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DOES QUANTITY-CONSTRAINED BEHAVIOR MAKE THE CONJECTURE FUNCTION KINKED?*

Takanobu IKEDA

Abstract: This paper studies a trader's conjecture-making behavior concerning the quantities and the prices of his net-trade. In the literature of monopolistic competition and quantity-constrained equilibria, it is known that asymmetry in traders' conjecture-making behavior causes price-rigidity. However, what causes such asymmetry has not yet been studied. We demonstrate that a quantity conjecture function, which represents the trader's conjecture-making behavior concerning quantities, has an intrinsic kinkedness when he forecasts his quantity constraints based on his market share.

1. INTRODUCTION

In a market where an agent has to explore his trading possibilities, he may consider that the market's response to a change in his behavior depends upon the direction of that change or that a slight change in the market state causes a drastic change in the outcome of his behavior. Such a perception is characterized by kinks in the agent's subjective excess demand functions, and it is argued that the existence of kinks in this sense significantly affects the performance of the market.[1] In the studies of quantity-constrained equilibrium models by Negishi (1979) and Hahn (1978), the kink is considered to cause quantity-adjustment rather than price-adjustment in a competitive price system.[2] However, what causes the kink has not been studied. The purpose of this paper is to identify a type of quantity-constrained perception which gives rise to kinkedness of the subjective excess demand function.

The quantity-constrained perception to be considered here is characterized by two features. First, for each good an agent receives quantity signals as well as a

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[1] In the study of imperfect competition, Sweezy (1939) originates the notion of kinkedness in a subjective demand curve. See Negishi (1979).

[2] For a survey of the literature, see Negishi (1979), Benassy (1982), and Ito (1985). Drèze (1975), Benassy (1975), and Younès (1975) introduce the notion of quantity-constrained equilibrium in terms of a rationing scheme with the fixed-price assumption. As for models of quantity-constrained equilibrium with a game theoretical framework, see Böhm-Lévine (1979) and Heller-Starr (1979).

price signal. Using the set of signals, the agent forecasts the actual trading price and the quantity constraints, which are the upper and lower limits of possible trade. Quantity constraints are described by forecast functions, which can take either finite or infinite values. When the values of a trader's forecast functions for a good are always plus and minus infinity, we say that the agent has perfect competitive perception. Otherwise, an agent has the trade-limit perception.

Secondly, an agent believes that trading an amount beyond the constraints is costly. This belief is called quantity-constrained conjecture and is described by a conjecture function. The cost may be prohibitive or may change gradually according to the difference between the desired amount and the limit.

When an agent has the forecasts and conjecture described above, we say that the agent has quantity-constrained perception. It is obvious that the behavior of an agent with perfect competitive perception is identical with that of a price-taker in textbooks. Thus the notion of quantity-constrained perception is a generalization of price-taking behavior.

The motivation behind the generalization is as follows: It is often argued that in a partially monopolistic market the group of small firms compete for the remaining market share at the price set by the monopolist. Such behavior may be conceivable in a market only with small traders. A tiny trader knows that he does not have the power to be the leader in price competition and tries to protect his market share. Or he may try to expand the share though he may win little. The market for such a trader may consist of himself and all the other traders. The trader may imagine that all the other traders form a coalition implicitly and behave like a monopolist. Thus he may behave like a small firm in a partially monopolistic market. The notion of quantity-constrained perception describes behavior of a trader who is a price-taker and is conscious of his market share.

It is known that the conjecture function considered by Negishi and Hahn may or may not have kinks.[3] To compare our result with theirs, we decompose a conjecture function into price conjecture functions and quantity conjecture functions. A price conjecture function describes a trader's conjecture of the actual trading price of a good. His conjectured amount of trade of a good is given by a quantity conjecture function. The conjecture function studied by Negishi and Hahn is a price conjecture function in this paper. When it is conjectured that the cost of trading an amount beyond the constraints changes gradually, the price conjecture function is smooth and does not have any kink.

Our main result is that a quantity conjecture function always has kinks even if

[3] Hahn (1978) did not refer to the role of kinkedness of the conjecture function. It was Gale (1978) who showed, in Hahn's model, that if conjecture functions are smooth, then equilibria are necessarily Walrasian; but if the conjecture functions are kinked, then there exist non-Walrasian equilibria. To be more precise, existence of kinkedness itself is not sufficient for existence of non-Walrasian (or Keynesian) equilibria, as Negishi (1979) noted in his theory of a firm in Chapters 6 and 7. Trujillo (1980) introduces the notion of sufficiently kinked. For details, see Trujillo (1980) and Ikeda (1984).

the conjectured cost of trading an amount beyond the constraints changes smoothly. The difference comes from the types of quantity-constrained perception. A price conjecture function embodies a type of quantity-constrained perception that a change in an agent's behavior may change the market's response. On the other hand, a quantity conjecture function describes a perception that the same behavior on the part of the agent may bring different amounts of trade depending on the market state. We show that the latter type of quantity-constrained perception indeed causes intrinsic kinks in the conjecture function.[4]

The organization of this paper is as follows: In section 2 we present our notation, assumptions, and the class of mechanisms to be considered for pure-exchange economies. In section 3 we show the uniqueness and smoothness of the conjecture function of quantity-constrained Walrasian mechanisms. In section 4 we characterized the conjecture function of quantity-constrained non-Walrasian mechanism in terms of kinkedness. All proofs are given in the appendix.

2. A QUANTITY-CONSTRAINED MECHANISM FOR A PURE-EXCHANGE ECONOMY

(2.1.) *A Class of Environments, E*

We consider a pure-exchange economy consisting of n persons ($N=\{1, 2, \cdots, n\}$) trading in l goods ($L=\{1, 2, \cdots, l\}$) and "money" (subscripted 0) which differs from goods in that no agent perceives any quantity constraints on money. Although we consider one period, it is one period in a multiperiod situation. So money is a store of wealth in an uncertain world and is in the utility function. E^i is the set of consumer i's characteristics. Its generic element is $e^i=\{C^i, u^i, \omega^i\}$, where C^i is his consumption set, $u^i: C^i \rightarrow \boldsymbol{R}$ is the utility function, and $\omega^i=(\omega_0^i, \omega_1^i, \cdots, \omega_l^i)$ is his initial endowment. Let $E=\prod_{k \ni N} E^k$ be the set of environments.

(2.2) *A Class of General Exchange Processes, Π*

In this subsection we present a class of general exchange processes which includes the Walrasian mechanism as a special case. In the Walrasian mechanism each agent believes that he can trade as much as he wants at the prevailing prices, regardless of other agents' excess demands. Such perception is called perfect competitive behavior or perception. It is the perfect competitive perception that makes the Walrasian mechanism bring the harmony between agents self-interest seeking behaviors and socially desired goals like efficiency and full-employment. On the other hand, it is known that the Walrasian mechanism does not provide

[4] Ikeda (1984, 1985) shows that a mechanism with non-manipulable quantity-constrained conjectures and trade-limit perceptions always generates non-Walrasian performance. Thus, the existence of kinkedness implies non-Walrasian performance, and in particular price-rigidity.

agents with enough incentives to take perfect competitive perception.[5] Thus from the viewpoint of a descriptive analysis, it is plausible to regard that an agent may or may not have perfect competitive perception.

We keep all the characteristics of the Walrasian mechanism, but exclude perfect competitive perception. Consumer i emits his message $m^i=(m_1^i, m_2^i, \cdots, m_l^i)\in M^i$ concerning his desired net-trade of goods except money. Let $M=\prod_{i\in N} M^i$ and $m\in M$. As in the Arrow-Debreu model, the existence of an auctioneer is assumed to make the mechanism embody the law of demand and supply: namely the prices are adjusted according to the aggregate excess demands. The auctioneer announces the price of money, $p_0=1$, and the price vector $p=(p_1, \cdots, p_l)\in M^0$. Thus, every agent observes the current message vector (p, m). We assume:

Assumption 1 (A.1)
$M^0=\boldsymbol{R}_+^l=\{p\in\boldsymbol{R}^l|p_j\geqq 0\}$ and for every $i\in N$, $M^i=\boldsymbol{R}^l$.

