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A MACROECONOMIC MODEL WITH THE RATE OF UNEMPLOYMENT AS A RISK PROBABILITY UNDER THE GOVERNMENT BUDGET RESTRAINT*

Fumimasa HAMADA

Abstract: This paper attempts to analyse the effectiveness of government fiscal policies through the bond financing when an economy's growth rate is slowing down. Firstly, by introducing the rate of unemployment, defined as a risk probability that consumers and firms bear in their decision making into a macroeconomic model under a government budget restraint, the performances of transitory equilibra are analysed, and it is found that the higher is the rate of unemployment, the smaller are the impact multipliers of governament expenditure.

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1. INTRODUCTION

It is widely accepted that the subjective formation of expectations about future economic circumstances plays an important role in economic decision-making. Economic theorists, however, have not yet found any useful hypotheses that can be used for modelling the formation of expectations. One exception is the rational expectations hypothesis. It can be argued that the rational expectation is one exception to this claim, but this hypothisis seems too restrictive to use, in the context of a simple macroeconomic model, for analysing the effects of fiscal pollcy on an economy.

Since the formation of expectations can be thought of as being based on past

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and present experiences, it may be a useful abstraction to imagine that the public use the latest information available on a few relevant variables that can express general market conditions most intensively. This approach has been taken by Meyer and Glauber (1964), who used the stock price index as an indicator announcing intermediate term market conditions in their investment equations for the U.S. economy, and by Darling and Lovell (1965), who introduced the ratio of unfilled orders to sales as an indicator for the short term expectations of the commodity market in an equation for investment in inventories. Others who have taken this approach include Bowen and Finegan (1965), and Tella (1964 and 1965), who introduced the rate of unemployment into an equation for labor force participation, as an indicator of the degree of willingness to find a job.

It is somethimes claimed that a high rate of unemployment has the psychological effect of weakening the will to search for a job, and so it suppersses labor supply. It may be inaccurate to assume this, but it is imporatnt to note that a rise in the rate of unemployment would cause a shift in the anticipated time path of income of consumers, so that the observed marginal propensity to consume from current and past incomes will turn out to be smaller. The expectations of firms are also much influenced by the rate of unemployment. Since a high rate of unemployment generally implies the existence of excess supply in commodity markets, firms will see a rising unemployment as a symptom of a decline in demand, and so thier investment behavior will be more cautious.

It is, however, very difficult to construct one theory about the unemployment rate's psychological impact on consumer behavior, and another theory about its psychological influence over firms' investment behavior. Instead, by regarding the rate of unemployment as a risk probability faced by the public whenerver they make any economic decision which involves paying some cost in advance, we can make theorizing about the public's decision making process more exact and more simple. In the next section, I will present a 'risk probability hypothesis', a wording to which the rate of unemployment as a risk probability depresses the marginal propensity to consume from current and past incomes, and retards the realization of firm's investment in plants and equipments. Assuming this risk probability hypothesis for consumers and firms, a macroeconomic model, under a government budget restaint, which follows Christ (1968, 1969, 1978), Bilnder and Solow (1973), and Tobin and Buiter (1976) is used in the following sections to analyse the effects of fiscal policies through bond financing of the government.[1]

2. A RISK PROBABILITY HYPOTHESIS

For simplicity's sake, we first describe the two period time preference theory of consumer behavior with a risk probability. It is assumed that the price level is

[1] Pyle and Turnavsky (1976) and Hayakawa (1979) analysed some cases of inflation dynamics, and it may be possible to extend this study in that direction also.

fixed at unity,[2] and the discount rate is also given at the beginning-of-period of planning horizon, at which the consumer stands. Let us define C_t as the volume of goods to be consumed in period t, and C_{t+1}, as the volume of goods which the consumer, at the beginning of period t, plan to consume in period $t+1$. Consumer utility estimated at the beginning of period t is then defined as:

$$(1) \qquad U=U(C_t, C_{t+1}),$$

where U is a well-behaved continuous function with continuous first and second order derivatives.

It is assumed that all wage earners are homogeneous, and they anticipate earning incomes of ${}_tY_1$ in period t with a probability of $1-PR_t$, and incomes of ${}_tY_2$ in period $t+1$ with the same probability, where PR_t is the probability of losing or failing to find a job in period t. Risk probability PR_t is assumed to be equal to the difference between the actual rate of unemployment and the equilibrium rate of unemployment that arises from job search and so forth. All the cases that a consumer can imagine at the beginning of period t are summarized in Table 1.

