Title	PRICE FLEXIBILITY AND UNEMPLOYMENT: MICROECONOMICS OF SOME OLD-FASHIONED QUESTIONS
Sub Title	
Author	MUKHERJI, Anjan SANYAL, Amal
Publisher	Keio Economic Society, Keio University
Publication year	1986
Jtitle	Keio economic studies Vol.23, No.2 (1986.) ,p.19- 35
JaLC DOI	
Abstract	The paper examines the effect of moneywage cut on employment - an issue on which traditional macrotheory and recent microtheory derive opposite conclusions. We show that if prices are flexible the effect of money-wage cut on employment is uncertain, and its direction depends on the nature of the house-holds' utility functions. Since this opposes the established microtheoretic result that the cut in such situations is always expansionary, we examine the standard demonstrations showing that they implicitly restrict the utility functions. This brings microtheoretic result on wage-cut in line with the standard macro-economic result that money-wage cut is generally ambiguous in effect, unless additional restrictions are imposed on the aggregate consumption function.
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19860002-0019

PRICE FLEXIBILITY AND UNEMPLOYMENT: MICROECONOMICS OF SOME OLD-FASHIONED QUESTIONS*

Anjan MUKHERJI and Amal SANYAL

Abstract: The paper examines the effect of moneywage cut on employment—an issue on which traditional macrotheory and recent microtheory derive opposite conclusions. We show that if prices are flexible the effect of money-wage cut on employment is uncertain, and its direction depends on the nature of the households' utility functions. Since this opposes the established microtheoretic result that the cut in such situations is always expansionary, we examine the standard demonstrations showing that they implicitly restrict the utility functions. This brings microtheoretic result on wage-cut in line with the standard macro-economic result that money-wage cut is generally ambiguous in effect, unless additional restrictions are imposed on the aggregate consumption function.

I. INTRODUCTION

In the recent literature which seeks to provide microtheoretic foundation of macroeconomics, two broad results stand out. The first (Malinvaud [4]) is that in a fix price set up, rationing of households in the labour market can be classified into configurations that can be labelled Keynesian or classical depending on whether only an expenditure policy or only a money wage cut is effective in expanding employment. The second result (Malinvaud [4, page 69]; Benassy [1, chapter 13]) seeks to establish that if prices are flexible, then both Keynesian and classical remedies work; thus, the analytical distinction between the two types of unemployment breaks down. An implication of the above two results taken together is that a fix price set up is necessary for the emergence of a distinctly Keynesian variety of unemployment, i.e. one which can be cured by an expenditure policy alone.

This however seems to disagree both with Keynes' own ideas as also with the subsequent macroeconomics literature. This latter literature had argued through aggregative reasoning that given an underemployment equilibrium in a flex price system, (i) expenditure rise is always expansionary; (ii) results of money-wage cut are *in general* ambiguous (e.g. Keynes [2, chapter 19]) and (iii) the money wage cut will lead to expansion and eventually to full employment only if specific

* An earlier version of this paper was presented at the Economic Theory Workshop at Indian Statistical Institute, New Delhi, March 1986. Comments received from Amitava Bose, Dipankar Dasgupta, Mihir Rakshit and the referee are gratefully acknowledged.

properties of the aggregate consumption function are assumed [e.g. the Pigou Effect].

We try to show below that *if* an underemployment equilibrium exists in a system with flex price and fixed money wage, and an autonomous element of demand, then (i) *a stable equilibrium exists*, and this stability implies (ii) *that an increase in autonomous demand is always expansionary*, but (iii) *a parametric cut in the money wage rate will in general lead to ambiguous results.*

In an appendix, we consider the conditions on utility functions and expectations which yield a consumption function with the appropriate properties. This allows us to locate the special features of the consumption function in Malinvaud [4] and Benassy [1]. That the comparative statics results of Malinvaud are special have been noted by Hildenbrand and Hildenbrand [2], as well. However their point of focus is somewhat different from our objectives. First of all, they conduct comparative statics exercises when money wage rate and price are both rigid; secondly they show that the comparative statics results change when expectations alter or when the distribution of initial money holdings are changed. In contrast, we consider the situation when money wage is rigid but the price is flexible. And we assume (as in Malinvaud) that agents are identical. In this set up, we obtain a necessary and sufficient condition for the effect of a money wage cut to be unambiguous: a condition which need not be satisfied in general.

II. THE MODEL

We consider as in Malinvaud [4] and Benassy [1] a model with three types of agents: households, firms and a government. There are three commodities—output, labour and money. We take up the behavioural hypothesis regarding agents in turn. First of all, the firms.

Firms:

The firms use labour to produce output which cannot be stored; their technology is specified by a production function

$$y=f(z)$$

where y stands for output produced and z is the amount of labour employed. $f(\cdot)$ satisfies $f'<0$, $f''(\ )<0$; $f(0)=0$; $\lim_{z\to\infty} f'(z)=0$ and $\lim_{z\to 0} f'(z)=+\infty$. Given a money wage w and price of output p, maximizing profits lead to the familiar condition

$$pf'(z)=w$$

Thus the profit maximizing demand for labour is given by

$$z=f'^{-1}(w/p)=h(w/p) \quad \text{say} \tag{1}$$

where $h'(\ )<0$. The profit maximising supply is given by

$$y = f(h(w/p)) \tag{2}$$

(1) and (2) thus express the response of firms to a given real wage rate w/p, if they are not constrained in any manner.