Observing the current messages sent by other agents, consumer i makes a conjecture concerning his possible net-trades. The conjecture is described by his conjecture function $\hat{h}^i\colon M^0\times M\to\boldsymbol{R}^{l+1}$.[6] Let $z^i\in\boldsymbol{R}^{l+1}$ be a conjectured net-trade vector, where $z^i=(z_0^i, \cdots, z_l^i)$. Then $\hat{h}^i(p, m)=z^i$. For each good, consumer i makes a conjecture with respect to the volume and price of net-trade which he expects to be realized. His conjecture concerning the volume of goods is given by a quantity conjecture function vector $X^i=(X_1^i, \cdots, X_l^i)$, where $X_j^i\colon M_j\to\boldsymbol{R}$, $m_j=(m_j^1, \cdots, m_j^n)\in M_j=\prod_{k=1}^n M_j^k$ and $z_j^i=X_j^i(m_j)$ for $j\in L$. Each quantity conjecture function represents the volume of the good he expects to trade. Consumer i also makes the conjecture concerning the price vector he expects to be the actual trading price vector when he observes the current message vector. The price vector he expects is given by a price conjecture function vector $P^i=(P_1^i, \cdots, P_l^i)$, where $P_j^i\colon M_j^0\times M_j\to\boldsymbol{R}_{++}$ and $p_j^i=P_j^i(p_j, m_j)$. For convenience, we use $m(i)=(m^1, \cdots, m^{i-1}, m^{i+1}, \cdots, m^n)\in M(i)=\prod_{k\neq i} M^k$ and $m_j(i)\in M_j(i)$.

The most essential feature of a price system for a pure-exchange economy is that each consumer must satisfy the budget constraint. Thus, we assume:

Assumption 2 (A.2)
For every $i\in N$, $z_0^i+\sum_{j\in L} p_j^i z_j^i=0$.

By (A.2), $\hat{h}^i(p, m)=(-\sum_{j\in L} P_j^i(p_j, m_j)X_j^i(m_j), X_1^i(m_1), \cdots, X_l^i(m_l))$.

In order to interpret his message vector as his excess demand vector, we assume that an agent believes that he will obtain as much as his desired quantity whenever the sum of all agents desired quantities is zero.

Assumption 3 (A.3)
For every $i\in N$ and every $j\in L$, $X_j^i(m_j)=m_j^i$ if $\sum_{k\in N} m_j^k=0$.

[5] This is known as the incentive compatibility problem. See Hurwicz (1972).

[6] A conjecture function in this paper is called an outcome function in both the literature of game theory and the study of resource allocation mechanisms.

Since we are interested in the form of conjecture function, a description of the agents behavior is brief. Every consumer is assumed to maximize his utility in the Nash way, where $v^i(p, m)=u^i \circ \hat{h}^i(p, m)$ is consumer i's objective function. The auctioneer is assumed to follow a behavior rule which represents the law of demand and supply. A natural equilibrium concept is the Nash equilibrium concept. So we define:

Definition 1 (D.1)

An *equilibrium* is $(p^*, m^*) \in M^0 \times M$ such that

(a) for every $i \in N$, $\hat{h}^i(p^*, m^*) \in F^i$ and
$v^i(p^*, m^*) \geqq v^i(p^*, m^*(i), m^i)$
for all $m^i \in S^i(p^*, m^*(i)) = \{m^i \in M^i | \omega^i + \hat{h}^i(p^*, m(i)^*, m^i) \in C^i\}$;

(b) $\sum_{i \in N} m_j^{i*} = 0$ for every $j \in L$; and

(c) $P_j^i(p_j^*, m_j^*) = p_j^*$ for every $i \in N$ and every $j \in L$.

Condition (a) requires that for every consumer i, the equilibrium net trade, $\hat{h}^i(p^*, m^*)$, is feasible and his equilibrium message is a solution of his utility maximization problem when $(p^*, m^*(i))$ is given. Condition (b) requires that p^* is given according to the law of demand and supply. Finally, condition (c) requires that at an equilibrium, "one-good, one-price" holds: prices conjectured by consumers are all equal.

We denote by $\mu(e)$ the set of equilibria for an environment e. We call $\mu: E \to M^0 \times M$ the equilibrium correspondence. We assume that the actual trade takes place only at an equilibrium. In this way, we have introduced the three most fundamental assumptions which characterize a tâtonnement process, (A.1)–(A.3), into Hurwicz's resource allocation mechanism framework.[7]

Definition 2 (D.2)

We call $\pi = \{\mu, M^0 \times M, (\hat{h}^i)_{i \in N}\}$ a *general exchange process* if (A.1)–(A.3) are satisfied. We denote the class of general exchange processes by Π.

A general exchange process is a tâtonnement process without the specification of the form of the conjecture function. Of course, this class Π includes the Walrasian mechanism in which (a) all agents price conjectures are identical with the prices called by the auctioneer; and (b) every agent believes that he can obtain more of the good by sending a larger message regardless of other agents messages. At the same time, this class Π is broad enough to include a variety of non-Walrasian mechanisms.

Definition 3 (D.3)

(i) We say that π is *Walrasian*, i.e., $\pi \in \Pi^W$, when, for every $i \in N$ and every $j \in L$, $P_j^i(p_j, m_j) = p_j$ for all $(p_j, m_j) \in M_j^0 \times M_j$ and $X_j^i(\cdot, m_j(i)): M_j^i \to \boldsymbol{R}$ is strictly increasing for all $m_j(i) \in M_j(i)$.

[7] As for the framework of the study of a resource allocation mechanism, see Hurwicz (1976) and (1979).

(ii) We say that π is *non-Walrasian*, i.e., $\pi \in \Pi^{NW}$, when $\pi \notin \Pi^{W}$.

(2.3) *A Class of Quantity-Constrained Exchange Process*, $\Pi_{QC}(\subset \Pi)$

We make a conjecture function embody quantity-constrained behavior and obtain the class of quantity-constrained exchange processes. It is a subclass of general exchange processes constructed in the previous subsection. A quantity conjecture function embodies absolute or non-manipulable quantity-constrained behavior such that a consumer's expected net-trade must be between the limits. On the other hand, a price conjecture function embodies relative or manipulable quantity-constrained perception such that if he wants to trade an amount beyond these limits, then he expects the actual trading price must be higher and lower than the prevailing one when he is buying and selling, respectively. Thus, the limits on net-trade work differently for quantity and price conjecture functions. This difference causes different answers to our question: Does quantity-constrained behavior make the conjecture function kinked? So we distinguish limits for a quantity conjecture function from a price conjecture function. We call the former the trade-limit vector of good j ($\beta^{ij}=(\beta^{ij}_{+}, \beta^{ij}_{-})$, where β^{ij}_{+} and β^{ij}_{-} are the upper and lower trade-limits, respectively). We call the latter the quantity signal vector of good j ($\gamma^{ij}=(\gamma^{ij}_{+}, \gamma^{ij}_{-})$, where γ^{ij}_{+} and γ^{ij}_{-} are the upper and lower quantity signals, respectively).

Both quantity and price conjecture functions are a composite of a perception function and a pair of forecast functions. For each conjecture function, we first describe by a perception function how the limits restrict a consumer's trading possibilities. Then we introduce forecast functions by which a consumer predicts upper and lower limits. Following this order, we introduce a quantity conjecture function first, and then a price conjecture function.

Consumer i feels that a trade-limit vector of good j is absolute; therefore, he cannot trade any amount beyond these limits no matter how much he wants to trade. This perception is described by consumer i's quantity perception function for good $j \to G^i_j$: $\boldsymbol{R}\times\boldsymbol{R}^{\infty}_{+}\times\boldsymbol{R}^{\infty}_{-} \to \boldsymbol{R}$.

Definition 4 (*D.4*)

$\boldsymbol{G}$ is a class of *quantity perception functions* such that $(\beta^{ij}_{+}, \beta^{ij}_{-})$ restricts the form of G^i_j only in the following way:

(i) G^i_j: $\boldsymbol{R}\times\boldsymbol{R}^{\infty}_{+}\times\boldsymbol{R}^{\infty}_{-} \to \boldsymbol{R}$ is continuous;

(ii) $G^i_j(m^i_j, \beta^{ij}_{+}, \beta^{ij}_{-})=\begin{cases}\beta^{ij}_{+} & \text{if } m^i_j \geqq \beta^{ij}_{+} \geqq 0\\ \beta^{ij}_{-} & \text{if } m^i_j \leqq \beta^{ij}_{-} \leqq 0\text{; and}\end{cases}$

(iii) $G^i_j(\cdot, \beta^{ij}_{+}, \beta^{ij}_{-})$: $M^i_j \to \boldsymbol{R}$ is a strictly increasing function of m^i_j if $\beta^{ij}_{-}<m^i_j<\beta^{ij}_{+}$.