TABLE 1. Possible Cases for Two Periods

Cases \ Periods	I	II	Probability
1	Employed $1-PR_t$	Employed $1-PR_t$	$(1-PR_t)^2$
2	Employed $1-PR_t$	Unemployed PR_t	$PR_t(1-PR_t)$
3	Unemployed PR_t	Employed $1-PR_t$	$PR_t(1-PR_t)$
4	Unemployed PR_t	Unemployed PR_t	$(PR_t)^2$
Total			1

The last column of Table 1 shows the probabilities associated with each case. Each of these probabilities corresponds to a term of the second order polynomial; $[(1-PR_t)+PR_t]^2$, and if the number of periods is, n, then the order of this polynomial turns out to be n.

The expected amount of real funds availbale to consumers (wage earners) at the beginning of period t, which is supposed to be the maximum consumption in period t, and the amount of real funds that consumers, at the beginning of period t, expect to have available in period $t+1$, which is supposed to be the maximum consumption in period $t+1$, can be written respectively as:

$$(2) \qquad {}_tE[W_t|\Omega_{t-1}]=(1-PR_t)\{{}_tY_1+{}_tY_2/(1+r)\}+PR_t\{SS_t+SS_{t+1}/(1+r)\},$$

$$(3) \qquad {}_tE[W_{t+1}|\Omega_{t-1}]=(1-PR_t)\{{}_tY_1(1+r)+{}_tY_2\}+PR_t\{SS_t(1+r)+SS_{t+1}\},$$
$$={}_tE[W_t|\Omega_{t-1}](1+r),$$

[2] This assumption will be dropped in a later section.

where ${}_tE$ is the expectation operator, W is the amount of real funds availble to consumers, Ω_{t-1} is the body of information available to consumers on market conditions at the end of period $t-1$, r is the discount rate, and SS is unemployment insurance benefits to be paid. Needless to say, equations (2) and (3) can be obtained by referring to Table 1 and by rearranging all of the terms.[3] From equations (2) and (3), the two period consumption frontier defined by consumers' budget constraint can be written as below:

$$(4) \qquad C_t=[(1-PR_t)\{(1+r)_tY_1+{}_tY_2\}+PR_t\{(1+r)SS_t+SS_{t+1}\}]/(1+r) \\ -1/(1+r)\cdot C_{t+1}\,.$$

Maximization of consumers' utility (1) subject to the budget frontier (4) leads to the following demand function for consumption goods in period t:

$$(5) \qquad C_t=g(r)\cdot[(1-PR_t)\{{}_tY_1+{}_tY_2/(1+r)\}+PR_t\{SS_t+SS_{t+1}/(1+r)\}]\,,$$

where $g(r)$ is a function of the discount rate, and can be derived from the first order conditions for utility maximization. Assuming the 'adaptive' expectations for the anticipated incomes ${}_tY_1$ and ${}_tY_2$, the above equation (5), can be rewritten as:

$$(6) \qquad C_t=g(r)\cdot[(1-PR_t)A(L)\{1+\mu_1/(1+r)\}Y_t+PR_t\{SS_t+SS_{t+1}/(1+r)\}]\,,$$

where $A(L)=\sum^{\infty}\mu_iL^i$, Y_t is the real income (and also net output) to be recognized socially in period t, L is the lag operator, and the μ_i are lag weights.

The risk probability that is relevant to the firm's inverstment is supposed to depend on the divergence of the actual rate of utilization of the capital stock, from its optimal rate. We assume that this risk probability is proportional to that of the labor market. Define f to be the firm's optimal capacity of output in the short-run, and ρ_t, to be the risk probability for the commodity market. Then,

$$(7) \qquad \rho_t=1-Y_t/f\,,$$

which implies that firms which are assumed to be homogeneous regard the ratio of excess capacity to output capacity, in the short-run, as a signal of risk-bearing to lose their market in the same ratio as the above when they expand production capacity. We assume ρ_t is PR_t, so that PR_t can be written as:

$$(8) \qquad PR_t=\zeta\cdot\rho_t=\zeta(1-Y_t/f)\,,$$

where ζ is a positive constant. Now, the expected demand for output can be written as:

$$(9) \qquad {}_tE[Y_t\,|\,\Omega_{t-1}]=(1-PR_t/\zeta)f\,.$$

Assuming the adaptive expectations for f, we can write down the following simple equation for a firm's investment behavior:

$$(10) \qquad I_t=I[(1-PR_t)B(L)Y_t,\ C(L)cc_t,\ K_{t-1}]\,,$$

[3] It is assumed that the anticipated income of non-wage-earners is proportional to that of wage-earners, and unemployment insurance benefits for non-wage earners is zero.

