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OPTIMAL LABOUR CONTRACT UNDER ASYMMETRIC INFORMATION

Masao TAKESHIMA

Hart [7] made an excellent survey of recent developments concerning labour contract theory under asymmetric information. However he generally confined his attention to situations which resulted in underemployment. In this paper, I will use a more general framework in dealing with this problem than [7]. My intention is to show the type of conditions required to produce underemployment or overemployment. As a byproduct of these main theorems, I have also shown that there is a case where informational asymmetry does not impose any welfare losses.

1. INTRODUCTION

Throughout last year, recent developments concerning labour contract theory under asymmetric information appeared in the Quarterly Journal of Economics, supplement. These studies dealt with the following situation; at ex-ante, firms and workers have the same probabilistic beliefs about the state of the world, but at ex-post the firms are better informed than the workers about the state realizations. In this case, the term "asymmetric information" is used. In the course of the studies, Azariadis [1] and Grossman-Hart [5] reached the conclusion that involuntary underemployment occurs under asymmetric information, whereas Chari [3] and Green-Kahn [4] proposed the result that under the same informational conditions involuntary overemployment occurs.

Although Hart [7] made an excellent study of the subject he paid little attention as to why two different conclusions were offered and concentrated instead on the underemployment result. The purpose of this paper is 1) to generalize the underemployment result discussed by Azariadis [1], Grossman-Hart [5] and Hart [7], 2) to prove rigorously the overemployment result discussed by Chari [3], 3) to show that there is a case where neither underemployment nor overemployment occurs at the optimal contract under asymmetric information. All of these results are established in a uniform framework.

In 1) we should note that while, in the papers already referred to, the underemployment results is confined to situation where the worker's leisure demand is independent of his income we can extend this result to a situation where leisure is an inferior good. In 2) we should note that since Chari's proof of the overemployment result is slightly ambiguous, we shall prove his main result by using the technique developed by Hart [7]. 3) is a direct corollary of the main theorems of this paper. However it has never been proven in any previous papers. By showing these results, we can clarify the conditions required to produce either underemployment or overemployment.

The paper is organized as follows. In Section 2, the basic assumptions used in this paper are outlined. In Section 3, the optimal contract under symmetric information is defined; in Section 4, the optimal contract under asymmetric information is defined. We prove the underemployment result in Section 4(i) and the overemployment result in Section 4(ii). In Section 5, we shall add some important remarks.

2. BASIC ASSUMPTIONS

For simplicity, we shall consider a labour contract between one worker and one firm. The basic structure of the model is analogous to Hart [7].

The model is two period one, namely 0 and 1. Period 0 is an initial date and real economic activity does not occur until period 1. In period 1, there are n potentially realizable states reflecting demand shock, e.g. a low price for the firm's output, or a supply shock, e.g. technological innovation. These are represented by $s_1, \cdots, s_n$ with associated probabilities $p_1, \cdots, p_n$. ($p_i > 0$ for $i = 1, \cdots, n$, $\sum_{i=1}^{i=n} p_i = 1$)

These probabilities are assumed to be known to both firm and worker at period 0. The contract is negotiated at the end of period 0 and specifies an employment level $L(s_i)$ and a wage income $w(s_i)$ corresponding to each state s_i $(i = 1, \cdots, n)$ in period 1. Further, we assume that the contract will not be broken at period 1.

The worker's preference is represented by von Neumann-Morgenstern utility function $U(w, L)$. We assume:

Assumption 1. $U(w, L)$: $R \times R_+ \to R$ is bounded, twice differentiable in both arguments, strictly quasi-concave. The following derivative conditions are satisfied; $U_w > 0$, $U_{ww} \leqq 0$, $U_L < 0$, $U_{LL} < 0$.[1]

At period 0, the worker has an alternative source of income that yields a utility of $\bar{U}$. Therefore, the worker must be guaranteed a utility level of at least $\bar{U}$, if he is to be induced to sign a contract.

The firm's revenue in state s_i is represented by $f(s_i, L(s_i))$ $(i = 1, \cdots, n)$, where $L(s_i)$ is the labour input in state s_i. We assume:

[1] Hart [7] uses the utility function of the form $U(w, L) = U(w - RL)$, where $U' > 0$, $U'' \leqq 0$, and R is an opportunity cost of labour supply, to derive his underemployment result.

Assumption 2. $f(s_i, L)$ is a twice differentiable function defined for all $L \geqq 0$ with

$$f(s_i, 0) \geqq 0\,, \qquad \frac{\partial f(s_i, L)}{\partial L} > 0\,, \qquad \frac{\partial^2 f(s_i, L)}{\partial L^2} < 0\,, \qquad \lim_{L \to \infty} \frac{\partial f(s_i, L)}{\partial L} = 0\,.$$

The firm's profit in each state is $f(s_i, L(s_i)) - w(s_i)$ $(i = 1, \cdots, n)$. The preference of the firm is assumed to be represented by von Neumann-Morgenstern utility function $V(f(s_i, L(s_i)) - w(s_i))$. We assume:

Assumption 3. V is defined and twice differentiable on an open interval of the real line $P = (a, +\infty)$ and $\lim_{y \to a} V(y) = -\infty$. In addition, $V' > 0$, $V'' \leqq 0$ on P.[2]

The firm selects $w(s_i)$ and $L(s_i)$ so as to maximize expected utility $\sum_{i=1}^{i=n} p_i V(f(s_i, L(s_i)) - w(s_i))$ subject to several constraints.

Each state is defined as follows:

Assumption 4. For all $L \geqq 0$, the following holds:

$$\frac{\partial f(s_n, L)}{\partial L} > \frac{\partial f(s_{n-1}, L)}{\partial L} > \cdots > \frac{\partial f(s_1, L)}{\partial L}\,.$$

Assumption 5. For all $L \geqq 0$, the following holds:

$$f(s_n, L) \geqq f(s_{n-1}, L) \geqq \cdots \geqq f(s_1, L)\,.$$

(For $L > 0$, strict inequality holds.)

Thus, each state is ranked according to the marginal and average revenue productivity of labour. And we shall assume for s_1;

Assumption 6. For all w, the following holds:

$$\frac{\partial f(s_1, 0)}{\partial L} > -\frac{U_L(w, 0)}{U_w(w, 0)}\,.$$

At period 0, both firm and worker are assumed to know the probabilities p_i $(i = 1, \cdots, n)$ but do not know which s_i will occur at period 1. On the other hand, when s_i is realized at period 1, we can distinguish two cases; (1) both firm and worker know the realization of s_i (we call this symmetric information). (2) only the firm knows the realization of s_i (we call this asymmetric information.)

At period 0, both firm and worker are assumed to know $f(s_i, L)$ $i = 1, \cdots, n$, utility functions U, V and $\bar{U}$.

