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DEMAND FOR DIFFERENTIATED BRANDS, ADVERTISING AND R & D ACTIVITY*

Yasuo KAWASHIMA

Abstract. This paper develops a model in which consumers pick a bundle consisting of the numeraire and one of the available brands on the basis of rational behavior of consumers. A quasiconvex utility function is introduced to deal with demand for brands. After the existence of a Nash-Cournot equilibrium in prices in differentiated markets is demonstrated, we take up interesting properties of the markets: a firm can capture a larger market share if it can succeed in developing new technologies or in undertaking more advertising. The welfare effects of R & D and advertising are more complicated: cost reductions of any firm lead to a welfare improvement, whereas advertising of a top quality brand does not improve welfare but that of a low quality brand does improve welfare.

1. INTRODUCTION

In traditional demand theory, goods that provide almost the same characteristics to consumers (e.g., brands of the same commodity) are treated in the same way as different commodities. However, a choice between bread and oranges is utterly different from that between differentiated brands: consumers usually choose just one of the available brands. The present paper seeks to explain this phenomenon and then to explore some interesting properties of differentiated market structure concerning R & D activity and advertising, both of which are the important characteristics of modern monopolistic markets.

In his pioneering work, Phlips [1964] proposed linear indifference curves to deal with the demand for differentiated brands. Recently, Gabszewicz and Thisse [1979] developed a simple model which enables them to derive interesting properties of differentiated markets. Their model not only provided deep insight into this subject, but also stimulated important subsequent studies by Gabszewicz, Shaked, Sutton and Thisse [1981] and Shaked and Sutton [1982]. A theory of demand for brands is further advanced by Novshek and Sonnenschein [1979]. All of these models, except Philips [1964], rest crucially upon the assumption that consumers purchase just one of the available brands.

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The present model tries to provide an explanation of why consumers choose just one of the available brands, when consumers' objectives are to maximize their respective utility function subject to budget constraints.[1] Indifference curves, which are not convex to the origin, are introduced to deal with the demand for brands[2] and then examine the equilibrium of a differentiated market. While Roberts and Sonnenschein [1977] demonstrated the absence of general conditions on preferences and technologies that will quarantee the existence of a non-cooperative Nash-Cournot equilibrium in prices, our specification of consumers' tastes and production technologies enables us to derive well-behaved reaction functions through which the existence of a Nash-Cournot equilibrium in prices is demonstrated in differentiated markets.

More importantly, the model also enables us to illustrate the effects of R & D activity and advertising on welfare and the market shares of the firms; cost reductions of a firm, due to its R & D activity, lead to a larger share of the firm. On the other hand, the advertising effect is more complicated; a firm which undertakes more advertising can capture a larger share, but more advertising does not necessarily provide welfare gain to the economy.

Our paper proceeds as follows. In Section 2 we set forth a general model of demand for a brand, which helps us explain why consumers pick just one of the available brands. In Section 3, a specification of the utility functions is introduced to derive individual demands, while market demand is defined as the integral of individual demands over consumers. In Section 4, these market demand functions are utilized to derive continuous reaction functions of firms and to demonstrate the existence of a Nash-Cournot equilibrium in prices. Section 5 explores the important roles played by R & D activity and advertising concerning welfare and the market shares of firms. Finally, we summarize our analysis in Section 6.

2. THE BASIC MODEL

Consider an economy in which there are a numeraire good labelled 1 and two differentiated brands i, where $i=2, 3$.[3] Let x_i stand for consumption of good i, for $i=1, 2, 3$. We assume that a consumer has a weakly separable utility function of the form

$$u(x_1, x_2, x_3) = V(v^1(x_1), v^2(x)) , \tag{1}$$

and

$$x = (x_2, x_3) ,$$

[1] Following Phlips [1964], we assume that indifference curves between differentiated brands are not covex to the origin.

[2] Phlips and Thesse [1982] point out that a choice between brands cannot be modelled using indifference curves that are convex to the origin.

[3] The case where there are many brands is discussed in Kuroiwa [1983].

where v^i denotes the specific satisfaction function. Assume further that

A1: V and v^i are differentiable and have the positive partial derivative with respect to their respective arguments except a point on coordinate axes,

A2: V is strictly quasi-concave with respect to v^i's,

A3: v^2 is quasi-convex with respect to x,

A4: $u(x_1, 0, 0) = u(0, x_2, 0) = u(0, 0, x_3) = 0$.