where I_t is investment in period t, cc_t is the real cost of capital services in period t, K_{t-1} is the capital stock at the end of period $t-1$, and $B(L)$ and $C(L)$ are lag polynomials.

A risk probability is also included in equation for money demand, along the same lines as for the consumption equation, but *I* presume that it is not necessary to describe the role of the risk probability on money demand in detail.

3. A BASIC MODEL

In this section, a macroeconomic model of the standard type used by Christ (1978) is presented in its relation to the risk probability hypothesis proposed in the previous section. For simplicity's sake, and to focus on the way risk probability affects the economic behavior of the public, the price level is assumed to depend on the money wage rate and the marginal productivity of labor, as seen in an ordinary Keynesian supply function, so that the characteristics of the model in this study turn out to be considerably different from those of Christ's model, but in other respects the model used here follows Christ (1978).

Following Christ, we assume that the government includes a monetary authority, and it can finance budget deficits by an increase in high-powered money, a bond issue, or by some combination of both. We also assume that there are only two types of financial assets: high-powered money, and government bonds of the "Consol" type.

A basic model incorporating these assumptions can be written as follows:

$$Y=X+G \tag{11}$$

$$T=V/p-B/p+tB/p+tY \tag{12}$$

$$X=a_0+a_1(u)Y-a_2T+a_3 1/r+a_4H/p+a_5B/p+a_6\pi^*+a_7DY\,; \tag{13}$$
$$\pi^*=Dp/p\,,\quad Dp=dp/dt\,,\quad DY=dY/dt\,,$$
$$a_1(u)=a_{10}+a_{11}u>0\,,\quad a_{10}>0\,,\quad a_{11}<0\,.$$
$$a_2>0\,,\quad a_3>0\,,\quad a_4>0\,,\quad a_5>0\,,\quad a_6>0\,,\quad a_7>0\,.$$

$$1/r=b_0-b_1(u)Y+b_2H/p-b_3B/p\,, \tag{14}$$
$$b_1(u)=b_{10}+b_{11}u>0\,,\quad b_{10}>0\,,\quad b_{11}>0\,,\quad b_2>0\,,\quad b_3>0\,.$$

$$G=T+DH/p+DB/rp\,,\quad DH=dH/dt\,,\quad DB=dB/dt\,. \tag{15}$$

$$u=c_0+c_1(1-Y/f)\,,\quad c_0>0\,,\quad c_1>0\,. \tag{16}$$

$$p=(1+\alpha\pi^*+\beta u)w/(e_0+e_1Y+e_2Y^2)\,;\quad \pi^*=Dp/p\,,\quad Dp=dp/dt\,. \tag{17}$$
$$\alpha>0\,,\quad \beta>0\,,\quad e_0>0\,,\quad e_1>0\,,\quad e_2<0\,.$$

As can be seen quite easily, this model is the same as that proposed by Christ (1978), except for equations (13) and (14), which include the rate of unemployment u(supposed to be a signal of expectations that influence the behavioral psychology of consumers and firms), and equation (17), which incorporates our assumption that expectations about inflation rate are static.

Equation (11) is the equilibrium condition for the market for final goods, under which real income, Y, is equal to the sum of the final demand of the private sector, X, and real government expenditure G, where we are implicitly adopting the "Dual Decision" hypothesis. Equation (12) states that real taxes less transfers of the government, T, is equal to the sum of real taxes less transfers which are not dependent on real income V/p, real taxes from real interest income, $t*B/p$ (where t is the tax rate, B is nominal interest income from government bonds, the unit of which is chosen so as to equate this to their stock, and p is the price-level.), and real income taxes, $t*Y$, less real interest payments for the stock of government bonds, B/p.

