MONOPOLY AND GENERAL EQUILIBRIUM*

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1. INTRODUCTION

In 1930s, economists paid much attention to the problems of monopoly and monopolistic competition. They dealt with these problems within the framework of partial equilibrium analysis. Although these studies brought about fruitful results, they have serious drawbacks in several respects. In particular, these theories did not make it clear who exploits whom. That is, it was not clear who gains and loses through monopoly. Monopoly exploitation simply meant real wage rate is less than marginal product of labor in monopolistic equilibrium. Moreover, these theories did not take account of the effect of an increase in monopolistic profits on consumers' demand. In other words, they did not consider that demand depends on supply. This paper presents a new general equilibrium model of monopoly, taking account of the interdependence of demand and supply.

Our model, enabling us to deal with income distribution, including profit, consistently within the framework of general equilibrium analysis, gives itself a merit in comparison with traditional ones. In our model, we see clearly that the production manager, considering shareholders' (capitalists') benefits, has an incentive to behave monopolistically. In other words, capitalists exploit laborers by controlling the production managers. We reach this result by comparing monopolistic equilibrium with competitive one in utility space, which is commonly used in welfare economics. Focusing our attention on the possession of means of production, we reexamine who gains and who loses through monopoly.

This paper is organized as follows. In Section 2, a general equilibrium model of monopoly is developed, and monopoly exploitation is formally defined. In Section 3, we compare monopolistic states with competitive ones by an example of the model described in the preceding section. In addition, we examine how the monopolistic equilibrium is attained. Section 4 is assigned to concluding remarks.

2. GENERAL MODEL

We deal with Arrow-Debreu economies throughout this section.[1] In an economy, there are l commodities, m consumers and n producers, when m and n

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[1] Almost all symbols and notations follow those in Debreu [1].

are both positive integers. Consumer i's consumption set is denoted by X_i. Producer j's production set is denoted by Y_j. They are both certain subsets of $\boldsymbol{R}^l$.[2] $\gtrsim_i$ is consumer i's preference, which is a complete preordering on X_i. ω_i is consumer i's initial endowment, which is a point in X_i. θ_{ij} denotes consumer i's share of the profit of firm j. Assume $\theta_{ij} \geqq 0$ for all j. Of course $\sum_j \theta_{ij} = 1$ for all i. We divide consumers into two categories. One is the *capitalist* class whose members receive profits, i.e., $\theta_{ij} > 0$. Another is the *laborer* class whose members receive no profit, i.e., $\theta_{ij} = 0$ for any j. Throughout this section, we often write x, y, ω and Y for $\sum_i x_i$, $\sum_j y_j$, $\sum_i \omega_i$ and $\sum_j Y_j$ respectively. Δ denotes a set, $\{p\colon p \in \boldsymbol{R}^l$ and $\sum_i p_i = 1\}$. Δ is called price simplex.

We assume:

Assumption 1. For every i,

(a) X_i is closed, convex and has a lower bound for $\leqq$,

(b.1) there is no satiation consumption in X_i,[3]

(b.2) for every x_i' in X_i, the sets $\{x_i \in X_i\colon x_i \gtrsim_i x_i'\}$ and $\{x_i \in X_i\colon x_i' \gtrsim_i x_i\}$ are closed in X_i,

(b.3) if x_i^1 and x_i^2 are two points of X_i and if t is a real number in $]0, 1[$, then $x_i^2 \gtrsim_i x_i^1$ implies $tx_i^2 + (1-t)x_i^1 \gtrsim_i x_i^1$,

(c) there is x_i^0 in X_i such that $x_i^0 < \omega_i$.

For every j,

(d.1) $0 \in Y_j$,

(d.2) Y_j is closed and convex;

(d.3) $Y \cap (-Y) \subset \{0\}$,

(d.4) $Y \supset (-\boldsymbol{R}^l_+)$.

We now define a competitive equilibrium which we compare with a monopolistic equilibrium to be defined later.