Figures 1 and 2 show the curve for the function $G^i_j(\cdot, \beta^{ij}_{+}, \beta^{ij}_{-})$ when the trade-limit vector is given. Condition (ii) requires that his expected net-trade is restricted by the trade-limits, while condition (iii) requires that any amount between the limits can be his net-trade. The consumer feels that in order to realize an amount beyond

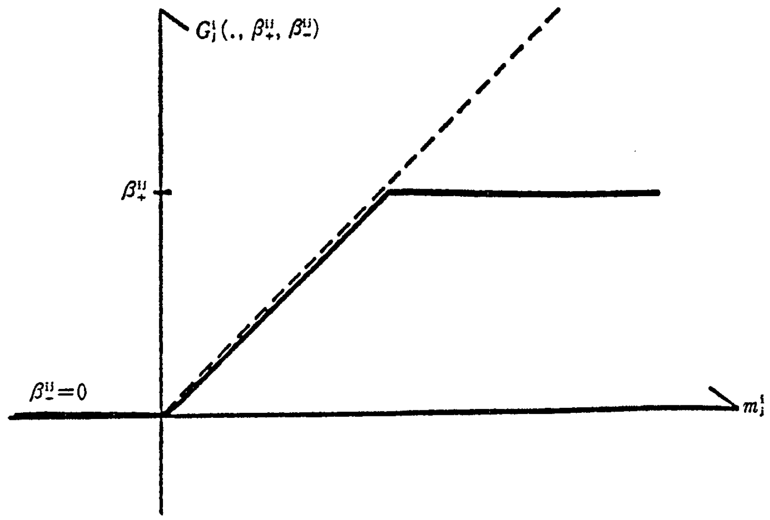

Fig. 1.

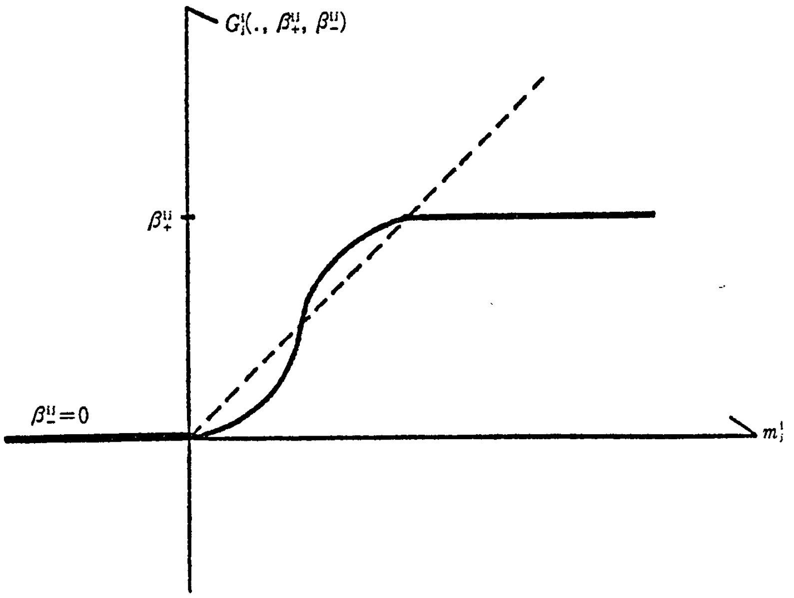

Fig. 2.

the trade-limits as his net-trade, he has to pay prohibitive costs. We assume:

Assumption 4 (A.4)
For every $i \in N$ and every $j \in L$, $G_j^i \in \boldsymbol{G}$.

Trade-limits are subjective limits which consumer i forecasts based on the current messages concerning net-trade of good j. His forecasting activity of a trade-limit vector is described by the forecast function vector (f_+^{ij}, f_-^{ij}). $f_+^{ij}: M_j(i) \rightarrow \boldsymbol{R}_+^\infty$ and $f_-^{ij}: M_j(i) \rightarrow \boldsymbol{R}_-^\infty$ are, respectively, consumer i's forecast functions of the upper and lower trade-limits of good j, where $M_j(i)=\prod_{k \neq i} M_j^k$ and $\boldsymbol{R}_+^\infty=\boldsymbol{R}_+ U\{+\infty\}$

and $R_{-}^{\infty}=R_{-}\cup\{-\infty\}$ are extended real half-lines. Then $f_{+}^{ij}(m_j(i))=\beta_{+}^{ij}$ and $f_{-}^{ij}(m_j(i))=\beta_{-}^{ij}$. Now we assume:

Assumption 5 (A.5)

For every $i\in N$ and every $j\in L$, consumer i's quantity conjecture function for good j, namely X_j^i, is of the form:

$$X_j^i(m_j)=G_j^i(m_j^i, f_{+}^{ij}(m_j(i)), f_{-}^{ij}(m_j(i))) \qquad \text{for all} \quad m_j\in M_j\,.$$

Assumption (A.5) requires a quantity conjecture function X_j^i to be a composite of a quantity perception function and a forecast function pair. Thus, consumer i forecasts the trade-limits and is constrained absolutely by the trade-limits.

Now let us introduce a price conjecture function, by which a consumer's net-trade is constrained by the quantity signals. As we did for a quantity conjecture function, we first introduce a perception function. We call K_j^i: $R_{+}\times M_j^i\times R_{+}^{\infty}\times R_{-}^{\infty}\to R_{+}$ consumer i's price perception function for good j.

Definition 5 (D.5)

$\boldsymbol{K}$ is a class of *price perception functions* such that $(p_j, \gamma_{+}^{ij}, \gamma_{-}^{ij})$ restricts the form of K_j^i only in the following way:

(i) K_j^i: $R_{+}\times R\times R_{+}^{\infty}\times R_{-}^{\infty}\to R_{+}$ is constinuous;

(ii) $K_j^i(p_j, m_j^i, \gamma_{+}^{ij}, \gamma_{-}^{ij})=p_j$ if $\gamma_{-}^{ij}\leqq m_j^i\leqq\gamma_{+}^{ij}$; and

(iii) $K_j^i(p_j,\cdot, \gamma_{+}^{ij}, \gamma_{-}^{ij})$: $R\to R_{+}$ is a strictly increasing function of m_j^i for all $p\in M^0$ if either $m_j^i>\gamma_{+}^{ij}$ or $m_j^i<\gamma_{-}^{ij}$.

See Figure 3 and 4 for the curve of the function $K_j^i(p_j, \cdot, \gamma_{+}^{ij}, \gamma_{-}^{ij})$ when $(p_j, \gamma_{+}^{ij}, \gamma_{-}^{ij})$ is given. Condition (ii) requires that as long as his message is within the quantity signals, he can trade the value of quantity conjecture functions at the

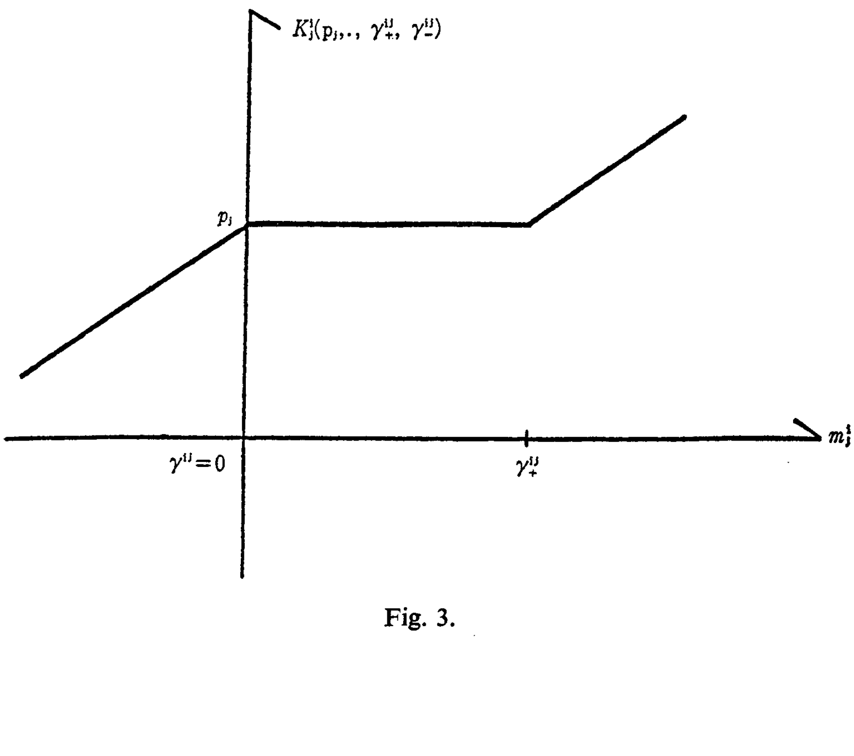

Fig. 3.

