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PORTFOLIO BALANCE AND DISEQUILIBRIUM GROWTH THEORY

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I. INTRODUCTION

Disequilibrium analysis in macroeconomic models has been the subject of extensive research recently.[1] In particular the introduction of disequilibrium into one sector growth models has been analysed by, among others, Ito (1978), (1980), Newbury and Atkinson (1972) and Uzawa (1973). Another line of research has been concerned with the incorporation of a monetary sector into neo-classical growth models.[2] It is well-known that in this neo-classical money and growth literature, where the monetary modelling is of the Tobin type, the steady states are generally unstable.

This paper is concerned with combining some aspects of these two strands in the literature and at the same time extending Ito's (1980) analysis. One of the shortcomings of *real* disequilibrium models (like Ito's) is the absence of a monetary economy. Our purpose is to extend these models by introducing money.[3] We concentrate on only one cause of disequilibrium, namely labour market disequilibrium.[4] This is brought about by assuming that the wage rate is not perfectly flexible. Thus depending at what level the wage rate is fixed, the labour market is under a regime of excess supply (unemployment), excess demand (overfullemployment), or full employment. The actual transactions are assumed to take place at the minimum of the demand and supply of the labour force.[5] Thus the amount of saving depends not only on the wage rate but also the regime under

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[1] The authors who pursued this line of reasearch include Barro and Grossman (1976), (1977), Clower (1965), Leijonhufvud (1968), Malinvaud (1977) and Muellbauer and Portes (1978).

[2] For a detailed exposition and list of references on the "money and growth" area, the reader is referred to Burmeister and Dobell (1970).

[3] Alternatively this paper can be seen as attempting to introduce labour marker disequilibrium into the monetary growth models, especially those of Burmeister and Dobell (1970) and Hadjimichalakis (1971).

[4] In the more general models analysed by authors mentioned in footnote one, the economy is classified into four regimes according to the excess demands and supplies in both the aggregate consumption good and the labour market.

[5] This is the assumption and adjustment mechanism used by Ito (1980). There are many other ways of introducing disequilibria into a growth model. One other way is to assume a slow adjustment of the distribution of capital stock between the two sectors resulting in profit rates in two sectors not necessarily being equal.

which the labour market operates. The amount of saving determines a path of capital accumulation while the wage rate may be partially adjusted according to the labour market regime. Movements of capital per capita and the wage rate will trace a disequilibrium path of the economy as growth occurs.

We then introduce a monetary sector into the model and examine the effects of monetary repercussions and portfolio balance in the growth process. Since in the 'real' sector of our economy we have disequilibrium, there will be repercussions in the 'money' sector depending under which labour market disequilibrium situation the economy is operating. In other words, disequilibrium in the labour market "spills over" to the money market. Thus the present study intends to examine the implications and consequences of disequilibrium in the growth process in which monetary repercussions are explicitly introduced.[6] Growth models can be distinguished from the usual disequilibrium macroeconomic model by an explicit consideration of capital accumulation.

Ito (1980), has shown recently that a disequilibrium system of the type considered here, excluding money, has a unique steady state and it is globally asymptotically stable. One of the conclusions of this paper is that, with the introduction of money, the disequilibrium system has a unique unstable steady state with positive real money balances.[7] In other words the result that is characteristic of neo-classical equilibrium growth theory with money, namely that the steady state (with positive real money balances) is unstable, carries over to our disequilibrium growth model with money.

II. THE MODEL

The model consists of an economy producing a single consumption/investment good Y using two factors, capital (K) and labour (L), under the usual neoclassical technology assumptions of constant returns to scale and diminishing marginal products. Thus

$$\begin{aligned} &Y = F(K, L) \\ &Y/L = f(K/L), \qquad f' > 0, \quad F'' < 0 \end{aligned} \tag{1}$$

which also satisfies the Inada conditions.

At each point in time, the capital stock is historically determined and the wage rate is also fixed. The (representative) firm maximises profit, ρ, with respect to the labour input. That is

$$\max_L \rho \equiv Y - wL$$

[6] Some work has been carried out by Drabicki and Takayama (1982) on examining the monetary repercussions in a one-sector growth model with a minimum wage rule. This is a particular type of labour market disequilibrium which results in excess demand for labour (unemployment). Their analysis is a partial extension of Sgro and Takayama (1981).