Definition 1. A *competitive equilibrium* is a point, $((x_i^*)_i, (y_j^*)_j, p^*, (\pi_j^*)_j)$ in $\prod_i X_i \times \prod_j Y_j \times \Delta \times \boldsymbol{R}^n_+$ such that;

(1) x_i^* is a greatest element of $\{x_i \in X_i\colon p^* \cdot x_i \leqq p^* \cdot \omega_i + \sum_j \theta_{ij} \pi_j^*\}$ for $\gtrsim_i$, for every i,

(2) $p^* \cdot y_j^* \geqq p^* \cdot y$ for every y in Y_j, for every j,

(3) $x^* - y^* = \omega$,

(4) $\pi_j{}^* = p^* \cdot y_j^*$ for every j.

Following Debreu [1, p. 83, (1)], we have:

THEOREM 1. *There is a competitive equilibrium under Assumption* 1.

Next, let us define a monopolistic equilibrium. At first, consider a special Arrow-Debreu economy which contains only one firm. Here, we omit the firm's index.

[2] $\boldsymbol{R}^l$ denotes an l-dimensional real space.

[3] That is, for every $x_i \in X_i$, there is $x_i' \in X_i$ such that $x_i \gtrsim_i x_i'$ does not hold.

Definition 2. A *monopolistic equilibrium* is a point, $((\tilde{x}_i)_i, \tilde{y}, \tilde{p}, \tilde{\pi})$ in $\prod_i X_i \times Y \times \Delta \times \boldsymbol{R}_+$ such that;

(5) $\tilde{x}_i$ is a greatest element of $\{x_i \in X_i\colon \tilde{p}\cdot x_i \leqq \tilde{p}\cdot \omega_i + \theta_i \tilde{\pi}_j\}$ for $\precsim_i$, for every i,

(6) $\tilde{p}\cdot\tilde{y} \geqq p\cdot y$ for all p and y such that $y=\hat{x}-\omega$, $y\in Y$ and $p\in\Delta$ for some $\hat{x}$ such that $\hat{x}$ is a sum of greatest elements, $\hat{x}_i$ of $\{x_i\in X_i\colon p\cdot x_i \leqq p\cdot\omega_i+\theta_i p\cdot y\}$ for $\precsim_i$,

(7) $\tilde{\pi}=\tilde{p}\cdot\tilde{y}$.

Let n be equal to one in Definition 1 and compare Definition 1 with Definition 2. Difference between them is in the behavior of the firm. (See (2) and (6).) On the one hand, in Definition 1, the firm maximizes its profit under a given price system. On the other hand, in Definition 2, the firm maximizes its profit under a given total demand function or correspondence.[4]

Our main theorem is:

THEOREM 2. *Arrow-Debreu economy which contains only one firm has a monopolistic equilibrium under Assumption* 1.

The following lemma is used for providing Theorem 2.

LEMMA. *An economy* $\mathscr{E}'=((X_i,\precsim_i)_i,\ -R^l_+,\ (\omega_i)_i,\ (\theta_i)_i)$ *has a competitive equilibrium under Assumption* 1.

The equilibrium of such an economy is called *competitive exchange equilibrium.* The proof is omitted here, since this lemma is a corollary of Theorem 1.

Proof of Theorem 2. Let A be a set of attainable states. $\hat{X}_i$ is defined as $\mathrm{proj}_{X_i} A$. Our attention is restricted to $\tilde{X}_i = X_i \cap K$, where K is a closed cube of R^l with center 0, containing $\mathrm{proj}_Y A$ and any $\mathrm{proj}_{X_i} A$ in its interior. Since A is compact under our assumption,[5] there is some K such that $\hat{X}_i \subset \tilde{X}_i$ for all i. Following Debreu [1, Ch. 4], demand correspondence of consumer i, which is upper hemi continuous, is defined. It is denoted by ξ_i. Define $S'=\bigcap_i S'_i$. And define $\tilde{\xi}_i(p,\pi)=\xi_i(p, p\cdot\omega_i+\theta_i\pi)$. Since $p\cdot\omega_i+\theta_i\pi$ is a continuous function of p and π, $\tilde{\xi}_i$ is upper hemi continuous in a proper domain. Clearly, $\tilde{\xi}_i$ is positive homogeneous of degree zero because ξ_i is so.