3. OPTIMAL CONTRACT UNDER SYMMETRIC INFORMATION

Under symmetric information, the optimal contract $(w(s_i), L(s_i))$ $i = 1, \cdots, n$

[2] In this case, "a" can be interpreted as the bankruptcy point of the firm. We allow for the possibility that $a = -\infty$.

solves the following problem:

$$\text{Max.} \quad \sum_{i=1}^{i=n} p_i V(f(s_i, L(s_i)) - w(s_i)) \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{i=n} p_i U(w(s_i), L(s_i)) \geqq \bar{U} \tag{2}$$

$$f(s_i, L(s_i)) - w(s_i) \in P \qquad i=1, \cdots, n \tag{3}$$

$$L(s_i) \geqq 0 \qquad i=1, \cdots, n \tag{4}$$

The first order conditions are as follows:

$$V'(f(s_i, L(s_i)) - w(s_i)) = \lambda U_w(w(s_i), L(s_i)) \qquad i=1, \cdots, n \tag{5}$$

$$\frac{\partial f(s_i, L(s_i))}{\partial L} = -\frac{U_L(w(s_i), L(s_i))}{U_w(w(s_i), L(s_i))} \qquad i=1, \cdots, n \tag{6}$$

At the solution, (2) holds equality. In (5) λ is the nonnegative Lagrangian multiplier corresponding to the constraint (2). Equation (6) represents that the equality holds between the marginal productivity of labour and the worker's marginal rate of substitution between labour and income in every state.

4. OPTIMAL CONTRACT UNDER ASYMMETRIC INFORMATION

Contrary to the above section, at period 1, only the firm can know the realization of s_i ($i=1, \cdots, n$). In this case, in general, the firm will have an incentive to lie, i.e. to announce s_j has occurred when in fact the true state is s_i. In fact, the firm will always lie if there is a positive gain from doing so. If, however, the following truth telling condition holds, the firm will not have such an incentive.

TRUTH TELLING CONDITION:
For all $i, j=1, \cdots, n$, the following holds:

$$f(s_i, L(s_i)) - w(s_i) \geqq f(s_i, L(s_j)) - w(s_j)$$

In what follows, we shall only concentrate on contracts satisfying this condition. Since any contract in which the firm has an incentive to lie is equivalent to another contract satisfying this condition, there is no loss of generality by this restriction. (See Azariadis [1] Lemma 1, or Harris-Townsend [6], Myerson [8].)

Then, the optimal contract under asymmetric information $(w(s_i), L(s_i))$ ($i=1, \cdots, n$) solves the following problem:

$$\text{Max.} \quad \sum_{i=1}^{i=n} p_i V(f(s_i, L(s_i)) - w(s_i)) \tag{7}$$

$$\text{s.t.} \quad f(s_i, L(s_i)) - w(s_i) \geqq f(s_i, L(s_j)) - w(s_j) \qquad i, j=1, \cdots, n \tag{8}$$

$$\sum_{i=1}^{i=n} p_i U(w(s_i), L(s_i)) \geqq \bar{U} \tag{9}$$

$$f(s_i, L(s_i)) - w(s_i) \in P \qquad i=1, \cdots, n \tag{10}$$

$$L(s_i) \geqq 0 \qquad i=1, \cdots, n \tag{11}$$

Equation (8) was not considered in Section 3. Under asymmetric information, the worker recognizes that the firm will have an incentive to lie.[3] Then, any contract that does not satisfy (8) is meaningless for the worker; the calculation of his expected utility $\sum_{i=1}^{i=n} p_i U(w(s_i), L(s_i))$ will become meaningless. Then, under asymmetric information, the worker will not sign a contract unless it satisfies (8) and (9) simultaneously.

Our main concern is the properties of the solution to (7)–(11).

(i) *Underemployment Result*

Since (8) contains so many equations, especially if n is large, it is difficult to solve (7)–(11) directly. We first replace (8) by the following, (12a) and (12b):

$$f(s_i, L(s_i)) - w(s_i) \geqq f(s_i, L(s_{i-1})) - w(s_{i-1}) \qquad i=2, \cdots, n \tag{12a}$$

$$L(s_i) \geqq L(s_{i-1}) \qquad i=2, \cdots, n \tag{12b}$$

In what follows, we will characterize the property of the solution to (7)–(11) by considering the solution to (7), (12), (9)–(11).

We can show that (12) follows from (8). That (12a) follows from (8) is trivial. Also, from (8), the following holds:

$$f(s_i, L(s_i)) - w(s_i) \geqq f(s_i, L(s_{i-1})) - w(s_{i-1})$$

$$f(s_{i-1}, L(s_{i-1})) - w(s_{i-1}) \geqq f(s_{i-1}, L(s_i)) - w(s_i)$$

Combining these two inequalities, we must have

$$f(s_i, L(s_i)) - f(s_i, L(s_{i-1})) \geqq f(s_{i-1}, L(s_i)) - f(s_{i-1}, L(s_{i-1})) .$$

From Assumption 4, we must have $L(s_i) \geqq L(s_{i-1})$. Hence, (12b) also follows from (8). In what follows, we prove some lemmas.

LEMMA 1. *Assumptions* 1–6 *hold, but we exclude the case where* $V''=0$ *and* $U_{ww}=0$. *Then, at the solution to* (7), (12), (9)–(11), *the following holds*:

$$\frac{\partial f(s_i, L(s_i))}{\partial L} \geqq -\frac{U_L(w(s_i), L(s_i))}{U_w(w(s_i), L(s_i))} \qquad i=1, \cdots, (n-1)$$

$$\frac{\partial f(s_n, L(s_n))}{\partial L} = -\frac{U_L(w(s_n), L(s_n))}{U_w(w(s_n), L(s_n))}$$

[3] We assumed that the worker knows the form of $f(s_i, L)$ $(i=1, \cdots, n)$. Hence, given $(w(s_i), L(s_i))$, the worker can calculate firm's profit in each state.

Proof. (In this proof, we consider the case where $V''<0$ and $U_{ww}=0$. It is straightforward to show that the same result holds under the case where $V''<0$ and $U_{ww}<0$, or $V''=0$ and $U_{ww}<0$.)

We distinguish two cases;

(I) the case where $L(s_i)>L(s_{i-1})$; Suppose

$$\frac{\partial f(s_i, L(s_i))}{\partial L}<-\frac{U_L(i)}{U_w(i)}\ ^4$$

holds at the solution to (7), (12), (9)–(11). Reduce $w(s_i)$, $L(s_i)$ a little to make $f(s_i, L(s_i))-w(s_i)$ constant. Then,

$$\frac{dw(s_i)}{dL(s_i)}=\frac{\partial f(s_i, L(s_i))}{\partial L}.$$

By Assumption 4, this does not violate (12). (By doing this, $f(s_{i+1}, L(s_i))-w(s_i)$ will be reduced.) On the other hand, the change in the worker's utility is

$$\Delta=U_w(i)dw(s_i)+U_L(i)dL(s_i).$$

Taking into account the above relations,

$$\Delta=U_w(i)dL(s_i)\left\{\frac{\partial f(s_i, L(s_i))}{\partial L}+\frac{U_L(i)}{U_w(i)}\right\}>0.$$

Hence, the worker will be better off. This Pareto improvement contradicts optimality. So, in this case, at the solution to (7), (12), (9)–(11),

$$\frac{\partial f(s_i, L(s_i))}{\partial L}\geqq-\frac{U_L(i)}{U_w(i)}.$$

(II) the case where $L(s_i)=L(s_{i-1})>L(s_{i-2})$; For the state s_{i-1}, in the same way as in (I), we can obtain

$$\frac{\partial f(s_{i-1}, L(s_{i-1}))}{\partial L}\geqq-\frac{U_L(i-1)}{U_w(i-1)}.$$