Households:

There are N identical households; the i-th household has a maximum supply of labour b which he is prepared to offer at any wage rate w. Aggregate supply of labour is thus Nb and is perfectly inelastic. In situations of unemployment the demand for labour z, is less than Nb; define

$$\begin{aligned} e &= z/Nb \quad \text{if} \quad z < Nb \\ &= 1 \quad \text{otherwise} \end{aligned}$$

then $e \leqq 1$ and we assume that in such situations, there is uniform rationing of all households, in the sense that each household supplies be. Apart from wages wbe, each individual has an initial supply of money m^0 and these two together limit the purchases of the i-th household. Regarding the household's tastes we postulate an utility function of the form

$$V(c_1, c_2) = v_1(c_1) + v_2(c_2)$$

where c_1 is current consumption and c_2 is future consumption. This is maximized subject to the budget constraints

$$\begin{aligned} p_1c_1 + m &= w_1be_1 + m^0 \\ p_2c_2 &= w_2be_2 + m \end{aligned}$$

where variables with subscript 2 refer to the future prices, consumption and wages etc. Money is the only good which can be stored and the amount of money to be held depends on the household's expectations regarding the future.

Allign

$$\begin{aligned} c_2 &= (w_2be_2 + m)/p_2 \\ &= \psi(w_1, e, p_1) + m/\varphi(p_1) \quad \text{say} \end{aligned}$$

where $\psi(\cdot)$, $\varphi(\cdot)$ denote that future expected prices, wages and employment level are related to current values of these parameters. Consequently, the utility function takes the form

$$\begin{aligned} & v_1(c_1) + v_2(\psi(w_1, e_1, p_1) + m/\varphi(p_1)) \\ & = u(c_1, m; p_1, e_1, w_1) \end{aligned}$$

and thus (c_1, m) have to be chosen so as to maximize the above subject to the current budget constraint (see Benassy [1, chapter 8], as well)

$$p_1c_1 + m = w_1be_1 + m^0$$

This gives rise to a consumption function of the form

$$c_1(p_1, w_1, e_1, m^0)$$

for the i-th household. We carry out such an exercise in the appendix and show that under certain reasonable conditions, we have

$$\frac{\partial c_1}{\partial p_1}<0\,, \quad \frac{\partial c_1}{\partial w_1}>0\,, \quad \frac{\partial c_1}{\partial e_1}>0\,, \quad \frac{\partial c_1}{\partial m^0}>0$$

Aggregate consumption is now defined by

$$C_1(p_1, w_1, e_1, m^0)=Nc_1(p_1, w_1, e_1, m^0) \tag{3}$$

and the above sign restrictions on partial derivatives hold for the aggregate as well.

Government:

The last agent to be discussed is the government; it has an autonomous real demand g. The government has no income, and buys g amount of goods in exchange of financial claim against itself, which we call 'money'. The firms keep their profits and the households their saving in the form of 'money'. Thus at the end of the period when the equilibrium is established, the government has made its real purchase g through increasing money supply by an amount $p.g$. Of this a part is acquired by firms in exchange of the current profits, and the other part acquired by households in exchange of their current savings from wage income. Thus the following equality will hold in equilibrium:

$$p.g.=\pi+(W-pC)$$

where π is total profit, W, the total wage bill and pC, the total consumption bill.

The following discussion can be easily modified to the case where government has a tax income, without changing any conclusion *qualitatively*.

III. RIGID MONEY WAGES

When (p_1, w_1) are fully flexible, we expect that markets in the current period should equilibrate i.e.,

$$g+C_1(p_1, w_1, l, m^0)=f(h({}_1w/p_1)) \tag{4}$$

and

$$Nb=h(w_1/p_1) \tag{5}$$

should hold. (4) guarantees equilibrium in the current output market whereas (5) guarantees current labour market equilibrium. We do not take up the question of the existence of an equilibrium (p_1^*, w_1^*) satisfying (4) and (5) as that is not our immediate concern. Rather, we assume that there is a rigidity in money wages $w_1=\bar{w}_1$ and the price p_1 is flexible. In these circumstances, we define an equilibrium in the current markets by a configuration $(\bar{p}_1, \bar{e}_1)$ satisfying

$$g+C_1(p_1, \bar{w}_1, e_1, m^0)=f(h(\bar{w}_1/p_1)) \tag{6}$$

$$Nbe_1=h(\bar{w}_1/p_1) \tag{7}$$

and

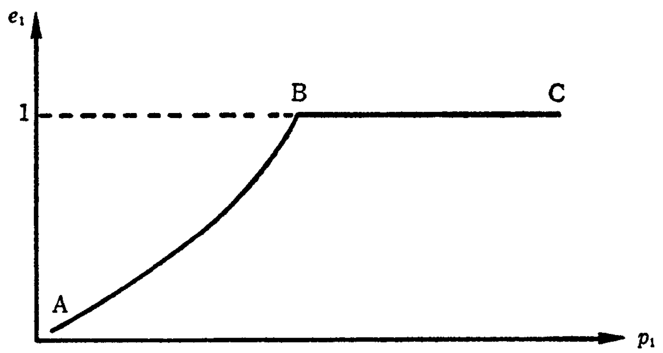
$$e_1 \leqq 1 \tag{7a}$$

If such a configuration exists with $\bar{e}_1<1$, then (6) assures us that the effective (constrained) demand is matched by supply and in the labour market, there is unemployment with firms being able to hire their profit maximizing demand. And at such a configuration, there would be

(i) no rationing of firms

(ii) households would be rationed in their supply of labour ($\bar{e}_1 b$ instead of b) while their constrained demands for output would be met.