Define $\tilde{\xi}(p,\pi)=\sum_i\tilde{\xi}_i(p,\pi)$ and $\lambda(p,\pi)=\tilde{\xi}(p,\pi)-\{\omega\}$. And define $D=\{(p,\pi)\colon (p,\pi)\in\mathrm{proj}_l S'\times\boldsymbol{R}_+$ and $\lambda(p,\pi)\cap Y\neq\varnothing\}$. Note that D is a cone not containing 0 because of positive homogenity of λ. From the above lemma and the fact that $0\in Y$, it follows that $(p^{**},0)\in D$, where p^{**} is a competitive exchange equilibrium price system of $\mathscr{E}'=((X_i,\precsim_i)_i,\ -\boldsymbol{R}^l_+,\ (\omega_i)_i,\ (\theta_i)_i)$. Hence, D is non-empty.

Let D' be $(\Delta\times\boldsymbol{R}_+)\cap D$. Since D is a non-empty cone, D' is non-empty. In addition, D' is closed because of the upper hemi continuity of λ. The boundness of D' is clear from the compactness of A and Δ. Hence, D' is compact.

[4] In case of multiple firm economies, it is difficult to define the equilibrium of monopolistic competition because of firms' intricate price setting behavior. Assuming sloped subjective demand function of firms, Negishi [2] considered monopolistic competition in a general equilibrium model.

[5] See Debreu [1, p. 77, (1) and (2)].

The projection which is a function on D' onto $\boldsymbol{R}_+$ has a maximum because of its continuity and the compactness of D'. Let $(\tilde{p}, \tilde{\pi}) \in D'$ be one of the maximizers. By definition of D, there is $\tilde{y}$ such that $\tilde{y} \in Y$ and $\tilde{y} \in \lambda(\tilde{p}, \tilde{\pi})$. There is a $(\tilde{x}_i)_i$ such that $\tilde{y} = \sum_i \tilde{x}_i - \omega$ and $\tilde{x}_i \in \tilde{\xi}_i(\tilde{p}, \tilde{\pi})$ for any i. Thus, $\tilde{p} \cdot \tilde{y} = \sum_i \tilde{p} \cdot \tilde{x}_i - \tilde{p} \cdot \omega$. On the other hand, $\tilde{p} \cdot \tilde{x}_i = \tilde{p} \cdot \omega_i + \theta_i \tilde{\pi}$ because $\tilde{x}_i \in \tilde{\xi}_i(\tilde{p}, \tilde{\pi})$ and (c) of Assumption 1. Hence, $\tilde{p} \cdot \tilde{x} = \tilde{p} \cdot \omega + \tilde{\pi}$. Then, $\tilde{\pi} = \tilde{p} \cdot \tilde{y}$. $((\tilde{x}_i)_i, \tilde{y}, \tilde{p}, \tilde{\pi})$ is a monopolistic equilibrium.

Q.E.D.

Remark 1. $\tilde{\pi} \geqq p^* \cdot y^*$, where p^* is a competitive equilibrium price system and y^*, a competitive producer's action in economy $\mathscr{E} = ((X_i, \succsim_i)_i, Y, (\omega_i)_i, (\theta_i)_i)$. It results from the fact that $(p^*, p^* \cdot y^*) \in D'$. In words, the firm's profit in monopolistic equilibrium is always not smaller than that in competitive equilibrium. It follows that if the production manager of the firm has sufficient information as to other agents' demand conditions, he must be induced to choose a monopolistic action whenever he tries to maximize profit.

Remark 2. The convexity of production set Y is not essential in the above proof if the compactness of attainable set A is assured. It follows that there possibly exists a monopolistic equilibrium under non-convex production environments.

Remark 3. Our system can be interpreted as a joint profit maximization model in a multiple firm economy. That is, forming a coalition, firms maximize joint profit under given consumers' demand conditions.

Now, let us consider a multiple firm economy where production decisions are not centralized. In order to define a monopolistic equilibrium in such an economy, we consider the notion of partial monopoly, which has been discussed in partial equilibrium analysis.[6] It is assumed here that there are n firms and that the n'th firm is a monopolist. Definition 2 is modified in the following manner.