Now suppose at the solution to (7), (12), (9)–(11) the following holds:

$$f(s_i, L(s_i))-w(s_i)>f(s_i, L(s_{i-1}))-w(s_{i-1}) \tag{13}$$

By $L(s_i)=L(s_{i-1})$, $w(s_i)<w(s_{i-1})$ follows, and by Assumption 5, the following must hold:

$$f(s_i, L(s_i))-w(s_i)>f(s_{i-1}, L(s_{i-1}))-w(s_{i-1}).$$

Hence, the firm's profit in state s_i is higher than that in state s_{i-1}. We then increase $w(s_i)$ a little and reduce $w(s_{i-1})$ a little so as to ensure that the firm's expected utility at period 0 remains constant. By (13) this does not violate (12). By assumption, the following holds:

[4] In the sequel, we will use $U_L(i)$, $U_w(i)$ etc. in place of $U_L(w(s_i), L(s_i))$, $U_w(w(s_i), L(s_i))$ etc.

$$\frac{dw(s_{i-1})}{dw(s_i)} = -\frac{p_i V'(f(s_i, L(s_i)) - w(s_i))}{p_{i-1} V'(f(s_{i-1}, L(s_{i-1})) - w(s_{i-1}))} > -\frac{p_i}{p_{i-1}} \tag{14}$$

The change in worker's utility is

$$\Delta = p_{i-1} U_w(i-1) dw(s_{i-1}) + p_i U_w(i) dw(s_i)$$

If we take the above relations and $U_{ww}=0$ into account:

$$\Delta = p_{i-1} U_w(i-1) dw(s_i) \left\{ \frac{dw(s_{i-1})}{dw(s_i)} + \frac{p_i U_w(i)}{p_{i-1} U_w(i-1)} \right\}$$

$$= p_{i-1} U_w(i-1) dw(s_i) \left\{ \frac{dw(s_{i-1})}{dw(s_i)} + \frac{p_i}{p_{i-1}} \right\} > 0 .$$

Hence, the worker will be better off. This contradicts optimality. Hence at the solution to (7), (12), (9)–(11), if $L(s_i) = L(s_{i-1})$, (13) does not hold. By (12a), if $L(s_i) = L(s_{i-1})$ we must have

$$f(s_i, L(s_i)) - w(s_i) = f(s_i, L(s_{i-1})) - w(s_{i-1}) \qquad \text{i.e.} \quad w(s_i) = w(s_{i-1}) .$$

After all, we can obtain the following chain of inequalities:

$$\frac{\partial f(s_i, L(s_i))}{\partial L} > \frac{\partial f(s_{i-1}, L(s_i))}{\partial L} = \frac{\partial f(s_{i-1}, L(s_{i-1}))}{\partial L}$$

$$\geqq -\frac{U_L(i-1)}{U_w(i-1)} = -\frac{U_L(i)}{U_w(i)} .$$

Hence, we have

$$\frac{\partial f(s_i, L(s_i))}{\partial L} > -\frac{U_L(i)}{U_w(i)} .$$

If $L(s_i) = L(s_{i-1}) = L(s_{i-2}) > L(s_{i-3})$, we can go the same way. Through (I) and (II), we have shown that at the solution to (7), (12), (9)–(11)

$$\frac{\partial f(s_i, L(s_i))}{\partial L} \geqq -\frac{U_L(i)}{U_w(i)} \qquad (i=1, \cdots, n)$$

hold. Suppose

$$\frac{\partial f(s_n, L(s_n))}{\partial L} > -\frac{U_L(n)}{U_w(n)} .$$

Then, increase $w(s_n)$, $L(s_n)$ a little to make $f(s_n, L(s_n)) - w(s_n)$ constant. This does not violate (12) but in the same way as in (I), we can show that the worker will be better off. Hence, at the state s_n,

$$\frac{\partial f(s_n, L(s_n))}{\partial L} = -\frac{U_L(n)}{U_w(n)} .$$

Q.E.D.

LEMMA 2. *Assumption 1–6 hold but the case where $V''=0$ and $U_{ww}=0$ is excluded. If $f(s_i, L(s_i))-w(s_i)>f(s_i, L(s_{i-1}))-w(s_{i-1})$ holds in the solution to* (7), (12), (9)–(11), *then*

$$\text{(I)} \qquad \frac{\partial f(s_{i-1}, L(s_{i-1}))}{\partial L}=-\frac{U_L(i-1)}{U_w(i-1)}.$$

In addition, if leisure is not a normal good, i.e.

$$U_{ww}\cdot\left(-\frac{U_L}{U_w}\right)+U_{wL}\geqq 0,$$

then, at the solution to (7), (12), (9)–(11),

$$\text{(II)} \qquad f(s_{i-1}, L(s_{i-1}))-w(s_{i-1}))\geqq f(s_{i-1}, L(s_i))-w(s_i).$$

Proof. (In this proof, we consider the case where $V''<0$ and $U_{ww}<0$. It is straightforward to show that the same result holds under the case where $V''=0$ and $U_{ww}<0$ or $V''<0$ and $U_{ww}=0$.)

(I) We have already obtained

$$\frac{\partial f(s_{i-1}, L(s_{i-1}))}{\partial L}\geqq-\frac{U_L(i-1)}{U_w(i-1)}$$

holds at the solution from Lemma 1. Suppose that strict inequality holds. We have shown in Lemma 1 (II) that in the solution to (7), (12), (9)–(11), $L(s_i)=L(s_{i-1}) \Rightarrow f(s_i, L(s_i))-w(s_i)=f(s_i, L(s_{i-1}))-w(s_{i-1})$. We have now, $f(s_i, L(s_i))-w(s_i)>f(s_i, L(s_{i-1}))-w(s_{i-1})$. Hence at the optimum, $L(s_i)>L(s_{i-1})$ must hold. So, without violating (12), we can increase $w(s_{i-1})$, $L(s_{i-1})$ a little to make $f(s_{i-1}, L(s_{i-1}))-w(s_{i-1})$ constant.

On the other hand, the change of worker's utility is

$$\Delta=U_w(i-1)dL(s_{i-1})\left\{\frac{\partial f(s_{i-1}, L(s_{i-1}))}{\partial L}+\frac{U_L(i-1)}{U_w(i-1)}\right\}>0.$$

This contradicts optimality. Hence for state s_{i-1},

$$\frac{\partial f(s_{i-1}, L(s_{i-1}))}{\partial L}=-\frac{U_L(i-1)}{U_w(i-1)}$$

must hold.

(II) By the assumption of this lemma, without violating (12a), we can increase $w(s_i)$ and reduce $w(s_{i-1})$ a little to make the firm's expected utility constant. Since the firm's profit in state s_i is higher than that in state s_{i-1}, the following holds;

$$\frac{dw(s_{i-1})}{dw(s_i)}=-\frac{p_iV'(f(s_i, L(s_i))-w(s_i))}{p_{i-1}V'(f(s_{i-1}, L(s_{i-1}))-w(s_{i-1}))}>-\frac{p_i}{p_{i-1}}$$

On the other hand, the change in worker's utility is

$$\Delta = p_{i-1}U_w(i-1)dw(s_{i-1}) + p_iU_w(i)dw(s_i)$$
$$= p_{i-1}U_w(i-1)dw(s_i)\left\{\frac{dw(s_{i-1})}{dw(s_i)} + \frac{p_iU_w(i)}{p_{i-1}U_w(i-1)}\right\}$$

Later on we shall show that if the second part of the lemma does not hold, $\Delta > 0$ follows; this contradicts optimality. To show $\Delta > 0$, it is sufficient to prove $U_w(i) > U_w(i-1)$.

Then, suppose the second part of the lemma does not hold:

$$w(s_i) < f(s_{i-1}, L(s_{i-1})) - f(s_{i-1}, L(s_i)) + w(s_{i-1}) . \tag{15}$$

Define the right hand side of (15) as w'. The situation is illustrated in Fig. 1.