Equation (13) determines real private demand, X, where $a_1(u)$ is the marginal propensity to spend, which is assumed to be naegatively dependent on rate of unemployment, u; and, following Christ, the negative effect of taxes on private demand is assumed to be smaller than $a_1(u)$. The inverse of the interest rate, $1/r$, has a positive effect on private demand, and real high-powered money, H/p, and the stock of real government bonds held by the private sector, B/p, also have positive effects on private demand. π^* is the rate of inflation, and Dy is the change in real income which is the accelerated demand factor influencing private investment. Equation (14) is the asset equilibrium condition, in which $b_1(u)$ captures the idea that the real demand for money is higher for a given Y, when the unemplyment rate (risk probability), is high, than it is low; that is $1/r$ is a negative function of the rate of unemployment u, and, needless to say, $b_1(u)$ itself is positive. An increase in the supply of real high-powered money has a positive effect on $1/r$, and an increase in the number of government bonds also has a negative effect on $1/r$.

Equation (15) is the government budget restraint, which states that real government expenditure is financed by real taxes less transfers, T, an increase in the supply of real high-powered money, DH/p, and/or an issue of government bonds, DB/rp, where the real in-flow from bond financing is the real market value of the new bonds issued. Equation (16) determines the rate of unemployment, u, where c_0 is the natural rate, and the second term is the rate of involuntary unemployment, in which f is defined to be the volume of full capacity output which is assumed to be constant in the short run. Equation (17) is a short-run supply equation of the Keynesian type, where the marginal productivity of labor is a decreasing function of real output Y, and the expected money wage rate is determined by a simple Phillips curve, given an exogenous inital money wage rate.

4. STATIONARY EQUILIBRIUM

It may be most useful to obtain the stationary equilibrium solutions for the relevant variables first. Since the focus of our analysis is on changes in the government deficit, its financing, and the performance of the economy as a whole, we must first obtain the partial solution, or the quasi-reduced form, for both the $IS=LM$ relation, and the government budget restraint.

The $IS=LM$ relation can be obtained by substituting equations (12), (13), (14), and (16) into equation (11). Thus eliminating T, X, r, and u, the aggregate demand equation (11), can be rewritten as:

$$(18) \qquad [M+NY]Y=S(t)+a_8Dp/p+a_7DY+G=F_t\,, \quad DY=dy/dt\,,$$

where

$$M=1-a_{10}-(c_0+c_1)(a_{11}-a_3b_{11})+a_2t-a_3b_{10}>0\,,$$
$$N=c_1(a_{11}-a_3b_{11})/f<0\,, \quad a_{11}<0\,, \quad \text{and}$$
$$S(t)=a_0+a_3b_0-a_2V/p+\{a_2(1-t)-a_3b_3+a_5\}B/p+(a_3b_2+a_4)H/p\,.$$

As can be easily seen in equation (18), the $IS=LM$ relation is a quadratic form with reaspect to net output, Y. If the effects of the rate of unemployment as a signal of expectations that consumers and firms form psychologically are neglected, or excluded, in the above equation, it turns out to be the ordinary form; that is,

$$(19) \qquad M=1-a_{10} \quad \text{and} \quad N=0\,,$$

so that the impact multiplier of government expenditure, G, on real net output, Y, is the inverse of M; that is, $1/(1-a_{10})$, where the effect of a change in r upon Y, in the LM equation (14), is neglected.

In equation (18), the third term of M can be rewritten as:

$$-(a_{11}-a_3b_{11})(c_0+c_1)\,,$$

which is the sum of the direct effect, and the indirect effect, through a change in the rate of unemployment, of a change in autonomous demand on net output or real income. These effects are positive, and so they reduce the impact multipliers. $S(t)$, the first term of the right hand side of equation (18), includes the effect of an increase in government bonds on the private demand for goods, the net effect of which is a reduction in taxes less transfers, $a_2(1-t)$, less the indirect effect of a rise in interest rates through the bond market, a_3b_3, plus the direct effect of an increase in bond, a_5, as a financial asset held by the private sector. The net effect of an increase in the real supply of high-powered money on private demand is the sum of the direct effect of an increase in high powered money, a_4, and its indirect effect through a decline of interest rates, a_3b_2.

