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# NON-MINORITY RULES: CHARACTERIZATION OF CONFIGURATIONS WITH RATIONAL SOCIAL PREFERENCES

Satish K. JAIN

*Abstract*: It is shown that for every non-minority rule a necessary and sufficient condition (i) for quasi-transitivity is that value-restriction or weakly conflictive preferences or unique-value restriction holds over every triple of alternatives and (ii) for transitivity is that conflictive preferences or extreme-value restriction holds over every triple of alternatives.

The purpose of this paper is to derive necessary and sufficient conditions for quasi-transitivity and transitivity of non-minority rules. One member of this class, namely the simple non-minority rule, also known as absolute (strict) majority rule, has been widely discussed in the literature. Several conditions on configurations of individual preferences have been formulated for the rationality of the social preference relation generated by the simple non-minority rule. Dummett and Farquharson [2] have shown that if in every triple of alternatives there exists an alternative which no individual regards as uniquely worst then the simple non-minority rule yields acyclic social preferences. Pattanaik [6] showed that the existence of an alternative in every triple which is regarded by none as uniquely best also guarantees acyclicity. In [3] Fine has derived necessary and sufficient conditions for the transitivity of the social preference relation.

We show that for every non-minority rule a necessary and sufficient condition for quasi-transitivity of the social preference relation is that the configuration of individual preferences satisfies, over every triple of alternatives, value-restriction (VR) or weakly conflictive preferences (WCP) or unique-value restriction (UVR). For every non-minority rule, satisfaction of extreme-value restriction (EVR) or conflictive preferences (CP) over every triple of alternatives is shown to be both necessary and sufficient for transitivity of the social preference relation. Of the four restrictions introduced in this paper, WCP and CP are partial antagonism conditions while UVR and EVR are in the same spirit as Sen's extremal restriction.

The interesting feature that emerges is that the necessary and sufficient conditions for quasi-transitivity or transitivity are same for all non-minority rules. This is in sharp contrast to the case of majority rules where conditions for transitivity are known to be different. While extremal restriction is both necessary and sufficient for transitivity of the social preference relation generated by the simple majority rule, it is not sufficient for transitivity of the social preference relation generated by the two-thirds majority rule.

*Restrictions on Preferences*

The set of social alternatives would be denoted by $S$. The cardinality $n$ of $S$ would be assumed to be finite and greater than 2. The set of individuals and the number of individuals are designated by $L$ and $N$ respectively. $N(\quad)$ would stand for the number of individuals holding the preferences specified in the parentheses and $N_k$ for the number of individuals holding the $k$-th preference ordering. Each individual $i \in L$ is assumed to have an ordering $R_i$ defined over $S$. The symmetric and asymmetric parts of $R_i$ are denoted by $I_i$ and $P_i$ respectively. The social preference relation is denoted by $R$ and its symmetric and asymmetric components by $I$ and $P$ respectively.

*Non-Minority Rules*: $\forall x, y \in S: xRy$ iff $N(yP_ix) \leqslant pN$, where $p$ is a fraction such that $1/2 \leqslant p < 1$. For $p = 1/2$ we obtain the familiar simple non-minority rule.

An individual is defined to be concerned with respect to a triple iff he is not indifferent over every pair of alternatives belonging to the triple; otherwise he is unconcerned. For individual $i$, in the triple $\{x, y, z\}$, $x$ is best iff $(xR_iy \wedge xR_iz)$; medium iff $(yR_ixR_iz \vee zR_ixR_iy)$; worst iff $(yR_ix \wedge zR_ix)$; uniquely best iff $(xP_iy \wedge xP_iz)$; uniquely medium iff $(yP_ixP_iz \vee zP_ixP_iy)$; and uniquely worst iff $(yP_ix \wedge zP_ix)$. Now, we define several restrictions which specify the permissible sets of individual orderings. All these restrictions are defined over triples of alternatives.

*Value-Restriction* (*VR*): VR holds over a triple iff there is an alternative in the triple such that all concerned individuals agree that it is not best or all concerned individuals agree that it is not medium or all concerned individuals agree that it is not worst.

*Weakly Conflictive Preferences* (*WCP*): Whenever an individual considers an alternative best in some strong ordering as worst, he regards the alternative worst in the strong ordering as best; or alternatively whenever an individual considers an alternative worst in some strong ordering as best, he regards the alternative best in the strong ordering as worst. Formally, WCP holds over $\{x, y, z\}$ iff $[\forall a, b, c \in \{x, y, z\}: [\exists i: (aP_ibP_ic) \rightarrow \forall i: ((bR_ia \wedge cR_ia) \rightarrow cR_ib)]] \vee [\forall a, b, c \in \{x, y, z\}: [\exists i: (aP_ibP_ic) \rightarrow \forall i: ((cR_ia \wedge cR_ib) \rightarrow bR_ia)]]$.