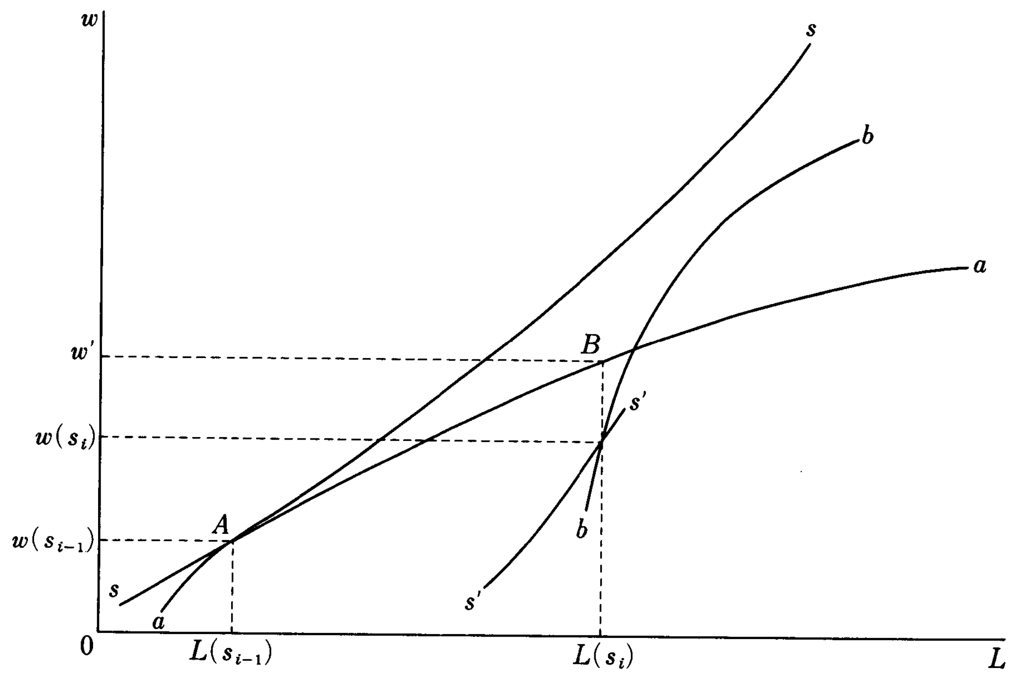

Fig. 1.

The curve ss and $s's'$ are the worker's indifference curves and aa and bb are the firm's iso-profit curves corresponding to state s_{i-1} and s_i respectively. According to (I), ss and aa are tangent at point A $(L(s_{i-1}), w(s_{i-1}))$. In what follows we will represent any point on ss as $(\hat{L}, \hat{w})$. The slope of ss is strictly increasing by Assumption 1 and the slope of aa is strictly decreasing by Assumption 2. Then, for all $\hat{L} \geqq L(s_{i-1})$ and $L \geqq L(s_{i-1})$ the following holds:

$$\frac{\partial f(s_{i-1}, L)}{\partial L} \leqq -\frac{U_L(\hat{w}, \hat{L})}{U_w(\hat{w}, \hat{L})} \tag{16}$$

(equality holds if and only if $L = \hat{L} = L(s_{i-1})$)

On the other hand, under the assumption that leisure is not a normal good, the following must hold:

$$\frac{\partial\left(-\dfrac{U_L}{U_w}\right)}{\partial w}=-\frac{1}{U_w}\left\{U_{ww}\cdot\left(-\frac{U_L}{U_w}\right)+U_{wL}\right\}\leqq 0 \tag{17}$$

Then choose any point and move downward. By (17), the slope of the indifference curve at the new point is at least as steep as that at the old point. Hence, if we consider $(\hat{L}, \hat{w})$ as a reference point, for all $w\leqq\hat{w}$, the following holds:

$$-\frac{U_L(w,\hat{L})}{U_w(w,\hat{L})}\geqq-\frac{U_L(\hat{w},\hat{L})}{U_w(\hat{w},\hat{L})} \tag{18}$$

Referring to these points, we then consider how the worker's marginal utility of income changes as we move from point A to point B on the iso-profit curve aa. (point B corresponds to $(L(s_i), w')$). Since for $L\geqq L(s_{i-1})$ aa lies below ss, the slope of the indifference curve on aa is at least as steep as that on ss for the same L, $L\geqq L(s_{i-1})$. As we move on aa, we have

$$\frac{dw}{dL}=\frac{\partial f(s_{i-1}, L)}{\partial L}.$$

Hence, by (16)–(18), for all $L\geqq L(s_{i-1})$, the following holds;

$$\begin{aligned}\left.\frac{dU_w(w,L)}{dL}\right|_{\text{s.t. } f(s_{i-1},L)-w=f(s_{i-1},L(s_{i-1}))-w(s_{i-1})} &=U_{ww}\frac{dw}{dL}+U_{wL}\\ &=U_{ww}\frac{\partial f(s_{i-1},L)}{\partial L}+U_{wL}\\ &\geqq U_{ww}\cdot\left(-\frac{U_L}{U_w}\right)+U_{wL}\geqq 0\end{aligned} \tag{19}$$

(in the former inequality, the equality holds if and only if $L=L(s_{i-1})$.)

From (19), we can see that the worker's marginal utility of income increases as we move from A to B. Hence,

$$U_w(w', L(s_i))>U_w(w(s_{i-1}), L(s_{i-1}))\,.$$

But from (15), $w'>w(s_i)$, then by $U_{ww}<0$,

$$U_w(w(s_i), L(s_i))>U_w(w', L(s_i))$$

Hence,

$$U_w(i)>U_w(i-1)$$

Then, the desired contradiction leads from (15). Q.E.D.

LEMMA 3. *Under the same assumptions as in Lemma* 2, (8) *holds at the solution to* (7), (12), (9)–(11).

Proof. If (12a) holds with equality, by Assumption 4 and (12b),

$$\begin{aligned} &f(s_{i-1}, L(s_{i-1})) - w(s_{i-1}) - (f(s_{i-1}, L(s_i)) - w(s_i)) \\ &\quad = f(s_{i-1}, L(s_{i-1})) - f(s_{i-1}, L(s_i)) + w(s_i) - w(s_{i-1}) \\ &\quad \geqq f(s_i, L(s_{i-1})) - f(s_i, L(s_i)) + w(s_i) - w(s_{i-1}) \\ &\quad = 0 \end{aligned} \tag{20}$$

And we have already shown in Lemma 2 (II) that if (12a) holds with strict inequality at the solution, the left hand side of (20) is nonnegative. Hence, in all cases, the left hand side of (20) is nonnegative at the solution. Then, to prove that (8) holds at the solution we can apply the same argument as in Hart [7].

Q.E.D.

By Lemma 3, it becomes clear that the solution to (7), (12) (9)–(11) is also the solution to (7)–(11) (Because we have shown that (12) is weaker than (8).). By summarizing the above results, we obtain the following theorem.