To understand the nature of the unemployment equilibrium, consider the following diagram:



The curve ABC depicts

$$e_1=\min(1,\ h(\bar{w}_1/p_1)/Nb)$$

on the AB portion both (7) and (7a) are met and

$$\left.\frac{de_1}{dp_1}\right|_{AB}=-\frac{1}{Nb}h'(\ )\cdot\bar{w}_1/p_1^2>0$$

Consider rewriting (6) as

$$\gamma_g(p_1, e_1)=C_1(p_1, \bar{w}_1, e_1, m^0)+g-f(h(\bar{w}_1/p_1))=0 \tag{6'}$$

Along (6′):

$$\left.\frac{de_1}{dp_1}\right|_{\gamma_g=0}=-\frac{C_{1_{p_1}}+f^1\cdot h^1\cdot\bar{w}_1/p^2}{C_{1_{e_1}}}<0$$

from our restrictions on the consumption function noted earlier. Thus $\gamma_g=0$ is upward rising too and it may intersect ABC at

a) a point below B: an unemployment equilibrium

or

b) the point B: even with rigidity in w, price flexibility by itself guarantees that (4) and (5) are satisfied—the Walrasian equilibrium or

c) between B and C: equation (6) and (7a) are met, the latter with an equality; i.e. there is no unemployment but firms are constrained to sell the full employment output.

or finally,

d) no point at all i.e. (6) lies wholly above or below ABC.

We neglect cases b) and c) from our discussion, noting them as logical possibilities, and confine our attention to cases a) and d). For this purpose, it would be more convenient to rewirte (6′) as

$$X(e_1, p_1)=g \tag{6''}$$

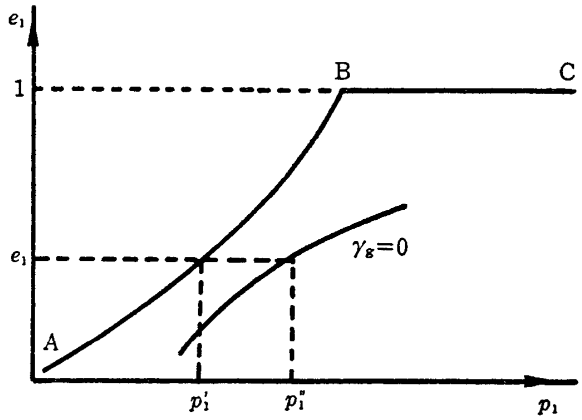
where $X(e_1, p_1)=f(h(\bar{w}_1/p_1))-C_1(p_1, \bar{w}_1, e_1, m^0)$

We may begin by noting that

$$\frac{\partial X(e_1, p_1)}{\partial p_1}>0\,, \qquad \frac{\partial X(e_1, p_1)}{\partial e_1}<0$$

and for any given e_1, $X(e_1, p_1) \to +\infty$ as $p_1 \to \infty$; also regardless of e_1, $X(e_1, p_1)\leq g\,\forall p_1\leq\bar{p}$ where $\bar{p}$ satisfies $f(h(\bar{w}_1/\bar{p}_1))=g$.

Thus for any e_1, there is an unique p_1 satisfying (6″); the locus of such (e_1, p_1) is naturally the curve $\gamma_g(p_1, e_1)=0$. And if $e_1=0$, there is $p_1>0$ such that $X(0, p_1)=g$. Since along (AB), $e_1 \to 0 \Rightarrow p_1 \to 0$, the curve $g=0$ must lie below AB for



small e_1. Hence the only possibility is shown if there is no equilibrium. Consider an employment level e_1; for this to be sustained in the labour market, p_1' is required; whereas to clear the output market $p_1''(>p_1')$ is necessary. Thus at (p_1', e_1), the effective demand $g-X(e_1, p_1')>0$ and this is true for every (p_1, e_1) on AB.

We have assumed that prices are flexible; but to specify what this means, we need an adjustment on prices; consider, then

$$\dot{p}_1=\theta(g-X(e_1, p_1)) \tag{8}$$

where

$$e_1 = \min\left(1, \frac{1}{Nb} h(\bar{w}_1/p_1)\right)$$

and θ is an increasing sign preserving function of its argument. In fact, one is assuming that (8) is such that p_1 adjusts instantaneously whenever there is effective excess demand. If this is so, then in the diagram above, p_1 rises without limit; e_1 of courses reaches 1 and no unemployment exhibits itself.

Alternatively, at B, $e_1=1$ and $p_1=\hat{p}_1$ say and $y=f(h(\bar{w}_1/\hat{p}_1))=y^*$ the full employment output. Moreover for the diagram drawn

$$g > y^* - C_1(\hat{p}_1, \bar{w}_1, l, m^0)$$

so that g is too large. Consequently, so long as

$$0 < g < y^* - C_1(\hat{p}_1, \bar{w}_1, l, m^0) \tag{9}$$

we are assured of an unemployment equilibrium where y^* is the full employment output and $\bar{w}_1/\hat{p}_1$ the unique real wage required to generate full employment [see (5) above]. So given (9) we have an unemployment equilibrium and an equilibrium where the

$$\text{slope of } (\gamma_g=0) > \text{slope of AB}$$

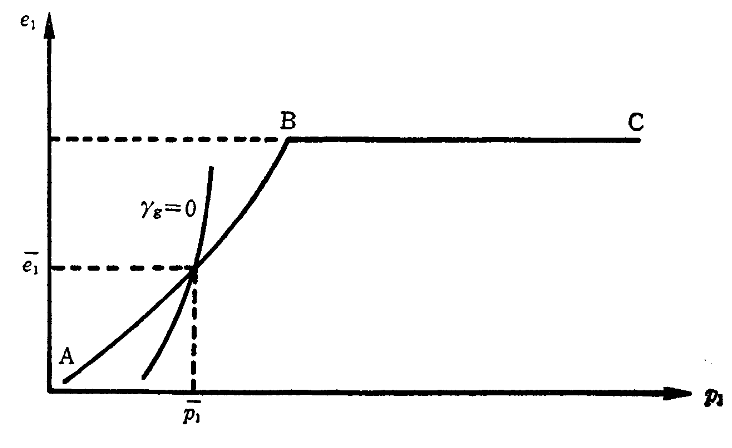
i.e.,

$$-\frac{C_{1_{p_1}} + f'h'\bar{w}_1/p_1^2}{C_{1_{e_1}}} > -\frac{1}{Nb} h'\bar{w}_1/p_1^2$$