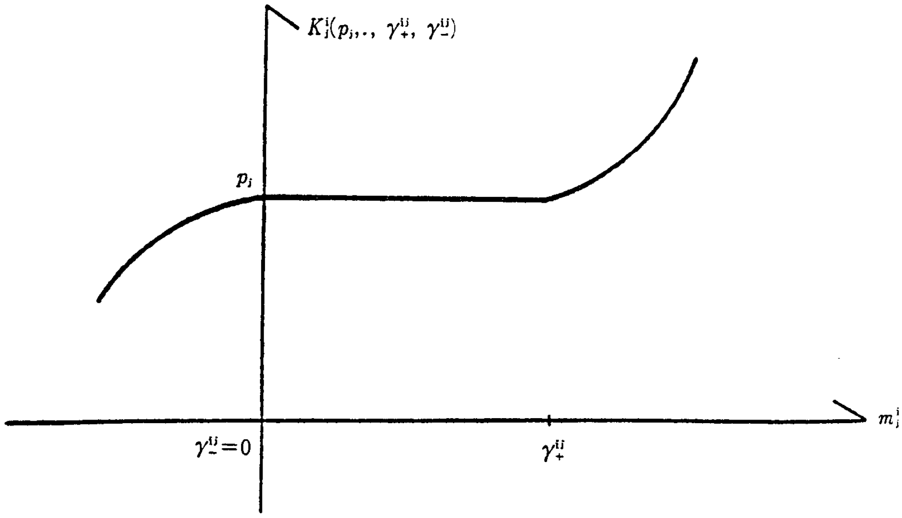

Fig. 4.

prevailing price p_j. Condition (iii) requires that if his message is beyond these quantity signals, then consumer i believes that he should offer his personal price p_j^i which should be higher (lower) than the prevailing price if his message is greater (smaller) than the upper (lower) quantity signal. This part of consumer i's perception truncates the set of his net trades of good j in terms of the price. We assume:

Assumption 6 (A.6)

For every $i \in N$ and every $j \in L$, $K_j^i \in \boldsymbol{K}$.

Now we introduce forecast functions of quantity signals of good j. Consumer i forecasts the upper and lower quantity signals by the forecast functions ($g_+^{ij}: M_j(i) \to \boldsymbol{R}_+^\infty$ and $g_-^{ij}: M_j(i) \to \boldsymbol{R}_-^\infty$, respectively). Then, $g_+^{ij}(m_j(i))=\gamma_+^{ij}$ and $g_-^{ij}(m_j(i))=\gamma_-^{ij}$. Now, we assume:

Assumption (A.7)

For every $i \in N$ and $j \in L$, consumer i's price conjecture function for good j, namely P_j^i, is of the form:

$$P_j^i(p_j, m_j)=K_j^i(p_j, m_j^i, g_+^{ij}(m_j(i)), g_-^{ij}(m_j(i)))$$

for all $p_j \in M_j^0$ and $m_j \in M_j$.

Assumption (A.7) requires a price conjecture function P_j^i to be a composite of a perception function and a forecast function pair. In this way, a price conjecture function embodies manipulable quantity-constrained perception so that if a consumer wants to trade beyond the quantity signals, then he might be able to do so by offering his price.

Consumer i forecasts his trade-limit vector $(\beta_+^{ij}, \beta_-^{ij})$ and quantity signal vector $(\gamma_+^{ij}, \gamma_-^{ij})$ for good j by the forecast functions $f^{ij}=(f_+^{ij}, f_-^{ij})$ and $g^{ij}=(g_+^{ij}, g_-^{ij})$, respectively. We assume that these forecast functions belong to the common class of functions denoted by $\boldsymbol{P}$; that is.

$$\boldsymbol{P}=\{f^{ij}|f^{ij}: M_j(i) \to \boldsymbol{R}_+^\infty \times \boldsymbol{R}_-^\infty\}=\{g^{ii}|g^{jj}: M_j(i) \to \boldsymbol{R}_+^\infty \times \boldsymbol{R}_-^\infty\}$$

We will assume that this has the following partition. To save us from repetition, we state Assumption 8 in terms of the forecast function of the trade-limits, f^{ij}.

Assumption 8 (*A.8*)

$$\boldsymbol{P}=\boldsymbol{P}^{PC} \cup \boldsymbol{P}^{TL}$$

where:

(a) $\boldsymbol{P}^{PC}$ is the class of forecast functions $f^{ij}: M_j(i) \to \boldsymbol{R}_+^\infty \times \boldsymbol{R}_-^\infty$ such that $f^{ij}=(f_+^{ij}, f_-^{ij})$ represents perfect competitive perception:

$$f_+^{ij}(m_j(i))=+\infty \quad \text{and} \quad f^{ij}(m_j(i))=-\infty \quad \text{for all } m_j(i)\in M_j(i) \text{ ; and}$$

(b) $\boldsymbol{P}^{TL}$ is the class of forecast functions $f^{ij}: M_j(i) \to \boldsymbol{R}_+^\infty \times \boldsymbol{R}_-^\infty$ such that $f^{ij}=(f_+^{ij}, f_-^{ij})$ represents trade-limit perception:

(i) $f_+^{ij}(m_j(i))<+\infty$ and $f_-^{ij}(m_j(i))>-\infty$ for all $m_j(i)\in M_j(i)$;
(ii) $f_+^{ij}(m_j(i))=0$ if $\sum_{s\neq i} m_j^s \geqq 0$ and $f_-^{ij}(m_j(i))=0$ if $\sum_{s\neq i} m_j^s \leqq 0$;
(iii) $f_-^{ij}(m_j(i)) \leqq -\sum_{s\neq i} m_j^s \leqq f_+^{ij}(m_j(i))$ for all $m_j(i)\in M_j(i)$; and
(iv) $f_+^{ij}: M_j(i) \to \boldsymbol{R}_+^\infty$ and $f_-^{ij}: M_j(i) \to \boldsymbol{R}_-^\infty$ are continuous.

The most important difference between $\boldsymbol{P}^{PC}$ and $\boldsymbol{P}^{TL}$ is that the forecast functions take as their values infinity or finite real numbers, respectively. As for f^{ij} in $\boldsymbol{P}^{TL}$, see Figures 5 and 6.

An agent who has quantity-constrained perception must want to forecast trade-

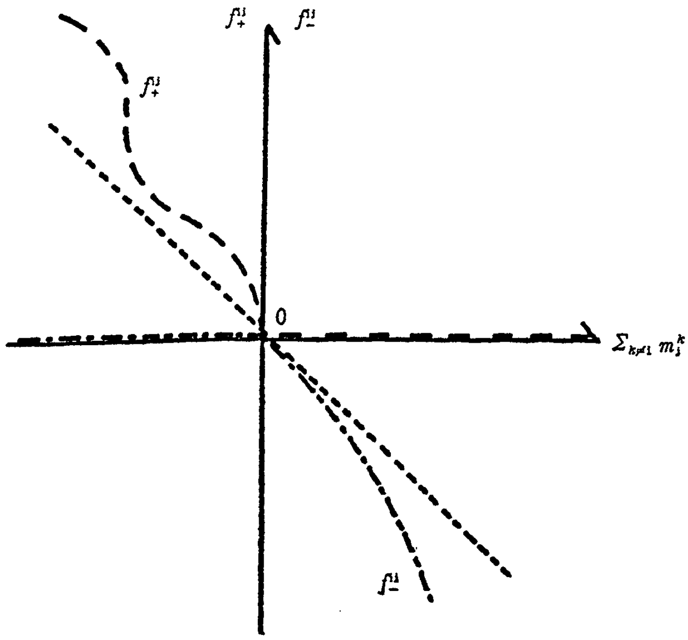

Fig. 5.

