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# THEORY OF INPUT DEMAND UNDER PRICE UNCERTAINTY*

Sandwip Kumar Das

*Abstract.* This paper raises doubts regarding the validity of a few well-established propositions describing the behaviour of a competitive firm that faces uncertainty in the product price. The conventional approach relates price uncertainty and risk-aversion with production loss and claims that price uncertainty does not affect the choice of a cost-minimising technique. It is shown here that a firm choosing some inputs after the selling price is known may produce a more-than-certainty output and select a non-optimum technique. In a two-state decision making framework it is revealed that a firm choosing capital *ex ante* and labour *ex post* may hire more capital than what it would under certainty in order to hedge against future capital shortage. Some comparative studies and properties of input demand functions are included in the paper. Answers to a large number of questions related with trade policy and public finance depend crucially on how a decision maker reacts to uncertainty. In view of this, the exercises worked out here are important.

Decision making under uncertainty has been the certerpiece in many of the recent developments in trade theory, general equilibrium theory, public economics, monetary economics and welfare economics. It has been found that conclusions of some of these studies depend crucially on how a typical decision maker reacts to uncertainty in product prices, factor prices or technology. In other words, the framework of a stochastic microtheory is useful in dealing with a large number of issues which must arise in several branches of economic theory and policy when the assumption of perfect knowledge is dropped.

The microtheoretic framework that has been widely used in the literature, particularly in trade theory,[1] is derived from the contributions of Sandmo (1971) and Batra and Ullah (1974). These authors have taken up the case of a competitive firm facing uncertainty in product price. The firm is assumed to choose *all* inputs (and hence output) before the product price is known. All input-hiring decisions are made by maximising expected utility of profit for a given probability density function of the product price. At the time of transactions the firm sells its predetermined output at the *ex post* market price with no possibility of last-minute

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[1] See Batra (1975 and 1974) for the application of microtheory in dealing with problems of international trade under uncertainty.

adjustments in the input decisions.

Implicit rigidity of these models has led to formulation of alternative frameworks of analysis. Turnovsky (1973) assumes that the firm formulates the initial production plans before the selling price is known, but makes later adjustments in output after the market price is revealed. Oi (1961) and, more recently, Epstein (1978) have done further work on production flexibility under uncertainty.

The extent to which output decisions are subject to revision in a firm facing price uncertainty is an empirical question. However, it is almost always true that a firm need not choose *all* inputs before the product price is known. In fact, in most cases labour inputs can be hired *ex post*, though capital stock must be chosen *ex ante* because of the time involved in negotiating a loan or raising share capital. In this paper we have developed the implications of a model of input choices under product price uncertainty in which one input is chosen *ex ante* while the other is chosen *ex post*.[2] This is a two-input model with a two-stage decision making process in which capital is chosen before the product price is known. Thus, the firm faces the risk of capital shortage in the *ex post* situation when it hires labour inputs by taking the selling price in the market as given. The firm may hedge against this probable capital shortage in the future by hiring more capital than it would otherwise have done. This interesting possibility is explored here.

It is well known in the literature that price uncertainty leads to production loss in a risk-averse firm. This is because of the fact that price uncertainty brings in an implicit cost of risk which is added to the firm's production cost. Since this risk premium is a variable cost for the risk-averse firm, it produces less. Therefore, one should ask whether the effect of uncertainty on the level of output is similar under the two-stage decision making rule. It turns out that the effect may be quite different. Also, the firm may reject the cost-minimizing technique.

It is also necessary to compare the impact of uncertainty on the levels of output under the two decision making rules: one requiring choice of both factors *ex ante* and the other allowing labour to be chosen *ex post*. The outcomes of these comparisons are counter-intuitive. The properties of the input demand functions under the new decision rule have also been explored here. Since capital is chosen *ex ante* and labour *ex post*, the factor-complementarity effect gains prominence in the analysis and this effect may either weaken or strengthen the risk-aversion effect.