or

$$h'\bar{w}_1/p_1^2\left[\frac{1}{Nb} C_{1_{e1}} - \bar{w}_1/p_1\right] - C_{1_{p1}} > 0 \tag{10}$$

the slope condition is guaranteed, at some equilibrium if an equilibrium exists because $\gamma_g=0$ is below AB when e_1 is small. One may check, that (10) guarantees the local stability of the process (8):



Notice that (10) may not hold at all (p_1, e_1) configurations; our analysis establishes that it will hold at some equilibrium provided an equilibrium exists and the existence of an equilibrium is guaranteed by a condition such as (9).

IV. EFFECTS ON UNEMPLOYMENT

We have obtained an unemployment equilibrium (p_1, e_1) and now wish to find how this is changed by variations in $\bar{w}_1$ and g.

(A) *Variations in g*

The conditions defining (p_1, e_1) are

$$\bar{e}_1 = 1/Nbh(\bar{w}_1/\bar{p}_1) \tag{11}$$

and

$$g + C_1(\bar{p}_1, \bar{e}_1, \bar{w}_1, m^0) = f(h(\bar{w}_1/\bar{p}_1)) \tag{12}$$

From (11):

$$\frac{\partial \bar{e}_1}{\partial g} = -\frac{1}{Nb} h' \cdot \bar{w}_1/\bar{p}_1^2 \frac{\partial \bar{p}_1}{\partial g}$$

and from (12)

$$1 + \frac{\partial C_1}{\partial p_1}\frac{\partial \bar{p}_1}{\partial g} + \frac{\partial C_1}{\partial e_1}\frac{\partial \bar{e}_1}{\partial g} = -f' \cdot h' \cdot \bar{w}_1/\bar{p}_1^2 \frac{\partial p_1}{\partial g}$$

so that substituting for $\partial \bar{e}_1/\partial g$ and collecting terms

$$\frac{\partial \bar{p}_1}{\partial g} = \frac{1}{h'\bar{w}_1/\bar{p}_1^2\left[\frac{1}{Nb}\frac{\partial C_1}{\partial e_1} - \bar{w}_1/\bar{p}_1\right]} > 0$$

by virtue of (10).

Hence

$$\frac{\partial \bar{e}_1}{\partial g} > 0 \quad \text{too.} \quad \text{Also since} \quad \bar{y}_1 = f(h(\bar{w}_1/\bar{p}_1))$$

$$\frac{\partial \bar{y}_1}{\partial g} = \frac{-f'h'\bar{w}_1/\bar{p}_1^2}{h'\bar{w}_1/\bar{p}_1^2\left[\frac{1}{Nb}\frac{\partial C_1}{\partial e_1} - \bar{w}_1/\bar{p}_1\right] - \frac{\partial C_1}{\partial p_1}} > 0$$

(B) *Variations in* $\bar{w}_1$

Now from (11)

$$\frac{\partial \bar{e}_1}{\partial \bar{w}_1} = \frac{1}{Nb} h'\left(\frac{1}{\bar{p}_1^2}\bar{p}_1 - \bar{w}_1 \frac{\partial \bar{p}_1}{\partial \bar{w}_1}\right)$$

and from (12)

$$\frac{\partial C_1}{\partial p_1}\frac{\partial \bar{p}_1}{\partial \bar{w}_1} + \frac{\partial C_1}{\partial \bar{w}_1} + \frac{\partial C_1}{\partial e_1}\frac{\partial \bar{e}_1}{\partial \bar{w}_1} = f'h'\left(\frac{1}{\bar{p}_1^2}\bar{p}_1 - \bar{w}_1 \frac{\partial \bar{p}_1}{\partial w_1}\right)$$

substituting for $\partial \bar{e}_1/\partial \bar{w}_1$,

$$\frac{\partial \bar{p}_1}{\partial \bar{w}_1}=\frac{\dfrac{\partial C_1}{\partial \bar{w}_1}+h'/p\left\{\dfrac{\partial C_1}{\partial e_1}\dfrac{1}{Nb}-\bar{w}_1/\bar{p}_1\right\}}{h'\bar{w}_1/\bar{p}_1^2\left[\dfrac{\partial C_1}{\partial e_1}\dfrac{1}{Nb}-\dfrac{\bar{w}_1}{\bar{p}_1}\right]-\dfrac{\partial C_1}{\partial p_1}} \tag{13}$$

where the denominator is positive by (10), once again.