THEOREM 1. *Assumptions* 1–6 *hold but the case where* $V''=0$ *and* $U_{ww}=0$ *hold simultaneously is excluded. If leisure is not a normal good, at the solutions to* (7)–(11), *the following holds*:

(I)
$$\frac{\partial f(s_i, L(s_i))}{\partial L} \geqq -\frac{U_L(w(s_i), L(s_i))}{U_w(w(s_i), L(s_i))} \qquad i=1, \cdots, (n-1)$$

$$\frac{\partial f(s_n, L(s_n))}{\partial L} = -\frac{U_L(w(s_n), L(s_n))}{U_w(w(s_n), L(s_n))}$$

(II)
$$\sum_{i=1}^{i=n} p_i U(w(s_i), L(s_i)) = \bar{U}$$

And if $V''=0$ *and* $U_{ww}<0$, *the inequality in* (I) *holds strictly for all* $i<n$.

Proof. (I) follows from Lemma 1–3. (II) is obvious. For the last part of the theorem, we shall use the first order condition to the problem (7), (12), (9)–(11);

$$-p_i V'(f(s_i, L(s_i)) - w(s_i)) - \lambda_i + \lambda_{i+1} + vp_i U_w(i) = 0 \tag{21}$$

$$\begin{aligned} &p_i V'(f(s_i, L(s_i)) - w(s_i)) \frac{\partial f(s_i, L(s_i))}{\partial L} + \lambda_i \frac{\partial f(s_i, L(s_i))}{\partial L} \\ &\quad - \lambda_{i+1} \frac{\partial f(s_{i+1}, L(s_i))}{\partial L} + m_i - m_{i+1} + vp_i U_L(i) \leqq 0 \end{aligned} \tag{22}$$

for all i where λ_i, m_i, v are nonnegative Lagrange multipliers corresponding to the constraint (12a), (12b) and (9) respectively and (22) holds equality when $L(s_i)>0$.

Suppose

$$\frac{\partial f(s_i, L(s_i))}{\partial L} = -\frac{U_L(i)}{U_w(i)}$$

holds for some i. Then, from (21) and (22), we obtain

$$m_i - m_{i+1} = \lambda_{i+1}\left\{\frac{\partial f(s_{i+1}, L(s_i))}{\partial L} - \frac{\partial f(s_i, L(s_i))}{\partial L}\right\} \quad \text{(By Assumption 6, } L(s_i)>0)$$

Then, $m_i \geqq m_{i+1}$. Suppose $m_i > 0$. Then $L(s_i) = L(s_{i-1})$. As we have shown in Lemma 1 (II), $w(s_i) = w(s_{i-1})$ at the solution. Then,

$$\frac{\partial f(s_{i-1}, L(s_{i-1}))}{\partial L} < \frac{\partial f(s_i, L(s_{i-1}))}{\partial L} = \frac{\partial f(s_i, L(s_i))}{\partial L} = -\frac{U_L(i)}{U_w(i)} = -\frac{U_L(i-1)}{U_w(i-1)}.$$

This is inconsistent with the result of Lemma 1. So, $m_i = m_{i+1} = 0$. Hence, $\lambda_{i+1} = 0$. Applying (21) at i and $i+1$, we obtain the following;

$$V'(f(s_i, L(s_i)) - w(s_i)) = vU_w(i) - \frac{\lambda_i}{p_i}$$

$$V'(f(s_{i+1}, L(s_{i+1})) - w(s_{i+1})) = vU_w(i+1) + \frac{\lambda_{i+2}}{p_{i+1}}$$

If $U_{ww} = 0$ and leisure is not a normal good, $U_{wL} \geqq 0$. Hence, by (12b), $U_w(i+1) \geqq U_w(i)$ must follow. On the other hand, we have $L(s_i) > 0$.

By (12a) and Assumption 5

$$f(s_{i+1}, L(s_{i+1})) - w(s_{i+1}) \geqq f(s_{i+1}, L(s_i)) - w(s_i)$$
$$> f(s_i, L(s_i)) - w(s_i)\,.$$

Combining this inequality with $V'' < 0$, the desired contradiction follows.

Q.E.D.

It is clear why the underemployment result holds in the contract defined by Theorem 1. If

$$\frac{\partial f(s_i, L(s_i))}{\partial L} > -\frac{U_L(i)}{U_w(i)}$$

holds for some s_i, the worker can be made better off by increasing $L(s_i)$, $w(s_i)$ a little so that the firm's profit in s_i remains constant.

In Fig. 2, this can be represented as a movement from point A to point B. (ss and $s's'$ represent the worker's indifference curves and aa represents the firm's iso-profit curve in s_i) We should say that the worker is underemployed at A compared to B in the sense that the allocation at A is dominated by the allocation at B. Since a point like A is obtained as a solution, condition (8) would be violated if we were to move from A to B.

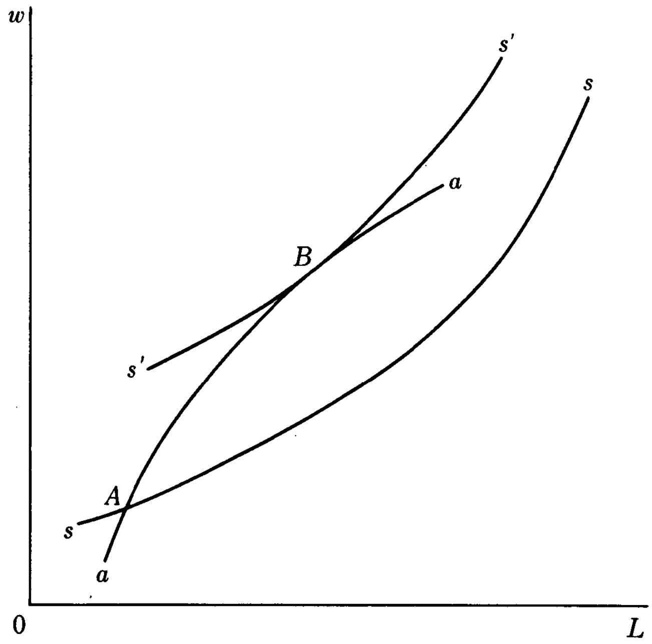

Fig. 2.

(ii) *Overemployment-result*

As in (i), we replace (8) by the following (23).

$$f(s_i, L(s_i)) - w(s_i) \geqq f(s_i, L(s_{i+1})) - w(s_{i+1}) \qquad i=1, \cdots, (n-1) \tag{23a}$$

$$L(s_{i+1}) \geqq L(s_i) \qquad i=1, \cdots, (n-1) \tag{23b}$$

We will consider the properties of the solution to (7), (23), (9)–(11) to obtain the solution to (7)–(11). We can show that (23) is weaker than (8) in the same way as we have shown (12) is weaker than (8) in (i). In what follows we prove some lemmas.

LEMMA 4. *Assume* $V''=0$ *and* $U_{ww}<0$ *and Assumptions* 1–6 *hold. Then, at the solution to* (7), (23), (9)–(11), *the following hold*:

$$\frac{\partial f(s_1, L(s_1))}{\partial L} = -\frac{U_L(w(s_1), L(s_1))}{U_w(w(s_1), L(s_1))}$$

$$\frac{\partial f(s_i, L(s_i))}{\partial L} \leqq -\frac{U_L(w(s_i), L(s_i))}{U_w(w(s_i), L(s_i))} \qquad i=2, \cdots, n$$

Proof. We can use the same technique as in Lemma 1, so we outline the proof.

(I) the case where $L(s_{i+1}) > L(s_i)$; Suppose

$$\frac{\partial f(s_i, L(s_i))}{\partial L} > -\frac{U_L(i)}{U_w(i)}.$$

Then, without violating (23), the worker will be made better off by increasing $w(s_i)$,

$L(s_i)$ a little so that $f(s_i, L(s_i))-w(s_i)$ remains constant. This contradicts optimality. Hence, at the optimum,

$$\frac{\partial f(s_i, L(s_i))}{\partial L} \leqq -\frac{U_L(i)}{U_w(i)}$$

must hold.

