ON THE DISPERSION OF EXPECTATIONS AND THE EQUILIBRIUM INTEREST RATE

Mariko YOSHIDA*

1. INTRODUCTION

In a monetary economy where there exist bonds in addition to money, the equilibrium interest rate will be influenced by two types of economic data. One is expectations of the future course of interest rates and the other is money supply. In terms of expectations we assume that individuals expect the future prices of bonds with perfect confidence, but that they have not unanimous expectations about them. Thus we shall consider the economy in which there exists uncertainty in the market as a whole.

First, we examine the movement of the equilibrium interest rates in response to a given change in the quantity of money. Namely we ask whether the liquidity preference function in Keynes' sense is decreasing in the economy with uncertainty in the above sense. Next we rigorously investigate how the equilibrium bond rate is affected by the degree of dispersion in expectations. For this purpose we use the fundamental framework which is employed by formal temporary equilibrium models, the recent survey of which has been supplied by Grandmont [3].

This paper is organized as follows. In Section 2, a monetary economy is described and the results of the comparative statics are stated. In Section 3, a concluding discussion is given. The formal proof of Theorem 2 is shown in the Appendix.

2. FORMULATION AND RESULTS

We consider a pure exchange economy. An individual will be indexed by $\omega \in \Omega$, where Ω is a probability space. We will omit the index ω as long as we study the behavior of the representative individual.

The relevant span of economic activity is two periods ($t = 1, 2$). Period 1 ($t = 1$), the present, is the date at which the decisions we study will be taken. Period 2 ($t = 2$) occurs in the future. In each period there are one type of perishable consumption goods, fiat money and a unique type of long-term bonds. Money is the unit of account and a store of value. The total quantity of money is constant over time. Bonds are perpetuities which pay one unit of money in each period after purchase.

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At the start of each period, the individual receives an initial endowment of consumption goods, $w_t \in R_{++}$, and has an initial portfolio of bonds and money, $(b_{t-1}, m_{t-1}) \in R^2_{++}$ as a result of his choice in previous period. They are assumed to be known with certainty. For simplicity we assume that

(A-1) (i) $w_1^\omega = w_2^\omega \equiv w$ for all $\omega \in \Omega$.

(ii) $(m_0^\omega, b_0^\omega) = (m_0, b_0)$ for all $\omega \in \Omega$.

Let $p \in R_{++}$, $r \in R_{++}$ and 1 be the money price of goods, interest rate of bonds and the price of money in period 1. We define $P \equiv R_{++} \times R_{++}$.

In order to choose his action at period 1, the individual ω has to forecast the price system which will prevail at period 2. For simplicity we assume that the individual forecasts with certainty a price vector in P. We denote the expected price system by $(p_e, r_e) \in P$. As we concentrate on expectations of the interest rate, we assume that

(A-2) $p_e^\omega = p$ for all $\omega \in \Omega$.

Furthermore we assume that

(A-3) the number of individuals whose expected rate of interest is $r_e \in R_{++}$ is measured by the probability density function $\mu(r_e)$.

Preferences are identical for all individuals and are defined over lifetime consumption bundles, (x_1, x_2), with $x_t \in R_{++}$ $(t = 1, 2)$ denoting consumption of goods in period t. These preferences are representable by a utility function u: $R^2_{++} \to R$. For simplicity the utility function is specified as follows.

DEFINITION. $u(x_1, x_2) = x_1 x_2$.

We describe the individual behavior. His problem is to choose an action $(x_1, m_1, b_1) \in R^3_+$ which maximizes $u(x_1, x_2)$ on the set of consumption streams which he expects to be feasible in response to an arbitrary price system (p, r, r_e), i.e. on the set of $(x_1, x_2) \in R^2_{++}$ which satisfies:

(C-1) $$px_1 + b_1/r + m_1 = pw + (1 + r)b_0/r + m_0,$$

and

(C-2) $$px_2 = pw + (1 + r_e)b_1/r_e + m_1.$$

Constraints (C-1) and (C-2) represent budget hyperplanes in period 1 and 2. The solution of this maximum problem gives a set of actions.