Thus

$$\frac{\partial \bar{p}_1}{\partial \bar{w}_1}>0 \quad \text{iff} \quad \frac{\partial c_1}{\partial \bar{w}_1}+h'/p_1\left(\frac{\partial C_1}{\partial e_1}\frac{1}{Nb}-\bar{w}_1/\bar{p}_1\right)>0 \tag{α}$$

A sufficient condition for (α), given our other sign restrictions, is

$$\frac{\partial C_1}{\partial e_1}\frac{1}{Nb}-\bar{w}_1/\bar{p}_1<0$$

i.e.

$$\frac{\bar{w}_1}{\bar{p}_1}\left[\frac{\partial C_1}{\partial e_1}\frac{1}{Nb}\frac{1}{f'(Nb\bar{e}_1)}-1\right]<0$$

or

$$\frac{\partial C_1(\cdot)}{\partial y}-1<0\,.$$

For a related result, see [2, Proposition 2].

$$\frac{\partial \bar{e}_1}{\partial \bar{w}_1}<0 \Leftrightarrow \bar{p}_1-\bar{w}_1\frac{\partial \bar{p}_1}{\partial \bar{w}_1}>0 \qquad \because h'<0$$

i.e.

$$1>\bar{w}_1/\bar{p}_1\frac{\partial \bar{p}_1}{\partial \bar{w}_1}\,.$$

Returning to (12), note that

$$\frac{\partial \bar{p}_1}{\partial \bar{w}_1}=\frac{\dfrac{\partial C_1}{\partial \bar{w}_1}+h'/\bar{p}_1\left(\dfrac{\partial C_1}{\partial e_1}\dfrac{1}{Nb}-\bar{w}_1/\bar{p}_1\right)}{\bar{w}_1/\bar{p}_1\left[h'/\bar{p}_1\left(\dfrac{\partial C_1}{\partial e_1}\dfrac{1}{Nb}-\dfrac{\bar{w}_1}{\bar{p}_1}\right)-\dfrac{\bar{p}_1}{\bar{w}_1}\dfrac{\partial C_1}{\partial p_1}\right]}\,.$$

Thus, under (α),

$$\bar{w}_1/\bar{p}_1\frac{\partial \bar{p}_1}{\partial \bar{w}_1}<1 \Leftrightarrow \frac{\partial C_1}{\partial \bar{w}_1}<-\bar{p}_1/\bar{w}_1\frac{\partial C_1}{\partial p_1}$$

or

$$\bar{w}_1\frac{\partial C_1}{\partial w_1}+\bar{p}_1\frac{\partial C_1}{\partial p_1}<0 \tag{β}$$

or

$$\bar{w}_1/C_1\frac{\partial C_1}{\partial \bar{w}_1}<-\frac{\bar{p}_1}{C_1}\frac{\partial C_1}{\partial p_1} \tag{β'}$$

(β') is just a restatement of (β) in the form of elasticities. As we have indicated in the appendix, there is no reason why one may expect condition (β) to be satisfied. Hence, in general, *the effect on employment of a money wage cut would be ambiguous. No such ambiguity pertains to the effect on empliyment of an increase in autonomous demand.*

V. CONCLUSION

We conclude by discussing the reasons why Malinvaud [4] and Benassy [1] obtain conclusions different from ours.

Given our assumption of an inelastic supply of labour, we naturally look at the Malinvaud model spelled out in the Appendix to his book [4, page 117]. One may note the form of the utility function used there viz. $u(x, m/p)$. As we have pointed out in our appendix, for such an utility function, condition (β) is satisfied. In particular, consumption is homogeneous of degree zero in p_1, w_1 and m^0 and this is why Malinvaud obtains the different results. Even in Malinvaud's slightly different model in the text, the consumption function [4, page 45] is

$$c(p, w, e, m^0)=\frac{2N}{3p}(1-e)^{m^0}+\frac{N}{2p}(m^0+wb)e$$

and it is easily seen, that this function too is homogeneous of degree zero in (p, w, m^0). It is not at all surprising therefore that Malinvaud finds a result 'quite contrary to the teaching of post-war Keynesians' [4, page 69].

Regarding the analysis in Benassy [1, Chapter 13 and Appendix P] it would be instructive to reconsider the matter once more. In [1, Chapter 13] the basic difference from our construction lies in the redistribution of profits to households. In our appendix, we show that such a formulation leads to a consumption function

$$C_1(p_1, \bar{w}, y_1, m^0)$$

where C_{1p_1} is of ambiguous sign. The equilibrium with rigid money wages is given by

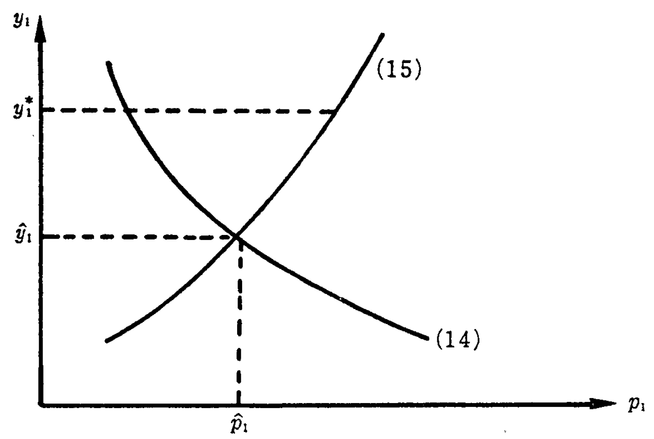
$$C_1(p_1, \bar{w}, y_1, m^0)+g=y_1 \tag{14}$$

$$y_1=f(h(\bar{w}/p_1)) \tag{15}$$

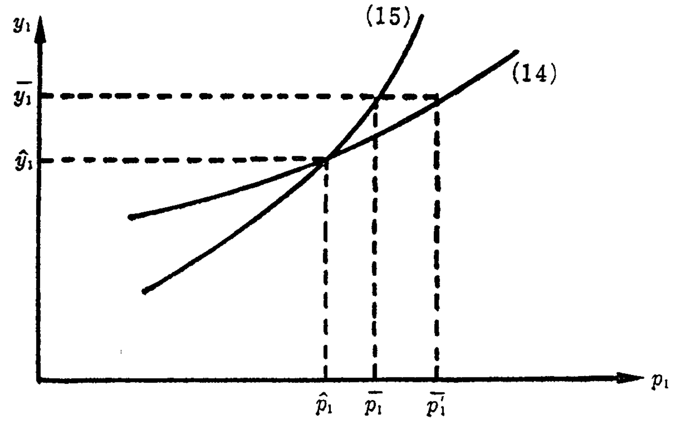
In case $(\partial C_1/\partial p_1)<0$, the case assumed by Benassy [1, page 123] we have the possibility of an unemployment equilibrium as depicted in the first diagram on p. 29, where y_1^* denotes the full employment output. The above nature of the curves follow since, along (14),

$$\frac{dp_1}{dy_1}=\frac{C_{1p_1}}{1-C_{1y_1}}<0$$

if $C_{1p_1}<0$. However, given that C_{1p_1} is of ambiguous sign, it could be possible



that at the intersection of (14) and (15) $C_{1_{p_1}}>0$. In such situations, we are likely to have the following situation:



To understand why the intersection should be as depicted, we have to return to the question of price flexibility. This should be interpreted to mean that price adjusts quickly to guarantee that (14) holds. To explain how prices adjust when there is disequilibrium in the output market, we choose the simple

$$\mathring{p}=\alpha[C_1(p_1, \bar{w}_1, s(p_1), m^0)-s(p_1)+g]$$

where $s(p_1)=f(h(\bar{w}_1/p_1))$, and $\alpha(\cdot)$ is some increasing sign preserving function.

At $\bar{y}_1, \bar{p}_1$ is required to guarantee that $\bar{y}_1$ is produced but $\bar{p}_1'$ is required to satisfy (14) and $\bar{p}_1'>\bar{p}_1$ so

$$C_1(\bar{p}_1, \bar{w}_1, \bar{y}_1, m^0)-\bar{y}_1+g<C_1(\bar{p}_1', \bar{w}_1, \bar{y}_1, m^0)-\bar{y}_1+g=0 \qquad \because C_{1_{p_1}}>0$$

Thus at $(\bar{p}_1, \bar{y}_1)$, price falls and this fall continues till equilibrium is attained.

Thus stability requires that at the equilibrium,

Slope of the curve (14)<Slope of the curve (15) or

$$\frac{C_{1_{p_1}}}{1-C_{1_{y_1}}}<-f'\cdot h'\cdot \bar{w}_1/p_1^2$$

at equilibrium i.e.

$$C_{1_{p_1}}+f'\cdot h'\cdot \bar{w}_1/p_1^2\cdot(1-C_{1_{y_1}})<0 \tag{16}$$

must hold at equilibrium; it may be noted that if $C_{1_{p_1}}<0$ the condition (16) holds, as well.

Returning to (14) and (15), the effect of a money wage cut may now be investigated:

$$C_{1_{p_1}}\frac{\partial p_1}{\partial \bar{w}_1}+C_{1_{\bar{w}_1}}+C_{1_{y_1}}\frac{\partial y_1}{\partial \bar{w}_1}=\frac{\partial y_1}{\partial \bar{w}_1}$$

and

$$\frac{\partial y_1}{\partial \bar{w}_1}=f'h'\frac{1}{p_1^2}\left(p_1-\bar{w}_1\frac{\partial p_1}{\partial \bar{w}}\right).$$

One may now see that

$$\begin{aligned}\frac{\partial y_1}{\partial \bar{w}_1}&=\frac{C_{1_{p_1}}[f'h'/p_1+f'\cdot h'\cdot \bar{w}_1/p_1^2]}{C_{1_{p_1}}+f'h'\bar{w}_1/p_1^2(1-C_{1_{y_1}})}\\&=C_{1_{p_1}}\cdot(a+ve \text{ term}) \text{ by virtue of (16)}.\end{aligned}$$

And since $C_{1_{p_1}}$ is of indeterminate sign and so too is $\partial y_1/\partial \bar{w}_1$.

It is of course clear that Benassy by assuming $C_{1_{p_1}}<0$ obtained the result that money wage cuts resulted in increased employment. Additionally, consider the assertion [1, page 228] that the existence of 'two differentiated class of income recipients with marginal propensity to consume out of profits being lower than that out of wages' would make the effects of money wage cuts ambiguous. It is clear now that the existence of such classes have no role to play in determining the effects of money wage cuts.

Appendix

Consider the problem

$$\begin{aligned}&\text{Max } V(c_1, c_2)=v_1(c_1)+v_2(c_2)\\&\text{s.t. } p_1c_1+m=\bar{w}be_1+m^0\\&\qquad p_2c_2=\bar{w}be_2+m\end{aligned}$$

where

- c_1: present consumption
- c_2: future consumption
- p_1: present price
- p_2: future price
- w: money wages rigid in both periods
- e_1: present level of employment
- e_2: future level of employment
- m^0: initial stock of money
- m: stock of money at the end of the current period

$p_2=\varphi(p_1)$, $e_2=\psi(e_1)$ are the expectations regarding the future price and employment. Also we shall assume

$$v_i'<0\,, \qquad v_i''<0 \quad i=1,2$$

Thus

$$\begin{aligned} c_2&=1/p_2[\bar{w}be_2+m] \\ &=\frac{\bar{w}b\psi(e_1)+m}{\varphi(p_1)} \end{aligned}$$

and

$$\begin{aligned} V(c_1, c_2)&=v_1(c_1)+v_2\left(\frac{\bar{w}b\psi(e_1)+m}{\varphi(p_1)}\right) \\ &=U(c_1, m; \bar{w}, p_1, e_1) \end{aligned}$$

depending on present period quantities, only. This is the method suggested by Benassy [1, Chapter 8, section 4]. So the problem reduces to

$$\begin{aligned} &\text{Max } U(c_1, m; p_1, e_1, w) \\ &\text{s.t. } \ p_1c_1+m=\bar{w}be_1+m^0 \end{aligned}$$