Definition 3. A *partially monopolistic equilibrium with one monopolist*, n, is a point, $((\check{x}_i)_i, (\check{y}_j)_j, \check{p}, (\check{\pi}_j)_j)$ in $\prod_i X_i \times \prod_j Y_j \times \Delta \times \boldsymbol{R}^n_+$ such that;

(8) $\check{x}_i$ is a greatest element of

$$\left\{ x_i \in X_i \colon \check{p} \cdot x_i \leqq \check{p} \cdot \omega_i + \sum_j \theta_{ij} \check{\pi}_j \right\}$$

for $\succsim_i$, for every i,

(9) $\check{p} \cdot \check{y}_j \geqq \check{p} \cdot y$ for all y in Y_j, for every j such that $j \neq n$,

(10) $\check{p} \cdot \check{y}_n \geqq p \cdot y_n$ for all p in Δ and y_n in Y_n such that

$$y_n = \hat{x} - \omega - \sum_{j \neq n} \hat{y}_j$$

for some $(\hat{x}_i)_i$ and $(\hat{y}_j)_{j \neq n}$ such that for every i, $\hat{x}_i$ is a greatest element of

[6] See Stigler [3].

$$\left\{x_i \in X_i \colon p \cdot x_i \leqq p \cdot \omega_i + \sum_{j \neq n} \theta_{ij} p \cdot \hat{y}_j + \theta_{in} p \cdot y_n \right\}$$

for $\precsim_i$ and $p \cdot \hat{y}_j \geqq p \cdot y_j$ for all y_j in Y_j for every $j \neq n$,

(11) $\check{\pi} = \check{p} \cdot \check{y}_j$ for every j.

This equilibrium is a simple extension of the monopolistic equilibrium of Definition 2. There are some competitive firms besides one monopolist in Definition 3, while there is none besides one monopolist in Definition 2. In both cases, the monopolist maximizes his profit under given demand of other agents. By using a technique similar to the one in the proof of Theorem 2, one can derive the following theorem:

THEOREM 3. *Arrow-Debreu economy has a partially monopolistic equilibrium under Assumption* 1.

Sketch of Proof. In this case, the total net demand function for the monopolist n is defined as

$$\lambda(p, \pi_n) = \sum_i \xi_i(p, p \cdot \omega_i + \sum_{j \neq n} \theta_{ij} p \cdot \eta_j(p) + \theta_{in} \pi_n) - \{\omega\} - \sum_{j \neq n} \eta_j(p),$$

where η_j is a producer j's supply correspondence when he maximizes his profit under a given price system. (See Debreu [1, Ch. 3].) Moreover, the set D is defined as a relation between p and π_n.

We are interested in what influence monopoly brings about on capitalists' and laborers' welfare positions. We now define monopoly exploitation. Monopoly exploitation intuitively means that in a monopolistic equilibrium of the economy, those who own means of production are able to take an advantageous position relative to those who own no means of production. It is one of the objectives of this paper to show that there is an economy where capitalists exploit laborers through making the production manager choose monopolistic action. Let the n'th firm be a monopolist. Define three sets as follow:

$$C_a = \{i \colon \theta_{ij} > 0 \text{ for some } j\},$$

$$C_m = \{i \colon \theta_{in} > 0\},$$

$$L_a = \{i \colon \theta_{ij} = 0 \text{ for every } j\}.$$

C_a (resp. L_a) is a set of capitalists (resp. laborers). C_m is a subset of C_a. Its member is called the *monopolistic capitalist*. Let $(\tilde{x}_i)_i$ be a consumption allocation in (partially) monopolistic equilibrium, and $(x_i^*)_i$, that in competitive equilibrium of an economy. For each i, denote $u_i(\tilde{x}_i)$ by $\tilde{u}_i$ and $u_i(x_i^*)$ by u_i^*, where $u_i(\cdot)$ is a utility function representing consumer i's preference preordering, $\precsim_i$.