In view of A1 and A3, the utility function is also quasi-convex with respect to x. Thus, our utility function is quite different from that of standard microeconomic theory; indifference curves between brands are not convex to the origin, while V has standard indifference curves between v^i's. So far a utility function of this form has always been assumed away, while Phlips [1964] proposed linear indifference curves to dead with product differentiation. Our model is more general than the Phlips model in that it includes indifference curves concave to the origin and that consumers are allowed to chose a bundle consisting of the numeraire and differentiated brands.

The budget constraint is given by

$$x_1 + p_2 x_2 + p_3 x_3 = m_1 + m_2 = m \,, \qquad \text{with} \quad x_1 = m_1 \,, \tag{2}$$

where m_i's stand for category expenditures for the numeraire and for a brand, respectively.

Given these assumptions, a consumer is shown to purchase one of the available brands in addition to the numeraire.

PROPOSITION 1.[4] *Suppose that consumers' preferences are represented by a weakly separable utility function which satisfies* A1 *through* A4. *Then, consumers almost always choose one of the available brands in addition to the numeraire.*

Proof. When indifference curves between brands are linear, a consumer usually chooses one of the brands for a given m_2 when indifference curves are concave to the origin.

Our next step is to determine category expenditure m_1 and m_2. For given m_1 and m_2 such that

$$m_1 + m_2 = m \,,$$

let

$$v(m_2) = \max\left[v^2\left(\frac{m_2}{p_2}, 0\right), v^2\left(0, \frac{m_2}{p_3}\right)\right].$$

$v(m_2)$ is continuous in m_2 because v^2 is also continuous in m_2. Therefore, for given m_1 and m_2 such that $m_1 + m_2 = m$, the utility function is reduced to

$$V^* = V(v^1(m_1), v(m_2)) \,.$$

Consumers' objectives are to maximize their respective utility subject to their

[4] This idea is first pointed out by M. Ohyama.

budget constraints. In view of Weierstrass's theorem, the maximum utility is achieved at $m_1 = m_1^*$ and $m_2 = m_2^*$. By virtue of A1 and A4, m_i^*'s are both positive.

When v^2 provides linear indifference curves, the curves may coincide with the budget constraints at the specific price ratio; both brands may be purchased. On the other hand, given that indifference curves derived from v^2 are concave to the origin, the maximum utility may be achieved at both corners simultaneously; we may get

$$v^2\left(\frac{m_2^*}{p_2}, 0\right) = v^2\left(0, \frac{m_2^*}{p_3}\right).$$

These two cases take place at the specific price ratios. These conditions define a zero-measure subset in the parameter space. Q.E.D.

Note that all essential aspects of this problem will not only generalize to any number of available brands, but also to any number of categories of brands. Consumers determine a choice of brands, category expenditure m_i^* and demand for brands simultaneously.

If consumers pick brand 2,

$$v^2\left(\frac{m_2^*}{p_2}, 0\right) > v^2\left(0, \frac{m_2^*}{p_3}\right),$$

from which u is rewritten as

$$u^* = u\left(m_1^*, \frac{m_2^*}{p_2}, 0\right),$$

where u^* is the maximum utility subject to the budget constraint. Hence, differentiating it with respect to m_1, an equilibrium satisfies

$$\frac{\partial u}{\partial x_1} - \frac{1}{p_2}\frac{\partial u}{\partial x_2} = 0.$$

It immediately follows from this equation that the marginal rate of substitution between the numeraire and a brand is equal to the price ratio. Our conclusions are thus consistent with what traditional microeconomic theory predicts.

3. MARKET DEMAND FOR BRANDS

In this section, we shall first derive market demand for brands, which is defined as the integral of individual demands over consumers, and then the existence of a Nash-Cournot equilibrium in prices will be demonstrated in the next section. When we seek to demonstrate the existence, we should bear in mind the results pointed out by Roberts and Sonnenschein [1977]. Therefore, to get over the difficulty posed by them we shall specify the utility function which satisfies A1 through A4. Thus, assume that

$$u=ax_1(x_2+bx_3) \tag{3}$$

and

$$b=\beta\alpha \tag{4}$$

where a and β are held fixed and α is an index of a consumer. Assume further that consumers are indexed by α in the closed unit interval [0, 1] and that they are evenly distributed.

