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OPTIMAL POPULATION IN AN OPEN ECONOMY

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Abstract: Though there is a considerable body of literature on the optimal population problem in a closed economy model there is no such discussion in an open economy setting. A beginning is made in this paper to consider the optimal population and the optimal tariff problems in an open economy model. The optimality rules for the population size and the tariff rate are derived under three alternative formulations of the social utility function. A distinction is made between (a) the small country case and the large country case, (b) the "full optimisation situation" where both the tariff rate and the population size can simultaneously find their respective optimal values and the "restricted optimisation situation" where one of the policy instruments is held fixed at an arbitrary level and (c) the reproduction-based population growth case and the migration-caused population growth case. The sensitivity of the optimality rules to each of these distinctions is explored.

I. INTRODUCTION

There exists a large body of literature on optimal population size in a closed economy, originating at least as early as in Plato [10]. The reader may refer to a discussion of the history of the concept and its development in Dasgupta [5]. By contrast, writing on the concept in an open economy setting is virtually, if not altogether, absent.[1] No doubt, trade theorists have studied implications of population growth in an open economy model but the analysis has fallen short of the concept of an 'optimal population'. In an open economy, population growth can come about in two ways: reproduction of existing national population or migration from abroad. The early trade-theoretic analysis of the implications of

* I am indebted to M. C. Kemp as the original impetus to investigate the problem arose out of my correspondence with him while I was visiting Monash University, Australia, in 1977–78. Murry Kemp also provided helpful comments on an earlier draft. Thanks are also due to Amitava Bose, Henry Wan, Makoto Yano and a referee for useful discussions and comments. An earlier draft of the paper was completed while I was visiting Cornell University in 1981–82.

[1] Pitchford [9, Ch. 5] is, perhaps, the only known exception. However, his model and the focus of his analysis are significantly different from ours. In particular, he assumes a small country and free trade so that the interdependence between tariffs and optimal population is not brought out in his analysis. Further, he concentrates only on the average utility criterion and does not consider any of the implications of the distinction between reproduction and immigration. Meade's [7, Ch. VI] treatment, though it constitutes a chapter of his 'Trade & Welfare', is confined to a one-good closed economy model and refers to international trade only in a footnote.

population growth considered population growth of the first variety (see, for example, Mundell [8]). In recent years, attention has also been devoted to the second type of population growth in connection with the discussion on the "brain drain" phenomenon (see Bhagwati [2] for a recent survey). But, in either case, the idea of an optimal population has not come up for investigation. In this paper, we shall attempt a beginning in this direction.

As a beginning, we shall keep the model as simple as possible by abstracting from all considerations of dynamic or intertemporal optimality. Our method of analysis would be comparative statics. We shall also keep aside the problem of (optimal) saving by simply assuming that all income is currently consumed. The stock of all factors of production, other than labor, and the state of technology are assumed unchanged. No distinction is made between population and work force. Many alternative concepts of optimal population can be (and have been) thought of. We shall consider only three out of the possible alternative concepts of optimal population: (a) one which maximises social utility which is a function of the aggregate consumption basket of the community (the traditional social utility function as used in trade-theoretic discussions), (b) one which maximises social tuility which is a function of per capita consumption (Cassel-Wicksell-Wolfe formulation, to use Dasgupta's [5] terminology) and (c) one which maximises social utility which is defined as the utility level of a representative individual (a function of per capita consumption) multiplied by the number of persons enjoying that utility level (Sidgwick-Meade formulation, to use Dasgupta's [5] terminology again). Two instruments to achieve the welfare optimum would be considered, namely, the size of the population and the tariff rate. A distinction would be made between the "full optimisation" situation where both the policy instruments can be simultaneously varied to reach their respective optimal levels and the "restricted optimisation" situation where one of the instruments is fixed at an arbitrary level. The results in each of these situations would be sensitive to (a) whether one assumes the country to be a "small" country (i.e., no influence on world prices) or a large country and (b) whether the population growth comes about through reproduction of the national population or through migration. As we shall see, a distinction can be introduced between the reproduction case and the immigration case in several possible ways even in our (admittedly restrictive) stationary-state models, with divergent implications.

II. UNDER TRADITIONAL UTILITY FUNCTION

Let us start with the traditional two commodity trade theoretic model where the social utility U is a function of the aggregate consumption basket of the community so that

$$U = U(C_1, C_2) \tag{1}$$

where C_i is the aggregate national consumption of commodity i ($i = 1, 2$).

The utility function is assumed to have both behavioral and welfare significance.[2] Therefore, from the first-order consumer equilibrium conditions

$$\frac{\partial U/\partial C_2}{\partial U/\partial C_1} = P \tag{2}$$

where P is the domestic price of commodity 2 in terms of commodity 1.

The community budget constraint (or, alternatively, the balance of trade equation) can be written as

$$C_1 + P^* C_2 = X_1 + P^* X_2 \tag{3}$$

where P^* is the international price of commodity 2 in terms of commodity 1.

Market clearing conditions give

$$C_1 = X_i + M_i \qquad i = 1, 2 \tag{4}$$

where X_i is the level of domestic production of commodity i and M_i (algebraic + or −) is the level of import of commodity i. It is assumed throughout that commodity 2 is the import commodity of the home country (hence $M_2 > 0$) and commodity 1 is the export commodity (hence $M_1 < 0$).

The domestic and international prices are linked through the tariff rate t such that

$$P = P^*(1 + t)\,. \tag{5}$$

To keep matters simple, we shall assume that there are no taxes or subsidies on exports so that positive (negative) t implies a positive (negative) import duty.

In contrast to the usual trade theory model, we shall not be assuming constant returns to scale which necessarily implies diminishing marginal product of labor. Rather, we shall be assuming that in each industry the marginal product of labor curve has a positively sloped initial stretch followed by an eventual negatively sloped stretch. The usual justification for this assumption is the existence of some 'optimal' (or most effective) input proportion. As we increase the quantity of labor, starting from a very low (say, zero) level, with fixed quantities of other factors of production, we gradually approach the 'optimal' proportion and the marginal product of labor increases. But the marginal product of labor falls as the 'optimal' combination is surpassed.

