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NASH-COURNOT EQUILIBRIUM FOR AN INDUSTRY WITH OLIGOPOLY AND A COMPETITIVE FRINGE

Koji OKUGUCHI

Abstract. The Nash-Cournot equilibrium for an industry with a cartel and a competitive fringe has been analyzed by economists interested in exhaustible resources. In this paper the existence and stability of the Nash-Cournot equilibrium for an industry with oligopolists behaving independently one another without any cooperation among themselves and with many small firms forming a competitive fringe are proven under a set of general conditions.

1. INTRODUCTION

In this paper we shall be concerned with the existence and stability of the Nash-Cournot equilibrium for an industry with large oligopolistic firms and many small competitive firms. In the world oil industry large oligopolistic firms forming a cartel coexist with a competitive fringe comprising of many small firms. The Nash-Cournot equilibrium for such an industry has been analyzed by Salant (1976) Ulph and Folie (1980) and Okuguchi (1982). A contrasting situation is where large oligopolistic firms behave independently without any binding agreement or co-operation among themselves. The case where oligopolists behave independently under Cournot expectations on other oligopolists' outputs as well as on those of the competitive fringe and where, in addition, the competitive fringe behaves competitively on the basis of Cournot expectation on the market price of the goods commonly produced by oligopolists and itself has been analyzed by Sherali, Soyster and Murphy (undated). They have defined the Nash-Cournot equilibrium for this situation and proven the equivalence of the Nash-Cournot equilibrium and the optimum solution for a *equilibrating problem.* Though the stability of the Nash-Cournot equilibrium has been taken up by Ulph and Folie (1980) and Okuguchi (1982) for an industry with a cartel and a competitive fringe, Sherali et al. have not been concerned with stability analysis for their model.

This paper will analyze systematically the existence and stability of the Nash-Cournot equilibrium for an industry with oligopolists and a competitive fringe. As a preliminary to stability analysis, we shall in Section 2 give a self-contained proof of the existence of the Nash-Cournot equilibrium. In Section 3 a global stability condition will be derived from the viewpoint of diagonal dominance for the Jacobian matrix of a general continuous adjustment process for actual outputs of oligopolists and of a competitive fringe. The stability condition will be interpreted in relation to marginal profit functions, marginal costs and the number of oligopolists. The often discussed Cournot oligopoly is identified as a case where a competitive fringe does not exist. A new stability condition is seen to emerge for the Cournot oligopoly. Section 4 concludes.

2. EXISTENCE OF THE NASH-COURNOT EQUILIBRIUM

Let there be n oligopolists and a competitive fringe which produce identical goods. Let x_i and $C_i(x_i)$, respectively, be the output and cost function of the i-th firm, $i=0, 1, \cdots, n$, where a competitive fringe is identified as the 0-th firm. The market demand function is $p \equiv f(Q) = f(Q_{-0}+x_0)$, where $Q \equiv \sum_{i=0}^{n} x_i$, $Q_{-0} = \sum_{i=1}^{n} x_i$ and p is the price of the goods. All functions will be assumed to be differentiable as many times as necessary. We assume further[1]:

Assumption 1. A positive number $\bar{Q}$ exists such that $p=f(Q)=0$ for $Q \geq \bar{Q}$ and $f'(Q)<0$ for $Q<\bar{Q}$.

Assumption 2. $C_0''>0$.

Assumption 3. $f'(Q)<C_i''(x_i)$, $i=1,2, \cdots, n$.

Assumption 4. $f'(Q)+x_i f''(Q)<0$, $i=1,2, \cdots, n$.

The economic meanings of Assumptions 1, 2 and 3 are clear. Under Assumption 4, the marginal revenue of any oligopolist with respect to change in its own output is decreasing with respect to change in any other oligopolist's as well as the competitive fringe's outputs.

Adapting Szidarovszky and Yakowitz (1977),[2] consider

$$f(Q_{-0}+x_0)+x_i f'(Q_{-0}+x_0)-C_i'(x_i)=0\,, \qquad i=1, 2, \cdots, n \tag{1}$$

to define the reaction functions for oligopolists

$$x_i(Q_{-0}+x_0) = \begin{cases} x_i & \text{for } x_i \geq 0 \text{ which satisfies (1)}\,, \\ 0\,, & \text{otherwise} \end{cases} \tag{2}$$

and

$$y(Q_{-0}+x_0) \equiv \sum_{i=1}^{n} x_i(Q_{-0}+x_0)\,. \tag{3}$$

Under Assumptions 3 and 4, $x_i(Q_{-0}+x_0)$, hence $y(Q_{-0}+x_0)$ also, can be shown to be strictly decreasing in Q_{-0} for given x_0, $i=1,2, \cdots, n$. Clearly,

[1] In their analysis of Cournot oligopoly without competitive fringe, Szidarovszky and Yakowitz (1977) assume $f'<0$, $f'' \leq 0$, $C_i'>0$, $C_i'' \geq 0$ which are contained in our Assumptions 3 and 4. The proof in Frank and Quandt (1963) is based on more general assumptions than Szidarovszky and Yakowitz, but it fails to ensure non-negative profit for firms.