*Unique-Value Restriction* (*UVR*): There exist distinct alternatives $a$ and $b$ in the triple such that $a$ is not uniquely medium in any $R_i$, $b$ is not uniquely best in any $R_i$, and whenever $b$ is best in an $R_i$, $a$ is worst in that $R_i$; or alternatively there exist distinct $a$ and $b$ in the triple such that $a$ is not uniquely medium in any $R_i$, $b$ is not uniquely worst in any $R_i$, and whenever $b$ is worst in an $R_i$, $a$ is best in that $R_i$. More formally, UVR holds over $\{x, y, z\}$ iff $[\exists$ distinct $a, b, c \in \{x, y, z\}:$ $\forall i: [((aR_ib \wedge aR_ic) \vee (bR_ia \wedge cR_ia)) \wedge (aR_ib \vee cR_ib) \wedge (bR_ia \wedge bR_ic \rightarrow cR_ia)]]$ $\vee$ $[\exists$ distinct $a, b, c \in \{x, y, z\}: \forall i: [((aR_ib \wedge aR_ic) \vee (bR_ia \wedge cR_ia)) \wedge (bR_ia \vee$

$bR_ic) \wedge (aR_ib \wedge cR_ib \rightarrow aR_ic)]]$.

*Extreme-Value Restriction* (*EVR*): If an alternative is uniquely best in some ordering then in no ordering can it be medium unless it is worst also; or alternatively if an alternative is uniquely worst in some ordering then in no ordering can it be medium unless it is best also, i.e., EVR holds over the triple $\{x, y, z\}$ iff $[\forall a, b, c \in \{x, y, z\}: [\exists i: (aP_ib \wedge aP_ic) \rightarrow \forall i: [(bR_iaR_ic \rightarrow cR_ia) \wedge (cR_iaR_ib \rightarrow bR_ia)]]] \vee [\forall a, b, c \in \{x, y, z\}: [\exists i: (bP_ia \wedge cP_ia) \rightarrow \forall i: [(bR_iaR_ic \rightarrow aR_ib) \wedge (cR_iaR_ib \rightarrow aR_ic)]]]$.

*Conflictive Preferences* (*CP*): A set of individual orderings satisfies CP over the triple $\{x, y, z\}$ iff there are $L_1$, $L_2$ such that $\phi \subseteq L_1 \subseteq L_c$, $\phi \subseteq L_2 \subseteq L_c$, $L_1 U L_2 = L_c$, where $L_c$ is the set of individuals concerned with respect to $\{x, y, z\}$, and (i) $\forall i \in L_1$ have the same $R$-ordering, say, $xR_iyR_iz$ and $\forall i \in L_2$ have the opposite $R$-ordering $zR_iyR_ix$ and (ii) either ($\forall i \in L_1$ consider $x$ to be uniquely best and $\forall i \in L_2$ consider $x$ to be uniquely worst) or ($\forall i \in L_1$ consider $z$ to be uniquely worst and $\forall i \in L_2$ consider $z$ to be uniquely best).

LEMMA 1. *Conditions of value-restriction, weakly conflictive preferences and unique-value restriction are logically independent of each other.*

*Proof.* The following 8 examples constitute a proof of complete logical independence of VR, WCP, and UVR.

(1) $xP_iyP_iz$
$xP_izP_iy$

All three restrictions are satisfied.

(2) $yP_izP_ix$
$zP_iyP_ix$
$yP_ixI_iz$
$zP_ixI_iy$

VR and WCP are satisfied and UVR is violated.

(3) $xP_iyP_iz$
$yP_izP_ix$

VR and UVR are satisfied but WCP is violated.