## I. THE MODEL OF INPUT CHOICE

Let us now describe the decision-making process of a competitive firm facing

[2] In a recent paper Holthausen (1976) has tried to work out a model with basically the same purpose in mind. But Holthausen's formulation has been shown to be inconsistent with his assumptions by Mayer (1978) who has pointed out that in Holthausen's model both inputs are in fact being chosen *ex ante*. Thus, the part of Holthausen's model dealing with a competitive firm is just another way of looking at the Batra-Ullah model.

product-price uncertainty. The firm's production function is written as

$$q = f(L, K)\,, \tag{1}$$

where $q$ is quantity of the product and $L$ and $K$ labour and capital inputs respectively. Let $p$ be the random price of the product having a known probability distribution, $w$ the wage rate and $r$ the rental on capital. The firm chooses labour after the market price is known by maximising the profit function, $\pi = pq - wL - rK^*$, where $K^*$ is chosen *ex ante*.

Thus the firm equates the given wage rate with the value of marginal product of labour. In other words,

$$w = pf_L(L, K^*)\,, \tag{2}$$

where $f_L = \partial q / \partial L$. Solving (2) for $L$ we get the labour demand function.

$$L^* = L^*(p, w, K^*) \qquad \text{(2a)}[3]$$

The firm chooses $K$ before price is known by maximising the expected utility of profit. At this stage $L^*$ is a random variable whose probability distribution is derived from that of $p$. The *ex ante* decision rule is

$$\operatorname*{Max}_K E[U(pf(L^*, K) - wL^* - rK)]\,,$$

where $U(\pi)$ is a von Neumann-Morgenstern utility function and $E$ is the expectation operator on the random price $p$.

The first and second order conditions for an interior maximum of the above objective function are respectively

$$E[U'(\pi)\{(pf_L - w)(\partial L^* / \partial K) + (pf_K - r)\}] = 0\,.$$

Or,

$$E[U'(\pi)(pf_K - r)] = 0\,, \quad \text{by using (2)} \tag{3}$$

and

$$D \equiv E[U''(\pi)(pf_K - r)^2 + U'(\pi)p\{f_{KK} + f_{KL^*}(\partial L^* / \partial K)\}] < 0\,, \tag{4}$$

where $f_{KK}$ and $f_{KL^*}$ are second order partial derivatives of the production function.

The impact of uncertainty on the level of output produced by the firm can be understood by expanding equation (3) into covariance terms. Thus we get

$$r = E[pf_K] + \frac{\operatorname{cov}(U'(\pi), pf_K)}{E(U'(\pi))} \tag{3a}$$

The profit function for the *ex ante* part of the firm's decision-making process is written as

[3] $K^*$ is a parameter when labour is chosen *ex post*. The relationship between $L^*$ and $K^*$ should be understood in the light of the fact that they belong to two different stages of decision making.

$$\pi(K;p)=pf(L^*,K)-wL^*-rK.$$

Defining the risk-premium function as

$$R(K)=E[\pi]-U^{-1}[E(U(\pi))],$$

and differentiating it with respect to $K$ we get the marginal risk-premium at the optimum level of capital input hired:

$$R'(K)=E[pf_K-r], \quad \text{by (2) and (3)}$$

$$=\frac{-\operatorname{cov}(U', pf_K)}{E[U']}, \qquad \text{by (3a)}$$

It should be mentioned at this point that $f_K$ is a random variable. Rewriting (3a) we get

$$r=\mu E[f_K]+\operatorname{cov}(p,f_K)-R'(K), \tag{3b}$$

where $\mu=E(p)$.

## II. INTERPRETATION OF COVARIANCE TERMS

The meaning of the covariance term defining marginal risk-premium can be found along traditional lines. The mathematical relationship between the value of the marginal product of capital $(pf_K)$ and profit is a positive one. Therefore, in the risk-aversion case $(U''<0)$ marginal risk-premium is positive and in the risk-neutrality case $(U''=0)$ it is zero. But there is another covariance term, $\operatorname{cov}(p,f_K)$, which never appears in the traditional analysis. And it is exactly this second covariance term which makes this analysis more interesting than the traditional one.