Our utility function shows that the marginal utility of brand 3 increases with an index of a consumer, so that consumers with a high index feel more satisfaction from consumption of brand 3. In addition, indifference curves between brands 2 and 3 are linear, following Phlips [1964].

In view of Proposition 1, consumers are supposed to purchase either of the available brands. Formally, consumers' demand is either

$$x_1^*=\frac{m}{2}, \quad x_2^*=\frac{m}{2p_2} \quad \text{and} \quad x_3^*=0,$$

or

$$x_1^*=\frac{m}{2}, \quad x_2^*=0 \quad \text{and} \quad x_3^*\frac{m}{2p_3}.$$

The resulting indirect utility functions are

$$V^2=ax_1^*x_2^*=\frac{am^2}{4p_2} \quad \text{and} \quad V^3=abx_1^*x_3^*=\frac{am^2}{4p_3}\beta\alpha,$$

where V^i denotes the indirect utility function when a consumer is constrained to purchase brand i. The index for the marginal consumers is determined by

$$V^2=V^3$$

which yields

$$\alpha^*=\frac{p_3}{\beta p_2} \tag{5}$$

if α^* is less than 1. When this index is equal to or larger than 1, V^2 is always larger than V^3 for all α in the interval. Then, all consumers purchase brand 2. In the analysis to follow, we center on the case in which the index is less than 1, but positive. Therefore, consumers can choose either of the available brands.

So far, we do not make any assumption about the income levels of consumers. In the Gabszewicz and Thisse [1979] model, all consumers share the same utility function while income levels differ among consumers. In Novshek and Sonnenschein [1979] income levels and utility functions are closely related. In the present model we shall follow Novshek and Sonnenschein and assume that

$$m = \mu\alpha \tag{6}$$

where μ is a positive constant. Therefore it follows from (3), (4) and (6) that the ranking in terms of preferences is similar to the income ranking. As index α increases, the income levels go up and preferences toward brand 3 become stronger. Thus, brand 3 is the so-called top quality brand which wealthy consumers typically prefer.

Given these assumptions we can establish:

PROPOSITION 2. *If consumers' tastes are represented by* (3) *and* (4) *and budget constraints are given by* (2) *and* (6), *then market demand for brands is expressed respectively as*

$$X^2 = \int_0^{\alpha^*} x_2^*(\cdot)\mathrm{d}\alpha = \frac{\mu p_3^2}{4\beta^2 p_2^3} \tag{7}$$

and

$$X^3 = \int_{\alpha^*}^1 x_3^*(\cdot)\mathrm{d}\alpha = \frac{\mu}{4p_3}\left[1 - \left(\frac{p_3}{\beta p_2}\right)^2\right]. \tag{8}$$

Proof. Noting that a consumer with an index less than α^* is supposed to choose brand 2, market demand for it is given by (7), where we make use of (5) and (6). Similar calculations yield market demand for brand 3 as (8).

Q.E.D.

In the case in which all consumers purchase brand 2, we have

$$X^2 = \int_0^1 x_2^*(\cdot)\mathrm{d}\alpha = \frac{\mu}{4p_2}, \tag{9}$$

and

$$X^3 = 0 .$$

4. A COURNOT EQUILIBRIUM IN PRICES

Firms are assumed to provide differentiated brands and each of them is assumed to have its own brand. Assume also that average costs of firm i are constant, but are different across firms. Let c_i be average costs of firm i, where $i = 2, 3$. While fixed costs play a crucial role in differentiated industries,[5] they are assumed away for simplicity of our analysis. Firms are non-cooperative and seek to maximize their respective profits with respect to their prices, taking the other firm's price as given.

Profits of firm i are defined as

[5] For instance, see Spence [1976] and Dixit [1979].

$$\pi^i=(p_i-c_i)X^i\,, \qquad \text{where} \quad i=2,3\,,$$

where p_i denotes price of brand i and is assumed to be larger than c_i. By virtue of Proposition 2, we can derive profit functions for both firms, which enable us to obtain the firms' optimal price.