Now, totally differentiating (1) and (3) and then making use of Eqs. (4) and (5), one can get $\mathrm{d}Y$, the change in social welfare expressed in terms of commodity 1, as

[2] For the alternative conditions under which this is valid one can see Chipman [4, pp. 690–8]. In any case, this assumption about the social utility function is invariably made in the literature on trade and welfare. For our present purposes, the social utility function in (1) may be interpreted as a Bergson-Samuelson type social welfare function with ideal lump-sum income distribution in the background to equate the marginal social utility of a dollar for all individuals. See Samuelson [11].

$$\text{(6)} \qquad \mathrm{d}Y \equiv \frac{\mathrm{d}U}{\partial U/\partial C_1}$$
$$= -M_2 \mathrm{d}P^* + P^* t \mathrm{d}M_2 + (\mathrm{d}X_1 + P\mathrm{d}X_2)\,.$$

The above expression has become a standard one in trade and welfare literature and hence the details are not repeated here.[3] Note that $(\mathrm{d}X_1 + P\mathrm{d}X_2)$ which is zero for movement along a given production possibility curve (around the production equilibrium point) is not necessarily so in the case of labour growth which shifts the production possibility curve. $(\mathrm{d}X_1 + P\mathrm{d}X_2)$ in such a case refers to the change in the value of the output bundle that would take place as a result of growth, if P is held constant.[4]

For an optimum $\mathrm{d}Y$ has to be zero. The optimum population size, for a given tariff rate, and the optimum tariff, for a given population size, would be obtained by setting the following equations

$$\text{(7)} \qquad \frac{\partial Y}{\partial L} = -M_2 \frac{\partial P^*}{\partial L} + P^* t \frac{\partial M_2}{\partial L} + \left(\frac{\partial X_1}{\partial L} + P\frac{\partial X_2}{\partial L}\right)$$

and

$$\text{(8)} \qquad \frac{\partial Y}{\partial t} = -M_2 \frac{\partial P^*}{\partial t} + P^* t \frac{\partial M_2}{\partial t} + \left(\frac{\partial X_1}{\partial t} + P\frac{\partial X_2}{\partial t}\right)$$

equal to zero.

The $(\cdot)$ expression in (8) is zero as it represents the price-weighted sum of changes in outputs along a fixed production possibility curve. The remaining expression in (8) is the familiar one in connection with the optimum tariff literature which leads to the standard optimum tariff formula

$$\text{(9)} \qquad t_{\mathrm{opt}} = \frac{1}{e^* - 1}$$

[3] The interested reader may refer to Caves and Jones [3, pp. 420–21], for example, for the details.

[4] Since $X_i = X_i(P, L)$

$$\mathrm{d}X_i = \frac{\partial X_i}{\partial P}\mathrm{d}P + \frac{\partial X_i}{\partial L}\mathrm{d}L.$$

$$\therefore\ \mathrm{d}X_1 + P\mathrm{d}X_2 = \left[\frac{\partial X_1}{\partial P} + P\frac{\partial X_2}{\partial P}\right]\mathrm{d}P + \left[\frac{\partial X_1}{\partial L} + P\frac{\partial X_2}{\partial L}\right]\mathrm{d}L$$
$$= \left[\frac{\partial X_1}{\partial L} + P\frac{\partial X_2}{\partial L}\right]\mathrm{d}L.$$

since

$$\left[\frac{\partial X_1}{\partial P} + P\frac{\partial X_2}{\partial P}\right] = 0$$

for variation along a fixed transformation curve.

where e^* is the foreign elasticity of demand for imports (or what is alternatively known as the elasticity of the foreign offer curve) at the optimum point.[5] This is not surprising since with L constant as in Eq. (8) one is returned to the traditional fixed-factor-endowments model of international trade. It also follows as a corollary that for a "small" country ($e^* = \infty$) the optimum tariff continues to be zero, irrespective of the size of the population. Note, however, the important point that though the optimum tariff *formula* or *rule* is invariant with respect to the population size, the *rate* of optimum tariff need not be insensitive to the choice of the population size, except for the small country case. The choice of L would affect the production possibility curve, hence the trade indifference map (in Meade's well-known terminology) and, consequently, the point of tangency between the foreign offer curve and the highest possible home trade indifference curve. e^* is the elasticity of the foreign offer curve at this tangency point. Therefore, as L affects the position of the tangency point, e^* and consequently the *rate* of the optimum tariff, as obtained from the formula in (9), would change. Further, if the population change is due to migration from abroad, the foreign offer curve itself would change, in addition to the changes in the home trade indifference map and this would be an additional factor affecting the optimum tariff rate.

Turning, now, to the optimum population size, let us first take up the small country case ($dP^* = 0$). Eq. (7) then reduces to

$$\frac{\partial Y}{\partial L} = P^* t \frac{\partial M_2}{\partial L} + \frac{\partial Q}{\partial L} \tag{7'}$$

where $\partial Q / \partial L$ is defined as

$$\frac{\partial X_2}{\partial L} + P \frac{\partial X_2}{\partial L}.$$

Alternatively, by writing $\partial Q^W / \partial L$ for

$$\frac{\partial X_1}{\partial L} + P^* \frac{\partial X_2}{\partial L},$$

Eq. (7′) can be written as

$$\frac{\partial Y}{\partial L} = P^* t \frac{\partial C_2}{\partial L} + \frac{\partial Q^W}{\partial L}. \tag{7''}$$

Now writing C_2, generally, as

$$C_2 = C_2(P, Y) \tag{10}$$

one gets

[5] One may again refer to Caves and Jones [3], for example, for the details of the derivation.