(A-4) $(m_1^\omega, b_1^\omega) \neq (0, 0)$ for all $\omega \in \Omega$.

Here we present a theorem on the relation between money supply and the equilibrium interest rate r^*.

THEOREM 1. *Suppose that* (A-1)–(A-4) *are satisfied, then* $dr^*/dm_0 < 0$.

Proof. First of all we show an individual demand function for money. We solve the following maximization problem: maximize $x_1 x_2$ subject to the condition (C-2), for any $(x_1, m_1, b_1) \in R^3_+$ and any $(p, r_e) \in P$. The solution is immediately given by $x_2 = w + m_1/p + (1+r_e)b_1/pr_e$. We define the indirect utility function by $u'(x_1, m_1, b_1, p, r_e) \equiv u[x_1, w + m_1/p + (1+r_e)b_1/pr_e]$. Next we solve the following maximization problem: maximize $u'(x_1, m_1, b_1, p, r_e)$ subject to (C-1), for any $(p, r, r_e) \in P \times R_{++}$. Since we have to take account of corner solutions with respect to m_1 and b_1, following conditions can be derived. Let λ be a Lagrange multiplier.

(i) $\partial u'/\partial x_1 - p\lambda = w + m_1/p + (1+r_e)b_1/pr_e - p\lambda = 0$.

(ii) $\partial u'/\partial m_1 - \lambda = x_1/p - \lambda = 0$. if $m_1 > 0$.

$\leqq 0$. if $m_1 = 0$.

(iii) $\partial u'/\partial b_1 - \lambda/r = (1+r_e)x_1/pr_e - \lambda/r = 0$. if $b_1 > 0$.

$\leqq 0$. if $b_1 = 0$.

When $m_1 > 0$ and $b_1 = 0$, from (ii) and (iii) we can obtain $r_e \geqq r/(1-r)$ and $0 < r < 1$. Furthermore we can get the individual demand for money from (C-1), (i) and (ii) as follows.

(iv) $m_1 = (m_0 + b_0 + b_0/r)/2$.

By (A-3), we can aggregate the above demand function for individuals whose expected rate of interest is no less than $r/(1-r)$. Let us denote by $M(r)$ the aggregate demand function for money.

(v) $M(r) = (m_0 + b_0 + b_0/r)f(r)/2$, where $f(r) \equiv \int_{r/(1-r)}^{+\infty} d\mu(r_e)$.

According to (A-1) (ii) the total money supply is expressed by $m_0 + b_0$. Hence the equilibrium equation for money is as follows.

(vi) $(m_0 + b_0 + b_0/r)f(r) = 2(m_0 + b_0)$.

$\lim_{r \to +0} M(r) = +\infty$ and $M(1/2) = 0$. Furthermore $M(r)$ is a decreasing function on (0, 1). Then, since $m_0 + b_0$ is positive, there exists a solution $r^* \in (0, 1)$. Now we totally differentiate with respect to r^* and m_0.

(vii) $dr^*/dm_0 = (f(r^*) - 2)/\{b_0 f(r^*)/r^{*2} - (m_0 + b_0 + b_0/r)f'(r^*)\}$,

Since $0 < f(r^*) < 1$ and $f'(r^*) < 0$. We can get $dr^*/dm_0 < 0$. Q.E.D.

Remark. Theorem 1 asserts that the liquidity preference function in Keynes' sense is decreasing in the economy which satisfies the assumptions (A-1)–(A-4).

Next we examine the effect of a change in the dispersion of expectations upon the equilibrium interest rate. For this purpose we consider two economies which differ only in their expectations. In particular, let us specify μ and μ' as follows.

(A-5)
(i) μ and μ' are unimodal and symmetric probability density functions.
(ii) μ and μ' intersect at only two points.