Consider the feasible set of prices of both firms. As we have taken up the case of duopoly, the index α^* is less than 1. Hence, we have

$$p_2>\frac{1}{\beta}p_3\,, \tag{10}$$

and for positivity of profits of both firms

$$p_i \geq c_i\,, \qquad \text{for} \quad i=2,3\,. \tag{11}$$

An equilibrium point should be in this set of prices.

We now establish:

PROPOSITION 3. *Given that* $p_i \geq c_i$, *for* $i=2,3$ *and that* $\beta>p_3/p_2$. *If market demand for brands is given by* (7) *and* (8), *then reaction curves of both firms are expressed respectively as*

$$p_2=\frac{3}{2}c_2 \qquad \textit{for firm } 2 \tag{12}$$

and as

$$p_2=\frac{p_3}{\beta}\sqrt{\frac{2p_3-c_3}{c_3}} \qquad \textit{for firm } 3\,. \tag{13}$$

Proof. In view of Proposition 2, the profit function of 2 is given by

$$\pi^2=(p_2-c_2)X^2=(p_2-c_2)\frac{\mu p_3^2}{4\beta^2 p_2^2}\,.$$

The first order condition simplifies to

$$\frac{\partial \pi^2}{\partial p_2}=X^2+(p_2-c_2)\frac{\partial X^2}{\partial p_2}=\frac{\mu p_3^2}{4\beta^2 p_2^3}[p_2-3(p_2-c_2)]=0\,.$$

As a result, the reaction curve for 2 may be rewritten as

$$p_2=\frac{3}{2}c_2\,. \tag{12}$$

Note that the reaction curve for 2 always satisfies the requirement that the optimal prices be larger than average costs c_2.

Similarly using (8) we get

$$\frac{1}{\pi^3}\frac{\partial \pi^3}{\partial p_3}=\frac{1}{p_3-c_3}-\frac{1}{p_3}+\frac{2\left(\frac{p_3}{\beta p_2}\right)^2}{1-\left(\frac{p_3}{\beta p_2}\right)^2}\frac{1}{p_3}=0\,,$$

which may reduce to

$$\frac{c_3}{p_3-c_3}-\frac{2\left(\frac{p_3}{\beta p_2}\right)^2}{1-\left(\frac{p_3}{\beta p_2}\right)^2}=0\,.$$

Solving this equation for p_3 yields the reaction curve for firm 3, but it is more convinient to express it as

$$p_2=\frac{p_3}{\beta}\sqrt{\frac{2p_3-c_3}{c_3}}\,, \tag{13}$$

Q.E.D.

Although both firms are followers in the sense of Cournot, the reaction function of firm 2 is independent of the price of firm 3. This result follows from the assumption that consumers' preferences are represented by the specific utility function (3).

Now we establish the main proposition of this section.

PROPOSITION 4. *Given that* $(3/2)c_2 \geq c_3/\beta$ *and that the reaction functions of both firms are represented by* (12) *and* (13), *then there exists a unique Nash-Cournot equilibrium in prices in a differentiated market.*

Proof. It immediately follows from (12) that

$$p_2^*=\frac{3}{2}c_2\,, \tag{12'}$$

which belongs to the feasible set of firm 2's prices. Equation (13) shows that the reaction curve of firm 3 starts from $p_3=(1/2)c_3$ at $p_2=0$ with approaching infinity as p_2 increases. As Fig. 1 illustrates, firm 3 can set its optimal price corresponding to $p_2^*=(3/2)c_3$.

Note that (13) and $p_2=p_3/\beta$ intersect at $p_3=c_3$ and $p_2=c_3/\beta$. Hence, if $c_3/\beta>(3/2)c_2$, firm 2's optimal price p_2^* leads to an optimal price p_3^* which is less than c_3. Thus, in this case there is no equilibrium in prices, whereas firm 3's optimal price p_3^* is in the feasible set of prices if $c_3/\beta \leq (3/2)c_2$. Furthermore, Fig. 1 shows that the equilibrium point is unique and stable. Q.E.D.

