well as w^2.

As we showed in section 3, in the case of full information, both types are offered full insurance and the second period payment to them is independent of the realization of w^1. In the presence of asymmetric information, however, the final allocation of risk-bearing is inefficient. This inefficiency stems from informational externalities.

The dependence, of an agent's second period payment, on the realization of w^1, has two opposite effects on his expected utility. Clearly, from a a risk-sharing point of view, it is undesirable to make the second period payment conditional on the realization of w^1, because this exposes the agent to additional risk. Thus such a scheme has an adverse effect on his expected utility. (In the case of full information, only this effect is present). On the other hand, it can help to relax the constraints imposed by asymmetric information and so it can operate favorably on the agent's expected utility. In particular, this relaxation can be effected by exploiting the difference in the agents' preferences over multiperiod contracts. Proposition 2 (2) indicates that the favorable effect dominates the adverse effect in the case of the multiperiod contract intended for the L-type agents.

5. WELFARE RESULTS

We will examine the welfare implications of using multiperiod contracts in the presence of asymmetric information. Specifically, market performance under multiperiod contracts will be compared with that under a regime in which one-period contracts are traded at the beggining of each period.

Since the same situation is repeated over time, we omit the superscript t on all of the variables. An one-period insurance contract is defined by an m-dimentional vector $x=(x_1, \cdots, x_m)$. x_i represents the number of units of consumption good transferred from the insurer to the agent given that $w=w_i$. The one-period insurance contracts which we will consider are confined to the set

$$X=\{x: w_i+x_i \geqq 0 \quad \text{for all} \quad i\}.$$

The s-type agent's single-period expected utility under a one-period insurance contract is defined by

$$v_s(x)=\sum_{i=1}^{m} q_i^s u(w_i+x_i).$$

The insurer's single-period expected profit when the one-period insurance contract x is purchased by the s-type agent, is defined by

$$r_s(x)=\sum_{i=1}^{m} q_i^s(-x_i).$$

One-period insurance contracts are traded on a competitive market at the beginning of each period. As before, we employ the concept of Wilson equilibrium.

We also assume that market participants behave in the same way as was described in section 3. Let x_H and x_L denote the one-period insurance contracts intended for the H-type agents and L-type agents, respectively. By applying arguments similar to those in section 3, we can show that a Wilson equilibrium, in each period, is a solution to the following maximization problem:

$$\text{(P.2)} \qquad \max_{(x_H, x_L) \in X \times X} v_L(x_L)$$

subject to

$$v_H(x_H) \geqq v_H(x_L) \tag{9}$$

$$v_H(x_H) \geqq v_H(\hat{x}_H) \tag{10}$$

$$\theta_H r_H(x_H) + \theta_L r_L(x_L) \geqq 0 \,. \tag{11}$$

The left hand side of (11) represents an insurer's per capita single-period expected profit when the contract structure (x_H, x_L) attracts the H-type agents (N_H) and the L-type agents (N_L) in the ratio $N_H/N_L = \theta_H/\theta_L$.

$\hat{x}_H$ is a solution to the following maximization problem:

$$\max_{x_H \in X} v_H(x_H) \qquad \text{subject to} \quad r_H(x_H) = 0 \,. \tag{12}$$

It is easy to check that $w_i + \hat{x}_i^H = \hat{c}_H$ for all i. Here $\hat{c}_H$ is the same as that in section 3. Hence, $v_H(\hat{x}_H) + v_H(\hat{x}_H) = V_H(\hat{\delta}_H)$.

Let (x_H^{**}, x_L^{**}) denote a solution to the problem (P.2). Then, x_H^{**} and x_L^{**} are purchased by high and low risk people, respectively, and $\theta_H r_H(x_H^{**}) + \theta_L r_L(x_L^{**}) = 0$. Moreover, using the first-order conditions, it can be shown that $w_i + x_j^{H**} = c_H^{**}$ for all i, and $w_i + x_i^{L**}$ is increasing in i, where c_H^{**} is some constant. Thus, the uncertainty about consumption of the L-type agent can not be fully eliminated by purchasing an insurance contract. In this way, low risk people are forced to incur costs in order to be distinguished from high risk people. That is, there are negative informational externalities running from high to low risk people.

As was shown by Proposition 2, informational externalities also cause some inefficiency under multiperiod contractual regime. However, in that case, the costs which the L-type agents incur may be reduced, compared with the sequence of one-period contracts, by making the second period payment to them depend on the realization of w^1. Consequently, low risk people will gain by using multiperiod contracts.