(II) the case where $L(s_{i+2})>L(s_{i+1})=L(s_i)$; In the same way as in Lemma 1, we can show if $L(s_i)=L(s_{i+1})$ at the solution, then, $f(s_i, L(s_i))-w(s_i)=f(s_i, L(s_{i+1}))-w(s_{i+1})$ i.e. $w(s_i)=w(s_{i+1})$ must hold. By (I), for state s_{i+1}, we can obtain

$$\frac{\partial f(s_{i+1}, L(s_{i+1}))}{\partial L} \leqq -\frac{U_L(i+1)}{U_w(i+1)}.$$

Then, we obtain the following chain of inequalities;

$$\frac{\partial f(s_i, L(s_i))}{\partial L} < \frac{\partial f(s_{i+1}, L(s_i))}{\partial L} = \frac{\partial f(s_{i+1}, L(s_{i+1}))}{\partial L} \leqq -\frac{U_L(i+1)}{U_w(i+1)} = -\frac{U_L(i)}{U_w(i)}.$$

Hence,

$$\frac{\partial f(s_i, L(s_i))}{\partial L} < -\frac{U_L(i)}{U_w(i)}.$$

This argument can be applied to the case where $L(s_{i+3})>L(s_{i+2})=L(s_{i+1})=L(s_i)$.

By (I) and (II), we showed that at the solution,

$$\frac{\partial f(s_i, L(s_i))}{\partial L} \leqq -\frac{U_L(i)}{U_w(i)} \qquad (i=1, \cdots, n)$$

holds. Suppose

$$\frac{\partial f(s_1, L(s_1))}{\partial L} < -\frac{U_L(1)}{U_w(1)}.$$

In this case, $L(s_1)>0$ by Assumption 6.

Then, without violating (23), the worker will be made better off by reducing $L(s_1)$, $w(s_1)$ a little so that $f(s_1, L(s_1))-w(s_1)$ remains constant. This contradicts optimality. Hence, at the solution,

$$\frac{\partial f(s_1, L(s_1))}{\partial L} = -\frac{U_L(1)}{U_w(1)}. \qquad \text{Q.E.D.}$$

LEMMA 5. *Assume* $V''=0$ *and* $U_{ww}<0$ *and Assumption* 1–6 *hold. If* $f(s_i, L(s_i))-w(s_i)> f(s_i, L(s_{i+1}))-w(s_{i+1})$ *at the solution to* (7), (23), (9)–(11),

(I) $$\frac{\partial f(s_{i+1}, L(s_{i+1}))}{\partial L} = -\frac{U_L(i+1)}{U_w(i+1)}.$$

In addition, if leisure is not an inferior good (i.e. $U_{ww}(-U_L/U_w)+U_{wL} \leqq 0$)

(II) $$f(s_{i+1}, L(s_{i+1})) - w(s_{i+1}) \geqq f(s_{i+1}, L(s_i)) - w(s_i)$$

must hold at the solution.

Proof. We can use the same technique as in Lemma 2, so we outline the proof.

(I) From Lemma 4

$$\frac{\partial f(s_{i+1}, L(s_{i+1}))}{\partial L} \leqq -\frac{U_L(i+1)}{U_w(i+1)}$$

hold at the solution. Suppose strict inequality holds. Then, the worker becomes better off by reducing $L(s_{i+1})$, $w(s_{i+1})$ a little so that $f(s_{i+1}, L(s_{i+1})) - w(s_{i+1})$ remains constant. As in Lemma 2, we can show this does not violate (23) by the assumption of this lemma. Hence, the desired contradiction holds.

(II) If $U_w(i) > U_w(i+1)$, the worker will be made better off by reducing $w(s_{i+1})$ a little and increasing $w(s_i)$ a little so that the firm's expected profit remains constant. (This does not violate (23) by the assumption of this Lemma) As in Lemma 2, we can show that if the second part of this lemma does not hold, $U_w(i) > U_w(i+1)$ follows under the assumptions that leisure is not an inferior good, firm is risk neutral and that $U_{ww} < 0$. Q.E.D.

LEMMA 6. *The same assumptions of Lemma 5 holds. Then,* (8) *holds at the solution to* (7), (23), (9)–(11).

Proof. We can apply the same argument as in Lemma 3. Q.E.D.

By Lemma 6, we conclude that the solution to (7), (23), (9)–(11) is also the solution to (7)–(11). By summarizing the above results, we obtain the following theorem.

THEOREM 2. *Assume $V''=0$ and $U_{ww}<0$ and Assumption* 1–6 *hold. In addition, if leisure is not an inferior good, at the solution to* (7)–(11) *the following holds*:

(I) $$\frac{\partial f(s_1, L(s_1))}{\partial L} = -\frac{U_L(w(s_1), L(s_1))}{U_w(w(s_1), L(s_1))}$$

$$\frac{\partial f(s_i, L(s_i))}{\partial L} \leqq -\frac{U_L(w(s_i), L(s_i))}{U_w(w(s_i), L(s_i))} \qquad i=2, \cdots, n$$

(II) $$\sum_{i=1}^{i=n} p_i U(w(s_i), L(s_i)) = \bar{U}$$

And if $U_{wL} \leqq 0$, equality in (I) *does not hold in continuous states through* 1 *to n.*

Proof. (I) follows from Lemma 4–6. (II) is obvious. For the last part of the theorem we will use the first order condition to the problem (7), (23), (9)–(11);

$$-p_i - \lambda_i + \lambda_{i-1} + v p_i U_w(i) = 0 \tag{24}$$

$$p_i \frac{\partial f(s_i, L(s_i))}{\partial L} + \lambda_i \frac{\partial f(s_i, L(s_i))}{\partial L} - \lambda_{i-1} \frac{\partial f(s_{i-1}, L(s_i))}{\partial L}$$

$$-m_i + m_{i-1} + vp_i U_L(i) \leqq 0 \tag{25}$$

for all i where λ_i, m_i, and v are nonnegative Lagrange multipliers corresponding to the constraint (23a), (23b) and (9) respectively and (25) holds with equality when $L(s_i) > 0$. Suppose

$$\frac{\partial f(s_i, L(s_i))}{\partial L} = -\frac{U_L(i)}{U_w(i)}$$

holds for some i. Then, from (24) and (25), we obtain

$$m_i - m_{i-1} = \lambda_{i-1} \left\{ \frac{\partial f(s_i, L(s_i))}{\partial L} - \frac{\partial f(s_{i-1}, L(s_i))}{\partial L} \right\} \quad \text{(By Assumption 6, } L(s_i) > 0)$$

In the same way as in Theorem 1, we can show $m_i = m_{i-1} = \lambda_{i-1} = 0$. Then, applying (24) at i and $i-1$, we obtain the following;

$$vU_w(i) = 1 + \frac{\lambda_i}{p_i} \tag{26}$$

$$vU_w(i-1) = 1 - \frac{\lambda_{i-2}}{p_{i-1}} \tag{27}$$

$v > 0$ follows from (25), for we have now $m_i = m_{i-1} = \lambda_{i-1} = 0$. Suppose

$$\frac{\partial f(s_{i-1}, L(s_{i-1}))}{\partial L} = -\frac{U_L(i-1)}{U_w(i-1)}$$

also holds at the solution. Then we obtain $L(s_i) > L(s_{i-1})$. For if $L(s_i) = L(s_{i-1})$, we can show that $w(s_i) = w(s_{i-1})$ must hold at the solution, then the contradiction to the above relation follows. If $L(s_i) > L(s_{i-1})$, $w(s_i) > w(s_{i-1})$ follows from (23a). Then, if $U_{wL} \leqq 0$, we have $U_w(i) < U_w(i-1)$. Hence, it contradicts (26) and (27). Then, we have shown that

$$\frac{\partial f(s_i, L(s_i))}{\partial L} = -\frac{U_L(i)}{U_w(i)} \Longrightarrow \frac{\partial f(s_{i-1}, L(s_{i-1}))}{\partial L} < -\frac{U_L(i-1)}{U_w(i-1)}$$

for all $i > 2$. Q.E.D.