[7] There is also a stable steady *without* money.

where w is the real wage rate. The labour demand, L^d, is the level of input which satisfies

$$w = F_L(K, L) \tag{2}$$

when $F_L \equiv \partial F/\partial L$. Since F is homogenous of degree one, F_L is homogenous of degree zero. Therefore, we have a separable form

$$L^d = \Omega(w) \cdot K, \qquad \Omega' < 0. \tag{3}$$

Like Ito (1980), we will assume that the labour supply per capita, l, is inelastic. Therefore, the aggregate labour supply L^s, is

$$L^s = lN \tag{4}$$

where N is the population. The transition rule in disequilibrium is the usual minimum of demand and supply. Define, $k \equiv K/N$, as the per capita capital. Note that this variable does not depend on the actual level of employment. By choosing appropriate measurement units, we set $l = 1$. Let $k^d \equiv K/L^d = 1/\Omega(w)$ denote the "desired" capital/labour ratio. This, of course, is a function of the current wage rate. From (2) and Euler's theorem,

$$w = f(k^d) - k^d f'(k^d) \tag{5}$$

There are three possible regions in the labour market. The labour market is at full employment, unemployment and over employment if $L^s = L^d$, $L^d < L^s$ and $L^s < L^d$ respectively. Thus in the full employment and unemployment regimes

$$K/L^d = k^d(w) \tag{6}$$

while in the full employment and overemployment regimes

$$K/L = k \tag{7}$$

The about model under full employment is well-known via the work of Solow (1956) and Swan (1956). Assume that consumption is a constant fraction of income Y, i.e. $C = (1-s)Y$ where s is the savings rate and to ease the exposition, we ignore depreciation. Then we have

$$\begin{aligned} \dot{K} &= sY \\ &= sF(K, L) \\ &= sLf(K/L) \end{aligned} \tag{8}$$

where $\dot{K} = dK/dt$, $0 < s < 1$. One of the characteristics of the disequilibrium growth model is that the capital accumulation equation (8) is different for each regime.

Assume further that the population grows at a constant rate, n, i.e. $\dot{N}/N = n$.

We use the following simple scheme of wage adjustment. Assume that the wage increases in the overemployment regime and falls in the unemployment regime. Furthermore, the wage adjustment is assumed to be proportional to the rate of unemployment or overemployment.

$$\dot{w}=\begin{cases}\varepsilon_1(L^d-L^s)/L^s & \text{if } L^d\geq L^s, \quad \varepsilon_1>0\\ \varepsilon_2(L^d-L^s)/L^s & \text{if } L^d<L^s, \quad \varepsilon_2>0\end{cases}$$

or

$$\dot{w}=\varepsilon(k/k^d(w)-1), \qquad \varepsilon=\varepsilon_1 \quad \text{if } L^s\leq L^d$$
$$\varepsilon=\varepsilon_2 \quad \text{if } L^s>L^d \tag{9}$$

(a) *Full employment regime*

In this case, the combination of per capita capital and the wage rate gives a state of full employment. That is, from (3) and (4) (with $l\equiv 1$)

$$\Omega(w)K=N \tag{10}$$

Solving (10) for the "full employment wage rate" and using (5),

$$w^*=\eta(k)=f(k)-kf'(k) \tag{11}$$

Furthermore;

$$dw^*/dk=\eta'(k)=-kf''(k)>0 \tag{12}$$

i.e. the full employment wage rate is an increasing function of the per capita capital ratio. Thus we retain full employment regime capital accumulation equation by substitutions (7) and (11) into (8).

$$\dot{k}\equiv k(\dot{K}/K-\dot{N}/N)$$
$$=sf(k)-nk \tag{13}$$

while

$$\dot{w}=0 \tag{14}$$

since the wage rate does not change at the steady state and when the labour market is in equilibrium. Thus denoting $\hat{k}$ as the steady state capital per capita and $\hat{w}$ as the associated wage rate, then

$$\hat{k}=\{k\,|\,sf(k)-nk=0\} \tag{15}$$

$$\hat{w}=f(\hat{k})-\hat{k}f'(\hat{k}) \tag{16}$$

From (15) it is clear that the full employment regime is purely transitory unless $k=\hat{k}$.[8]

(b) *Unemployment regime*

In this case, the current wage rate is higher than the full employment wage rate,

[8] Burmeister and Dobell (1970) give a useful summary of the appropriate stability condition of such a system.