Now define the s-type of agent's two-period expected utility under a sequence of equilibrium one-period insurance contracts, (x_s^{**}, x_s^{**}), as

$$V_s^{**} = v_s(x_s^{**}) + v_s(x_s^{**})$$

and define an insurer's per capita two-period expected profit under a sequence of equilibrium one-period contract structure, $\{(x_H^{**}, x_L^{**}), (x_H^{**}, x_L^{**})\}$, as

$$R^{**} = \theta_H(r_H(x_H^{**}) + r_H(x_H^{**})) + \theta_L(r_L(x_L^{**}) + r_L(x_L^{**})) \,.$$

Then we have the following results:

Proposition 3: (1) if constraint (10) is binding at the optimum, then $V_H(\delta_H^*)\geqq V_H^{**}$; (2) $V_L(\delta_L^*)>V_L^{**}$; (3) $\theta_H R_H(\delta_H^*)+\theta_L R_L(\delta_L^*)=R^{**}$.

Proof: Since $V_H(\delta_H^*)\geqq V_H(\hat{\delta}_H)$, and $V_H^{**}\geqq V_H(\hat{\delta}_H)$, if constraint (10) is binding at the optimum, we get (1).

Consider $\delta_s^{**}=(x_i^{1s**}, x_{ij}^{2s**})$ $(s=H, L)$ such that $x_i^{1s**}=x_i^{s**}$ for all i and $x_{ij}^{2s**}=x_j^{s**}$ for all i and j. Note that $V_s(\delta_s^{**})=V_s^{**}$. Clearly, $(\delta_H^{**}, \delta_L^{**})$ satisfies the constraints (2)–(4). Thus, by the uniqueness of the solution and Proposition 2(2), we get (2). (3) is obvious. Q.E.D.

Whether constraint (10) is binding at the optimum depends, among other things, on the fractions θ_H and θ_L. As was shown by Rothschild and Stiglitz (1976), when there are relatively more high risk people i.e. the ratio θ_H/θ_L is relatively high, constraint (10) tends to be binding at the optimum. Therefore, the above results indicate that if the economy has relatively more high risk people, the multiperiod contractual regime is (ex ante) Pareto superior to the one-period contractual regime, so that there is a rationale for introducing multiperiod insurance contracts in a market that must cope with asymmetric information.

Figures 1 and 2 describe equilibrium one-period insurance contracts and equilibrium multiperiod insurance contracts, for the case $m=2$, respectively, where $x_s^{1*}=(x_1^{1s*}, x_2^{1s*})$, $x_s^{21*}=(x_{11}^{2s*}, x_{12}^{2s*})$ and $x_s^{22*}=(x_{21}^{2s*}, x_{22}^{2s*})$ for $s=H, L$. We are assuming here that constraints (2) and (10) are binding at the optimum. As is shown in Figure 2, we may have that $v_H(x_L^{22*})>v_H(x_H^{22*})$. Clearly, in the context of one-period contarcts, it is impossible to offer (x_H^{22*}, x_L^{22*}) because high risk

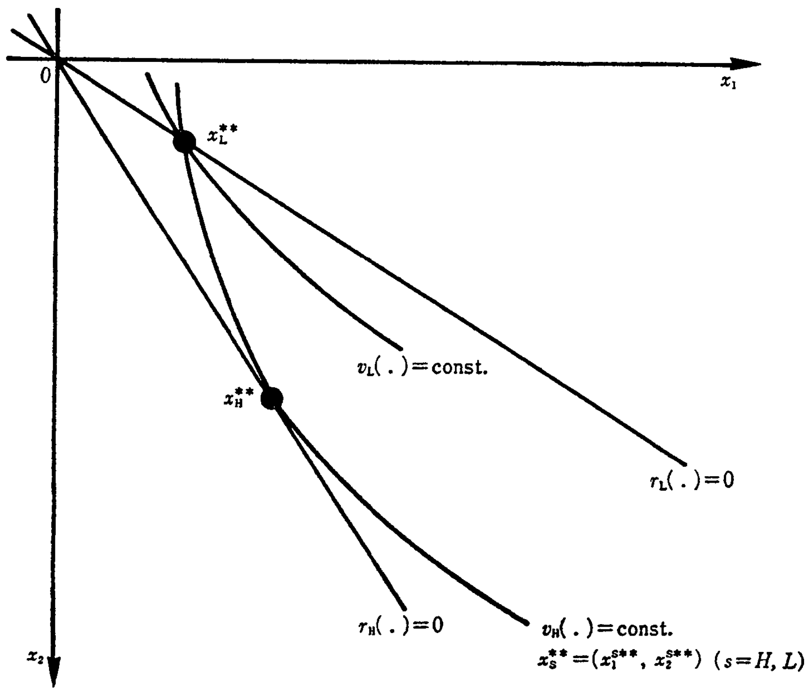

Fig. 1.