Given p_1, e_1, m^0 and $\bar{w}$, if c_1^*, m^* solve the above problem then there is λ^* such that the following necessary conditions at an optimum hold

$$\begin{aligned} \frac{\partial U^*}{\partial c_1}&\equiv U_{c_1}^*=\lambda^* p_1 \\ \frac{\partial U^*}{\partial m}&\equiv U_m^*=\lambda^* \end{aligned} \tag{1}$$

and

$$p_1c_1^*+m^*=\bar{w}be_1+m^0$$

The $*$s denote that all partial derivatives are evaluated at (c_1^*, m^*). If in addition to (1)

$$\det\begin{pmatrix} U_{c_1c_1}^* & U_{c_1m}^* & -p_1 \\ U_{mc_1}^* & U_{mm}^* & -1 \\ -p_1 & -1 & 0 \end{pmatrix}>0 \tag{2}$$

then (c_1^*, m^*) solves the problem formulated above. We shall assume that both (1) and (2) hold. Thus we have the functions

$$\begin{aligned} c_1^*&=c_1(p_1, e_1, \bar{w}, m^0) \\ m^*&=m(p_1, e_1, \bar{w}, m^0) \end{aligned}$$

and we wish to determine the signs of the partial derivatives of the former function: the consumption function. To this end, differentiate (1) totally to obtain

$$\begin{pmatrix} U^*_{c_1c_1} & U^*_{c_1m} & -p_1 \\ U^*_{mc_1} & U^*_{mm} & -1 \\ -p_1 & -1 & 0 \end{pmatrix} \begin{pmatrix} dc_1 \\ dm \\ d\lambda \end{pmatrix} = \begin{bmatrix} (\lambda^* - U^*_{c_1p_1})dp_1 - U^*_{c_1w}dw - U^*_{c_1e_1}de_1 \\ -U^*_{mp_1}dp_1 - U^*_{mw}dw - U^*_{me_1}de_1 \\ -c_1^*dp_1 - e_1bdw - \bar{w}bde_1 - dm^0 \end{bmatrix}$$

Thus (2) in effect, also guarantees that the various partial derivatives exist. Before computing these note

$$U^*_{c_1} = v_1'(c_1^*)\ ; \qquad U^*_{c_1m} = 0\ ; \qquad U^*_{c_1w} = 0\ ; \qquad U^*_{c_1e_1} = 0\ ;$$
$$U^*_{c_1p_1} = 0\ ; \qquad U^*_m = v_2'(c_2^*)\cdot 1/\varphi(p_1)$$

where

$$c_2^* = \frac{wb\phi(e_1) + m^*}{\varphi(p_1)}$$

$$U^*_{mp_1} = -\frac{\varphi'(p_1)}{\varphi^2(p_1)}[v_2'(c_2^*) + v_2''(c_2^*)c_2^*]\ ; \qquad U^*_{mm} = v_2''(c_2^*)\cdot 1/\varphi(p_1)^2\ ;$$

$$U^*_{me_1} = v_2''(c_2^*)\frac{1}{\varphi^2(p_1)}\bar{w}b\phi'(e_1)\ ;$$

$$U^*_{mw} = v_2''(c_2^*)\frac{1}{\varphi^2(p_1)}b\phi(e_1)\ .$$

Now writing the determinant in (2) as Δ

$$\frac{\partial c_1^*}{\partial p_1}\cdot\Delta = -\frac{1}{\varphi(p_1)}v_2'(c_2^*)\left[1 - \frac{p_1\varphi'(p_1)}{\varphi(p_1)}\right] + \frac{v_2''(c_2^*)}{\varphi^2(p_1)}[c_1^*p_1 + c_2^*p_1\varphi'(p_1)]\ ;$$

$$\frac{\partial c_1^*}{\partial m^0}\cdot\Delta = -p_1U^*_{mm}\ ;$$

$$\frac{\partial c_1^*}{\partial w}\cdot\Delta = -p_1\left[\frac{1}{\varphi^2(p_1)}v_2''(c_2^*)b\phi(e_1)\right] - be_1p_1U^*_{mm}\ ;$$

$$\frac{\partial c_1^*}{\partial e_1}\cdot\Delta = -p_1\left[\frac{1}{\varphi^2(p_1)}v_2''(c_2^*)\bar{w}b\phi'(e_1)\right] - b\bar{w}p_1U^*_{mm}\ .$$

Thus if $\varphi'(p_1)>0$, $\phi'(e_1)>0$ and

$$\frac{p_1\varphi'(p_1)}{\varphi(p_1)} \leqq 1$$

then we have,

$$\frac{\partial c_1^*}{\partial p_1} < 0\ ; \qquad \frac{\partial c_1^*}{\partial m^0} > 0\ ; \qquad \frac{\partial c_1^*}{\partial w} > 0\ ; \qquad \frac{\partial c_1^*}{\partial e_1} > 0$$

the signs assumed in the text.

Finally, to check whether (β) holds, we need to check the sign of $p_1(\partial c_1^*/\partial p_1) + w(\partial c_1^*/\partial w)$. From the expressions derived earlier

$$\Delta\cdot\left(p_1\frac{\partial c_1^*}{\partial p_1} + w\frac{\partial c_1^*}{\partial w}\right)$$
$$= -p_1\left[v_2'(c_2^*)/\varphi(p_1)\left(1 - \frac{p_1\varphi'(p_1)}{\varphi(p_1)}\right) + \frac{v_2''(c_2^*)}{\varphi^2(p_1)}c_2^*p_2\left(1 - \frac{p_1\varphi'(p_1)}{\varphi(p_1)}\right) - m^0\right]$$

where the term inside the brackets is made up of a positive term (given our assumption regarding expectations) and a term whose sign is ambiguous. Consequently there is no guarantee that (β) holds.