Definition 4. There is monopoly exploitation if and only if

$$\begin{array}{lll} \tilde{u}_i \geqq u_i^* & \text{for any } i \text{ in } C_m, & \\ \tilde{u}_i \leqq u_i^* & \text{for any } i \text{ in } L_a, & \\ \tilde{u}_i > u_i^* & \text{for some } i \text{ in } C_m & \text{and} \\ \tilde{u}_i < u_i^* & \text{for some } i \text{ in } L_a. & \end{array}$$

Note that if there is monopoly exploitation, a change from a monopolistic (resp. competitive) equilibrium to a competitive (resp. monopolistic) one is not Pareto improving.[7] It should be noted that our concept of exploitation has no relation to Marx's theory. In the next section, it is suggested that there is an abstract economy where capitalists exploit laborers under homogeneous preferences and initial endowments.

3. EXAMINATION BY AN EXAMPLE

Though we wish to examine the economy where monopolistic capitalists exploit laborers in our sense, it is not easy to do so generally. However, it can be shown that in the class of Arrow-Debreu economies, there exist some economies where monopolistic capitalists exploit laborers.

Suppose some homogeneous preferences which are represented by a Cobb-Douglas utility function and that the initial endowment of commodities is the same for all consumers. Suppose also that the number of elements of the set C_a is equal to that of L_a. We may consider only two representative consumers in the situation described above: a capitalist and a laborer. For simplicity, assume that there is only one firm. Thus, indices indicating the firm are omitted throughout this section. The representative capitalist (resp. laborer) is indexed by 1 (resp. 2) in the following part of this section. Hence, $\theta_1 = 1$ and $\theta_2 = 0$. Note here that $C_a = C_m$. Moreover, let there be only two commodities labelled by 1 and 2.

$$Y = \{(y_1, y_2) \colon y_1 \leqq 0 \text{ and } y_2 \leqq (-y_1)^{1/2}\}.$$

Utility function for the two consumers are:

$$u_1(x_{11}, x_{12}) = x_{11} \cdot x_{12} \qquad \text{and} \qquad u_2(x_{21}, x_{22}) = x_{21} \cdot x_{22}.$$

Consumption sets are both non-negative orthant in R^2. Initial endowments for the two consumers are:

$$\omega_1 = (\omega_{11}, \omega_{12}) = (2, 1) = (\omega_{21}, \omega_{22}) = \omega_2.$$

To begin with, we consider two competitive equilibria, with and without production. Competitive equilibrium allocation without production is the initial

[7] According to this definition of monopoly exploitation, there is no such exploitation if $\tilde{u} > u_i^*$ for some i in C_m while $\tilde{u}_k < u_k^*$ for some k in C_m.

endowment allocation itself. The utility level of two representative consumers is equal to 2 in equilibrium.

$$(u_1^{**}, u_2^{**}) = (u_1(\omega_1), u_2(\omega_2)) = (2, 2) .$$

This notation will be used in the following part.

When production is allowed for, demand and supply functions are obtained by the usual Lagrangean multiplier method. For the capitalist,

$$(1) \qquad x_{11} = \frac{1}{2}\left(2 + \frac{p_2}{p_1} + \frac{p_2^2}{4p_1^2}\right) = \frac{1}{2}\left(2 + q + \frac{1}{4}q^2\right) \qquad \text{and}$$

$$(2) \qquad x_{12} = \frac{1}{2}\left(\frac{2p_1}{p_2} + 1 + \frac{p_2}{4p_1}\right) = \frac{1}{2}\left(\frac{2}{q} + 1 + \frac{1}{4}q\right).$$

For the laborer,

$$(3) \qquad x_{21} = \frac{1}{2}\left(2 + \frac{p_2}{p_1}\right) = \frac{1}{2}(2 + q) \qquad \text{and}$$

$$(4) \qquad x_{22} = \frac{1}{2}\left(\frac{2p_1}{p_2} + 1\right) = \frac{1}{2}\left(\frac{2}{q} + 1\right).$$

Note here that q is relative price system, p_2/p_1. In the following part of this section, the above functions are denoted by $x_{11}(p_1, p_2)$ or $x_{11}(q)$, etc. For the producer,

$$(5) \qquad y_1 = -\frac{p_2^2}{4p_1^2} = -\frac{1}{4}q^2 \qquad \text{and}$$

$$(6) \qquad y_2 = \frac{p_2}{2p_1} = \frac{1}{2}q .$$

The profit of the firm is a function of prices:

$$\pi = \frac{p_2^2}{4p_1} .$$

We derive conditions for competitive equilibrium with production from the equalities (1) through (6). The values of unknowns in competitive equilibrium are:

$$(7) \qquad \begin{array}{lllll} x_{11}^* = \frac{17}{9}, & x_{12}^* = \frac{17}{12}, & x_{21}^* = \frac{5}{3}, & x_{22}^* = \frac{5}{4} & \\ y_1^* = -\frac{4}{9}, & y_2^* = \frac{2}{3}, & q^* = \frac{4}{3} & \text{and} & \pi^* = \frac{4}{21}, \end{array}$$

where the profit is evaluated under the condition that $p_1 + p_2 = 1$.