$w^* = \eta(k)$, (therefore $L^d < L^s$).

The firms demand for labour is satisfied so that (2) applies. That is, the capital-employment ratio is determined by (6), and is equal to the desired capital-labour ratio $k^d(w)$.

$$w = f(k^d(w)) - k^d(w) f'(k^d(w)) \tag{17}$$

Using (8), the capital accumulation equation reduces to

$$\dot{k} = k[sf(k^d(w))/k^d(w) - n] \tag{18}$$

Denoting $[\cdot]$ in (18) by $g(w)$, we can note that $\dot{k} \gtreqless 0$ as $g(w) \gtreqless 0$. Also note that $g(w)$ is independent of k. Thus we now have the following differential equation

$$\dot{k} = kg(w) \tag{19}$$

$$\dot{w} = \varepsilon_2 \{k/k^d(w) - 1\} \tag{20}$$

Note that, as an obvious point, $\dot{k} = 0$ at $(\hat{k}, \hat{w})$. Furthermore it is clear that

$$dw/dk\,|_{\dot{k}=0} = -(\partial \dot{k}/\partial k)/(\partial \dot{k}/\partial w)\,|_{\dot{k}=0} = 0 \tag{21}$$

since $k^d(w)$ is independent of k. We should also note that

$$\begin{aligned} \partial \dot{k}/\partial w &= k(dg/dk^d) \cdot (dk^d/dw) \\ &= -\{ks[f'(k^d) \cdot k^d - f(k^d)]\}/(k^d)^2 k^d f''(k^d) < 0 \end{aligned} \tag{22}$$

Thus, we can set out the signs of the paths of the state variables.[9]

$$\begin{array}{lll} \dot{k} < 0\,, & \dot{w} < 0 & \text{if} \quad \hat{w} < w \\ \dot{k} = 0\,, & \dot{w} < 0 & \text{if} \quad w = \hat{w} \\ \dot{k} > 0\,, & \dot{w} < 0 & \text{if} \quad w < \hat{w}\,. \end{array} \tag{23}$$

(c) *Overemployment regime*

In this case $L^s < L^d$ or $w < w^* = \eta(k)$. Substituting (7) into (8), and using the intensive form

$$\dot{k} = sf(k) - nk\,. \tag{24}$$

Since the demand for labour is quantity-constrained, the marginal condition (2) is not satisfied, i.e. $w \neq f(k) - kf'(k)$. The appropriate wage adjustment equation is therefore equation (9). Thus (9) and (24) yield

$$\dot{k} = sf(k) - nk \tag{25}$$

$$\dot{w} = \varepsilon_1 \{k/k^d(w) - 1\} \tag{26}$$

Furthermore from (24), $\dot{k} \gtreqless 0$ as $sf(k) \gtreqless k$. Thus defining the wage rate which gives

[9] Note that $g(w) = 0$ and $g' < 0$.

the stationary movement of k in the overemployment regime as $\phi = \hat{k}$, then

$$\begin{aligned} &\dot{k}=0\,, \quad \dot{w}>0 \quad \text{iff} \quad w=\phi(k) \\ &\dot{k}>0\,, \quad \dot{w}>0 \quad \text{if} \quad k<\hat{k} \\ &\dot{k}<0\,, \quad \dot{w}>0 \quad \text{if} \quad k>\hat{k} \end{aligned} \tag{27}$$

In Fig. 1, the full employment regime is described by a curve $\eta(k)$.[10] The neoclassical growth model is a special case here in that it is restricted to the curve $\eta(k)$. The unemployment regime is anywhere above the curve, and the overemployment regime is anywhere below the curve.

While the equilibrium (neo-classical) path exhibits global stability to the long-run steady $\hat{k}$, what changes occur when we allow disequilibria? It is possible, fortunately, to assert global, and therefore local, stability by examining Fig. 1. It is clear from the diagram that a solution path never approaches the horizontal axis. Therefore, a solution path has to converge to the steady state eventually.[11] Thus, even if there is slugishness in wage adjustment, an economy eventually converges to the long-run full employment equilibrium. However, during the adjustment process, excess demand or supply will be observed.