(11)
$$\frac{dC_2}{dL}=\frac{\partial C_2}{\partial P}\cdot\frac{dP}{dL}+\frac{\partial C_2}{\partial Y}\cdot\frac{dY}{dL}$$
$$=\frac{\partial C_2}{\partial P}\cdot\frac{dP}{dL}+\frac{b}{P}\cdot\frac{dY}{dL}$$

where b is defined as the marginal propensity to import (or, what is the same thing, the marginal propensity to consume the import commodity). Substituting (11) in (7″) and noting that for the small country case $dP/dL=0$, we obtain

(12)
$$\alpha\frac{\partial Y}{\partial L}=\frac{\partial Q^W}{\partial L}$$

where

$$\alpha=\left(1-\frac{t}{1+t}b\right)>0$$

if inferiority in consumption is ruled out (which is assumed throughout). It is clear, then, that, for the small country case, welfare would be increasing if and only if $\partial Q^W/\partial L>0$, i.e., the value of output at *world prices* is increasing with the increase in the population size. Therefore, for the small country, population reaches the optimal size (call it L_{opt}) where an additional man can no longer cause an increase in the value of output at world prices.[6] Note that this optimum population *rule* is quite independent of whether the country is following an optimum tariff policy (which is free trade for a small country) or not. However, the *size* of the optimum population is not invariant to the choice of the tariff rate. Given L, as t changes, P changes, (X_1, X_2) changes, hence $(X_1+P^*X_2)$ changes and the optimum population size L_{opt} which maximises $(X_1+P^*X_2)$ changes as well. To see that clearly we have depicted the effect of population growth from L_1 to L_2 under two alternative situations: (i) where $t=0$ (optimum tariff for the small country) and (ii) where t (fixed) >0 (inoptimal for the small country) in Fig. 1. In our diagram, L_2 gives a higher value of $(X_1+P^*X_2)$ as compared to L_1 when $t=0$ but the reverse is the case for $t>0$. The production point shifts from 1 to 2 with $t=0$ and from 1′ to 2′ with $t>0$. This implies, as shown in Fig. 2, that OB, the optimum population with $t=0$ is greater than OA, the optimum population with $t>0$.[7]

[6] It can be easily checked that

$$\frac{\partial Q}{\partial L}=\frac{\partial X_1}{\partial L_1}=P\cdot\frac{\partial X_2}{\partial L_2}$$

where $\partial X_i/\partial L_i$ is the marginal product of labor in industry i (holding constant all other factor inputs used in industry i). Therefore, $\partial Q^W/\partial L=0$ condition is equivalent to zero marginal product of labor if $P=P^*$ (free trade) but not otherwise.

[7] We should emphasize that this result is not generally valid. It is simply an outcome of the way we have chosen to draw Fig. 1.

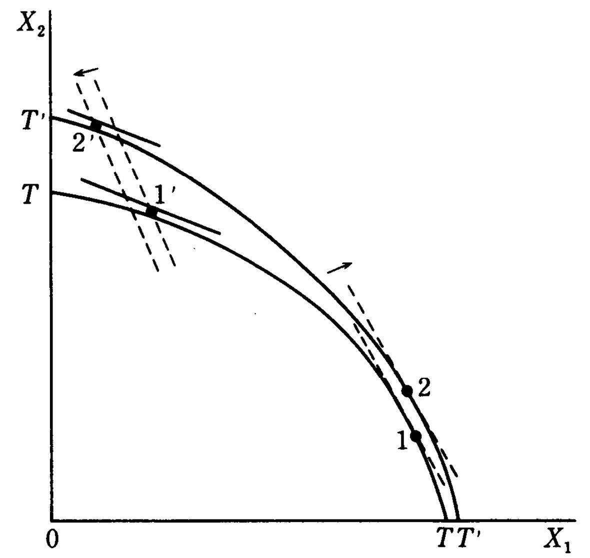

Fig. 1. TT and TT' are the production possibility curves corresponding to L_1 and L_2, respectively. Broken price lines refer to P^* and solid price lines refer to P. Commodity 2 is assumed to be labor-intensive.

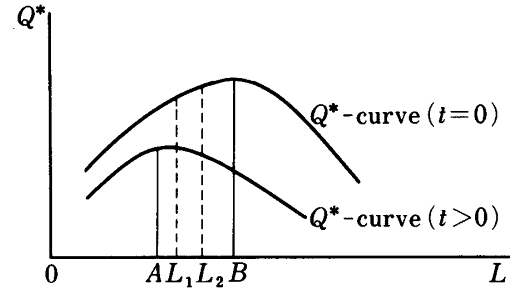

Fig. 2.

It may also be mentioned here that the optimum population rule for the *small country* as mentioned above and the size of the optimum population do not depend on whether the population growth is due to internal reproduction or migration from abroad. This is because P^* for a small country would be the same in both cases and no other difference is postulated between immigration and reproduction. One can, however, introduce a difference in 'cost of production of population' through reproduction relative to immigration. If one compares the real resource cost of maternity hospitals, child care and education with the cost of assisted migration (such as in Australia)[8] one may be inclined to think that the marginal 'cost of production' of population through natural means is substantially higher than that through immigration.[9] Under such a situation the size of L_{opt},

[8] Murry Kemp has drawn my attention to this difference.

[9] We are abstracting from the difference in terms of speeds of adjustment of population following from the fact that population can be almost instantaneously adjusted through migration but not through reproduction.

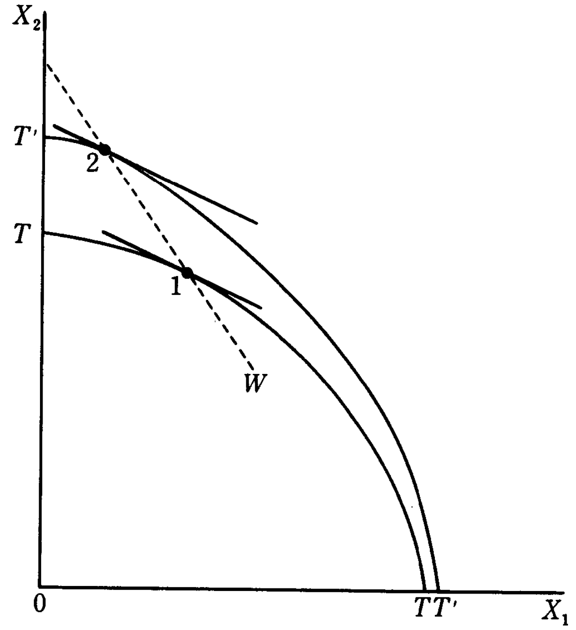

Fig. 3. The solid lines refer to P. The broken price line WW refers to P^*. Commodity 2 is assumed to be labor-intensive.