Having found competitive equilibrium, let us now consider monopolistic equilibrium. Remember that the profit of the firm is a function of production plan and price system, and that $\theta_1 = 1$. Thus, the capitalist's demand function has to be modified:

$$\hat{x}_{11}=\frac{1}{2}\left(2+\frac{p_2}{p_1}+\left(-y_2^2+\frac{p_2}{p_1}y_2\right)\right) \tag{8}$$

$$=\frac{1}{2}(2+q+(-y_2^2+qy_2)) \quad \text{and}$$

$$\hat{x}_{12}=\frac{1}{2}\left(\frac{2p_1}{p_2}+1+\left(-\frac{p_1}{p_2}y_2^2+y_2\right)\right) \tag{9}$$

$$=\frac{1}{2}\left(\frac{2}{q}+1+\left(-\frac{1}{q}y_2^2+y_2\right)\right).$$

Note that, for simplicity, we have focused our attention on the boundary of the production set Y. Thus, we used the equality $y_1+y_2^2=0$. (See the definition of Y.)

At this stage, we have to know the inverse demand function, which can easily be found in this simple case by solving the following equation with respect to y_2.

$$\hat{x}_{11}(q, y_2)+x_{21}(q)=-y_2^2+4 \tag{10}$$

or

$$\hat{x}_{12}(q, y_2)+x_{22}(q)=y_2+2\,. \tag{10$'$}$$

From (10) or (10′), the inverse demand function is found to be:

$$q=2-y_2\,. \tag{11}$$

Note here that the notation, $\hat{x}_{11}(q, y_2)$ in (10) indicates $\hat{x}_{11}$ in (8).

Now, we can find the monopolistic equilibrium solution. It should be noted that monopolistic equilibrium varies as the numéraire changes. If the normalization of prices in the previous section is adopted, that is, if composite commodity $(1, 1)\in R^2$ is taken as the numéraire, then, profit function becomes:

$$\hat{\pi}=\frac{2(1-y_2)y_2}{3-y_2}\,. \tag{12}$$

On the other hand, if the first commodity is taken as the numéraire, then, profit function becomes:

$$\check{\pi}=2(1-y_2)y_2\,. \tag{13}$$

Maximizer of $\hat{\pi}$ or $\check{\pi}$ is monopolistic production plan in the economy. Equilibrium price is obtained by substituting the maximizer for y_2 in (11). With profit function $\hat{\pi}$, monopolistic production plan and monopolistic relative price system in equilibrium are:

$$\hat{y}_2=3-\sqrt{6}\,, \qquad \hat{y}_1=-15+6\sqrt{6}\,, \qquad \hat{q}=\sqrt{6}-1\,. \tag{14}$$

With profit function $\check{\pi}$, they are:

$$(15) \qquad \check{y}_2=\frac{1}{2}, \qquad \check{y}_1=-\frac{1}{4}, \qquad \check{q}=\frac{3}{2}.$$

Let us compare states of the above equilibria. The consumption allocations corresponding to each equilibrium are derived from Equations (3), (4), (8), (9), (11), (14) and (15). Define

$$u_i^*=u_i(x_{i1}^*, x_{i2}^*) \qquad \text{for} \quad i=1,2,$$

$$\hat{u}_1=u_1(\hat{x}_{11}(\hat{q}, \hat{y}_2), \hat{x}_{12}(\hat{q}, \hat{y}_2)),$$

$$\hat{u}_2=u_2(x_{21}(\hat{q}), x_{22}(\hat{q})),$$

$$\check{u}_1=u_1(\hat{x}_{11}(\check{q}, \check{y}_2), \hat{x}_{12}(\check{q}, \check{y}_2)),$$

$$\check{u}_2=u_2(x_{21}(\check{q}), x_{22}(\check{q})).$$

u_i^{**}, u_i^*, $\hat{u}_i$ and $\check{u}_i$ represent repsectively the utility levels of consumer i in the four equilibria. Note that u_1^{**} and u_2^{**} are already defined. We can calculate approximate values for u_i^{**}, u_i^*, $\hat{u}_i$ and $\check{u}_i$ by using (7), (14) and (15).