It is also obvious why the overemployment result holds in the contract defined by Theorem 2. If

$$\frac{\partial f(s_i, L(s_i))}{\partial L} < -\frac{U_L(i)}{U_w(i)}$$

holds for some s_i, the worker can be made better off by decreasing $L(s_i)$, $w(s_i)$ a little so that the firm's profit in s_i remains constant.

In Fig. 3, this can be represented as a movement from point A to point C. (the notations are the same as in Fig. 2) We should say that the worker is overemployed at A compared to C in the sense that the allocation at A is dominated by the allocation at C. Since point A is obtained as a solution condition (8) would be violated if we were to move from A to C.

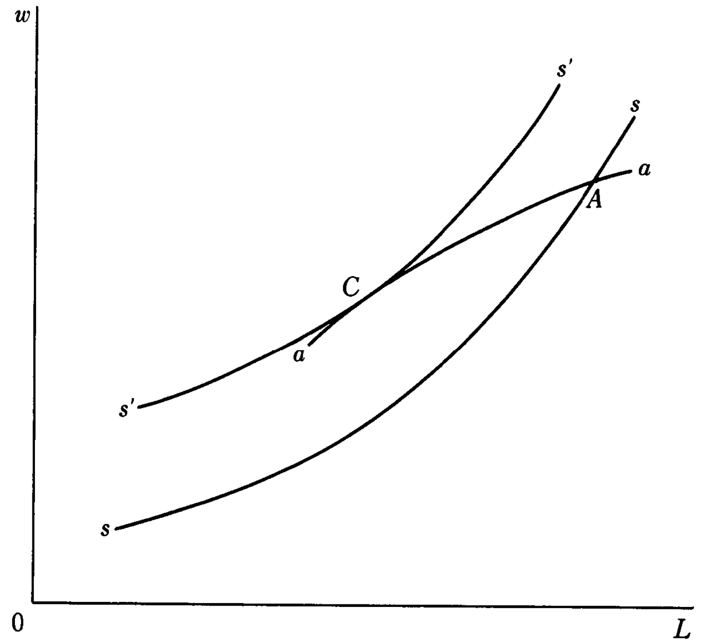

Fig. 3.

5. FURTHER REMARKS

(1) The case where neither underemployment nor overemployment occurs under asymmetric information:

As a result of Theorem 1 and Theorem 2, we obtain the following corollary:

COROLLARY 1. *Assumptions* 1–6 *hold. If* $V''=0$, $U_{ww}<0$ *and the leisure demand is independent of income* (*i.e.* $U_{ww}(-U_L/U_w)+U_{wL}=0$), *then at the solution to* (7)–(11), *the following equality satisfied*:

$$\frac{\partial f(s_i, L(s_i))}{\partial L} = -\frac{U_L(w(s_i), L(s_i))}{U_w(w(s_i), L(s_i))} \qquad i=1,\cdots,n.$$

Then we consider the example satisfying the assumption of this corollary. In the example, we can show that the solution to (1)–(4) satisfies (8).

Example 1. Let $V''=0$ and $U(w, L)=U(w-h(L))$ where $U'>0$, $U''<0$ and $h'>0$, $h''>0$. For simplicity, we consider a two state case. Problem (1)–(4) can be expressed as follows;

$$\text{Max.} \quad \sum_{i=1}^{i=2} p_i(f(s_i, L(s_i)) - w(s_i)) \tag{1$'$}$$

$$\text{s.t.} \quad \sum_{i=1}^{i=2} p_i U(w(s_i) - h(L(s_i))) \geqq \bar{U} \tag{2$'$}$$

$$f(s_i, L(s_i)) - w(s_i) \in P \qquad i=1,2 \tag{3$'$}$$

$$L(s_i) \geqq 0 \qquad i=1,2 \tag{4$'$}$$

Then, the first order conditions corresponding to (5) and (6) are as follows;

$$\frac{\partial f(s_i, L(s_i))}{\partial L} = h'(L(s_i)) \qquad i=1,2 \tag{5$'$}$$

$$1 = \lambda U'(w(s_i) - h(L(s_i))) \qquad i=1,2 \tag{6$'$}$$

By (5)$'$, we have $L(s_1) < L(s_2)$, and by (6)$'$, the following holds; $w(s_1) - h(L(s_1)) = w(s_2) - h(L(s_2))$. Then, we can show that $(w(s_i), L(s_i))$ $(i=1,2)$ which satisfies (5)$'$ and (6)$'$ also satisfies (8);

$$\begin{aligned} & f(s_1, L(s_1)) - w(s_1) - (f(s_1, L(s_2)) - w(s_2)) \\ & \quad = f(s_1, L(s_1)) - w(s_1) - f(s_1, L(s_2)) + w(s_1) - h(L(s_1)) + h(L(s_2)) \\ & \quad = f(s_1, L(s_1)) - h(L(s_1)) - (f(s_1, L(s_2)) - h(L(s_2))) > 0 \end{aligned}$$

(From (5)$'$, $L(s_1)$ is a unique maximizer of $f(s_1, L(s_1)) - h(L(s_1))$ and we have $L(s_1) < L(s_2)$.) In the same way, we can show $f(s_2, L(s_2)) - w(s_2) > f(s_2, L(s_1)) - w(s_1)$. so, in this case, (5)$'$ (and (6)$'$) hold at the solution under asymmetric information.

(2) Comparison of employment levels corresponding to the solution (1)–(4) and the solution to (7)–(11):

In Section 4, we showed that informational asymmetry causes inefficient underemployment or overemployment under some assumptions. But we did not discuss the comparison of employment levels corresponding to the solution to (1)–(4) and the solution to (7)–(11). In general, we cannot say much about this. The problem concerns the relationship between $w(s_i)$ and $L(s_i)$ satisfying (6). In what follows we shall define the solution to (1)–(4) as $(w_1(s_i), L_1(s_i))$ and the solution to (7)–(11) as $(w_2(s_i), L_2(s_i))$. We can classify three cases as Fig. 4 shows. In each case, *aa*, *bb* and *cc* represent the locus of $w(s_i)$ and $L(s_i)$ which satisfies (6) for some state s_i. In other words, the locus of the points at which the firm's iso profit curves and worker's indifference curves are tangent. After lengthy calculation, we can say: Case (A) follows if leisure is an inferior good. Case (B) follows if and only if the leisure demand is independent of income. Case (C) follows if