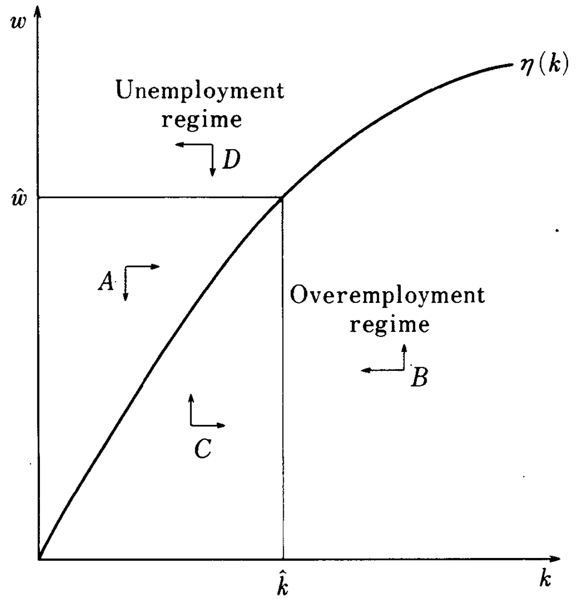

Fig. 1.

III. INTRODUCTION OF MONEY

(a) *Full employment regime*

Assume that there are only two assets in the economy, (outside) money and

[10] Our Fig. 1 is different from Figs. 1 and 2 of Ito (1980) in that we are using a different (and simpler) savings function rule.

[11] A mathematical proof of this is shown in Ito (1980), for the more general case of a classical savings function.

physical capital.[12] Letting M and p, respectively denote the money supply and the price level, the public's real wealth (W) can be written as

$$W \equiv K + M/p \tag{28}$$

Let π ($\equiv \dot{p}/p$) denote the rate of inflation or price change. We can specify the monetary equilibrium relation as

$$M = Q(pY, r, pW)\,, \qquad Q_1 > 0\,, \quad Q_2 < 0\,, \quad 0 < Q_3 < 1 \tag{29}$$

where $r \equiv f'(k) + \pi$, and where $Q_1 \equiv \partial Q/\partial(pY)$, $Q_2 \equiv \partial Q/\partial r$, $Q_3 = \partial Q/\partial(pW)$.

Using the homogeneity property of Q and assuming *full employment* one may rewrite (29) as

$$m = Q[f(k), f'(k) + \pi, k + m] \tag{30}$$

The quantity m signifies the per capita real cash balances.

From (30) one can determine the equilibrium value of π as

$$\pi = \pi(k, m) \tag{31}$$

where $\pi_k \equiv \partial\pi/\partial k > 0$, $\pi_m \equiv \partial\pi/\partial m < 0$ since $0 < Q_3 < 1$.

In a monetary economy, the public's income is affected by monetary transfers and capital losses or gains from holding money. Let $\theta = \dot{M}/M$ (the rate of monetary expansion) and assume θ is a positive constant. Then the equation for real disposable income (Y^D), using (28) is

$$Y^D = Y + (\theta - \pi)M/p \tag{32}$$

Given our fixed savings assumption, we can write

$$c = (1 - s)\{y + (\theta - \pi)m\} \tag{33}$$

where c denotes per capita consumption. Also we may write the capital accumulation equation as[13]

$$\begin{aligned}\dot{k} &= y - nk - c \\ &= sf(k) - nk - (1 - s)[\theta - \pi(k, m)]m \equiv \dot{k}(k, m)\end{aligned} \tag{34}$$

Differentiating (30) and utilizing (31) we may also obtain

$$\dot{m} = [\theta - \pi(k, m) - n]m \equiv \dot{m}(k, m) \tag{35}$$

Equation (34) and (35) constitute the basic dynamic equations with full employment. From (34) and (35) the steady state values k_s and m_s are defined by

$$\dot{k}(k_s, m_s) = 0 \qquad \text{and} \qquad \dot{m}(k_s, m_s) = 0 \tag{36}$$

and at steady state,

[12] Here we are following the usual "money and growth" literature of Tobin (1965) and others.