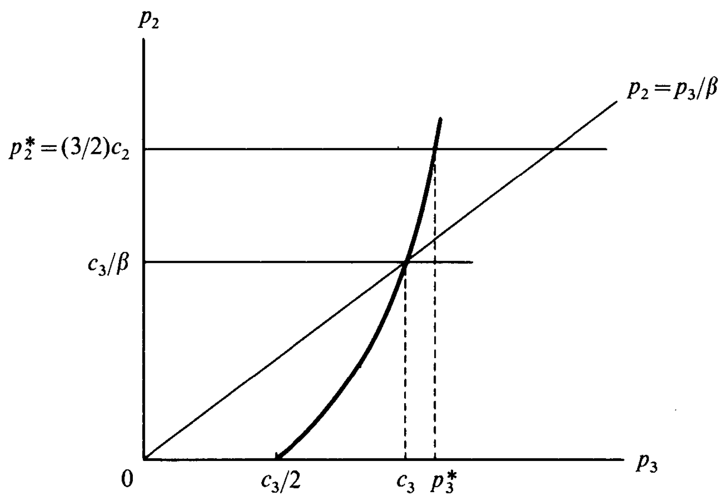

Fig. 1. Cournot-Nash equilibrium.

Before proceeding with the analysis, we shall examine barriers to entry in the present model. Bain [1956] specified three sources of entry barriers: absolute cost advantages, economies of scale, and product differentiation. Our focus is on the effects of product differentiation. To simplify the analysis, suppose that firm 2 is an incumbent firm and firm 3 an entrant.

In our model product differentiation is closely concerned with taste parameter β. Equation (3) implies that, for given u and x_1 indifference curves between brands reduce to

$$x_2+\beta\alpha x_3=u/ax_1\ . \tag{3'}$$

Thus, the marginal rate of substitution between the brands depends soley upon the value of β; if β increases one unit of brand 2 is compensated by less units of brand 3 so that the former becomes less differentiated and the latter becomes more differentiated. On the other hand, if a firm can succeed in lowering β by some means, then brand 2 is more differentiated from brand 3. Hence, β is an indicator of a degree of product differentiation.

Changes in the value of β have a profound effect on barriers to entry. Suppose that $(3/2)c_2\geq(c_3/\beta)$ holds so that an entrant can enter this market. After β declines by some means, it will be more probable that this inequality will not be satisfied; when taste parameter β is smaller than some critical value, we encounter the case in which $(3/2)c_2\leq(c_3/\beta)$, so that the entrant has to exit from the market. However, the entrant can enter the market if he counteracts it and succeeds in increasing β sufficiently. Hence, product differentiation can play a key role in barriers to entry. These are summarized as:

PROPOSITION 5. *An incumbent firm can block entry by more product differentiation, while an entrant can counteract this by differentiating his own brand.*

5. ANALYSIS OF DIFFERENTIATED MARKETS

Non-price competition is considered to be crucial in differentiated markets. Firms can capture a larger market share by various means; they may undertake advertising and invest money to facilitate technological improvements which enable them to reduce their production costs. In this section, we turn our attention to the roles played by technological improvements and advertising. Market structure is expressed by market shares, which are defined as

$$\sigma_i = \frac{X^i}{X^2 + X^3}, \qquad \text{for} \quad i = 2, 3,$$

which in turn depends soley upon the ratio (X^2/X^3); for instance, if the ratio goes up σ_2 increases whereas σ_3 declines. In the analysis to follow, we shall restrict attention to this ratio and to the cases in which (10) and (11) hold.

By virtue of Proposition 2, the ratio is

$$X^2/X^3 = \frac{\left(\dfrac{p_3}{p_2}\right)^3}{\beta^2\left\{1 - \left(\dfrac{p_3}{\beta p_2}\right)^2\right\}}, \tag{14}$$

from which we can show that the ratio is increasing with an increase in p_3, but it declines with rising p_2.

Before going further, we require some results concerning the effects of parameters c_i and β on equilibrium prices.

LEMMA. *The effects of c_i and β on the equilibrium prices p_2^* and p_3^* are expressed as*

$$\begin{aligned} \frac{\partial p_2^*}{\partial z} &= 3/2 \qquad \textit{for} \quad z = c_2, \\ &= 0 \qquad \textit{for} \quad z = c_3 \quad \textit{and} \quad \beta, \end{aligned} \tag{15}$$

and

$$\frac{\partial p_3^*}{\partial y} > 0 \qquad \textit{for} \quad y = c_2, c_3 \quad \textit{and} \quad \beta. \tag{16}$$

Proof. From (12′), we can demonstrate (15).