Now, solving equation (18) with respect to real net income, Y, and putting $DY=0$ and $Dp=0$, the partial solution for the aggregate demand equation (18), can be expressed as:

$$(20) \qquad Y=(2N)^{-1}[-M-\{M^2+4NA(t)\}^{-0.5}]\,; \quad A(t)=S(t)+G\,.$$

Taking the Taylor's expansion of the right-hand side of this equation near the equilibrium point, and neglecting the terms of higher degree than the first, an approximation to equation (20) can be written, after some rearrangement, as:

(21) $$Y=\partial Y/\partial A\cdot A(t)+\text{Const.},$$

where

$$\partial Y/\partial A=[M^2+4NA(t)]^{-0.5}>0 .$$

To be clear, the derivative $\partial Y/\partial A$ is an impact multiplier of government real expenditure, G, so this should be greater than unity. If the value of $A(t)$ is not large enough to off-set the effect of the rate of unemployment included in M, then since the value of M itself is greater than the marginal leakage rate, $1-a_{10}$, this multiplier turns out to be smaller than in the case where the effects of the rate of unemployment as a signal of expectations formed psychologically are neglected. By further rearranging equation (21), we can obtain the following relation between net output and the real stock of governement bonds, which is still an aggreagte demand equation:

(22) $$B/p=\Phi_2 Y+\Phi_1\Phi_2 a_2 V/p-\Phi_1\Phi_2\Phi_3 H/p-\Phi_1\Phi_2 G+\text{Const.},$$

where

$$\Phi_1=\partial Y/\partial A , \qquad \Phi_2=1/[\Phi_1\{a_2(1-t)-a_3b_3+a_5\}] , \qquad \Phi_3=a_3b_2+a_4 .$$

As we have already stated, the value of Φ_1 is positive, and so the sign of Φ_2 depends on whether or not the positive effect of government bonds on the private demand for goods is smaller than the negative effect of interest rates through increasing bond-issues. If the negative effects dominate the positive effects, then the value of Φ_2 turns out to be negative, just as it is in the ordinary case.

The next step is to rewrite the government budget restraint as another relation between net output and the real stock of government bonds. Substituting equations (12) and (14) into equation (15), to eliminate T and r, and putting $DH=DB=0$, a relation between net output Y and the real stock of government bonds, B/p, when tax revenues are equated to government expenditure, is obtained:

(23) $$B/p=t\Phi_4 Y+\Phi_4 V/p-\Phi_4 G ,$$

where

$$\Phi_4=1/(1-t) .$$

This is a balanced budget, which is a special case of the government budget retraint in Christ (1978) and others, but as a first step, equation (23) is important.

Since Φ_4 and t should be positive, the relation between net output Y and the real stock of government bonds should be positive. The simulataneous solution of equations (17), (22) and (23) gives the stationary equilibrium of the markets for commodities and money, and consequently for the market for government bonds under the government budget restraint.

Figure 1 shows this partial static equilibrium, with the price level p treated as given. The vertical axis measures real stock of government bonds, and the horizontal axis measures the net output of the whole economy. The cureve YM is

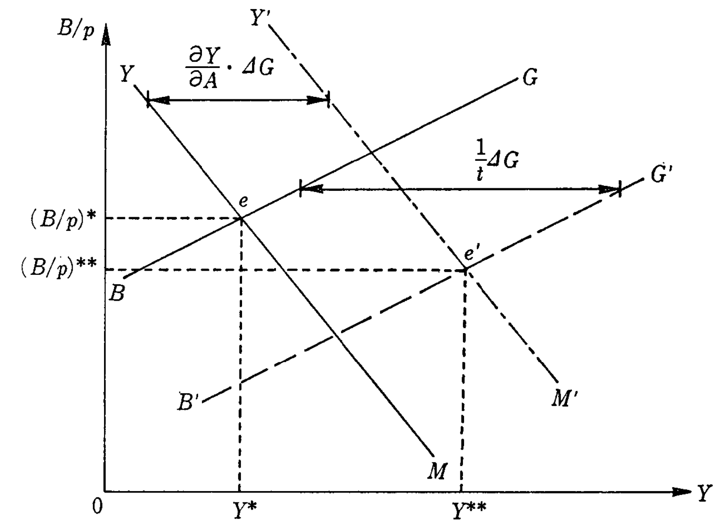

Fig. 1. Partial Static Equilibrium with p Fixed.