though not the optimal population rule, would generally differ in the two cases. First, suppose the small country is pursuing free trade ($t=0$). Then, at any given value of L, the outward shift in the production possibility curve (net of 'cost of production' of population) and hence $\partial Q^W/\partial L$ would be less if the marginal man is an offspring (of already existing national population) rather than an immigrant. The L_{opt} point where $\partial Q^W/\partial L=0$ would be reached at a lower value of L with reproduction relative to immigration. However, this is not certain if the country has non-zero tariffs ($t\neq 0$). In Fig. 3, the production possibility curve has shifted from TT to $T'T'$ as a result of the entry of one additional immigrant. Production point moves from 1 to 2 but the value of output at world prices (given by the broken line WW) remains the same—so L_{opt} with immigration is reached. If the marginal man were instead an offspring, the corresponding production possibility curve would lie somewhere in between TT and $T'T'$ (not drawn)—however the production point may be either to the left or to the right of the WW line. $\partial Q^W/\partial L$ may be positive or negative. Hence no ranking of optimum population under reproduction and immigration is possible when the small country is departing from free trade.

Consider, now, the large country case. We shall first find the expressions for $\partial M_2/\partial L$ and $\partial P^*/\partial L$ and insert them in Eq. (7).

Writing the import demand function for commodity 2 by the home country as

$$M_2=M_2(P^*, t, L) \tag{13}$$

one can obtain, after some manipulations, two alternative expressions

$$\left.\frac{\mathrm{d}M_2}{\mathrm{d}L}\right|_{t\,\mathrm{const.}} = -\frac{M_2}{P^*}e\frac{\partial P^*}{\partial L} + \frac{b}{\alpha P}\frac{\partial Q}{\partial L} - \frac{1}{\alpha}\frac{\partial X_2}{\partial L} \tag{14}$$

and

$$\left.\frac{\mathrm{d}M_2}{\mathrm{d}L}\right|_{t\,\mathrm{const.}} = -\frac{M_2}{P^*}e\frac{\partial P^*}{\partial L} + \frac{b}{\alpha P}\frac{\partial Q^W}{\partial L} - \frac{\partial X_2}{\partial L} \tag{14'}$$

where

$$e \equiv -\frac{\partial M_2}{\partial P^*} \cdot \frac{P^*}{M_2}$$

is the elasticity of demand for imports for the home country along the home country's offer curve.

Similarly, for the rest of the world (quantities with * refer to those for the rest of the world)

$$\left.\frac{\mathrm{d}M_1^*}{\mathrm{d}L}\right|_{t,\,L^*\,\mathrm{const.}} = \frac{M_1^*}{P^*}e^*\frac{\partial P^*}{\partial L} \tag{15}$$

where

$$e^* \equiv -\frac{\partial M_1^*}{\partial(1/P^*)}\,\frac{1}{P^* M_1^*}$$

is the elasticity of demand for imports by the rest of the world along the rest of the world's offer curve. Evidently, the expression for $\mathrm{d}M_1^*/\mathrm{d}L$ would be more complicated than that given in (15) if L^* is allowed to change with a change in L, such as in the case of migration from abroad.

Totally differentiating the balance of payments equilibrium condition

$$P^* M_2 = M_1^* \tag{16}$$

one gets

$$\frac{\mathrm{d}P^*}{\mathrm{d}L} = \frac{1}{M_2} \cdot \frac{\mathrm{d}M_1^*}{\mathrm{d}L} - \frac{P^*}{M_2} \cdot \frac{\mathrm{d}M_2}{\mathrm{d}L}. \tag{17}$$

Substituting (14) and (15) in (17) leads to

$$\frac{\mathrm{d}P^*}{\mathrm{d}L} = \frac{1}{M_2\Delta}\left[\frac{b}{\alpha(1+t)}\frac{\partial Q}{\partial L} - \frac{P^*}{\alpha}\frac{\partial X_2}{\partial L}\right] \tag{18}$$

whereas the insertion of (14′) and (15) in (17) yields

$$\frac{\mathrm{d}P^*}{\mathrm{d}L} = \frac{1}{M_2\Delta}\left[\frac{b}{\alpha(1+t)}\frac{\partial Q^W}{\partial L} - P^*\frac{\partial X_2}{\partial L}\right] \tag{19}$$

where $\Delta = (e + e^* - 1) > 0$ is the familiar Marshall-Lerner stability condition.

Now, by substituting (14) and (18) in Eq. (7)

$$\frac{\partial Y}{\partial L}=\beta\frac{\partial Q}{\partial L}+P^*\frac{\partial X_2}{\partial L}\left[\frac{1-t(e^*-1)}{\Delta\alpha}\right] \tag{20}$$

where

$$\beta=\frac{et(1-b)+\bar{e}+(e^*-1)(1+t)}{\Delta\alpha(1+t)} \tag{21}$$

In the above expression we have made use of the Slutsky decomposition

$$e=\bar{e}+b \tag{22}$$

where

$$\bar{e}\equiv\frac{\partial M_2}{\partial P^*}\frac{P^*}{M_2}\bigg|_{\bar{Y}}>0$$

is the pure substitution elasticity of demand for imports by the home country along the home country's offer curve.

Suppose, $t=t_{\text{opt}}$. It is well known that at the optimum tariff point $e^*>1$ which implies that $\beta>0$. It further follows, after substituting the value of t_{opt} from (9) into (20), that $[\cdot]$ in (20) reduces to zero. Therefore, at the optimum tariff point, $\partial Y/\partial L>0$ if and only if $\partial Q/\partial L>0$. Optimal tariff rules out the possibility of immiserising (labour) growth.[10] It also follows from (20) that for any country, large or small (since the small country is obviously a special case of the large country), the population size is less than optimal so long as $\partial Q/\partial L>0$, or equivalently, the marginal product of labour is positive,[11] provided an optimum tariff policy is simultaneously pursued. In other words, for a country following optimal tariff policy, optimal population is characterized by zero marginal value product at domestic prices or equivalently, zero marginal product of labour in each industry. No such characterisation of L_{opt} is possible, in general, if the country is following a suboptimal tariff policy.