[13] Since we are in the full employment regime, from (14), $\dot{w} = 0$.

$$\pi_s = \theta - n \qquad \text{where} \quad \pi_s \equiv \pi(k_s, m_s) \tag{37}$$

From this and (34) we may observe:

$$k_s < \hat{k} \tag{38}$$

where $\hat{k}$ is defined as in (15) and

$$w_s < \hat{w} \tag{39}$$

where $\hat{w}$ is defined as in (16) and w_s is defined analogously to k_s and m_s.[14]

Equation (38) shows that the capital-labour ratio of a monetary economy is lower than that of a non-monetary one. This has been noted in the literature.

The dynamic paths of (k, m) that satisfy (34) and (35) are illustrated in Fig. 2.

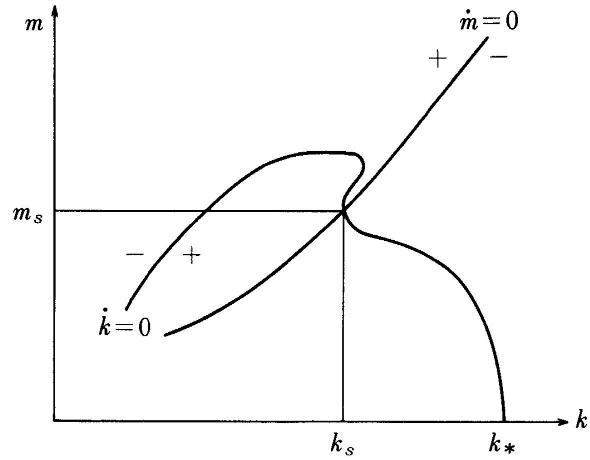

Fig. 2.

The slopes of the $(\dot{k}=0)$ and the $(\dot{m}=0)$ curves are easily obtained from (34) and (35) by suitable differentiation.

$$dm/dk\,|_{\dot{k}=0} = ((sf'(k) - n) + (1-s)m\pi_k)/((1-s)(\theta - \pi) - m\pi_m) \tag{40}$$

$$dm/dk\,|_{\dot{k}=0} = -\pi_k/\pi_m > 0 \tag{41}$$

where π_k and π_m are defined as in (31). The reader is referred to Burmeister and Dobell [3] for an extensive discussion of the $(\dot{k}=0)$ and $(\dot{m}=0)$ curves as illustrated in Fig. 2.

Concerning stability of the steady state (k_s, m_s), we proceed as follows. By setting $\dot{k}$ (from (34)) and $\dot{m}$ (from (35)) equal to zero, we determined the stationary point (k_s, m_s). By taking the Taylor linear approximation at this stationary point (k_s, m_s) we derive the matrix of the dynamic system (34) and (35)[15]

[14] These results illustrate the non-neutrality of money in the sense that the capital-labour ratio of the monetary economy is lower than in the non-monetary economy.

[15] This method is also used by Hadjimichalakis (1971).

$$A=\begin{bmatrix} sf'-n+(1+s)m\pi_k & (1-s)(\pi_m m-n) \\ -\pi_k m & -\pi_m m \end{bmatrix} \tag{42}$$

A necessary and sufficient condition for stability of the system is that the determinant of A be positive and its trace negative. From A we get

$$\det A = -\pi_m m(sf'-n) - \pi_k mn(1-s) \tag{43}$$

Since $\pi_m < 0$ and assuming $(sf'-n) > 0$, the first term on the right hand side of (43) is positive while the second term is negative since $\pi_k > 0$. The sign of the $\det A$ is therefore ambiguous. If the $\det A$ is negative, then the steady state (k_s, m_s) is a saddle-point. The trace of A is

$$\operatorname{trace} A = sf'-n+m[-\pi_m+(1-s)\pi_k] \tag{44}$$

Assuming $(sf'-n) > 0$, $\pi_k > 0$ and $\pi_m < 0$, the trace is positive and this is sufficient for local instability. If the $\det A$ is positive, then we would also have an unstable equilibrium because the trace of A is positive. Thus the long-run path described by the dynamic equations (34) and (35) is unstable.[16] Note that there is one more steady state, i.e. the point $(k_*, 0)$, which is locally asymptotically stable, although money disappears in this steady state.

(b) *Unemployment regime*

In this case using (19) and (20) we can define the following set dynamic equations.

$$\dot{k} = kg(w) - (1-s)[\theta - \pi(x, w)]m \equiv \phi(x, w) \tag{45}$$

$$\dot{m} = [\theta - \pi(x, w) - n]m \equiv \psi(x, w) \tag{46}$$

$$\dot{w} = \varepsilon_2[k/k^d(w) - 1] \tag{20}$$

where $x \equiv m/k \equiv M/pK$. Equations (45) and (46) represent the capital accumulation and money accumulation equation respectively. Equation (20) is the wage adjustment equation in the unemployment regime defined previously.