However suppose that $p_1\varphi'(p_1)/\varphi(p_1)=1$ then the above expression reduces to

$$p_1 m^0 v_2''(c_2^*)/\varphi^2(p_1)<0$$

and (β) holds. Finally, if in addition, $\psi(e_1)=0$ (i.e., the consumer expects to be unemployed in the future) then

$$V(c_1, c_2)=v_1(c_1)+v_2(m/p_1)=u(c_1, m/p) .$$

It may also be pointed out that in such situations

$$c_1^*(p_1, e_1, w, m^0)=c_1^*(\mu p_1, e_1, \mu w, \mu m^0) \qquad \text{for all scalars} \quad \mu>0 .$$

An Alternative Form of the Budget Constraint

In the above, profit incomes have played no role in determining the nature of the consumption function. It has been suggested by Benassy [1, page 228] that this fact plays a major role in determining whether money wage cuts are effective in curbing unemployment.

Consider then

$$p_1c_1+m=p_1y_1+m^0$$
$$p_2c_2=p_2y_2+m$$

where y_i are the outputs, reflecting a complete redistribution of profits to the households. Now

$$y_2=f(h(\bar{w}/p_2))$$

from our notation in the text. Thus if

$$p_2=\varphi(p_1) , \qquad y_2=f(h(\bar{w}/\varphi(p_1)))$$

and

$$c_2^*=y_2+m/p_2$$

and accordingly,

$$\begin{aligned} V(c_1, c_2)&=v_1(c_1)+v_2(f(h(\bar{w}/\varphi(p_1)))+m/\varphi(p_1)) \\ &=U(c_1, m; p_1) \end{aligned}$$

where we are retaining the assumption that money wages are expected to be rigid in the future too. So the present period problem reduces to

$$\begin{aligned} &\max\ U(c_1, m; p_1) \\ &\text{s.t.}\ \ p_1c_1+m=p_1y_1+m^0 . \end{aligned}$$

The first order conditions are:

$$\begin{aligned} &U_{c_1}^*=\lambda^* p_1 \\ &U_m^*=\lambda^* \\ &p_1c_1^*+m=p_1y_1+m^0 \end{aligned} \tag{3}$$

and the second order conditions which together with (3) form sufficient conditions is

$$\det\begin{pmatrix} U^*_{c_1c_1} & U^*_{c_1m} & -p_1 \\ U^*_{mc_1} & U^*_{mm} & -1 \\ -p_1 & -1 & 0 \end{pmatrix} > 0 . \tag{4}$$

To facilitate further calculations:

$$U^*_{c_1c_1} = v_1''(c_1^*) ; \qquad U^*_{c_1m} = U^*_{c_1p_1} = U^*_{mc_1} = 0 ;$$
$$U^*_m = v_2'(c_2^*)\cdot 1/\varphi(p_1) ;$$
$$U^*_{mm} = v_2''(c_2^*)\frac{1}{\varphi^2(p_1)} ; \qquad U^*_{mw} = \frac{1}{\varphi^2(p_1)} v_2''(c_2^*) f'\cdot h' ;$$
$$U^*_{mp_1} = -\frac{\varphi'(p_1)}{\varphi^2(p_1)}[v_2' + v_2''/\varphi^2(p_1)(f'h'\bar{w} + m^*)] .$$

Now from (3)

$$\begin{pmatrix} U^*_{c_1c_1} & 0 & -p_1 \\ 0 & U^*_{m_1m_1} & -1 \\ -p_1 & -1 & 0 \end{pmatrix}\begin{pmatrix} dc_1 \\ dm \\ d\lambda \end{pmatrix} = \begin{pmatrix} \lambda^* dp_1 \\ -U^*_{mp_1}dp_1 - U^*_{mw}dw \\ (c_1^* - y_1)dp_1 - dm^0 - p_1dy_1 \end{pmatrix}$$

and writing Δ' for the determinant of the matrix on the right (positive by (4)),

$$\frac{\partial c_1^*}{\partial p_1}\Delta'$$
$$= -v_2'(c_2^*)/\varphi(p_1)\left[1 - \frac{p_1\varphi'(p_1)}{\varphi(p_1)}\right]$$
$$+ v_2''(c_2^*)/\varphi^2\left[m^0 + \frac{p_1\varphi'(p_1)}{\varphi(p_1)}f'h' - m^*\left(1 - \frac{p_1\varphi'(p_1)}{\varphi(p_1)}\right)\right].$$

Notice now that $1 - \{p_1\varphi'(p_1)/\varphi(p_1)\} \geqq 0$ is not enough to sign the above expression. Thus the consumption function

$$c_1^*(p_1, w, y_1, m^0)$$

is such that

$$\frac{\partial c_1^*}{\partial p_1} \quad \textit{could be of either sign.}$$

One may show that the following hold:

$$\frac{\partial c_1^*}{\partial w} > 0 , \qquad 0 < \frac{\partial c_1^*}{\partial y_1} < 1 .$$

Centre for Economic Studies and Planning
Jawaharlal Nehru University
New Delhi 110 067

REFERENCES

[1] Benassy, J. P. (1982). *The Economics of Market Disequilibrium*, New York: Academic Press.

[2] Hildenbrand, K. and W. Hildenbrand. On Keynesian Equilibria with Unemployment and Quantity Rationing, *Journal of Economic Theory*, 18, 255–277 (1978).

[3] Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*, London: Macmillan.

[4] Malinvaud, E. (1977). *The Theory of Unemployment Reconsidered*, Oxford: Basil Blackwell.