drawn fromy equation (22), where the sign of Φ_2 is assumed to be negative.[4] The curve GB is the government budget restraint, with the price level as fixed, derived from equation (23). The point of intersection, e, determines the stationary equilibrium solution for net output, Y^*, and the real stock of government bonds, $(B/p)^*$. As easily seen from equations (22) (23), the equilibrium point, e, is stable, if and only if,

$$(1-t)/t<|\partial Y/\partial A\{a_2(1-t)-a_3b_3+a_5\}| .$$

Figure 1 also shows the size of the shifts of both curves in response to an increase in real government expenditures, ΔG. It should be noted that the impact multiplier, $\partial Y/\partial A$, is smaller than it is when the negative effect of the rate of unemployment is neglected, and this detracts from the effectiveness of fiscal policy as a tool for bringing about economic expansion on a larger scale.

Multiplying both sides of equations (22) and (23) by p, taking the Taylor's expansion of the right hand sides near the equilibrium point, and neglecting terms of higher degree than the first, approximations to equations (22) and (23) can be written as follows:

$$(24) \qquad B=\Phi_2p^*Y+[C_1+\Phi_2\{Y^*-\Phi_1G\}]p+\Phi_1\Phi_2a_2V-\Phi_1\Phi_2\Phi_3H+\text{Const.}$$

$$(25) \qquad B=t\Phi_4p^*Y+\Phi_4\{tY^*-G\}p+\text{Const.},$$

where C_1 is a positive constant, and p^* and Y^* are the equilibrium values for p and Y respectively.

[4] This may not be the ordinary case, where the net effect of an increase in government bonds on final demand is positive, because of the small negative effect of a government bond increase through a rise of interest rates and the large direct effect of a government bond increase on final demands. In the next section, the case for a positive slope will be discussed in its relation to dynamic stability of the model.

Proceeding in the same way, and setting $\pi^*=0$, the linearized approximation to equation (17) can be written as:

$$(26)\quad p=[-(2e_2p^*+\beta c_1w/f)/\{e_0+e_1Y^*+e_2(Y^*)^2\}]Y+C_2/\{e_0+e_1Y^*+e_2(Y^*)^2\}\,,$$

where C_2 is a positive constant, since c_1 is positive and β is negative, and the nominal wage rate, w, is exogenous and positive.

As is easily seen by reviewing equations (22) and (23) and the descriptions following them, the aggregate demand equation (24) tells us that the partial derivative of B with respect to Y is negative, and the government budget restraint equation (25) shows that the partial dervative of B with respect to Y is positive. It may also be clear from equation (24) that, ceteris paribus, a rise in the price level, p, will reduce the stock of government bonds, B, and from equation (25), that, ceteris paribus, a rise in the price level, p, will also reduce the stock of government bonds, B, when there exists a government deficit; that is, $tY^*-G<0$. If these results are borne in mind when evaluating a shift of the equilibrium point from e to e', it can easily be seen that the equilibrium point, after an increase in government expenditure ΔG takes place, is somewhere between points e and e', when the price level is not fixed. To be clear, equation (26) shows that the partial derivative of p with respect to Y is positive.

The simultaneous solution of equations (24), (25) and (26), for Y, B and p, can be written as:

$$(27)\qquad Z=W\cdot U\,,$$

where

$$Z=[Y\quad B\quad p]'\,,$$

$$\Phi_5=-(2e_2p^*+\beta c_1w/f)/\{e_0+e_1Y^*+e_2(Y^*)^2\}\,,$$

$$W=\begin{bmatrix}\Phi_2p^* & -1 & C_1+\Phi_2(Y^*-\Phi_1G)\\ t\Phi_4p^* & -1 & \Phi_4(tY^*-G)\\ \Phi_5 & 0 & -1\end{bmatrix}^{-1}\,,\quad\text{and}$$

$$U=\begin{bmatrix}\Phi_1\Phi_2a_2V-\Phi_1\Phi_2\Phi_3H+\text{Const.}\\ \text{Const.}\\ \text{Const.}\end{bmatrix}.$$