One can also explain easily why the optimal population rule in the form of zero marginal value product at world prices applies to a small country, irrespective of

[10] Though this is an oft-mentioned proposition, the proof of the result as customarily given in the literature is wrong. For example, Mundell's pioneering article [8] considers a situation of growth with no tariffs ($t=0$) and gets an expression for change in welfare due to growth. He then argues that under optimal tariffs $e^*>1$ which makes the expression positive proving that optimal tariff rules out immiserizing growth. But this proof is invalid since the expression whose positivity he establishes under an optimal tariff was in the first place derived under the assumption of zero tariffs-hence there is an inconsistency in the proof. Our proof is free from that defect. Further, Mundell's analysis might give one the impression that $e^*>1$ is sufficient to rule out immiserizing growth. But as Eq. (20) shows one needs the stronger condition of $t=t_{\text{opt}}$ for this purpose unless one confines to the special case of $dX_2=0$ which Mundell did by assuming complete specialisation. Incidentally, we have not encountered any correct general (in the sense of permitting simultaneous existence of tariffs and monopoly power in trade) *algebraic* proof of the proposition that optimal tariff precludes immiserizing growth, though Bhagwati [1, footnote 1] provided a neat *geometric* proof in terms of Baldwin envelopes in a footnote.

[11] See footnote 6.

whether an optimal tariff policy is followed or not. The reason is simply that a change in population brings about no change in the terms of trade for a small country. To see the point clearly, rewrite (7) as

$$\alpha\frac{\partial Y}{\partial L}=\frac{\partial Q^W}{\partial L}-M_2\frac{\partial P^*}{\partial L} \tag{7a}$$

and (19) as

$$\frac{\partial P^*}{\partial L}=\frac{1}{\alpha M_2\Delta}\left[\frac{b}{1+t}\frac{\partial X_1}{\partial L}-(1-b)P^*\frac{\partial X_2}{\partial L}\right]. \tag{19a}$$

in view of these equations, one can get

$$\alpha\frac{\partial Y}{\partial L}=\frac{\partial Q^W}{\partial L}-\frac{1}{\alpha(1+t)\Delta}\left[b\frac{\partial X_1}{\partial L}-(1-b)P\frac{\partial X_2}{\partial L}\right]. \tag{20a}$$

For a small country $\partial P^*/\partial L=0$ which implies that the $[\cdot]$ in Eq. (20a) vanishes. On the other hand, if for a (large) country $[\cdot]>0$, an increase in population leads to a terms of trade deterioration, and the optimal population is realized at a point where the marginal value product of labour at world prices is positive. It follows that the size of optimal population is smaller for a large country than for a small country in this particular instance. Obviously, the opposite is the case if $[\cdot]<0$.[12]

The analysis so far has assumed population growth only through reproduction of existing national population. Let us turn to the other type of population growth, namely, through migration from abroad. As already noted, Eq. (15) would now take a more complicated form

$$\left.\frac{\mathrm{d}M_1^*}{\mathrm{d}L}\right|_{t\ \mathrm{const.}}=\frac{M_1^*}{P^*}e^*\frac{\partial P^*}{\partial L}+b^*\frac{\partial Q^*}{\partial L}-\frac{\partial X_1^*}{\partial L} \tag{15'}$$

since

$$\frac{\mathrm{d}L}{\mathrm{d}L^*}=-1,\quad \frac{\partial Q^*}{\partial L^*}=-\frac{\partial Q^*}{\partial L}\quad \text{and}\quad \frac{\partial X_1^*}{\partial L^*}=-\frac{\partial X_1^*}{\partial L}$$

in the case of pure migration. In the above expression b^* is the marginal propensity to import (or, the marginal propensity to consume commodity 1), L^* is the size of the population, X_i^* is the level of output of commodity i and $\mathrm{d}Q^*$ represents $(\mathrm{d}X_1^*+P^*\mathrm{d}X_2^*)$, for the rest of the world. If one now solves for $\partial P^*/\partial L$ using (15′) and then works out the expression for $\partial Y/\partial L$ one can get

[12] A sufficient condition for $[\cdot]>0$ is $\mathrm{d}X_1/\mathrm{d}L>0$ and $\mathrm{d}X_2/\mathrm{d}L<0$. In the standard Swedish-Samuelson 2-commodity 2-factor trade theory model, this can be guaranteed by the well-known Rybczynski Theorem if commodity 1 is labour-intensives. This, however, requires constant returns to scale assumption.

$$\frac{\partial Y}{\partial L}=\beta\frac{\partial Q}{\partial L}+P^*\frac{\partial X_2}{\partial L}\left[\frac{1-t(e^*-1)}{\Delta\alpha}\right] + \frac{1}{\Delta}\left\{\frac{\partial X_1^*}{\partial L}(b^*-1)+b^*P^*\frac{\partial X_2^*}{\partial L}\right\} \tag{20'}$$

It follows that our previous result that immiserizing (labour) growth is impossible under an optimal tariff policy is not necessarily valid in the case of migration-caused population growth since $\{\cdot\}$ in (20′) does not necessarily vanish under an optimal tariff. The earlier result that for a country following optimal tariff policy population should be encouraged as long as labor's marginal product is positive also ceases to be valid in the case of migration. One can, of course, think of sufficient conditions under which $\{\cdot\}$ is positive such as $\partial X_1^*/\partial L<0$ and $\partial X_2^*/\partial L>0$.[13] In such a situation, population growth through migration should be encouraged so long as labor's marginal product is positive and the home country is following an optimal tariff policy. This is because the above sufficient conditions imply that the terms of trade tend to go in favor of the home country as a result of output changes (at constant P^*) in the foreign country due to migration, and hence the result obtained in the reproduction case is further reinforced.