(4) $yP_ixP_iz$
$yP_izP_ix$
$zP_ixP_iy$
$zP_iyP_ix$

VR is satisfied and both WCP and UVR are violated.

$$\text{(5)} \quad \begin{array}{l} xP_iyP_iz \\ yI_izP_ix \\ zI_ixP_iy \end{array}$$

Both WCP and UVR are satisfied and VR is violated.

$$\text{(6)} \quad \begin{array}{l} yI_izP_ix \\ zI_ixP_iy \\ xI_iyP_iz \end{array}$$

VR and UVR are violated and WCP is satisfied.

$$\text{(7)} \quad \begin{array}{l} xP_iyP_iz \\ yP_izP_ix \\ xP_izP_iy \\ zI_iyP_ix \\ yI_ixP_iz \end{array}$$

UVR is satisfied and VR and WCP are violated.

$$\text{(8)} \quad \begin{array}{l} xP_iyP_iz \\ yP_izP_ix \\ zP_ixP_iy \end{array}$$

All three restrictions are violated.

LEMMA 2. *Extreme-value restriction and conflictive preferences conditions are logically independent of each other.*

*Proof.* The proof consists of the following 4 examples:

$$\text{(1)} \quad \begin{array}{l} xP_iyP_iz \\ zP_iyP_ix \end{array}$$

Both EVR and CP are satisfied.

$$\text{(2)} \quad \begin{array}{l} xP_iyP_iz \\ zP_iyP_ix \\ xP_iyI_iz \\ zI_iyP_ix \end{array}$$

CP is satisfied and EVR is violated.

$$\text{(3)} \quad \begin{array}{l} xP_iyP_iz \\ xP_izP_iy \end{array}$$

EVR is satisfied but CP is violated.

$$\text{(4)} \quad \begin{array}{l} xP_iyI_iz \\ xI_iyP_iz \end{array}$$

Both CP and EVR are violated.

*Necessary and Sufficient Conditions for Quasi-Transitivity*

LEMMA 3. *A set of individual orderings violates all three restrictions VR, WCP and UVR over a triple $\{x, y, z\}$ iff it contains one of the following four 3-ordering sets, except for a formal interchange of alternatives*;

$$\begin{array}{llll} \text{(A)} & xP_iyP_iz & \text{(B)} & xP_iyP_iz \\ & yP_izP_ix & & yP_izP_ix \\ & zP_ixP_iy & & zP_ixI_iy \\[6pt] \text{(C)} & xP_iyP_iz & \text{(D)} & xP_iyP_iz \\ & yP_izP_ix & & yP_izI_ix \\ & zI_ixP_iy & & zI_ixP_iy \end{array}$$

*Proof.* It can be easily checked that WCP is violated iff the set of orderings contains one of the following four sets, except for a formal interchange of alternatives.

$$\begin{array}{llll} \text{(i)} & xP_iyP_iz & \text{(ii)} & xP_iyP_iz \\ & yP_izI_ix & & yP_izP_ix \\ & zI_ixP_iy & & \\[6pt] \text{(iii)} & xP_iyP_iz & \text{(iv)} & xP_iyP_iz \\ & xP_izP_iy & & yP_ixP_iz \\ & zI_ixP_iy & & zI_ixP_iy \\ & zP_iyI_ix & & xP_izI_iy \end{array}$$

(i) It is the same set as (D).

(ii) This configuration does not violate either VR or UVR. In this configuration $z$ is the only alternative which is never uniquely best, $x$ is the only alternative which is never uniquely medium, and $y$ is the only alternative which is never uniquely worst. Define,

$$T_1 = \{R_i\colon x \text{ uniquely medium}\} = \{zP_ixP_iy,\ yP_ixP_iz\}\ ,$$

$$T_2 = \{R_i\colon z \text{ uniquely best}\} = \{zP_ixP_iy,\ zP_ixI_iy,\ zP_iyP_ix\}\ ,$$

$$T_3 = \{R_i\colon y \text{ uniquely worst}\} = \{xP_izP_iy,\ xI_izP_iy,\ zP_ixP_iy\}\ ,$$

$$T_4 = \{R_i\colon z \text{ best and } x \text{ not worst}\} = \{zP_ixP_iy,\ zI_ixP_iy\}\ ,$$

$$T_5 = \{R_i\colon y \text{ worst and } x \text{ not best}\} = \{zP_ixP_iy,\ zP_ixI_iy\}\ .$$