5. STABILITY OF THE SYSTEM

In section 3, we analysed the stationary equilibrium of our model, assuming that the government sector's budget is balanced, and the price level is fixed. This simple case, however, does not prevail when the so-called "Government Budget Restraint" is explicitly considered in relation to the money and bond markets. In order to throw some light on the economic implications of the government budget restraint, government bond financing should be taken into consideration, with the government deficit accompanied by fiscal policies. This implies that equation (15) must be taken into consideration in the dynamic analysis of the whole system of equations (11)–(17). Substituting equations (12) and (14) into equation (15), eliminating T and r, and assuming that high-powered money is exogenous, we have a situation where bond financing is endogenous, $DH=0$, and H is constant, and we can obtain the following equation:

(28) $$DB=\{V+tpY-Gp-(1-t)B\}/\{(K+LY)Y-b_0-b_2H/p+b_3B/p\}\,,\quad DB=dB/dt\,,$$

where

$$K=b_{10}+b_{11}(c_0+c_1)>0 \quad \text{and} \quad L=-b_{11}c_1/f<0\,, \quad \text{and} \quad K+LY^*>0\,.$$

Equation (18) can then be rewritten as:

(29) $$DY=(a_7)^{-1}[(M+NY)Y-A(t)-(a_6/\alpha w)\{p(e_0+e_1Y+e_2Y^2)-\{1+\beta(c_0+c_1)-\beta c_1Y/f\}w\}]\,;\quad DY=dY/dt\,.$$

Finally, equation (17) can be solved for Dp; that is,

(30) $$Dp=(p^2/\alpha w)(e_0+e_1Y+e_2Y^2)-(p/\alpha)\{1+\beta(c_0+c_1)-\beta c_1Y/f\}\,.$$

Linearizing equations by means of the Taylor series expansion of the right hand sides near the equilibrium values Y^*, B^* and p^*, the following triad of differential equations in DB, DY, and Dp can be obtained:

(31) $$\begin{aligned}DY=&(a_7)^{-1}[M+2NY^*-(a_6/\alpha w)\{p^*(e_1+2e_2Y^*)+\beta c_1w/f\}](Y-Y^*)\\&-[\{a_2(1-t)-a_3b_3+a_5\}/(a_7p^*)](B-B^*)\\&-\{(p^*)^{-2}/a_7\}[a_2V-\{a_2(1-t)-a_3b_3+a_5\}B^*-(a_3b_2+a_4)H\\&+a_6(p^*)^2(e_0+e_1Y^*+e_2Y^{*2})](p-p^*)+\text{const.}\end{aligned}$$

(32) $$\begin{aligned}DB=&-\{tp^*/r^*-(K+2LY^*)\Pi\}(Y-Y^*)+\{(1-t)/r^*-b_3\Pi/p^*\}(B-B^*)\\&+\{(G-tY^*)/r^*+(\Pi/p^*)(b_2H-b_3B^*)/p^*\}(p-p^*)+\text{const.}\end{aligned}$$

(33) $$\begin{aligned}Dp=&(p^*/\alpha)\{p^*(e_1+e_2Y^*)/w+\beta c_1/f\}(Y-Y^*)+(1/\alpha)[2p^*\{e_0+e_1Y^*\\&+e_2(Y^*)^2\}/w-\{1+\beta(c_0+c_1)-\beta c_1Y^*/f\}](p-p^*)+\text{const.}\,,\end{aligned}$$

where

$$\Pi=p^*G+(1-t)B^*-V-tpY^*=p^*G-p^*T>0\,, \quad \text{and}$$
$$K=b_{10}+b_{11}(c_0+c_1)\,, \quad L=-b_{11}c_1/f\,.$$

The determinant of the characteristic equation of this linear system can be written as:

(34) $$\begin{vmatrix}E_{11}-\lambda & E_{12} & E_{13}\\ E_{21} & E_{22}-\lambda & E_{23}\\ E_{31} & E_{32} & E_{33}-\lambda\end{vmatrix}=0\,,$$