Differentiating (13) with respect to c_2 and using (15) we get

$$\frac{3}{2} = \frac{p_2^*}{p_3^*}\frac{\partial p_3^*}{\partial c_2} + \frac{\dfrac{\partial p_3^*}{\partial c_2}p_2^*}{2p_3^* - c_3},$$

implying that

$$\frac{\partial p_3^*}{\partial c_2}=\frac{\frac{3}{2}}{\frac{1}{\beta}\sqrt{\frac{2p_3^*-c_3}{c_3}}+\frac{p_2^*}{2p_3^*-c_3}}>0\,. \tag{16a}$$

Similarly, we could get

$$\frac{\partial p_3^*}{\partial c_3}=\frac{p_3^{*2}}{c_3(3p_3^*-c_3)}>0\,, \tag{16b}$$

where it should be noted that p_2^* is independent of changes of c_3. Finally, differentiating (13) with respect to β we get

$$\frac{\partial p_3^*}{\partial \beta}=\frac{1}{\beta}\,\frac{p_3^*(2p_3^*-c_3)}{3p_3^*-c_3}>0 \tag{16c}$$

Q.E.D.

As shown in Section 4, brand 3 becomes more differentiated the larger β becomes. This lemma implies that the price of brand 3 goes up as it becomes more differentiated and *vice versa*. On the other hand, optimal price of brand 2 is independent of taste parameter β. The former statement is quite consistent with Scherer's [1980, p. 381] assertion, where he suggests positive relationship between product differentiation and price levels of differentiated brands.

This lemma also shows that technological improvements, which cause c_i to go down, and taste changes effect equilibrium prices so that they exert some influence on equilibrium market shares for both firms. Their importance is clearly shown in the following proposition.

PROPOSITION 6. *Market share for a firm goes up with a decrease in its production costs and with an increase in its competitor's production costs. Formally,*

$$\begin{aligned}\frac{\partial \sigma_k}{\partial c_i}&<0 \quad \textit{for} \quad k=i\,,\\ &>0 \quad \textit{for} \quad k\neq i\,,\end{aligned} \tag{17}$$

where $k,\ i=2,3$.

Proof. Taking the natural logarithm of (14) and differentiating it yields

$$\frac{\partial w}{\partial \varepsilon}\,\frac{1}{w}=\frac{3}{2}+\frac{\frac{2\varepsilon}{\beta^2}}{1-\left(\frac{\varepsilon}{\beta}\right)^2}>0\,, \tag{18}$$

where $w=(X^2/X^3)$ and $\varepsilon=(p_3^*/p_2^*)$. Differentiating ε with respect to c_2 yields

$$\frac{\partial \varepsilon}{\partial c_2}=\frac{\left(\dfrac{\partial p_3^*}{\partial c_2}\Big/\dfrac{\partial p_2^*}{\partial c_2}-\dfrac{p_3^*}{p_2^*}\right)\dfrac{\partial p_2^*}{\partial c_2}}{p_2^*}. \tag{19}$$

From (13), we have

$$\frac{p_2^{*\prime}}{p_2^*}=\frac{p_3^{*\prime}}{p_3^*}+\frac{1}{2p_3^*-c_3}p_3^{*\prime},$$

which is reduced to

$$\frac{p_3^{*\prime}}{p_2^{*\prime}}=\frac{p_3^*}{p_2^*}\frac{2p_3^*-c_3}{3p_3^*-c_3}<\frac{p_3^*}{p_2^*}, \tag{20}$$

where $p_i^{*\prime}$ denotes the partial derivative of p_i^* with respect to c_2. By virtue of (19) and (20), we can conclude that

$$\frac{\partial \varepsilon}{\partial c_2}<0. \tag{21}$$

Finally, together with (18) and (21), we get

$$\frac{\partial w}{\partial c_2}=\frac{\partial w}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial c_2}<0. \tag{22}$$

As a result, σ_2 is a decreasing function of c_2 and σ_3 an increasing of c_2.
Similarly the effect of c_3 on ε is given by Lemma; that is,

$$\frac{\partial \varepsilon}{\partial c_3}>0,$$

from which, by virtue of (18), we have

$$\frac{\partial w}{\partial c_3}=\frac{\partial w}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial c_3}>0.$$

This equation implies that

$$\frac{\partial \sigma_2}{\partial c_3}>0 \quad \text{and} \quad \frac{\partial \sigma_3}{\partial c_3}<0$$

Q.E.D.