Furthermore we denote the mean and variance of each probability density function by (σ, δ) and (σ', δ'). Since our aim is to describe two economies which differ only in their dispersions of expectations, it is formally assumed that

(A-6)
(i) $\sigma=\sigma'$.
(ii) $\delta<\delta'$.

Now, two economies which satisfy (A-1)–(A-6) can be completely described as follows.

$$E_\delta = E(\Omega, u, m_0, b_0, w, \sigma, \delta)\,,$$

and

$$E_{\delta'} = E'((\Omega, u, m_0, b_0, w, \sigma, \delta')\,.$$

Let r^* and $r^{*\prime}$ be respectively the equilibrium interest rates in E_δ and $E_{\delta'}$. Then we can present a theorem on the relation between r^* and $r^{*\prime}$.

THEOREM 2. *Suppose that* (A-1)–(A-6) *are satisfied, then* $\sigma/(1+\sigma)\leqq r^*\leqq r^{*\prime}$ *or* $\sigma/(1+\sigma)>r^*>r^{*\prime}$.

Remark. The formal proof of Theorem 2 is given in the Appendix. However, for example, let us consider the probability functions presented in Fig. 1 which satisfy (A-5)–(A-6). Figure 1 tells us that the number of individuals whose expected rate of interest is more than $r^*/(1-r^*)$ is greater in $E_{\delta'}$ than in E_δ. Therefore, from (vi) in the proof of Theorem 1, it is shown that every equilibrium

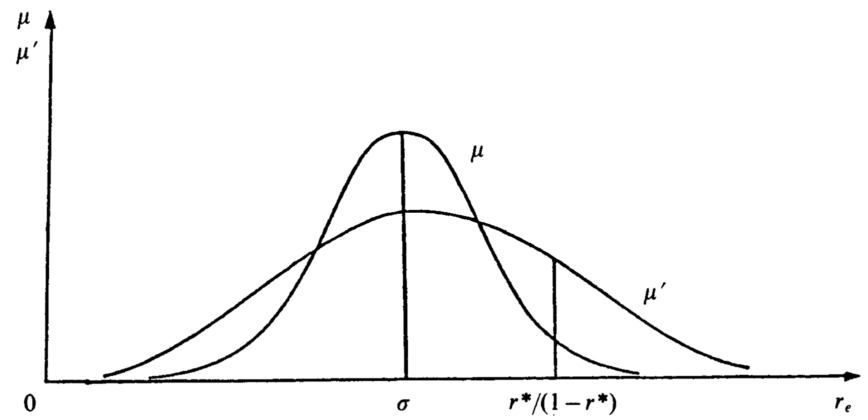

Fig. 1.

interest rate in E_δ which satisfies $r^*/(1-r^*)>\sigma$ creates an excess demand for money in $E_{\delta'}$. Then, using (v) in the proof of Theorem 1, we can obtain $\sigma/(1+\sigma)\leqq r^*\leqq r^{*\prime}$. Similarly, it is proved that $r^{*\prime}<r^*<\sigma/(1+\sigma)$.

3. CONCLUSION

This section gives some economic interpretations of Theorem 2. Let us consider the individual who forecasts σ. He expects with perfect confidence that everything invested in bonds today will earn not only the interest but also a capital gain or loss described by $(r/\sigma)-1\equiv k$, over the period ahead. As is clear from the proof of Theorem 1, the division of his savings into money and bonds is an all-or-nothing choice. When $r+k$ is positive, he will demand bonds. But when $r+k$ is negative, he will demand money. A critical level for his decision will be expressed by $\sigma/(1+\sigma)$. In this way, if it is possible to know the mean of a dispersion of expectations, then $\sigma/(1+\sigma)$ can be calculated.

Theorem 2 asserts that an increase in the dispersion of expectations widening the gap of "bull-bear positions" continuously raises or lowers the interest rate according as the existing rate is greater or less than $\sigma/(1+\sigma)$.