Is any ranking possible between L_{opt} under migration and L_{opt} under reproduction for a large country at least in the situation where an optimal tariff policy is being pursued? The answer is: No. Now there are two differences to reckon with—a 'cost of production' difference and a difference in terms of impact on terms of trade. The cost of production difference, under an optimal tariff policy, would tend to make L_{opt} greater with immigration relative to reproduction, by following the same kind of reasoning as used in the small country case. But since, in general, the terms of trade may go either way when the marginal man is an immigrant rather than an offspring, the higher value of output contribution of the immigrant need not imply a higher level of welfare if it causes a terms of trade deterioration through output changes in the foreign country as a result of migration from that country. Hence L_{opt} under immigration need not necessarily be higher than L_{opt} under reproduction, even with an optimal tariff policy.

III. UNDER NON-TRADITIONAL UTILITY FUNCTIONS

Specifically, we shall consider two variants of a social utility function which would be different from the traditional one as is customarily used (and considered in our previous section) in trade theoretic literature. First, consider a social utility

[13] This would be the case if, for example, commodity 1 (the import commodity for the foreign country) is the relatively labour-intensive commodity in the foreign country so that as L is rising,(and correspondingly L^* is falling), X_1^* is falling and $X_2{}^*$ is rising, in accordance with the Rybczynski logic. Note, however, that to apply the Rybczynski theorem, one has to assume constant returns to scale in the foreign country.

function which depends on the per capita or average levels of consumption of the two commodities such as

$$u = u(c_1, c_2) \tag{22}$$

where u is the level of social utility and $c_i = (C_i/L)$ is the average level of consumption of commodity i. This is known as the Cannan-Wicksell-Wolfe (CWW) formulation in the literature.[14] A set of sufficient conditions under which (22) would have both behavioral and welfare significance would be: (i) every individual within the country has identical utility function, and (ii) the income level of all individuals within a country is identical so that the per capita consumption is also the actual consumption of each individual. Maximising the traditional utility function would appear to be rational if the population growth is through reproduction of already existing national population or, in the case of migration, if the immigrants are considered a part of the national population and in addition, the national government is interested in maximizing the total size of the cake available to its national population. On the other hand, if the welfare of the *already* existing national population is the prime concern of a national government, one can, then, reasonably expect the authorities to permit or encourage population growth to the extent it improves the (average)[15] lot of the pre-existing national population. This may particularly be the case if the contemplated population increase is through further immigration rather than reproduction of existing population. The national policymakers, even when they are interested in the welfare of their offsprings at the cost of some reduction in their own level of well being, may simply be reluctant to permit further immigration if it causes any reduction in their own standard of living. From that point of view, maximizing the utility function as in (22) may be considered a more attractive exercise specially for the immigration case. There is, of course, a third alternative, namely, to maximise the average happiness multiplied by the number of individuals enjoying that level of happiness, in other words, to maximise $u \cdot L$. This is known as Sidgwick-Meade (SM) formulation in the literature. As already explained, this may be specially relevant for the reproduction case rather than the immigration case.[16]

Equations (2), (3), (4) would now be replaced by

$$\frac{\partial u/\partial c_2}{\partial u/\partial c_1} = P \tag{23}$$

$$c_1 + P^* c_2 = x_1 + P^* x_2 \tag{24}$$

$$c_i = x_i + m_i \tag{25}$$

[14] See Dasgupta [5].

[15] Note that the average lot is the same as the actual lot of a representative individual under the specification (22).

[16] See Dasgupta [5], [6] and Meade [7] for a more detailed discussion on the relative merits of the CWW and SM formulations in a closed economy setting.

where the lower-case letters c_i, x_i, m_i, refer to per capita values (C_i/L), (X_i/L) and (M_i/L), respectively. Equation (5) would remain intact. (6) would take the form of

$$\text{(26)} \qquad \mathrm{d}y = \frac{\partial u}{\partial u/\partial c_1} = -m_2\mathrm{d}P^* + P^*t\mathrm{d}m_2 + (\mathrm{d}x_1 + P\mathrm{d}x_2)\,.$$

setting $\mathrm{d}y\,|_{L\,\text{const.}} = 0$, one gets

$$\text{(27)} \qquad -m_2\mathrm{d}P^* + P^*t\mathrm{d}m_2 = 0$$

which gives

$$\text{(28)} \qquad t_{\text{opt}} = \frac{m_2}{\mathrm{d}m_2}\frac{\mathrm{d}P^*}{P^*} = \frac{\hat{P}^*}{\hat{m}_2}$$

where hat ($\hat{}$) over a variable denotes proportionate change in the variable (such as $\hat{A} \equiv \mathrm{d}A/A$).

Total differentiation of the balance of payments equilibrium condition (16) yields

$$\text{(29)} \qquad \begin{aligned} \hat{P}^* &= \hat{M}_1^* - \hat{M}_2 \\ &= \hat{M}_1^* - \hat{m}_2 \end{aligned}$$

since $\hat{m}_2 = \hat{M}_2 - \hat{L}$ and $\hat{L} = 0$ by assumption.
Substituting (29) in (28)

$$\text{(30)} \qquad t_{\text{opt}} = \frac{1}{e^* - 1}\,.$$

Thus, one gets the usual optimum tariff *formula* even when one is working in terms of the per capita magnitudes since L is being held fixed. Of course, as already explained in the previous section, the *rate* of optimum tariff would vary depending on the value at which L is held constant.

Again, following the same type of analysis as in Section II, one can obtain, for the small country case ($\mathrm{d}P^* = 0$),

$$\text{(31)} \qquad \alpha\frac{\partial y}{\partial L} = \frac{\partial q^W}{\partial L}$$

where

$$\frac{\partial q^W}{\partial L} = \frac{\partial x_1}{\partial L} + P^*\frac{\partial x_2}{\partial L}\,.$$

Therefore,

$$\frac{\partial y}{\partial L} > 0 \qquad \text{if and only if} \quad \frac{\partial q^W}{\partial L} > 0\,.$$

The optimal population for a small country should be such as to maximise the value of per capita output at *world* prices. Using the well known relationship between average and marginal magnitudes, one can equivalently say that population attains the optimal level when q^W ($\equiv x_1 + P^* x_2$), the value of per capita output at world prices, is equal to $\partial Q^W/\partial L$, the value of incremental output (at world prices) due to an additional worker.