[2] See also Friedman (1981).

$$
\begin{aligned}
y(Q_{-0}+x_0) &\geq 0 \qquad \text{for} \quad Q_{-0}=0\,, \\
&= 0 \qquad \text{for} \quad Q_{-0}=\bar{Q}-x_0\,.
\end{aligned}
$$

As we consider a situation where at least one oligopolist is engaged in positive production in addition to the competitive fringe, we may reasonably assume that $y(x_0)=\sum_{i=1}^{n} x_i(x_0)>0$ for any given x_0. This requirement is equivalent to

Assumption 5. $f(0)>C_i'(0)$ for at least one i for $i=1, 2, \cdots, n$.

Under Assumption 5 there exists a unique Q_{-0}^* such that

$$y(Q_{-0}^*+x_0)=Q_{-0}^*\,, \tag{4}$$

yielding

$$Q_{-0}^* \equiv Q_{-0}^*(x_0)\,. \tag{5}$$

Since $x_i(Q_{-0}+x_0)$ is strictly decreasing in Q_{-0} and x_0, consideration of $y(Q_{-0}^*+x_0)=\sum_{i=1}^{n} x_i(Q_{-0}^*+x_0)$ and total differentiation of (4) lead to

$$-1<dQ_{-0}^*/dx_0<0\,. \tag{6}$$

Now consider

$$f(Q_{-0}^*+x_0)=C_0'(x_0)\,. \tag{7}$$

To consider a situation where the competitive fringe coexists with oligopolists, we introduce

Assumption 6. $C_0'(0)<f(Q_{-0}^*(0))$.

From (6) and $f'<0$, the LHS of (7) is strictly decreasing in x_0, while the RHS is strictly increasing in the same variable due to Assumption 2. In view of Assumption 6, a unique $x_0=x_0^*$ exists which satisfies (7). This is the output for the competitive fringe at the Nash-Cournot equilibrium. The corresponding outputs for oligopolists are $x_i^* \equiv x_i(Q_{-0}^*(x_0^*)+x_0^*)$, $i=1, 2, \cdots, n$.

3. STABILITY

In this section we shall analyze the global stability of the Nash-Cournot equilibrium for a continuous system of differential equations for adjustment of outputs for oligopolists and the competitive fringe. Let x_i and x_i^* be the i-th firm's actual and profit maximizing outputs, respectively, for $i=0, 1, \cdots, n$. If all oligopolists form expectations on outputs of other oligopolists and of the competitive fringe a la Cournot, x_i^*'s are implicitly determined by

$$
\begin{aligned}
&f(x_0+x_1+\cdots+x_i^*+\cdots+x_n)+x_i^* f'(x_0+x_1+\cdots+x_i^*+\cdots+x_n) \\
&\qquad -C_i'(x_i^*)=0\,, \qquad i=1, 2, \cdots, n
\end{aligned} \tag{8}
$$

when interior maximum is assumed. Solving (8),

$$x_i^* = \psi_i(x_0 + \cdots + x_{i-1} + x_{i+1} + \cdots + x_n)\,, \qquad i = 1, 2, \cdots, n\,, \tag{9}$$

where

$$\partial\psi_i/\partial x_j = -(f' + x_i^* f'')/(2f' + x_i^* f'' - C_i'')\,, \tag{10}$$
$$i \neq j\,, \quad i = 1, 2, \cdots, n\,, \quad j = 0, 1, \cdots, n\,.$$

The competitive fringe is assumed to have expectation on the market price a la Cournot. Hence its profit maximizing output is determined by

$$p = f(x_0 + x_1 + \cdots + x_i + \cdots + x_n) = C_0'(x_0^*)\,, \tag{11}$$

or

$$x_0^* = \psi_0(x_0 + x_1 + \cdots + x_n)\,, \tag{12}$$

where

$$\partial\psi_0/\partial x_j = f'/C_0''\,, \qquad j = 0, 1, \cdots, n\,. \tag{13}$$