where

$$E_{11}=(1/a_7)[M+2NY^*-(a_6/\alpha w)\{p^*(e_1+2e_2Y^*)+\beta c_1 w/f\}]\,,$$
$$E_{12}=-\{a_2(1-t)-a_3b_3+a_5\}/a_7p^*\,,$$
$$E_{13}=-(p^*)^{-2}[a_2V-\{a_2(1-t)-a_3b_3+a_5\}B^*-(a_3b_2+a_4)H$$
$$+(a_6/\alpha w)\{e_0+e_1Y^*+e_2(Y^*)^2\}(p^*)^2]/a_7\,,$$
$$E_{21}=-\{tp^*/r^*-(K+2LY^*)\Pi\}\,,$$
$$E_{22}=\{(1-t)/r^*+\Pi b_3/p^*\}\,,$$
$$E_{23}=\{(G-tY^*)/r^*+(\Pi/p^*)(b_2H-b_3B^*)/p^*\}\,,$$
$$E_{31}=(p^*/\alpha)\{p^*(e_1+2e_2Y^*)/w+\beta c_1/f\}\,,$$
$$E_{32}=0\,,\quad \text{and}$$
$$E_{33}=(1/\alpha)[(2p^*/w)\{e_0+e_1Y^*+e_2(Y^*)^2\}-\{1+\beta(c_0+c_1)-\beta c_1Y^*/f\}]\,.$$

Thus, if equation (34) has a real negative root, this three-equation dynamic system will have solutions for the three variables, Y, B, and p. A sufficient condition for stability of the system consisting of equations (31), (32) and (33), is:

$$(35)\qquad \begin{vmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{vmatrix}>0\,,\qquad \begin{vmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{vmatrix}>0\,,\qquad \text{and}\quad E_{11}+E_{22}+E_{33}<0\,.$$

Upon evaluating Π and the derivatives of these equations at the equilibrium point, and making use of our assumptions about the values of variables and the signs of parameters, we found that whether the sign of the determinant is positive or not depends on the signs of some factors in the determinatnt; that is, if

$$(36)\qquad (1-t)/r^*+\Pi b_3/p^*>0\,,$$

$$(37)\qquad -tp^*/r^*-(K+2LY^*)\Pi>0\,,\qquad \text{and}$$

$$(38)\qquad \{a_2(1-t)-a_3b_3+a_5\}/a_7p>0\,,$$

then, this system is stable in the neighborhood of equilibrium. This point deserves further analysis.

As one probable case, if the government deficit is so large that the value of Π turns out to be large enough to make the value of E_{21} positive, and if the net effect of an increase in government bond issues on real demand for final goods is positive; that is, $\{a_2(1-t)-a_3b_3+a_5\}/a_7p>0$, then this system could be stable under the following strict condition:

$$(39)\qquad E_{11}\cdot E_{22}<E_{12}\cdot E_{21}\;;\quad E_{11}<0\,,\quad E_{12}<0\,,\quad E_{21}>0\,,\quad E_{22}>0\,,$$
$$E_{13}\cdot E_{22}<E_{12}\cdot E_{23}\;;$$
$$E_{13}<0\,,\quad E_{23}>0\,,\quad E_{31}<0\,,\quad E_{33}>0\,,\quad \text{and}\quad E_{11}+E_{22}+E_{33}<0\,.$$

In this case, the determinant is positive, and the trace is negative, so that the real parts of roots of this characteristic equation are negative. This clearly satisfices the necessary and sufficient conditions for stability in the neighborhood of transitory equilibrium. Otherwise, the system turns out to be unstable as in the cases investigated by Christ and others.

6. CONCLUDING REMARKS

In this paper, *I* proposed the concept of "Risk Probability", and analysed its depressing effect on the behavior of consumers and firms. The main conclusions are as follows. Firstly, it was found that the higher is the rate of unemployment as a risk probability, the smaller are the impact multipliers of government expenditure. Secondly, making a dynamic analysis with this model, it was concluded that the smaller is the government deficit, near a transitory equilibrium, the higher is the possibility that the conditions for dynamic stability of an economy will be satisfied. Thirdly, it was shown that an increase in government expenditure through bond financing will result in a larger government deficit in an economy which has already suffered from a large deficit.

The last of the stability conditions (39), seems very unlikely to be satisfied, because E_{33} may be greater than unity. Therefore an economy that can be adequately represented by this type of dynamic model seems to be unstable against the impact of fiscal policies accompanied by bond financing. Though the reaction speed of the economy is very slow (reflecting a high rate of unemployment is a high risk probability), an increase in government expenditures may result in a larger government deficit which brings about a larger amount of government bond issues, which will result in a rise in interest rates in the economy.

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