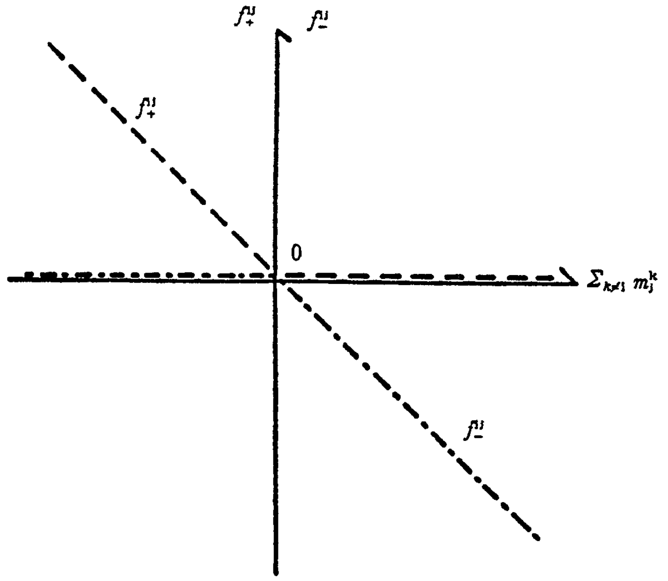

Fig. 6.

limits which will sensitively reflect the market situation. A forecast function pair in P^{PC} is completly insensitive, because it does not let the agent know what is happening in the market. A forecast function pair in P^{TL} is slightly more sensitive. The sensitivity of forecast functions depends on the volume of communication a consumer has in the market. The volume is determined by the existing communication network or the agent's market share. The agent achieves the market share through his past trading activities. In other words, the existing communication network is a capital good, and the volume of communication is the amount of the capital service. Since an agent in a competitive market is small, his market share is also small. But, an agent may perceive that his market share is at least as large as the minimum one; that is, of a trader who is ranked last in a queing system. This requires condition (iii) above, which states that the excess demand aggregated over all except for the consumer with the opposite sign is always the consumer's possible trade. When the market, excluding a consumer, is in excess demand (or supply), the consumer sees no possibility of buying (or selling) the good. This is condition (ii) above.

By adding the features of quantity-constrained behavior, (A.4)–(A.8), to the class of general exchange processes, we obtain a class of quantity-constrained exchange processes. In a mechanism of this class, consumer i's set of feasible allocations is always a subset of his (Walrasian) budget set. This is why we call this type of conjecture quantity-constrained behavior, in contrast with price-taking behavior.

Definition 6 (D.6)

A mechanism π is called a *quantity-constrained exchange process*, i.e., $\pi \in \Pi_{QC}$, if $\pi \in \Pi$ and (A.4)–(A.8) are satisfied.

We noted that a quantity conjecture function embodies non-manipulable quantity-constrained behavior, while a price conjecture function embodies manipulable quantity-constrained behavior. We will study how this difference causes the contrast of answers to the question: Does quantity-constrained behavior make the conjecture function kinked? So it is convenient to name the special cases.

Definition 7 (D.7)

A quantity-constrained mechanism π is *of the quantity type*, i.e., $\pi \in \Pi^{q}_{QC}$, if $\pi \in \Pi_{QC}$ and, for all $i \in N$ and all $j \in L$, $g^{ij} \in \boldsymbol{P}^{PC}$.[8]

This is, each consumer has perfectly competitive perceptions with respect to the prices of all goods. So, studying a non-manipulable quantity-constrained mechanism or studying quantity conjecture functions of a mechanism $\pi \in \Pi_{QC}$ is equivalent to the study of a mechanism $\pi \in \Pi^{q}_{QC}$.

Definition 8 (D.8)

A quantity-constrained mechanism π is *of the price type*, i.e., $\pi \in \Pi^{p}_{QC}$, if $\pi \in \Pi_{QC}$ and, for all $i \in N$ and all $j \in L$, $f^{ij} \in \boldsymbol{P}^{PC}$.[9]

That is, each consumer has perfectly competitive perception with respect to his net-trade of all goods. For the study of a manipulable quantity-constrained mechanism or the study of price conjecture functions of $\pi \in \Pi_{QC}$, we focus on the study of a mechanism $\pi \in \Pi^{p}_{QC}$. This is the advantage gained from the introduction of these special cases.

3. THE CLASS OF QUANTITY-CONSTRAINED WALRASIAN MECHANISMS

In this section, we show that the class Π^{W}_{QC} of quantity-constrained Walrasian mechanisms is a singleton set, and that its conjecture function is continuously differentiable. These results are not surprising. However, they raise interesting questions: Is the Walrasian mechanism unique as a mechanism which achieves the Walrasian performance? Is continuous differentiability of conjecture functions a necessary condition for Walrasian performance?[10]

A class of general exchange processes is defined without specifying the forms of conjecture functions. Our definition of a Walrasian mechanism is, therefore, quite general and is consistent with a variety of forms of conjecture functions. However, since the form of conjecture functions is given by perception functions, consumers' perfectly competitive perceptions single out the form of the conjecture

[8] An agent's set of feasible net-trades in a quantity-constrained mechanism of the quantity type is the same as that in Drèze (1975).

[9] A price conjecture function in this mechanism restricts an agent's net-trade in the same way that it does in Hahn's model. In Hahn (1979), quantity constraints are signals from the market, while, in this paper, they are forecasted by agents.

[10] The importance of the Walrasian mechanism comes from the fact that it realizes a Walrasian allocation (that is, a Pareto optimal and full-employment allocation) at an equilibrium.

functions. The unique form is exactly the description of a perfect competitor's behavior: price-taking behavior or the perfect elasticity of his subjective demand curve.

Lemma 1

(i) If $\pi \in \Pi^{p}_{QC}$, then for all $i \in N$ and all $j \in L$, $X^i_j(m^i_j)=m^i_j$ for all $(p_j, m_j) \in M^0_j \times M_j$.

(ii) If $\pi \in \Pi^{q}_{QC}$, then for all $i \in N$ and all $j \in L$, $P^i_j(p_j, m_j)=p_j$ for all $(p_j, m_j) \in M^0_j \times M_j$.

(iii) If $\pi \in \Pi^{W}_{QC}$, then for all $i \in N$ and all $j \in L$, $P^i_j(p_j, m_j)=p_j$ and $X^i_j(m_j)=m^i_j$ for all $((p_j, m_j) \in M^0_j \times M_j$. Thus, the class Π^{W}_{QC} is a singleton set.

Proof

See Appendix.

As a corollary of this lemma, we obtain a necessary condition for a quantity-constrained Walrasian mechanism.

Theorem 1

If $\pi \in \Pi^{W}_{QC}$, then $\hat{h}=(\hat{h}^i)_{i \in N}$: $M^0 \times M \rightarrow \boldsymbol{R}^{n(l+1)}$ is continuously differentiable.

Proof

See Appendix.

Gale (1978) shows that Hahn's conjectural equilibrium model, a non-Walrasian mechanism, yields only Walrasian equilibria if conjecture functions are differentiable. His result invites us to explore the degree or the type of smoothness of conjecture functions that is compatible with quantity-constrained non-Walrasian performance.

4. THE CLASS OF QUANTITY-CONSTRAINED NON-WALRASIAN MECHANISMS

Knowing that the conjecture function of the quantity-constrained Walrasian mechanism is continuously differentiable, our question is whether one can characterize the conjecture function $\hat{h}$ of a quantity-constrained non-Walrasian mechanism in terms of (non-)differentiability. In other words, does a conjecture function have kinks of any kind intrinsically, if the forecast functions are of the class $\boldsymbol{P}^{TL}$? We show a keen contrast of answers to this question, depending on which, a quantity or a price, conjecture function is considered.

The kinkedness of a forecast function arises from two conditions in (A.8-b): (ii) the sum of all the other consumers' messages with the opposite sign is always a credible trade-offer; and (iii) whenever the sum with the opposite sign is negative (respectively, positive), the upper limit (the lower limit) is zero. (See Figures 5 and 6).[11]

[11] Note that either the upper or the lower limit is zero. In figures, we use the case in which the lower limit is zero; that is, agent i's "market" is a seller of good j.

Lemma 2

If $f^{ij} \in \boldsymbol{P}^{TL}$, then f^{ij}_{+} and f^{ij}_{-} are non-differentiable with respect to m^s_j, $s \neq i$ at $m_j(i)$ when $\sum_{k \neq i} m^k_j = 0$. Similarly, if $g^{ij} \in \boldsymbol{P}^{TL}$, then g^{ij}_{+} and g^{ij}_{-} are non-differentiable with respect to m^s_j, $s \neq i$, at $m_j(i)$ when $\sum_{k \neq i} m^k_j = 0$.

Proof

See Appendix.