Let us turn again to the question of immigration versus reproduction. For a small country P^*, the terms of trade, is the same in both cases but there is the cost of production difference. Assume we are at $\tilde{L}$, with the marginal man an offspring of existing national population, where $q^W = \partial Q^W/\partial L$. Therefore, $\tilde{L}$ is the same as L_{opt} under reproduction. Now, suppose, the marginal man is instead an immigrant. Because of the lower cost of production of population through immigration the shift in the production possibility curve caused by the marginal man would now be greater. With free trade ($P = P^*$), $\partial Q^W/\partial L$ would then be higher. q^W, the value of average output, would also be higher but it would go up by a smaller amount than the increase in $\partial Q^W/\partial L$. This is because the given increase in $\partial Q^W/\partial L$ would be distributed over all intramarginal units of labor to produce a much smaller increase in q^W. So $\partial Q^W/\partial L$ would exceed q^W at $\tilde{L}$, implying that optimal population with immigration would be higher than that with reproduction so long as free trade policy is followed. With restricted trade ($P \neq P^*$), as already explained in Section II, larger shift in the production possibility curve need not imply a higher value of $\partial Q^W/\partial L$ and hence no ranking of optimal population under the two alternatives modes of population increase is generally possible.

Similarly, for the large country case, one can obtain

$$\frac{\partial y}{\partial L} = \beta \frac{\partial q}{\partial L} + P^* \frac{\partial x_2}{\partial L}\left[\frac{1 - t(e^* - 1)}{\Delta \alpha}\right] \tag{32}$$

which would imply, following an analogous logic to that used in Section II, that under an optimal tariff policy the optimal population should be such as to maximise the value of per capita output at domestic prices. In other words, for a country pursuing optimal tariff policy population reaches optimum where the value of the marginal product of labor is equal to the value of per capita output at domestic prices.[17]

For (analogous) reasons already given in Section II, L_{opt} under immigration may be greater or smaller than L_{opt} under reproduction, even under an optimal tariff policy. To avoid repetition, the details are omitted here.

Let us now turn to the SM-formulation of the social utility function defined as

$$W = u \cdot L \tag{33}$$

[17] Note that

$$\frac{\partial q}{\partial L} = \frac{1}{L}\left[\frac{\partial X_1}{\partial L_1} - q\right]$$

where $q \equiv x_1 + P x_2$ and $\partial X_1/\partial L_1$ is as defined in footnote 6.

where u is as defined in (22).[18]

It would follow from (33) that dz, defined as the change in social utility in terms of commodity 1, would be

$$\text{(34)} \qquad dz \equiv \frac{dW}{(\partial u/\partial c_1)} = L \cdot dy + y dL$$

where y is defined as $u/(\partial u/\partial c_1) > 0$.

Substituting for dy, as obtained in (26), into (34)

$$\text{(35)} \qquad dz = L[-m_2 dP^* + P^* t dm_2 + dx_1 + P dx_2] + y dL$$

Now, setting

$$\left.\frac{dz}{dt}\right|_{L\,\text{const.}} = 0\,,$$

the solution for the optimal tariff can be seen to be the usual one

$$\text{(36)} \qquad t_{opt} = \frac{1}{e^* - 1}$$

Though the optimum tariff formula is the same under all formulations of the social utility function, the rate of optimum tariff need not be the same. More specifically, with the same value of L, u and uL (but not necessarily U) would reach their maximum at the same value of t since both demand and supply conditions and hence the home "trade indifference map" would be the same under each formulation.[19] On the other hand, if under each formulation L is held at the corresponding L_{opt} level (which is different under different formulations), t_{opt} would also differ because now the supply conditions (the production possibility curve) would be different under different formulations.

The solution for the optimal population would now be given by setting

[18] Following Meade [7], one can also introduce the concept of a 'welfare subsistence level' of consumption (which may be above the physical subsistence level) for a representative individual which is so "low" that for him life is just about worth living, yielding zero utility. In our 2-commodity model, this may take the form of specifying some (positive) minimum levels of consumption of the two commodities $(\bar{c}_1, \bar{c}_2)$ such that

$$u(c_1, c_2) = 0 \qquad \text{if} \quad c_1 = \bar{c}_1 \quad \text{or} \quad c_2 = \bar{c}_2$$

and

$$u(c_1, c_2) < 0 \qquad \text{if} \quad c_1 < \bar{c}_1 \quad \text{or} \quad c_2 < \bar{c}_2\,.$$

Note, however, that at L_{opt} consumption must be above the welfare subsistence level—otherwise social utility becomes zero or negative which obviously cannot be an optimum. Hence we have not explicitly introduced the welfare subsistence level in the algebra of the paper.

[19] Note that with fixed L, uL is a monotonic transformation of u.

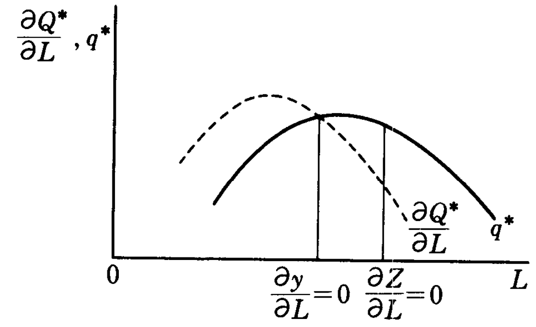

Fig. 4.

$$\left.\frac{\mathrm{d}z}{\mathrm{d}L}\right|_{t\ \text{const.}}=0$$

which yields

(38) $$\frac{\mathrm{d}y}{\mathrm{d}L}=-\frac{y}{L}<0$$

at the optimum point.