Now, UVR would be violated iff (a) there does not exist an alternative which is never uniquely medium, i.e., an $R_i \in T_1$ is included, or (b) there exists neither an alternative which is never uniquely best nor an alternative which is never uniquely worst, i.e., we include an $R_i \in T_2$ and an $R_i \in T_3$, or (c) there does not exist an alternative which is never uniquely best and there exists an $R_i$ in which $y$ is worst but $x$ is not best, i.e., an $R_i \in T_2$ and an $R_i \in T_5$ are included, or (d) there does not exist an alternative which is never uniquely worst and there exists an $R_i$ in which $z$ is best but $x$ is not worst, i.e., an $R_i \in T_3$ and an $R_i \in T_4$ are included, or (e) there exists an $R_i$ in which $z$ is best but $x$ is not worst and an $R_i$ in which $y$ is worst but $x$ is not best, i.e., an $R_i \in T_4$ and an $R_i \in T_5$ are included. Now ((a) or (b) or (c) or (d) or (e)) implies that UVR is violated iff we include $[zP_ixP_iy \vee zP_ixI_iy \vee zI_ixP_iy \vee yP_ixP_iz \vee (zP_iyP_ix \wedge xP_izP_iy)]$. In the first three cases VR is also violated as the set of $R_i$ forms a Latin Square. In each of these three cases one of the four sets (A)–(D) is contained in the set of $R_i$. In the cases of inclusion of $zP_ixP_iy$, $zP_ixI_iy$ and $zI_ixP_iy$ the sets contained are (A), (B) and (C) respectively. For violating VR, if $yP_ixP_iz$ is included to violate UVR, we have to include [concerned $R_i$: $zR_ixR_iy$ $\vee$ (concerned $R_i$: $xR_izR_iy$ $\wedge$ concerned $R_i$: $zR_iyR_ix$)]; and in case $(zP_iyP_ix \wedge xP_izP_iy)$ is included to violate UVR, we have to include (concerned $R_i$: $zR_ixR_iy$ $\vee$ concerned $R_i$: $yR_ixR_iz$). It is easy to see that with the required inclusion the set of $R_i$ contains one of the sets (A)–(D).

(iii) Niether VR nor UVR is violated. VR would be violated iff a concerned ordering in which $y$ is best is included. Excepting the case when we include $yI_izP_ix$, in all other cases UVR is also violated and one of the four sets (A)–(D) is contained in the set of $R_i$. In the case of inclusion of $yI_izP_ix$, UVR is violated iff we include $[yP_ixP_iz \vee zP_ixP_iy \vee yP_izP_ix \vee yP_izI_ix \vee yI_ixP_iz]$. In all cases one of (A)–(D) is contained in the set of $R_i$.

(iv) Again, neither VR nor UVR is violated. VR would be violated iff a concerned ordering in which $x$ is worst is included. In all cases other than the case of inclusion of $zP_iyI_ix$, UVR is also violated and one of (A)–(D) is contained in the set of $R_i$. In the case of inclusion of $zP_iyI_ix$, UVR is violated iff we include $[xP_izP_iy \vee yP_izP_ix \vee zP_iyP_ix \vee yI_izP_ix \vee yP_izI_ix]$. In all cases we see that one of the four sets (A)–(D) is contained in the set of $R_i$.

The proof of the lemma is completed by noting that all the four sets (A)–(D) violate all three restrictions.

THEOREM 1. *For every non-minority rule, a necessary and sufficient condition for quasi-transitivity of the social preference relation is that* $(VR \vee WCP \vee UVR)$ *holds over every triple of alternatives.*

*Proof.*

*Sufficiency*

Suppose quasi-transitivity is violated. Then for some $x, y, z \in S$ we must have $xPy \wedge yPz \wedge \sim(xPz)$.

$$xPy \rightarrow N(xP_iy) > pN \tag{1}$$

$$yPz \rightarrow N(yP_iz) > pN \tag{2}$$

$$\sim(xPz) \rightarrow N(xP_iz) \leqslant pN$$

$$\rightarrow N(zR_ix) \geqslant (1-p)N \tag{3}$$

$$(1) \wedge (2) \rightarrow \exists i: xP_iyP_iz\,, \qquad \text{as} \quad 1/2 \leqslant p < 1 \tag{4}$$

$$(2) \wedge (3) \rightarrow \exists i: yP_izR_ix \tag{5}$$

$$(1) \wedge (3) \rightarrow \exists i: zR_ixP_iy \tag{6}$$

(4), (5) and (6) imply that the set of individual orderings must contain one of the following 4 sets of orderings,

$$\begin{array}{llll}
\text{(a)} & xP_iyP_iz & \text{(b)} & xP_iyP_iz \\
 & yP_izP_ix & & yP_izP_ix \\
 & zP_ixP_iy & & zI_ixP_iy \\
\text{(c)} & xP_iyP_iz & \text{(d)} & xP_iyP_iz \\
 & yP_izI_ix & & yP_izI_ix \\
 & zP_ixP_iy & & zI_ixP_iy
\end{array}$$

As each one of these sets violates all 3 restrictions VR, WCP and UVR it follows that (VR $\vee$ WCP $\vee$ UVR) is sufficient for quasi-transitivity.