The actual outputs are assumed to be adjusted according to

$$dx_i/dt = g_i(x_i^* - x_i)\,, \qquad i = 0, 1, \cdots, n \tag{14}$$

where we assume[3]

$$g_i(0) = 0\,, \qquad g_i' > 0\,, \qquad i = 0, 1, 2, \cdots, n\,.$$

By a simple calculation,

$$\partial G_0/\partial x_0 = g_0'(f'/C_0'' - 1)\,, \quad \partial g_0/\partial x_j = g_0' f'/C_0''\,, \quad j = 1, 2, \cdots, n$$
$$\partial g_i/\partial x_i = -g_i'\,, \quad \partial g_i/\partial x_j = g_i' \partial\psi_i/\partial x_j\,, \tag{15}$$
$$i \neq j\,, \quad i = 1, 2, \cdots, n\,, \quad j = 0, 1, \cdots, n\,.$$

Hence diagonal elements of the Jacobian matrix of the RHS of (14) are negative. Assume that the Jacobian matrix has diagonal dominance with respect to rows everywhere[4]:

Assumption 7.

$$|\partial g_i/\partial x_i| > \sum_{\substack{j=0 \\ j \neq i}}^{n} |\partial g_i/\partial x_j|\,, \qquad i = 0, 1, \cdots, n\,.$$

In view of (10) and (15), this can be rewritten as Assumptions 7′a and 7′b.

[3] This adjustment hypothesis is less general than that in Okuguchi (1964) for a purely oligopolistic industry, where g_i's are assumed to be only sign-preserving.

[4] We assume here diagonal dominance for the Jacobian matrix with respect to rows. The diagonal dominance may also be assumed with respect to columns.

Assumption 7′a. $C_0'' > (1-n)f'$.

Assumption 7′b. $2f' + x_i^* f'' - C_i'' < n(f' + x_i^* f'')$, $i = 1, 2, \cdots, n$,

where the left hand side of Assumption 7′b is negative by virtue of Assumptions 3 and 4. Since the diagonal elements of the Jacobian matrix for (14) are all negative, a similar reasoning as Arrow and Hurwicz (1958) or Gandolfo (1980) can be applied to establish the global stability of the Nash-Cournot equilibrium under Assumption 7′. Alternatively, we may prove the stability by showing that the Jacobian matrix is quasi-negative definite under the same assumption.[5]

4. INTERPRETATION OF THE STABILITY CONDITION

Let us now interprete our global stability condition comprising of Assumptions 7′a and 7′b. If $n = 1$, Assumption 7′a is always satisfied.[6] If $n > 1$, the assumption is less likely to be satisfied, the larger the number of oligopolists. From (9),

$$dx_i^* = \sum_{\substack{j=0 \\ j \neq i}}^{n} \partial\psi_i/\partial x_j dx_j, \qquad i = 1, 2, \cdots, n. \tag{16}$$

Suppose here that any oligopolists decrease the profit maximizing outputs less than one unit when all other firms including the competitive fringe simultaneously increase their outputs by one unit. In the light of (10) and (16), Assumption 7′b is satisfied in this case.

Consider next a pure Cournot oligopoly without competitive fringe. Assumption 7′a is then redundant, and the stability condition reduces to

$$2f' + x_i^* f'' - C_i'' < (n-1)(f' + x_i^* f''), \qquad i = 1, 2, \cdots, n. \tag{17}$$

Other things being equal, (17) is more likely to be satisfied, the larger C_i'' is. The stabilizing effect of increase in the rate of change of the marginal cost has been noted by Fisher (1961) whose analysis was based on linear demand and quadratic cost functions. This stabilizing effect, however, is to be contrasted with the destabilizing one for a price-adjusting Cournot oligopoly with product differentiation.[7]

5. CONCLUSION

We have proven, under a set of reasonable assumptions, the existence of a

[5] See Namatame and Tse (1981).

[6] This contrasts with the result for a discrete system in Okuguchi (1982), where the stability is asserted under a stronger condition $C_i'' > -2f'$.

[7] See Okuguchi (1976). See also Furth (1982) and Al-Nowaihi and Levine (1985) for some aspects of the most recent developments in the Cournot oligopoly theory.

unique interior Nash-Cournot equilibrium for an industry comprising of oligopolists and many small firms forming a competitive fringe. The equilibrium is stable under Assumption 7 (or Assumptions 7′a and 7′b), that is, under the assumption of diagonal dominance for the Jacobian matrix for output adjustment functions. We have also interpreted our stability condition in relation to the properties of the reaction and cost functions as well as to the number of oligopolists. In the absence of the competitive fringe, (17) gives the stability condition for the Cournot equilibrium for a purely oligopolistic industry without product differentiation.

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