For the small country, we know from (31) that

$$\frac{\mathrm{d}y}{\mathrm{d}L}=\frac{1}{\alpha}\frac{\partial q^W}{\partial L}$$

which has then to be equal to $-(y/L)<0$ at the optimum point. It follows that for the optimum $(\partial z/\partial L=0)$

(39) $$\frac{\partial q^W}{\partial L}=-\frac{y}{L}\cdot\alpha<0\,.$$

Since L_{opt} was characterised by $\partial q^W/\partial L=0$ under the CWW-formulation but by $\partial q^W/\partial L<0$ under the SM-formulation it is clear that L_{opt} under the SM-formulation must be greater than L_{opt} under the CWW-formulation, as depicted in Fig. 4.

One can further characterize the optimal population under the SM-formulation as one where [from Eq. (34)]

$$\frac{\partial z}{\partial L}=L\frac{\partial y}{\partial L}+y=0$$

which would lead to

(40) $$y=\frac{1}{\alpha}\left[q^W-\frac{\partial Q^W}{\partial L}\right]$$

at the optimum point. This is readily amenable to simple economic interpretation.[20] y is the gain in welfare to an additional man. q^W is the value (at world prices) of consumption of the additional man (the same as the value of average consumption or average output at world prices) and $\partial Q^W/\partial L$ is his contribution to the value of national output. Therefore,

$$\frac{1}{\alpha}\left[q^W - \frac{\partial Q^W}{\partial L}\right]$$

is the welfare loss the additional man is inflicting on the existing population by reducing their consumption by $(q^W - (\partial Q^W/\partial L))$. At the optimal population size, the welfare gain of an additional man must be the same as the welfare loss of the existing population due to the entry of the additional man.

Again, take up the case of immigration versus reproduction. Suppose, we are at $\tilde{L}$, with the marginal man an offspring of existing population, where (40) holds. So, $\tilde{L}$ is L_{opt} with reproduction. Now suppose, the marginal man at $\tilde{L}$ is instead an immigrant. Thèn, with $t=0$, $\partial Q^W/\partial L$ increases (due to difference in 'cost of production'), q^W increases, though by a smaller amount than the increase in $\partial Q^W/\partial L$, and y increases.[21] Therefore, the gain to the marginal man goes up and the loss to the existing population (as measured by the R.H.S. expression in (40)) goes down. The marginal immigrant at $\tilde{L}$ is clearly desirable implying L_{opt} with immigration greater than L_{opt} with reproduction. With $t \neq 0$, as already explained in connection with CWW-formulation, $\partial Q^W/\partial L$ need not necessarily go up when the marginal man is an immigrant rather than offspring and hence, no general ranking in such a situation is possible.

We can now establish the result that for a large country following optimal tariff policy, L_{opt} under SM-formulation would be larger than that under the CWW-formulation. To see that, note

$$\frac{\partial z}{\partial L} = L\frac{\partial y}{\partial L} + y$$

or, substituting for $\partial y/\partial L$ from (32)

(41) $$\frac{\partial z}{\partial L} = L\left[\beta\frac{\partial q}{\partial L} + P^*\frac{\partial x_2}{\partial L}\left\{\frac{1-t(e^*-1)}{\Delta\alpha}\right\}\right] + y\,.$$

Setting $\partial z/\partial L = 0$ and assuming $t = t_{\text{opt}}$, which implies $\beta > 0$ and $t = 1/(e^*-1)$, one can obtain

(42) $$\frac{\partial q}{\partial L} = -\frac{1}{\beta}\cdot\frac{y}{L} < 0$$

[20] Meade [7] provided a similar interpretation for the more simplified version of Eq. (40) which he obtained in his one-good closed economy model. In fact, (40) can be considered a generalisation of the Meade condition in an open economy setting.

[21] From (31), y increases whenever q^W-increases.

at the optimal population point. Since the same assumptions under the CWW-formulation led to $\partial q/\partial L=0$ at the optimum point, the result is immediately established.

For reasons already given in Section II, no general ranking would be possible between L_{opt} under immigration and L_{opt} under reproduction for a large country even when it is simultaneously adjusting its tariff rate to the optimal level.

Finally, let us consider another possible difference between the immigration case and the reproduction case, apart from the differences in terms of 'costs of production' and terms of trade. Due to considerations already mentioned, the 'average utility criterion' (maximising u) may be appropriate for optimal population via immigration but the 'total utility criterion' (maximising uL) may be relevant for determining optimal population through reproduction. Since we have already proved that L_{opt} under SM-formulation is greater than L_{opt} under CWW-formulation for a small country pursuing free trade, it would follow that for such a country (if we ignore the 'cost of production' difference)[22] L_{opt} under immigration would be less than L_{opt} with reproduction. However, the cost of production difference under a given criterion would tend to make L_{opt} greater under immigration relative to reproduction. Therefore, L_{opt} with immigration may be greater or smaller than L_{opt} with reproduction, even for a small country practicing free trade, when we consider at the same time all three types of differences between the immigration case and the reproduction case. Needless to say, the same would be true for a large country, even with an optimal tariff policy, with the added complication of a terms of trade differential which may go either way.

IV. CONCLUDING REMARKS

So, we have considered the optimal population and the optimal tariff problems in an open economy setting under three alternative formulations of the social utility function. Throughout, all income has been assumed to be immediately consumed and all considerations of intertemporal optimality have been assumed away. If variability of saving is allowed in a model involving time then a solution of the optimal population/saving problems would require a variational analysis. As a beginning, we have abstracted from all these complications which may be incorporated in a subsequent exercise. The important question of adjustment, over time, of actual population to its optimal level under both immigration and reproduction can also be investigated. Other formulations of the social utility function, apart from the three explicitly considered in this paper, may also be taken up. Finally, one can consider the population size to be an "intermediate" instrument in that it is itself influenced by more basic factors such as the gap between the national wage rate the wage rate prevailing abroad, the birth rate

[22] Note, again, that P^* is the same under immigration and reproduction for a small country.

which may in turn be influenced by the excess of the wage rate over a subsistance level etc. and can work out the analysis in terms of such more basic determinants.

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