Note that $g(w) = (sf(k^d(w) - n)/k^d$ is not a function of k and that

$$\pi = \pi(x, w)$$

Furthermore, $\pi_x \equiv \partial\pi/\partial x > 0$ while $\pi_w \equiv \partial\pi/\partial w \gtreqless 0$.

From (45).

$$\begin{aligned} \partial\dot{k}/\partial w &= k\cdot(dg/dk^d)\cdot(dk^d/dw) + (1-s)\pi_w m \\ &= -ks[f'(k^d)\cdot k^d - f(k^d)]/(k^d)^3 f''(k^d) + (1-s)\pi_w m \end{aligned} \tag{47}$$

[16] As Hadjimichalakis (1971, p. 473) noted "as far as positive (as distinct from normative, optimal) models are concerned, it is immaterial whether we have a saddlepoint or a completely unstable equilibrium; neither can be useful."

From (47), $\partial\dot{k}/\partial w < 0$ if $\pi_w < 0$ since the term in the $[\cdot]$ is negative. If $\pi_w > 0$, the sign of $\partial\dot{k}/\partial w$ is ambiguous.

From (46),

$$\partial\dot{m}/\partial w = -m\pi_w \tag{48}$$

which, since $\pi_w \gtreqless 0$, is of ambiguous sign. We are familiar with the behaviour of equation (45) from (23).

Since the sign of $\partial\dot{k}/\partial w$ is ambiguous for $\pi_w > 0$, we derive the signs of the paths of the state variables for $\pi_w < 0$ as follows

$$\begin{aligned} &\dot{m} > 0, \quad \dot{k} < 0 \quad \text{if} \quad w_s < w \\ &\dot{m} = 0, \quad \dot{k} = 0 \quad \text{if} \quad w_s = w \\ &\dot{m} < 0, \quad \dot{k} > 0 \quad \text{if} \quad w_s > w \end{aligned} \tag{49}$$

In Fig. 3, we have combined Figs. 1 and 2 to enable us to represent conditions (49) in a geometrical fashion. In quadrant I of Fig. 3, we have reproduced Fig. 2 while in quadrant II, we have the relationship between k and w as discussed in Fig. 1.

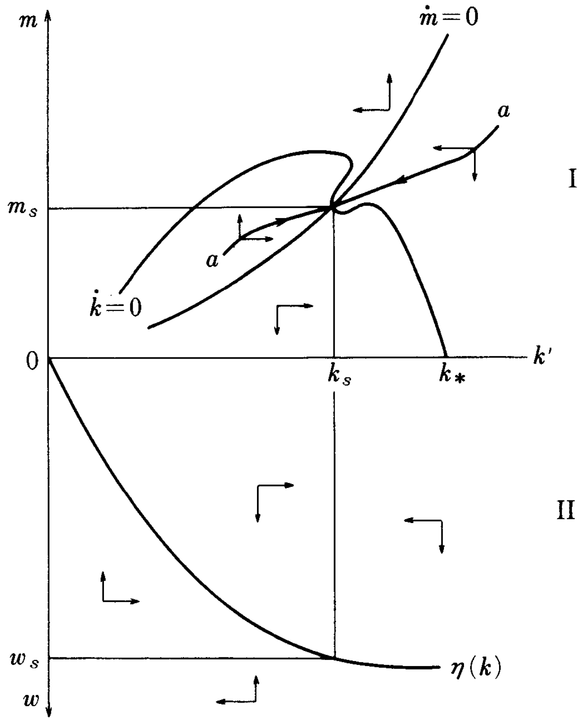

Fig. 3.

(c) *Overemployment regime*

In this case, using (34), (35) and (26) we can define the following sets of dynamic equations

$$\dot{k} = sf(k) - nk'(1-s)[\theta - \pi(k, m)]m \equiv \dot{k}(k, m) \tag{50}$$

$$\dot{m} = [\theta - \pi(k, m) - n]m \equiv \dot{m}(k, m) \tag{51}$$

$$\dot{w} = \varepsilon_1[k/k^d(w) - 1] \tag{26}$$

Equations (50) and (51) represent the capital accumulation and money accumulation equations. These two differential equations are identical to those that apply for the full employment case. Equation (26) is the wage adjustment equation in the overemployment regime defined previously.