$$u_1^{**}=2, \qquad u_2^*=2,$$

$$u_1^*=\frac{289}{108}\doteqdot 2.676, \qquad u_2^*=\frac{25}{12}\doteqdot 2.083,$$

$$\hat{u}_1\doteqdot 2.683, \qquad \hat{u}_2\doteqdot 2.052,$$

$$\check{u}_1=\frac{8}{3}\doteqdot 2.667, \qquad \check{u}_2=\frac{49}{24}\doteqdot 2.042.$$

Comparing (u_1^*, u_2^*) with $(\hat{u}_1, \hat{u}_2)$, one finds that there is monopoly exploitation in our sense. But, there is no such relation between (u_1^*, u_2^*) and $(\check{u}_1, \check{u}_2)$. It should be noted that any change from the competitive allocation without production to some equilibrium allocation with production is Pareto improving. Furthermore, $\hat{u}_1$ is not the maximum utility level for the capitalist. The reader can easily check this fact by using the capitalist's indirect utility function of y_2, which is to be derived from the equations (8), (11) and the utility function $u_1(\cdot)$. In fact, the capitalist's utility level attains its maximum when $y_2=(17-\sqrt{97})/12\doteqdot 0.5959$.[8] It has now been shown that there exists monopoly exploitation in some Arrow-Debreu economy.

We now examine briefly how a monopolistic equilibrium is attained. Since every consumer maximizes his utility under given prices, and since demand and supply for goods are always balanced, no agent but the production manager is capable of

[8] Professor M. Ohyama suggested that this issue depends not only on the choice of numéraire but also on the distribution of initial endowment over consumers. As to the above example, the reader can find a numéraire and a distribution of initial endowment such that the capitalist's utility level attains its maximum in the monopolistic equilibrium.

recognizing whether the current state is in competitive equilibrium. Now, suppose a sequence of states of a single firm economy and suppose that the production manager does not know the demand conditions except for the history of past economic states. If the production manager, on the basis of his knowledge of the history, tries to maximize the profit of the firm, it is expected that any sequence of economic states converges to monopolistic equilibrium through the manager's learning of the total demand conditions.

Of course, the above discussion strongly depends on the assumption that there is only one firm. In case of a multiple firm economy, however, a similar result holds if one supposes monopolist as defined in Definition 3 in the previous section. But if one supposes that each of the production manager is a "monopolist," the sequence of economic states may not converge to any monopolistic equilibrium of Definition 3 or Remark 3 in the previous section.

4. CONCLUDING REMARKS

This paper has shown the existence of a monopolistic equilibrium of the private ownership economy. In the monopolistic equilibrium, the firm's profit is not smaller than that in the competitive one. Thus, the production manager, maximizing the profit, has a weak incentive to behave monopolistically when he knows the demand conditions of others.

In addition, we have examined the relation between the monopolistic equilibrium and the competitive one in a simple example. Comparing the two equilibria, we have seen that capitalists are able to take an advantageous position relative to laborers through monopoly. However, it is not clear, in general, whether or not the monopolistic action of the firm is the most beneficial to the shareholders (capitalists) of the firm.

As Debreu pointed out, the private ownership economies are economies where consumers own the resources and control the producers.[9] Considering a production manager in such economies who behaves so as to benefit the shareholders only, several economists have thought that he sees the profit or the value of the firm as an aggregative welfare index of the shareholders. As we saw, however, maximizing the monopolistic profit might fail to *maximize* the shareholders' utility even when their preferences are homogeneous. Furthermore, it is not necessarily supported unanimously by shareholders with heterogeneous preferences.

[9] See Debreu [1, p. 74].

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