The result seems very plausible; technological improvements lead a firm to reduce its production costs, which in turn cause prices of the firm to go down providing a larger market share.

Firms usually undertake advertising for their products, which is expected to change consumers' preferences in favor of their products. In our model, taste changes are expressed in terms of changes in a taste parameter in the utility function. To discuss the advertising effect, assume that total costs consist of variable costs and fixed advertising costs denoted by F_i. Assume further that

$$\begin{aligned} \frac{\partial \beta}{\partial F_i} &< 0 \quad \text{for} \quad i=2, \\ &> 0 \quad \text{for} \quad i=3. \end{aligned} \tag{23}$$

In view of (3′), (23) implies that more advertising by firm 2 leads taste parameter β to go down so that the marginal rate of substitution between brands 2 and 3 changes in favor of brand 2, and *vice versa.*

Profits for firm i are rewritten as

$$\pi^i = (p_i - c_i)X^i - F_i\,, \qquad \text{for} \quad i=2,3\,,$$

where F_i is held fixed. The advertising effect on the market share is summarized as:

PROPOSITION 7. *An increase in firm's advertising enables the firm to capture a larger market share, and vice versa. Formally, we get*

$$\frac{\partial \sigma_i}{\partial F_i} > 0 \qquad \textit{for} \quad i=2,3\,, \tag{24}$$

provided that optimal profits for both firms are positive.

Proof. From (14), we have

$$\frac{\partial w}{\partial \beta}\frac{1}{w} = -\frac{2}{\beta} + \frac{3p_3^{*\prime}}{p_3^*} + \frac{\dfrac{2p_3^{*2}}{\beta^3 p_2^{*2}}\left(\dfrac{p_3^{*\prime}}{p_3^*}\beta - 1\right)}{1 - \left(\dfrac{p_3^*}{\beta p_2^*}\right)^2}\,, \tag{25}$$

where $p_3^{*\prime}$ denotes the partial derivative of p_3^* with respect to β. From (16c), the first two terms in (25) are reduced to

$$\frac{3p_3^{*\prime}}{p_3^*} - \frac{2}{\beta} = \frac{2p_3^* - c_3}{3p_3^* - c_3}\frac{3}{\beta} - \frac{2}{\beta} = -\frac{1}{\beta}\frac{c_3}{3p_3^* - c_3} < 0\,. \tag{26}$$

Similarly, by virtue of (15), the numerator of the last term in (25) is negative because

$$\frac{p_3^{*\prime}}{p_3^*}\beta - 1 = \frac{2p_3^* - c_3}{3p_3^* - c_3} - 1 < 0\,. \tag{27}$$

Substituting (26) and (27) into (25) yields

$$\frac{\partial w}{\partial \beta} < 0\,. \tag{28}$$

On account of (23) and (28), we can conclude that

$$\frac{\partial \sigma_i}{\partial F_i}=\frac{\partial \sigma_i}{\partial \beta}\frac{\partial \beta}{\partial F_i}>0 \tag{24}$$

Q.E.D.

This conclusion seems contrary to recent studies of the advertising effect on market shares. Lambin [1976] and Martin [1979] fail to find a significant advertising effect on market concentration in their analysis. In the real world, however, each firm usually advertises so that the advertising effect by each firm cancels out. Hence, the net effect of a firm's advertising is more likely to be ambiguous, whereas its effect by one firm's advertising is clearly visible in the present model.