From equation (51),

$$\partial\dot{m}/\partial k = -m\pi_k < 0 \tag{52}$$

since $\pi_k > 0$ as defined in (31). Hence using (52) along with (27) we have the following time paths of the critical variables

$$\begin{array}{lll} \dot{m}=0\,, & \dot{k}=0 & \text{iff} \quad w=\phi(k) \\ \dot{m}>0\,, & \dot{k}>0 & \text{if} \quad k<k_s \\ \dot{m}<0\,, & \dot{k}<0 & \text{if} \quad k>k_s \end{array} \tag{53}$$

We can also represent condition (53) in Fig. 3.

Concerning stability when we are introducing disequilibria, we can see from Fig. 3, the longrum steady state is a saddlepoint so that $\lim_{t\to\infty}(k, m) = (k_s, m_s)$ only from initial positions (k^0, m^0) lying on the curve labelled *aa*.[17] The other steady state equilibria is at $(k_*, 0)$ although money disappears in this steady state.

IV. COMPARATIVE STATICS

We now examine the effects on the steady state values of k, m and $\dot{p}/p$ $(\equiv\pi)$ when the government increases (say) the rate of monetary expansion θ. At the longrun equilibrium, π_s $(=\dot{p}/p) = \theta - n$, and $m_s = \hat{Q}(k_s, \theta - n)$. Setting (34) equal to zero, having substituted for m_s and differentiating totally, we have

$$dk_s/d\theta = (1-s)n\hat{Q}_2/(sf'(k_s) - (1-s)n\hat{Q}_1(k_s, \theta - n) - n) > 0 \tag{54}$$

since the denominator is negative.[18] Note that $\hat{Q}_1 = -\pi_k/\pi_m$. The effects on $\dot{p}/p$ is positive since π_s $(=\dot{p}/p) = \theta - n$ and

$$d\pi_s/d\theta = 1 > 0 \tag{55}$$

The effects on m_s is negative

[17] That is, unless the initial point happens to be on the stable saddle, the economy diverges from the steady state.

[18] The negativity of the denominator is well-known and quite easy to show and comes from the proof that the equilibrium is a saddlepoint. See for example, Burmeister and Dobell (1970) and Hadjimichalakis (1971). In particular, Hadjimichalakis (1971) has shown the equilibrium is a saddlepoint if the elasticity of demand for real money balance in greater than one.

$$dm_s/d\theta = (sf' - n)dk_s/d\theta < 0$$

when $(sf' - n)$ is negative (i.e. when the equilibrium point occurs on the downward portion of the $\dot{k} = 0$ curve).[19] Thus monetary expansion decreases both k_s and m_s.[20]

V. CONCLUSION

This paper has analysed the stability properties of two models under the assumption that the labour market does not clear automatically. The first is a neoclassical growth model of the Swan-Solow type; the second is a monetary growth model of the Tobin type. It is known that these two types of models have stable and unstable steady states, respectively. The central result is that there exist two equilibrium points for the system, a saddle-point equilibrium with positive real balances and a stable equilibrium with zero real balances. These same properties carry over for these two types of models when there is labour market disequilibrium. The disequilibrium labour market determines the actual employment at the minimum of demand and supply. This in turn determines the actual output which is exactly absorbed either as consumption, capital accumulation or money balances.

We also examine the effects on the steady state values of the capital/labour ratio and per capita real cash balances of monetary expansion. Our results that monetary expansion decreases both k_s and m_s reinforces the comparative static results in full employment money and growth literature. The complexity that the introduction of money adds to equilibrium growth models, carries over to disequilibrium growth models of the type considered in this paper.

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[19] All of these comparative static results are in line with Tobin (1965), Hadjimichalakis (1971) and others. However, as Hadjimichalakis (1971) has pointed out, these results cannot be used as an operating rule since the equilibrium is unstable.

[20] This result has been termed the "non-neutrality of money" in the sense that the economy steady-state capital intensity will change as monetary expansion or contraction occurs. Also the actual long-run capital intensity for the monetary economy is different from that of the non-monetary economy.

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