The assumptions which cause the kinkedness of a forecast function seem fundamental in the context of quantity-constrained behavior. The most essential condition is (iii): only when the "market," which consists of all other consumers, wants to trade with the consumer, can the consumer trade. Since this condition causes such a big kink, one might expect that non-existence of cross-derivatives of a conjecture function is an intrinsic property of the quantity-constrained behavior. From now on, we examine this surmise.

A consumer's conjecture function is constructed with factor functions, i.e., price and quantity conjecture functions. So we will examine factor functions separately; first we will examine a quantity conjecture function and then consider a price conjecture function.

Example 1 is a quantity conjecture function which has its own-derivatives but not cross-derivatives.

Example 1

Let

$$G^i_j(m^i_j, \beta^{ij}_{+}, \beta^{ij}_{-}) = \begin{cases} \beta^{ij}_{+} & \text{if } 0 \leqq \beta^{ij}_{+} < m^i_j \\ N^i_j(m^i_j, \beta^{ij}_{+}, \beta^{ij}_{-}) & \text{if } \beta^{ij}_{-} \leqq m^i_j \leqq \beta^{ij}_{+} \\ \beta^{ij}_{-} & \text{if } m^i_j < \beta^{ij}_{-} \leqq 0 \,, \end{cases}$$

where

$$N^i_j(m^i_j, \beta^{ij}_{+}, \beta^{ij}_{-}) = [(\beta^{ij}_{+} - \beta^{ij}_{-})/2] \sin [(\pi/2)\{m^i_j - (\beta^{ij}_{+} - \beta^{ij}_{-})/2\}/\{(\beta^{ij}_{+} - \beta^{ij}_{-})/2\}] + (\beta^{ij}_{+} + \beta^{ij}_{-})/2 \,.$$

We note that

$$N^i_j \text{ is of } C^1 \text{ class} \,,$$
$$N^i_j(\beta^{ij}_{+}, \beta^{ij}_{+}, \beta^{ij}_{-}) = \beta^{ij}_{+} \,, \qquad N^i_j(\beta^{ij}_{-}, \beta^{ij}_{+}, \beta^{ij}_{-}) = \beta^{ij}_{-} \,,$$
$$(\partial N^i_j/\partial m^i_j)(\beta^{ij}_{+}, \beta^{ij}_{+}, \beta^{ij}_{-}) = (\partial N^i_j/\partial m^i_j)(\beta^{ij}_{-}, \beta^{ij}_{+}, \beta^{ij}_{-}) = 0 \,,$$

and

$$(\partial N^i_j/\partial m^i_j)(m^i_j, \beta^{ij}_{+}, \beta^{ij}_{-}) > 0 \qquad \text{if } \beta^{ij}_{-} < m^i_j < \beta^{ij}_{+} \,.$$

Then it is easily seen that $G^i_j \in \boldsymbol{G}$. Set $X^i_j(m_j) = G^i_j(m^i_j, f^{ij}_{+}(m_j(i)), f^{ij}_{-}(m_j(i)))$ for all $m_j \in M_j$. The smoothness of G^i_j implies existence everywhere of own-derivatives of the quantity conjecture function. At the same time, the kinkedness of f^{ij}_{+} and f^{ij}_{-} generates the non-existence of cross-derivatives of this quantity con-

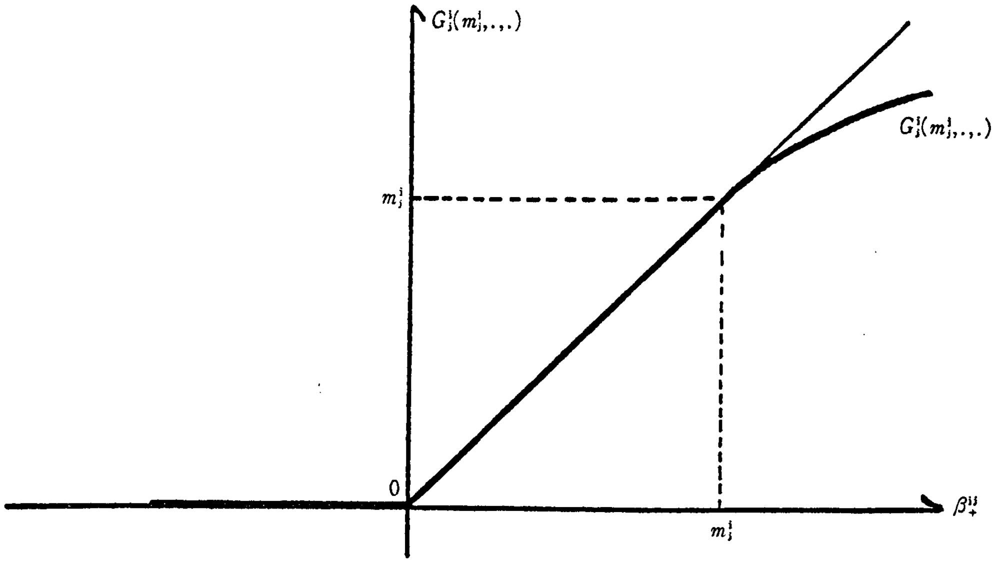

Fig. 7.

jecture function when $\sum_{k\neq i} m^k_j=0$.[12]

We can transform the kinked curve in Figure 1 to the smooth curve in Figure 2 without violating any assumptions of a quantity-constrained mechanism. However, any quantity perception function cannot cancel the kinkedness of forecast functions shown in Lemma 2. Thus, no quantity conjecture functions has cross-derivatives at m_j when $\sum_{k\neq i} m^k_j=0$. (See Figure 7.)

Theorem 2

If $\pi\in\Pi_{QC}$ and $f^{ij}\in \boldsymbol{P}^{TL}$, then $(\partial X^i_j/\partial m^s_j)(m_j)$, $i\neq s$, does not exist at m_j when $\sum_{k\neq i} m^k_j=0$.

Proof

See Appendix.

This theorem provides a characterization of a quantity-constrained non-Walrasian mechanism of the quantity type: the consumer who has the trade-limit perception has a conjecture function the cross-derivatives of which do not exist. Thus, for a mechanism of the quantity type, existence or non-existence of cross-derivatives informs us of the type of mechanism. Since we know the performance of the mechanism is non-Walrasian when cross-derivatives of its conjecture function do not exist, their non-existence also informs us of the type of its performance.

Now, we examine a price conjecture function. In example 2, we show the existence of a price perception function which cancels the kinkedness of forecast

[12] Note that if a perception function G^i_j is kinked, we can pair various forms of forecast functions with the perception function. However, if a perception function G^i_j is smooth, then the set of forecast functions is a singleton set. See Figure 6 for the form of the forecast function. For the proof of this result, see Ikeda (1984).

functions. As a result, there is a continuously differentiable price conjecture function.

Example 2

Let

$$K_j^i(p_j, m_j^i, \gamma_+^{ij}, \gamma_-^{ij}) = \begin{cases} p_j^{+h((m_j^i-\gamma_+^{ij})/m_j^i)H(m_j^i)} & \text{if } m_j^i > \gamma_+^{ij} \geqq 0 \\ p_j & \text{if } \gamma_-^{ij} \leqq m_j^i \leqq \gamma_+^{ij} \\ p_j^{-h((m_j^i-\gamma_-^{ij})/m_j^i)H(m_j^i)} & \text{if } m_j^i < \gamma_j^i \leqq 0 \,, \end{cases}$$

where $H(m_j^i) = e^{(m_j^i)^2} - 1$ and

$$h(t) = f(t)/\{f(t)+f(1-t)\} \quad \text{and} \quad f(t) = \begin{cases} e^{-1/t^2} & \text{if } t<0 \\ 0 & \text{if } t \leqq 0 \,. \end{cases}$$

(It is well known that $f(t)$ is of C^∞ class; so is $h(t)$.) One can show that $K_j^i \in \boldsymbol{K}$; furthermore K_j^i is continuously differentiable with respect to $(m^i, \gamma_+^{ij}, \gamma_-^{ij})$. By setting

$$P_j^i(p_j, m_j) = K_j^i(p_j, m_j^i, g_+^{ij}(m_j(i)), g_-^{ij}(m_j(i))) \quad \text{for all } p \in M^0 \text{ and } m_j \in M_j \,,$$

one obtains a price conjecture function which has its own-derivatives everywhere. It is easily seen that K_j^i is constructed so as to cancel out the kinkedness of g^{ij}. So, the price conjecture function has cross-derivatives everywhere. Furthermore, one can show that these derivatives are continuous everywhere. Thus, this price conjecture function is continuously differentiable even if $g^{ij} \in \boldsymbol{P}^{TL}$.