From now on, we turn attention to the welfare analysis of cost reductions and advertising activity. Although the indirect utility function V^i depends upon p_i, m and index α, in view of (6) it is reduced to a function of p_i and α. Hence, we get

$$V^i=V^i(p_i, m, \alpha)=g^i(p_i; \alpha)\,.$$

Suppose that welfare is defined as

$$W=\int_0^{\alpha^*} g^2(p_2^*; \alpha)\mathrm{d}\alpha+\int_{\alpha^*}^1 g^3(p_3^*; \alpha)\mathrm{d}\alpha\,.$$

Then, we may establish:

PROPOSITION 8. *Cost reductions of a firm lead to welfare improvements. More advertising by firm 2 provides welfare improvements, while that by the other firm decreases welfare. Then,*

$$\frac{\partial W}{\partial x}<0 \qquad \textit{for} \quad x=c_2, c_3 \quad \textit{and} \quad \beta\,, \tag{29}$$

and

$$\begin{aligned}\frac{\partial W}{\partial F_i}=\frac{\partial W}{\partial \beta}\frac{\partial \beta}{\partial F_i}&>0 \qquad \textit{for} \quad i=2\,,\\ &<0 \qquad \textit{for} \quad i=3\,.\end{aligned} \tag{30}$$

Proof. Differentiating welfare W yields

$$\begin{aligned}\frac{\partial W}{\partial x}&=\int_0^{\alpha^*} g_p^2(\cdot)\frac{\partial p_2^*}{\partial x}\mathrm{d}\alpha+\int_{\alpha^*}^1 g_p^3(\cdot)\frac{\partial p_3^*}{\partial x}\mathrm{d}\alpha+g^2(p_2^*, \alpha^*)\frac{\partial \alpha^*}{\partial x}\\ &\quad -g^3(p_3^*, \alpha^*)\frac{\partial \alpha^*}{\partial x}=\int_0^{\alpha^*} g_p^2(\cdot)\frac{\partial p_2^*}{\partial x}\mathrm{d}\alpha+\int_{\alpha^*}^1 g_p^3(\cdot)\frac{\partial p_3^*}{\partial x}\mathrm{d}\alpha\,,\end{aligned} \tag{31}$$

where g_p^i denotes the partial derivative of g^i with respect to prices. It should be noted that by the definition of α^*

$$g^2(p_2^*, \alpha^*)=g^3(p_3^*, \alpha^*)\,.$$

Together with the lemma and (31), we can conclude that

$$\frac{\partial W}{\partial x}<0 \qquad \text{for} \quad x=c_2, c_3 \quad \text{and} \quad \beta, \tag{29}$$

where g_p^i is negative. Furthermore, bearing (23) and (29) in mind, we have

$$\begin{aligned} \frac{\partial W}{\partial F_i}=\frac{\partial W}{\partial \beta}\frac{\partial \beta}{\partial F_i} &>0 \qquad \text{for} \quad i=2, \\ &<0 \qquad \text{for} \quad i=3 \end{aligned} \tag{30}$$

Q.E.D.

As expected, cost reductions provide higher welfare to an economy through a decline of equilibrium prices. Their effect is symmetric in that cost reductions by any firm cause welfare to improve, while the advertising effect is asymmetric. Proposition 8 states that welfare increases with a decline of F_3, but it does not with that of F_2. Hence, we may conclude that it is socially favorable for firm 2 to undertake more advertising, while the opposite is true for firm 3's advertising activity. Noting that consumers with a high index choose brand 3 by virtue of (6), advertising of the so-called low quality brand leads to socially favorable situations, whereas that by a top quality brand is unfavorable. Note, however, that this difference comes from asymmetry of the advertising effect on a taste parameter.

6. CONCLUSION

This paper has provided a rational explanation of why consumers purchase just one of the available brands. Although the quasiconvex utility function has always been assumed away in traditional demand theory, it is essential to deal with demand for brands as was pointed out by Phlips [1964]. To procceed with our analysis, we specified a utility function with linear indifference curves between brands. Our assumptions about consumers' preferences enable us to demonstrate the existence of a Nash-Cournot equilibrium in prices in differentiated markets.

Our next step was to examine the effects of R & D and advertising activity of firms on the equilibrium market shares and welfare. Cost reductions of any firm, due to the firm's R & D activity, do not only enable the firm to increase its market shares, but also to provide larger welfare to the economy. The advertising effect is more complicated. Advertising by a firm is supposed to vary consumers' tastes in favor of advertised brands so that it may contribute to an increase in its market shares. However, the advertising effect is not symmetric in that advertising by a firm which produces a low quality brand causes welfare to improve, while the other firm which produces a top quality brand decreases welfare instead.

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