$$U_{wL} \leqq U_{LL} \frac{U_w}{U_L}$$

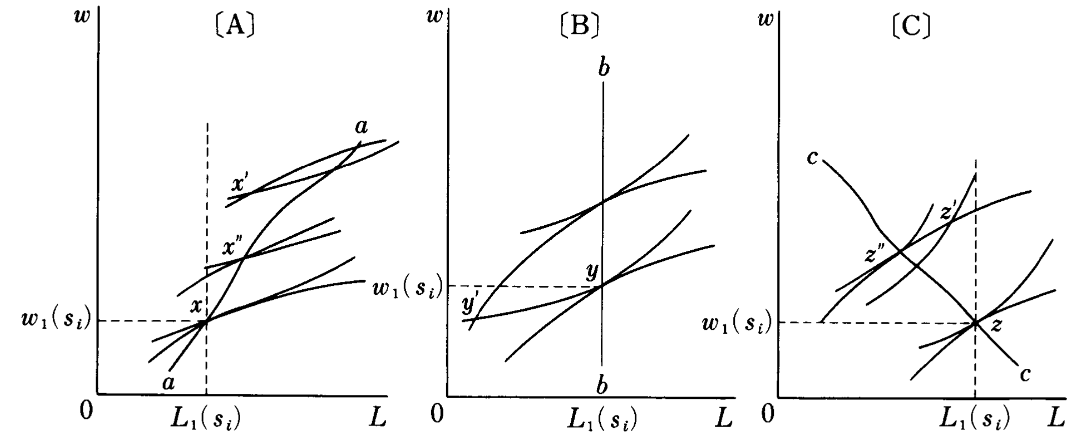

Fig. 4.

The solution we obtained at Theorem 1 concerns case (A) and case (B). (strictly speaking, the range which excludes the right side of *aa* and *bb*) In case (A), even if

$$\frac{\partial f(s_i, L_2(s_i))}{\partial L} > -\frac{U_L(i(2))}{U_w(i(2))}$$ [5]

we cannot say $L_2(s_i) < L_1(s_i)$. Instead we can say: If

$$\frac{\partial f(s_i, L_2(s_i))}{\partial L} > -\frac{U_L(i(2))}{U_w(i(2))}$$

and $L_2(s_i) \geqq L_1(s_i)$ hold, $U(w_1(s_i), L_1(s_i)) < U(w_2(s_i), L_2(s_i))$ follows: this is represented by the comparison of utility levels corresponding to point x and x' in Fig. 4(A). And if

$$\frac{\partial f(s_i, L_2(s_i))}{\partial L} = -\frac{U_L(i(2))}{U_w(i(2))}$$

and $L_2(s_i) \geqq L_1(s_i)$ holds, $U(w_1(s_i), L_1(s_i)) \leqq U(w_2(s_i), L_2(s_i))$ follows: this is represented by the comparison of utility levels corresponding to point x and x'' in Fig. 4(A). Of course, there are numerous points which satisfy the conditions for x, x' and x''.

On the other hand, in case (B),

$$\frac{\partial f(s_i, L_2(s_i))}{\partial L} > -\frac{U_L(i(2))}{U_w(i(2))}$$

holds if and only if $L_2(s_i) < L_1(s_i)$: this is represented by the comparison of employment levels corresponding to point y and y' in Fig. 4(B). We can treat case (C) as being the opposite of case (A). (Compare the employment levels and utility levels corresponding to the points z, z' and z''.) Combining these facts with the results of Theorem 1 and Theorem 2, we obtain the following theorem.

[5] We will use $U_L(i(2))$, $U_w(i(2))$ etc. in place of $U_L(w_2(s_i), L_2(s_i))$, $U_w(w_2(s_i), L_2(s_i))$ etc.

THEOREM 3. *Assumption* 1–6 *hold.*
(I) *If* $V''<0$, $U_{ww}=0$ *and* $U_{wL}>0$, $L_2(s_i)<L_1(s_i)$ *holds for at least one state.*
(II) *If* $V''<0$, $U_{ww}=0$ *and* $U_{wL}=0$, $L_2(s_i)<L_1(s_i)$ *holds for* $i=1,\cdots,(n-1)$.
(III) *If* $V''=0$, $U_{ww}<0$ *and* $U_{wL}\leqq 0$, $L_2(s_i)>L_1(s_i)$ *holds for at least one state.*

Proof.
(I) By Theorem 1, we have

$$\frac{\partial f(s_i, L_2(s_i))}{\partial L} > -\frac{U_L(i(2))}{U_w(i(2))} \quad \text{for} \quad i=1,\cdots,(n-1)\,.$$

From inferiority of leisure, Fig. 4(A) applies here. Then, if $L_2(s_i)\geqq L_1(s_i)$ holds for all i,

$$U(w_2(s_i), L_2(s_i)) > U(w_1(s_i), L_1(s_i)) \qquad \text{for} \quad i=1,\cdots,(n-1)$$

and

$$U(w_2(s_n), L_2(s_n)) \geqq U(w_1(s_n), L_1(s_n))$$

holds.
It follows that;

$$\bar{U}=\sum_{i=1}^{i=n} p_i U(w_2(s_i), L_2(s_i)) > \sum_{i=1}^{i=n} p_i U(w_1(s_i), L_1(s_i))=\bar{U}$$

Then, there is a contradiction. ($\bar{U}$ was treated as an exogenous variable.)
(II) As in (I), we have

$$\frac{\partial f(s_i, L_2(s_i))}{\partial L} > -\frac{U_L(i(2))}{U_w(i(2))} \quad \text{for} \quad i=1,\cdots,(n-1)$$

from Theorem 1. Since leisure is independent of income in this case, the result is obvious from Fig. 4(B).
(III) We can apply the same argument as in (I). Q.E.D.

As Theorem 3 shows, even if the under(over)employment result is obtained at the solution to (7)–(11), we cannot say unambiguously that the employment level is less (larger) than that at the solution to (1)–(4). Then, it is adequate to interpret simply the under(over)employment result discussed here as a kind of misallocation of resources rather than as a variation of an employment level which is not optimal.

On the other hand, since the underemployment result established by Azariadis [1], Grossman-Hart [5] and Hart [7] is restricted to the case where (II) of Theorem 3 can be applied, we can say unambiguously that in an optimal contract, the employment level under asymmetric information is less than that under symmetric information. Then, we might say that since their main concern is the comparison of employment levels, they construct a model where this comparison is possible. None of them deal with a case where leisure is an inferior good.

6. CONCLUSION

In this paper, we have tried to clarify the logic of the labour contract theory under asymmetric information. We have shown that the direction of inefficiency, underemployment or overemployment, depends to a critical extent on the preference of firm and worker. Also, it was shown that when the preference of firm and worker satisfies certain conditions, neither underemployment nor overemployment occurs at the optimal contract under asymmetric information.

Finally, we should note several points. Firstly, since employment adjustment is done in terms of man hours in this paper, underemployment does not mean lay off unemployment. Moreover, wage rigidity, the optimality of which was the main result in an earlier implicit contract theory, is not shown in this paper. Then, I should say that it is inadequate to interpret the underemployment result established here as a microeconomic foundation of Keynesian Unemployment. Secondly, this theory suggests the view that the observed employment level is inefficient. The importance of the view in explaining observed employment fluctuations in macro economy remains to be seen. To make this assessement possible, we will need a general equilibrium framework. Thirdly, the underemployment or overemployment results established in this paper represent an inefficiency of sorts caused by asymmetric information. However we have only considered one short term contract. A question which desires some consideration is this: can we avoid this inefficiency by extending the length of the contract?

Further work should be done concerning these problems.

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