In spite of big kinks in forecast functions, there may not be any kinks in a price conjecture function. In this sense, a quantity-constrained mechanism of the price type does not have any intrinsic kinkedness. Hence, in addition to the existence of consumers with trade-limit perceptions, a condition which causes kinks in price conjecture functions is required for a quantity-constrained mechanism of the price type to yield non-Walrasian performance.

The difference of the results between quantity and price conjecture functions comes from the difference of quantity-constrained perception these conjecture functions embody. A quantity conjecture function embodies non-manipulable quantity-constrained behavior through a quantity perception function, while a price conjecture function embodies manipulable quantity-constrained behavior through a price perception function. A consumer takes non-manipulable quantity-constrained behavior because he feels that developing new communication channels or trading partnerships is impossible in the short-run. On the other hand, a consumer with manipulable quantity-constrained behavior perceives that such development is possible although it may cost very much.

Which approach, non-manipulable or manipulable, is more reasonable? We consider the following analogy. A consumer's communication network is a capital good, and the volume of communication is the capital service. Since the existing communication network is the result of trading activities in the past, today's trading activity is the investment. Since today's investment does not affect today's capital service, we argue that non-manipulable quantity-constrained behavior is more reasonable and robust in the circumstance considered here. From this viewpoint, we conclude that quantity-constrained behavior does make the conjecture function kinked.

Appendix : The Results and Proofs

Proposition 1

Let $\pi \in \Pi_{QC}$. Then,

(i) $\pi \in \Pi^{W}_{QC}$ if and only if for all $i \in N$ and all $j \in L$, $f^{ij} \in \boldsymbol{P}^{PC}$ and $g^{ij} \in \boldsymbol{P}^{PC}$; and

(ii) $\pi \in \Pi^{NW}_{QC}$ if and only if there exist $i \in N$ and $j \in L$ such that either $f^{ij} \in \boldsymbol{P}^{TL}$ or $g^{ij} \in \boldsymbol{P}^{TL}$.

Proof

Assume that for all $i \in N$ and all $j \in L$, $f^{ij} \in \boldsymbol{P}^{PC}$ and $g^{ij} \in \boldsymbol{P}^{PC}$. Then for every $i \in N$ and $j \in L$,

$$\begin{array}{l} g^{ij}_{+}(m_j(i)) = +\infty \quad \text{and} \quad g^{ij}_{-}(m_j(i)) = -\infty \\ f^{ij}_{+}(m_j(i)) = +\infty \quad \text{and} \quad f^{ij}_{-}(m_j(i)) = -\infty \end{array} \qquad \text{for all} \quad m_j(i) \in M_j(i) \, .$$

Since $\pi \in \Pi_{QC}$, for every $i \in N$ and $j \in L$,

$$P^i_j(p_j, m_j) = p_j \qquad \text{for all} \quad (p_j, m_j) \in M^0_j \times M_j$$

amd

$$X^i_j(\cdot, m_j(i)) \colon M^i_j \to \boldsymbol{R} \quad \text{is strictly increasing} \, .$$

Hence, $\pi \in \Pi^{W}_{QC}$.

For the converse, follow the above argument backward. Thus, we have shown (*i*).

(ii) follows from (*i*) by (A.10). Q.E.D.

Lemma 1

(i) If $\pi \in \Pi^{p}_{QC}$, then for all $i \in N$ and all $j \in L$, $X^i_j(m_j) = m^i_j$ for all $(p_j, m_j) \in M^0_j \times M_j$.

(ii) If $\pi \in \Pi^{q}_{QC}$, then for all $i \in N$ and all $j \in L$, $P^i_j(p_j, m_j) = p_j$ for all $(p_j, m_j) \in M^0_j \times M_j$.

(iii) If $\pi \in \Pi^{W}_{QC}$, then for all $i \in N$ and all $j \in L$, $P^i_j(p_j, m_j) = p_j$ and $X^i_j(m_j) = m^i_j$ for all $(p_j, m_j) \in M^0_j \times M_j$.

Proof

(i) By the definition of a mechanism of the price type, for all $i \in N$ and all $j \in L$, $f^{ij} \in \boldsymbol{P}^{PC}$, that is, $f^{ij}_{+}(m_j(i)) = +\infty$ and $f^{ij}_{-}(m_j(i)) = -\infty$ for all $m_j(i) \in M_j(i)$. Since $\pi \in \Pi_{QC}$, by (A.7),

$$\begin{aligned} X^i_j(m_j) &= G^i_j(\alpha^i_j(m^i_j), f^{ij}_{+}(m_j(i)), f^{ij}_{-}(m_j(i))) \\ &= G^i_j(\alpha^i_j(m_j), +\infty, -\infty) \qquad \text{for all} \quad m_j \in M_j \, . \end{aligned}$$

In other words, $X_j^i(m_j)$ is independent of $m_j(i)$. And by (A.5), $X_j^i(m_j)=m_j^i$ if $\sum_{k\in N} m_j^k=0$. Hence $X_j^i(m_j)=m_j^i$ for all $m_j\in M_j$.

(ii) By the definition of a mechanism of the quantity type, for all $i\in N$ and all $j\in L$, $g^{ij}\in \boldsymbol{P}^{PC}$, that is, $g_+^{ij}(m_j(i))=+\infty$ and $g_-^{ij}(m_j(i))=-\infty$ for all $m_j(i)\in M_j(i)$. Since $\pi\in\Pi_{QC}$, by (A.9) and (A.8),

$$\begin{aligned}P_j^i(p_j, m_j)&=K_j^i(p_j, \alpha_j^i(m_j^i), g_+^{ij}(m_j(i)), g_-^{ij}(m_j(i)))\\&=K_j^i(p_j, \alpha_j^i(m_j^i), +\infty, -\infty)\\&=p_j \qquad \text{for all} \quad p_j\in M_j^0 \quad \text{and} \quad m_j\in M_j\,.\end{aligned}$$

(iii) Since $\pi\in\Pi_{QC}^W$ if and only if $\pi\in\Pi_{QC}^q\cap\Pi_{QC}^p$, (iii) follows from (i) and (ii).

Q.E.D.

Theorem 1

If $\pi\in\Pi_{QC}^W$, then $\hat{h}=(\hat{h}^i)_{i\in N}\colon M^0\times M\to \boldsymbol{R}^{n(l+1)}$ is continuously differentiable.

Proof

By Lemma 1, $\hat{h}^i(p, m)=(-\sum_{i\in N} p_j m_j^i, m_1^i, \cdots, m_l^i)$. Hence, $\hat{h}$ is obviously continuously differentiable. Q.E.D.

Lemma 2

If $f^{ij}\in \boldsymbol{P}^{TL}$, then f_+^{ij} and f_-^{ij} are non-differentiable with respect to m_j^s, $s\neq i$, at $m_j(i)$ when $\sum_{k\neq i} m_j^k=0$. Similarly, if $g^{ij}\in\boldsymbol{P}^{TL}$, then g_+^{ij} and g_-^{ij} are non-differentiable with respect to m_j^s, $s\neq i$, at $m_j(i)$ when $\sum_{k\neq i} m_j^k=0$.

Proof

Here, we use $m_j(i, s)=(m_j^1, \cdots, m_j^{i-1}. m_j^{i+1}, \cdots, m_j^{s-1}, m_j^{s+1}, \cdots, m_j^n)$. By the assumption, $f_+^{ij}(m_j(i))=0$ if $\sum_{k\neq i} m_j^k\geqq 0$ and $f_-^{ij}(m_j(i))\leqq -\sum_{k\neq i} m_j^k\leqq f_+^{ij}(m_j(i))$ for all $m_j(i)\in M_j(i)$. So, the left-hand-side derivative at $m_j(i)$ when $\sum_{k\neq i} m_j^k=0$ is

$$\left.\frac{\partial f_+^{ij}}{\partial m_j^s}\right|_- (m_j(i, s), m_j^s)\leqq -1 \qquad \text{for all} \quad s\neq i\,.$$

The right-hand-side derivative at $m_j(i)$ when $\sum_{k\neq i} m_j^k=0$ is

$$\left.\frac{\partial f_-^{ij}}{\partial m_j^s}\right|_+ (m_j(i, s), m_j^s)=0 \qquad \text{for all} \quad s\neq i\,.$$

Since the right-hand-side derivatives and the left-hand-side derivatives do not coincide at $m_j(i)$ when $\sum_{k\neq i} m_j^k=0$, f_+^{ij} is not differentiable at $m_j(i)$ when $\sum_{k\neq i} m_j^k=0$. Similarly, f_-^{ij} is kinked at $m_j(i)$ when $\sum_{k\neq i} m_j^k=0$. Also, the same argument shows that g_+^{ij} and g_-^{ij} are non-differentiable at $m_j(i)$ when $\sum_{k\neq i} m_j^k=0$. Q.E.D.