4. APPENDIX

Proof of Theorem 2. By (A-5) (ii), we can define $\{r_e\in S\,|\,\mu(r_e)=\mu'(r_e)\}\equiv\{\alpha,\beta\}$, $(\alpha<\beta)$. Furthermore without loss of generality we define that for any $r_e\in S/\{r_e\,|\,\mu(r_e)\neq 0\}$, $\mu(r_e)\equiv 0$, and that for any $r_e\in S/\{r_e\,|\,\mu'(r_e)\neq 0\}$, $\mu'(r_e)\equiv 0$. Now let us consider two possible cases.

Case (1): $\mu(r_e)>\mu'(r_e)$ for all $r_e\in(\alpha,\beta)$

and $\mu(r_e)\leqq\mu'(r_e)$ for all $r_e\in(0,\alpha]\cup[\beta,1]$.

Case (2): $\mu(r_e)<\mu'(r_e)$ for all $r_e\in(\alpha,\beta)$

and $\mu(r_e)\geqq\mu'(r_e)$ for all $r_e\in(0,\alpha]\cup[\beta,1]$.

We shall show that the case (2) contradicts (A-6) (ii). In the case (2),

$$\begin{aligned}(\delta-\delta')/2&=\int_\sigma^1|r_e-\sigma|^2d\mu(r_e)-\int_\sigma^1|r_e-\sigma|^2d\mu'(r_e)\\&=\int_\sigma^\beta|r_e-\sigma|^2\{d\mu(r_e)-d\mu'(r_e)\}+\int_\beta^1|r_e-\sigma|^2\{d\mu(r_e)-d\mu'(r_e)\}\\&>|\beta-\sigma|^2\int_\sigma^\beta\{d\mu(r_e)-d\mu'(r_e)\}+|\beta-\sigma|^2\int_\beta^1\{d\mu(r_e)-d\mu'(r_e)\}\\&=|\beta-\sigma|^2\left\{\int_\sigma^1 d\mu(r_e)-\int_\sigma^1 d\mu'(r_e)\right\}\\&=0\,.\end{aligned}$$

This result contradicts (A-6) (ii). Then from now on let us consider the case (1).

As is clear from the proof of Theorem 1, there exist r^* and $r^{*\prime}$. Now we shall show that if $\sigma \leqq r^*/(1-r^*) \equiv k^*$, then $f(r^*) \leqq g(r^*)$, where $f(r^*) \equiv \int_{k^*}^1 d\mu(r_e)$ and $g(r^*) \equiv \int_{k^*}^1 d\mu'(r_e)$. In the case (1), if $\sigma \leqq k^* \leqq \beta$, then $\int_\sigma^{k^*} d\mu(r_e) \geqq \int_\sigma^{k^*} d\mu'(r_e)$. Since $\int_\sigma^1 d\mu(r_e) = \int_\sigma^1 d\mu'(r_e) = 1/2$, we get $f(r^*) \leqq g(r^*)$. On the other hand, if $k^* > \beta$, it is obvious that $f(r^*) < g(r^*)$. Analogously we can show that if $\sigma > k^*$, then $f(r^*) > g(r^*)$.

By using the aggregate demand function for money in the proof in Theorem 1, we can define the aggregate excess demand functions for money in E_δ and $E_{\delta'}$ denoted by $M(r)$ and $M'(r)$.

(i) $M(r) \equiv (m_0 + b_0 + b_0/r) f(r)/2 - (m_0 + b_0)$,

(ii) $M'(r) \equiv (m_0 + b_0 + b_0/r) g(r)/2 - (m_0 + b_0)$.

Let $\sigma \leqq k^*$. Then since $f(r^*) \leqq g(r^*)$ from the above result, $M'(r^*) \geqq 0$. Furthermore, since $\partial M'(r)/\partial r$ is negative, we get $r^* \leqq r^{*\prime}$. Hence it is obtained that $\sigma/(1+\sigma) \leqq r^* \leqq r^{*\prime}$.

Conversely let $\sigma > k^*$. Then $f(r^*) > g(r^*)$ from the above result. By using the same method as above, we can get $r^{*\prime} < r^* < \sigma/(1+\sigma)$.

Q.E.D.

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