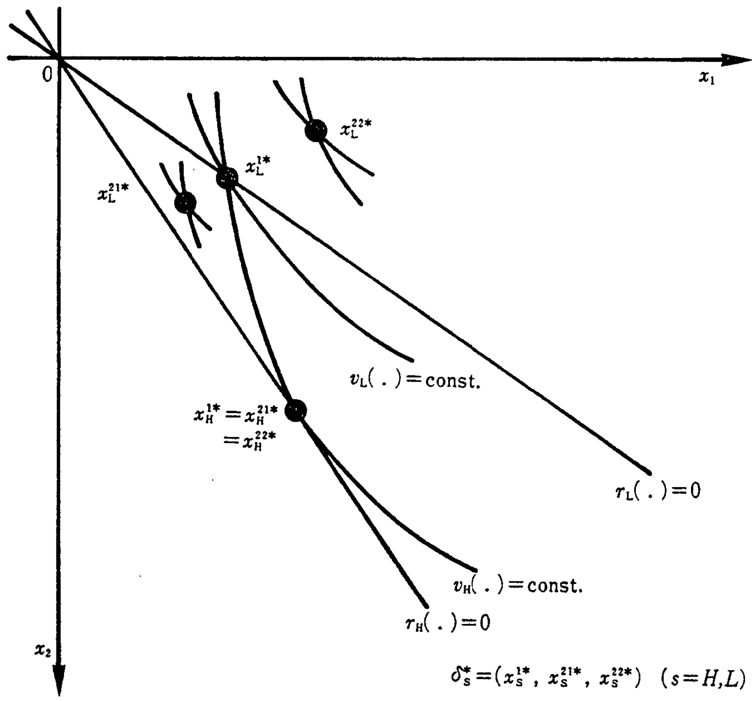

Fig. 2.

people purchase x_L^{22*} rather than x_H^{22*}. This observation shows that under multiperiod contracts, the constraint imposed by asymmetric information (constraint (2)) can be relaxed by making the second period payment to L-type agents depend on the realization of w^2 (as compared with the sequence of one-period contracts) and thus there is a gain from introducing multiperiod insurance contracts.

Now suppose that the agents' type are public information. Then, multiperiod contracts for both types are equivalent to the sequence of one-period contracts. Thus, in the case of full information, there is no gain from introducing multiperiod contracts.

6. OPTIMAL DESIGN OF MULTIPERIOD SOCIAL INSURANCE SYSTEM

So far our discussion has been confined to the private insurance market. In this section, we will consider a multiperiod social insurance system designed by the government.[4]

δ will be referred to as multiperiod social insurance programme. x_i^1 and x_{ij}^2 represent the transfer payment from the government to the agent in the first period, given that $w^1=w_i$, and in the second period, given that $w^1=w_i$ and $w^2=w_j$, respectively. An negative transfer payment is interpreted as the income tax pay-

[4] Spence (1978) discussed the implications of introducing distributional considerations into the model in a single period context. Diamond and Mirrlees (1978) considered a multiperiod social insurance system in the situation where the government is faced with a moral hazard problem, but they did not explicitly consider distributional issues.

ment from the agent to the government. Let δ_H and δ_L denote the multiperiod social insurance programme intended for the H-type agents and L-type agents, respectively. (δ_H, δ_L) will be referred to as a multiperiod social insurance system.

The government knows everything about the agents except for their risk types. We assume that the objective of the government is to select the multiperiod social insurance system which maximizes the weighted sum of the two-types' two-peviod expected utilities. There is no private insurance market. We assume that the agents are offered a choice of two social insurance programmes and they must select the progrmme that they prefer at the beggining of the first period.