*Necessity*

If a set of orderings violates all 3 restrictions VR, WCP and UVR then by Lemma 3 it must contain one of the four sets A, B, C and D, except for a formal interchange of alternatives. Therefore, for proving the necessity of (VR $\vee$ WCP $\vee$ UVR) it suffices to show that for each of the four sets there exists an assignment of individuals which results in violation of quasi-transitivity. For (A), (C) and (D) choose $N_1 = pN$, $N_2 = N_3 = (1-p)N/2$. For this assignment we have $N(xP_iy) > pN$, $N(yP_iz) > pN$ and $N(xP_iz) = pN$. So $xPy \wedge yPz \wedge \sim(xPz)$. For (B) choose $N_2 = pN$, $N_1 = N_3 = (1-p)N/2$. As $N(yP_iz) > pN$, $N(zP_ix) > pN$ and $N(yP_ix) = pN$, this results in $yPz \wedge zPx \wedge \sim(yPx)$.

*Necessary and Sufficient Conditions for Transitivity*

LEMMA 4. *A set of individual orderings violates both CP and EVR iff it includes one of the following four 2-ordering sets, except for a formal interchange of alternatives,*

$$\begin{array}{llll}
\text{(A)} & xP_iyP_iz & \text{(B)} & xP_iyP_iz \\
 & yP_izP_ix & & yP_izI_ix \\
\text{(C)} & xP_iyP_iz & \text{(D)} & xP_iyI_iz \\
 & zI_ixP_iy & & xI_iyP_iz
\end{array}$$

*Proof.* It can be easily checked that a set of individual orderings violates CP iff it contains one of the following 8 sets of orderings, except for a formal interchange of alternatives:

$$\begin{array}{llll}
\text{(1)} & xP_iyP_iz & \text{(2)} & xP_iyP_iz \\
 & yP_izP_ix & & yP_izI_ix
\end{array}$$

| | | | |
|---|---|---|---|
| (3) | $xP_iyP_iz$<br>$zI_ixP_iy$ | (4) | $xP_iyI_iz$<br>$xI_iyP_iz$ |
| (5) | $xP_iyP_iz$<br>$xP_izP_iy$ | (6) | $xP_iyP_iz$<br>$yP_ixP_iz$ |
| (7) | $xP_iyI_iz$<br>$yP_ixI_iz$ | (8) | $yI_izP_ix$<br>$zI_ixP_iy$ |

The first four sets are the same as (A), (B), (C) and (D) respectively, so it suffices to consider the remaining four sets.

(v) In this configuration $z$ is uniquely worst in the first ordering and medium without being best in the second ordering, so the second part of the definition of EVR does not hold. EVR, however, is satisfied as the first part of the definition holds because whenever an alternative is uniquely best in some $R_i$ it is not medium unless it is worst also. Therefore, EVR would be violated iff we include (a) an $R_i$ in which $x$ is medium without being worst, i.e., $(yP_ixP_iz \vee yI_ixP_iz \vee zP_ixP_iy \vee zI_ixP_iy)$; or (b) an $R_i$ in which $y$ is uniquely best, i.e., $(yP_ixP_iz \vee yP_ixI_iz \vee yP_izP_ix)$; or (c) an $R_i$ in which $z$ is uniquely best, i.e., $(zP_ixP_iy \vee zP_ixI_iy \vee zP_iyP_ix)$. (a) or (b) or (c) implies that EVR is violated iff we include $(yP_ixP_iz \vee yI_ixP_iz \vee zP_ixP_iy \vee zI_ixP_iy \vee yP_ixI_iz \vee yP_izP_ix \vee zP_ixI_iy \vee zP_iyP_ix)$. In each case one of the four sets (A)–(D) is contained in the set of $R_i$. The sets contained are (A) with an interchange of $x$ and $y$, (C) with an interchange of $y$ and $z$, (A) with a substitution of $x$, $y$, $z$ for $z$, $x$, $y$ respectively, (C), (B), (A), (B) with an interchange of $y$ and $z$, and (A) with an interchange of $y$ and $z$, respectively.

(vi) EVR would be violated iff an ordering is included in which $z$ is medium without being best or $x$ is uniquely worst or $y$ is uniquely worst. With the inclusion of the required ordering the set of $R_i$ contains one of the sets (A)–(D).