Theorem 2

If $\pi\in\Pi_{QC}$ and $f^{ij}\in\boldsymbol{P}^{TL}$, then $\partial X_j^i/\partial m_j^s(m_j)$, $i\neq s$, does not exist at m_j when $\sum_{k\neq i} m_j^k=0$.

Proof

Since $\pi\in\Pi_{QC}$,

$$(*)\qquad G^i_j(m^i_j,\ \beta^{ij}_+,\ \beta^{ij}_-)=\begin{cases}\beta^{ij}_+ & \text{if } \beta^{ij}_-\leqq m^i_j\\ \beta^{ij}_- & \text{if } m^i_j\leqq \beta^{ij}_- .\end{cases}$$

First, consider $m^i_j=0$. Notice that $f^{ij}_+(m_j(i))f^{ij}_-(m_j(i))=0$ for all $m_j(i)\in M_j(i)$. Since $m^i_j=\beta^{ij}_+$ or $m^i_j=\beta^{ij}_-$, $X^i_j(m_j)=G^i_j(0, f^{ij}_+(m_j(i)),\ f^{ij}_-(m_j(i)))=0$ for all $m_j(i)\in M_j(i)$. Hence, $\partial X^i_j/\partial m^s_j(0, m_j(i))=0$, so $\partial X^i_j/\partial m^s_j$ exists at $m_j=(0, m_j(i))$ where $\sum_{k\neq i} m^k_j=0$.

Now, fix $m^i_j=\rho>0$. Since $f^{ij}\in \boldsymbol{P}^{TL}$, $f^{ij}_+(m_j(i))=f^{ij}_-(m_j(i))=0$, if $\sum_{k\neq i} m^k_j=0$. So by (*) above, at $(m^i_j, m_j(i))=(\rho, m_j(i))$ where $\sum_{k\neq i} m^k_j=0$, $\partial G^i_j/\partial\beta^{ij}_+(\rho, 0, 0)=1$. Since $\alpha^i_j(m^i_j)=m^i_j=\rho>0$, $G^i_j(\rho, 0, \beta^{ij}_-)=0$ for all β^{ij}_- by (*). So

$$\left.\frac{\partial G^i_j}{\partial \beta^{ij}_-}\right|_-(\rho, 0, 0)=0 .$$

As shown in Lemma 2, at $m_j(i)$ where $\sum_{k\neq i} m^k_j=0$,

$$\left.\frac{\partial f^{ij}_+}{\partial m^s_j}\right|_-(m_j(i, s), m^s_j)\leqq -1 ,\qquad \left.\frac{\partial f^{ij}_-}{\partial m^s_j}\right|_+(m_j(i, s), m^s_j)\leqq -1$$

and

$$\left.\frac{\partial f^{ij}_+}{\partial m^s_j}\right|_+(m_j(i, s), m^s_j)=0 ,\qquad \left.\frac{\partial f^{ij}_-}{\partial m^s_j}\right|_-(m_j(i, s), m^s_j)=0 .$$

Since for all $m_j(i)$ with $\sum_{k\neq i} m^k_j\leqq 0, f^{ij}_-(m_j(i))=0$,

$$\left.\frac{\partial X^i_j}{\partial m^s_j}\right|_-(\rho, m_j(i, s), m^s_j)=\left.\frac{\partial G^i_j}{\partial \beta^{ij}_+}\right|_+(\rho, 0, 0)\left.\frac{\partial f^{ij}_+}{\partial m^s_j}\right|_-(m_j(i, s), m^s_j)\leqq 1 .$$

On the other hand, at $(m^i_j, m_j(i))$ with $\sum_{k\neq i} m^k_j=0$,

$$\left.\frac{\partial X^i_j}{\partial m^s_j}\right|_+(\rho, m_j(i, s), m^s_j)=0$$

because $G^i_j(\rho, 0, \beta^{ij}_-)=0$ for all β^{ij}_- so $X^i_j(\rho, m_j(i))=0$ for all $m_j(i)$ with $\sum_{k\neq i} m^k_j\geqq 0$.

Hence,

$$\left.\frac{\partial X^i_j}{\partial m^s_j}\right|_-(\rho, m_j(i, s), m^s_j)\leqq -1<0=\left.\frac{\partial X^i_j}{\partial m^s_j}\right|_+(\rho, m_j(i, s), m^s_j) .$$

Similarly, for $m^i_j=-\rho<0$,

$$\left.\frac{\partial X^i_j}{\partial m^s_j}\right|_-(-\rho, m_j(i, s), m^s_j)\leqq -1<0=\left.\frac{\partial X^i_j}{\partial m^s_j}\right|_+(\rho, m_j(i, s), m^s_j) .$$

Hence,

$$\frac{\partial X^i_j}{\partial m^s_j}(m^i_j,\ m_j(i, s),\ m^s_j)$$

does not exist when $\sum_{k\neq i} m^k_j=0$. Q.E.D.

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REFERENCES

[1] Benassy, J. P., "Neokeynesian Disequilibrium in a Monetary Economy," *Review of Economic Studies*, 42 (1975), 502–23.

[2] ——, *The Economics of Market Disequilibrium*, New York: Academic Press, 1982.

[3] Böhm, V. and P. Lévine, "Temporary Equilibria with Quantity Rationing," *Review of Economic Studies*, 46 (1979), 361–78.

[4] Drèze, J., "Existence of an Equilibrium under Price Rigidity and Quantity Rationing," *International Economic Review*, 16 (1975), 301–20.

[5i Gale, D., "A Note on Conjectural Equilibria," *Review of Economic Studies*, 45 (1978), 33–38.

[6] Hahn, F. H., "On Non-Walrasian Equilibria," *Review of Economic Studies*, 45 (1978), 1–17.

[7] Heller, W. P. and R. M. Starr, "Unemployment equilibria with Myopic Complete Information," *Review of Economic Studies*, 46 (1979), 339–59.

[8] Hurwicz, L., "On informationally decentralized systems," in *Decision and Organization*, C. B. McGuire and R. Radner, eds., Amsterdam and London: Northholland, 1972, pp. 297–336.

[9] ——, "On Informational Requirements for Non-Wasteful Resource Allocation Systems," presented at the Conference on Mathematical-Economic Models, Moscow, June, 1976.

[10] ——, "On Allocations Attainable Through Nash Equilibria," *Journal of Economic Theory*, 21 (1979), 140–65.

[11] Ikeda, T., "On the Highest Degree of Smoothness of Outcome Functions Compatible with Quantity Constrained Equilibrium Non-Walrasian Performance," unpublished Ph. D. Thesis, University of Minnesota, 1984.

[12] ——, "On Quantity-Constrained Perception Equilibria—The Pure-Exchange Economy Case," (mimeo) November, 1985.

[13] Ito, T., *Fukinko no Keizai-bunseki* (Japanese), Tokyo, Japan: Toyo-Keizai, 1985.

[14] Keynes, J. M., *The General Theory of Money, Interest, and Employment*, New York: Harcourt, Brace, 1936.

[15i Negishi, T., "Involuntary Unemployment and Market Imperfection," *Economic Studies Quarterly*, 25 (1974), 32–41.

[16] ——, *Microeconomic Foundations of Keynesian Macroeconomics*, New York: North-Holland, 1979.

[17] Stigler, G. J., "The Kinky Oligopoly Demand Curve and Rigid Prices," *Journal of Political Economy*, 55 (1948), 432–49.

[18] Sweezy, P., "Demand under Conditions of Oligopoly," *Journal of Political Economy*, 47 (1939), 568–73.

[19] Trujillo, Jose, "An Alternative Concept of Rationality for Conjectural Equilibria," unpublished Ph. D, Thesis, University of Minnesota, 1980.

[20] Younès, Y., "On the Role of Money in the Process of Exchange and the Existence of a Non-Walrasian Equilibrium," *Review of Economic Studies*, 42 (1975), 489–501.