An optimal multiperiod social insurance system is generated by the following maximization problem:

(P.3) $$\max_{(\delta_H,\delta_L)\in\Delta\times\Delta} \pi_H V_H(\delta_H)+\pi_L V_L(\delta_L)$$

subject to

$$V_H(\delta_H)\geqq V_H(\delta_L) \tag{12}$$

$$V_L(\delta_L)\geqq V_L(\delta_H) \tag{13}$$

$$\theta_H R_H(\delta_H)+\theta_L R_L(\delta_L)\geqq 0\,, \tag{14}$$

where $\pi_s\geqq 0$ for $s=H, L$ and $\pi_H+\pi_L=1$.

Constraints (12)–(13) require that neither risk group prefers the programme intended for the other group to their own. Thus, it can be guranteed that δ_H and δ_L will be selected by high and low risk people, respectively. These constraints reflect the government's inability to observe the agent's risk type. Eq. (14) is interpreted as the goverment's budget constraint. This constraint means that the per capita expected expenditure on social insurance benefits does not exceed the per capita expected revenue.

It is easy to check that the problem (P.3) has a solution. The first-order conditions for a solution, after some manipulation, are:

$$[(\pi_H/\theta_H)+(\lambda_H/\theta_H)-(\lambda_L/\theta_H)(q_i^L/q_i^H)]u'(w_i+x_i^{1H})=\mu \quad \text{for all } i \tag{15}$$

$$[(\pi_H/\theta_H)+(\lambda_H/\theta_H)-(\lambda_L/\theta_H)(q_i^L q_j^L/q_i^H q_j^H)]u'(w_j+x_{ij}^{2H})=\mu \quad \text{for all } i \text{ and } j \tag{16}$$

$$[(\pi_L/\theta_L)+(\lambda_L/\theta_L)-(\lambda_H/\theta_L)(q_i^H/q_i^L)]u'(w_i+x_i^{1L})=\mu \quad \text{for all } i \tag{17}$$

$$[(\pi_L/\theta_L)+(\lambda_L/\theta_L)-(\lambda_H/\theta_L)(q_i^H q_j^H/q_i^L q_j^L)]u'(w_j+x_{ij}^{2L})=\mu \quad \text{for all } i \text{ and } j \tag{18}$$

wher λ_H, λ_L and μ are the non-negative Lagrange multipliers correponding to the constraints (12), (13) and (14), respectively. We have also the comelmentary slackness conditions.

We are in a position to show the following results.

Proposition 4: An optimal multiperiod social insurance system satisfies the following properties: (1) if $\pi_H/\theta_H > \pi_L/\theta_L$ (resp. $\pi_H/\theta_H < \pi_L/\theta_L$), then (i) low (resp. high) risk people are offered full insurance and the second period transfer payment to them is independent of the realiztion of w^1; (ii) high (resp. low) risk people are not offered full insurance and the second period trasfer payment to them depends on the realization of w^1 as well as w^2; (2) if $\pi_H/\theta_H = \pi_L/\theta_L$, then both high and low risk people are offered a single programme that provides full insurance and the second period transfer payment to them is independent of the realization of w^1.

Proof: Let (δ_H^*, δ_L^*) denote a solution to the problem (P.3). Consider the case $\pi_H/\theta_H > \pi_L/\theta_L$. We will show that $\lambda_L > 0$. Suppose that $\lambda_L = 0$. Then, from (15)–(18) we have that $w_i + x_i^{1H*} > w_i + x_i^{1L*}$, for all i, and $w_j + x_{ij}^{2H*} > w_j + x_{ij}^{2L*}$ for all i and j. Since $V_L(\delta_H^*) < V_L(\delta_L^*)$, constraint (13) is violated. Hence we must have $\lambda_L > 0$. If $\lambda_L > 0$, from (15)–(16) and Assumptions 1-2, we have that $w_i + x_i^{1H*}$ is decreasing in i, and $w_j + x_{ij}^{2H*}$ is decreasing in i and j. Moreover, form (17)–(18) and Assumptions 1-2, we have that $w_i + x_i^{1L*}$ is non-decreasing in i, and $w_j + x_{ij}^{2L*}$ is non-decreasing in i and j. Thus, by first-order stochastic dominance, $V_H(\delta_H^*) > V_L(\delta_L^*)$ and $V_L(\delta_L^*) \geqq V_H(\delta_L^*)$. Since constraint (13) is binding at the optimum, we have $V_H(\delta_H^*) > V_H(\delta_L^*)$. Hence $\lambda_H = 0$ by complementary slackness. Then, from (17)–(18) and Assumption 1-2, we have $w_i + x_i^{1L*} = c_L^*$, for all i, and $w_j + x_{ij}^{2L*} = c_L^*$ for all i and j, where c_L^* is some constant. From the above arguments, we get our results for the case $\pi_H/\theta_H > \pi_L/\theta_L$. Similarly, we can show our results for the case $\pi_H/\theta_H < \pi_L/\theta_L$.