(vii) EVR would be violated only if an $R_i$ in which some alternative is uniquely worst is included. With the inclusion of an ordering in which some alternative is uniquely worst, excepting the cases when $xP_izP_iy$ or $yP_izP_ix$ is included, EVR is violated and one of (A)–(D) is contained in the set of $R_i$. If $xP_izP_iy$ or $yP_izP_ix$ is included then EVR is violated iff an ordering is included in which $x$ is medium without being worst or $y$ is medium without being worst or $z$ is uniquely best. In each of these cases the set of $R_i$ contains one of the sets (A)–(D).

(viii) EVR would be violated only if an $R_i$ in which some alternative is uniquely best is included. Again we see as in case (vii) that with the inclusion of required ordering, excepting the cases when $xP_izP_iy$ or $yP_izP_ix$ is included, EVR is violated and the set of $R_i$ contains one of the sets (A)–(D). If $xP_izP_iy$ or $yP_izP_ix$ is included then EVR is violated iff an ordering is included in which $x$ is medium without being best or $y$ is medium without being best or $z$ is uniquely worst. In each of these cases the set of $R_i$ contains one of the sets (A)–(D).

The proof is completed by noting that each of the four sets (A) (D) violates both CP and EVR.

THEOREM 2. *For every non-minority rule, a necessary and sufficient condition for transitivity of the social $R$ is that ($CP \vee EVR$) holds one every triple of alternatives.*

*Proof.*

*Sufficiency*

Let transitivity be violated. Then for some $x, y, z \in S$ we must have $xRy \wedge yRz \wedge zPx$.

$$xRy \rightarrow N(yP_ix) \leqslant pN$$
$$\rightarrow N(xR_iy) \geqslant (1-p)N \tag{1}$$

Similarly,

$$yRz \rightarrow N(yR_iz) \geqslant (1-p)N \tag{2}$$
$$zPx \rightarrow N(zP_ix) > pN \tag{3}$$
$$(1) \wedge (3) \rightarrow \exists i: zP_ixR_iy \tag{4}$$
$$(2) \wedge (3) \rightarrow \exists i: yR_izP_ix \tag{5}$$

(4) and (5) imply that the set of individual orderings must contain one of the following four sets of orderings,

| | | | |
|---|---|---|---|
| (a) | $zP_ixP_iy$<br>$yP_izP_ix$ | (b) | $zP_ixP_iy$<br>$yI_izP_ix$ |
| (c) | $zP_ixI_iy$<br>$yP_izP_ix$ | (d) | $zP_ixI_iy$<br>$yI_izP_ix$ |

As each of these sets violates both CP and EVR it follows that (CP $\vee$ EVR) is sufficient for transitivity.

*Necessity*

Let both EVR and CP be violated. Then, by Lemma 4, the set of $R_i$ must contain one of the four sets (A), (B), (C) and (D) of Lemma 4, except for a formal interchange of alternatives. Therefore it suffices to show that for each of the four sets there exists an assignment of individuals which results in intransitive social preferences. For each case take $N_1 = N_2$. For (A) and (B) this results in $xIy \wedge yPz \wedge xIz$, for (C) in $xPy \wedge yIz \wedge xIz$ and for (D) in $xIy \wedge yIz \wedge xPz$. This establishes the necessity of (EVR $\vee$ CP) for transitivity of the social $R$ generated by a non-minority rule.

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REFERENCES

[1] Arrow, K. J. (1963): Social Choice and Individual Values, Wiley, N. Y., Second edition.

[2] Dummett, M. and Farquharson, R. (1961): "Stability in voting," *Econometrica*, 29.

[3] Fine, K. (1973): "Conditions for the existence of cycles under majority and non-minority rules," *Econometrica*, 41.

[4] Inada, K. (1964): "A note on the simple majority decision rule," *Econometrica*, 32.

[5] Inada, K. (1969): "The simple majority decision rule," *Econometrica*, 37.

[6] Pattanaik, P. K. (1968): "A note on democratic decisions and the existence of choice sets," *Review of Economic Studies*, 35.

[7] Pattanaik, P. K. (1971): Voting and Collective Choice, Cambridge University Press, Cambridge.

[8] Sen, A. K. and Pattanaik, P. K. (1969): "Necessary and sufficient conditions for rational choice under majority decision," *Journal of Economic Theory*, 1.

[9] Sen, A. K. (1970): Collective Choice and Social Welfare, Holden-Day, San Francisco.