Next we will prove (2). We will show that $\lambda_L = 0$. Suppose that $\lambda_L > 0$. Then, by the arguments similar to those used in the proof of (1), we have $\lambda_H = 0$. Since $\pi_H/\theta_H = \pi_L/\theta_L$, from (15)–(18) we have $w_j + x_{ij}^{1L*} > w_j + x_{ij}^{1H*}$ for all i, and $w_j + x_{ij}^{2L*} > w_j + x_{ij}^{2H*}$ for all i and j. Hence we have $V_H(\delta_L^*) > V_H(\delta_H^*)$ and so constraint (12) is violated. Thus, we must have $\lambda_L = 0$. Similarly, we can show that $\lambda_H = 0$. Consequently, we can see that $\delta_H^* = \delta_L^*$ and $w_i + x_i^{1H*} (= w_i + x_i^{1L*}) = c^*$ for all i, and $w_j + x_{ij}^{2H*} (= w_j + x_{ij}^{2L*}) = c^*$ for all i and j, where c^* is some constant.

Q.E.D.

From the above results we get two important conclusions. First, from a distributional standpoint, it is in general desirable for the government to offer two distinct progerammes that induce the two types to self-select. In the prsence of asymmetric information, however, some risk-spreading must be sacrificed. The choice of an optimal social insurance system therefore involves a trade-off between redistributing (ex ante) welfare and reducing the variance of individual income streams, except in the special case where $\pi_H/\theta_H = \pi_L/\theta_L$. Second, the second period transfer payment depends on first period earnings in the case where a multiperiod social insurance programme is designed for a risk groupe who receives a higher weight in the social welfare function than its fraction of the population. Thus, except in the special case where $\pi_H/\theta_H = \pi_L/\theta_L$, when the government faces an information problem, there is a rationale for using a multiperiod social insurance system to improve social welfare over time.

The qualitative properties for the case $\pi_H/\theta_H<\pi_L/\theta_L$ are similar to those of a market equilibrium. The case, $\pi_H/\theta_H>\pi_L/\theta_L$, is of particular interest because the qualitative properties are different from those of a market equilibrium. Since the high risk group receives a higher weight in the social welfare function than its fraction of the population, the constraint (13) may be binding in this case. Note that there are negative informational externalities running from low to high risk people. Thus, the constraint imposed by asymmetric information can relaxed, as compared with a sequence of one-period social insurance programmes, by making the second period transfer payment depend on first period earnings.

In the first best world with full information, both types are offered full insurance and the second period transfer payment to them is independent of first period earnings. Thus, there is no reason to design a multiperiod social insurance system.

7. CONCLUDING REMARKS

With a simple model incorporating two-types and two-periods, we have considered the qulitative properties of equilibrium multiperiod private insurance contracts, and an optimal multiperiod social insurance system, under asymmetirc information. We have seen that the cost of informational externalities can be reduced, as compared with a sequence of one-period contracts, by making the second period payment to some risk group depend on the realization of w^2 as well as w^1. In the first-best world with full information, the optimal multiperiod insurance contract is a sequence of one-period insurance contracts. The effect of asymmetric information is that even when the enviornment is separable over time, it is desirable to design a mechanism that ties future benefits to present outcomes.

The results in this paper can be extended easily to cases more than two periods so long as the number is finite. A more interesting problem is posed by the following question: can the inefficiencies caused by asymmetric information be fully eliminated when the contract length is infinite?

It should be remembered that the behavior rule built into the Wilson concept of equilibrium is arbitary. Thus, in order to test the validity of the results obtained in the present paper, there is a need to investigate whether market behavior of the Wilson-type is relevant.

So far we have considered private insurance and public insurance separately. Both from theoretical and practical standpoints, it may be useful to inquire into the relationship between private insurance and public insurance. In particular, such an inquiry is indispensable if we are to obtain effectual recommendations on the design of a social insurance system. This issue deserves further consideration.

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