To determine the sign of $\operatorname{cov}(p,f_K)$ let us go back to Eq. (2) and differentiate it totally. Thus we get

$$pf_{LL}dL^*+pf_{LK^*}dK^*+f_Ldp-dw=0$$

It follows from the above equation that

$$\frac{\partial L^*}{\partial p}=-\frac{f_L}{pf_{LL}}>0^4 \tag{5a}$$

$$\frac{\partial L^*}{\partial w}=\frac{1}{pf_{LL}}<0 \tag{5b}$$

$$\frac{\partial L^*}{\partial K^*}=-f_{LK^*}/f_{LL}\gtrless 0 \qquad \text{according as} \quad f_{LK^*}\gtrless 0^5 \tag{5c}$$

[4] $f_{LL}<0$ assuming that the *ex post* profit function has an interior maximum.

[5] This derivative refers to the effect of a change in $K^*$ chosen *ex ante* on $L^*$ chosen *ex post*. To clarify this point let us assume that firms' utility functions are different but their technologies and subjective

Finally,

$$\text{sign}\,[\text{cov}\,(p,f_K)] = \text{sign}\left[\frac{\partial f_K}{\partial L^*}\,\frac{\partial L^*}{\partial p}\right] = \text{sign}\,[f_{KL^*}]\,,$$

by (5a).

Therefore, the sign of $\text{cov}\,(p,f_K)$ depends upon factor-complementarity and substitutability relations. If labour and capital are complements ($f_{LK}>0$) the covariance is positive. If, however, labour and capital are substitutes ($f_{LK}<0$)[6] the covariance term takes negative values. Now the interpretation of (3b) becomes quite clear. Since the risk-averse firm faces uncertainty at the stage of choosing capital input, the firm adds the marginal cost of risk with the marginal cost of capital and equates the sum of these two costs with the expected marginal revenue product of capital.

## III. OUTPUT COMPARISONS

Let us now compare the amount of capital hired by the risk-averse firm with the amount of capital hired by a firm which knows that the price is $\mu$ with certainty and that the marginal product of capital is equal to $E(f_K)$ at the profit maximisation point.[7] So far as the effect of risk-aversion is concerned the risk-averse firm would undoubtedly hire less capital than the firm knowing price with certainty. However, to the extent that factors have a complementary relation ($f_{LK}>0$) the risk-averse firm would hire more capital. For the firm facing a certain price the market price cannot go higher than $\mu$. For the risk-averse firm the market price may lie above $\mu$ with some probability. Therefore, the latter has to take account of the probable gain from choosing a larger capital stock (which raises marginal product of labour in the *ex post* situation) in the event of market price exceeding $\mu$.[8] This gain adds to the *ex ante* value of capital's marginal product,

probability assessments are identical. This would mean that different firms would choose different levels of capital stocks *ex ante* and, if $f_{LK}\neq 0$, would hire different levels of labour inputs in the *ex post* situation.

[6] $f_{LK}$ is unlikely to be negative in a two-factor case. But this only strengthens the conclusions reached in the following section However, the relevance of a negative cross-partial derivative arises when one takes up the case of two factors chosen *ex post* and one factor chosen *ex ante*. This three-factor case is not worked out here.

[7] These ground rules for comparison are quite commonly used in the literature. See, for example, Sandmo (1971) and Batra and Ullah (1974).

[8] If the actual market price is revealed to be less than $\mu$, the value of the marginal product of labour in the *ex post* situation falls below the value of the marginal product of labour in a certainty firm which takes $\mu$ as the market price. But if the factor complementarity effect dominates the risk-aversion effect, the fact that the firm chose a capital stock higher than that chosen by the certainty firm simply makes up for the shortfall in the *ex post* value of the marginal product of labour. Thus a risk-averse firm may hire more labour inputs and, therefore, produce more output than the certainty firm in the *ex post* situation even if the market price turns out to be less than $\mu$. If the market price is greater than $\mu$, then the risk-averse firm will definitely produce more output. All this, however, is based on the assumption that in the *ex ante* situation the firm hired more capital due to a stronger factor complementarity effect.

$\mu E[f_K]$.

The outcome of the comparison now depends upon the relative strength of the factor complementarity effect, measured by $\text{cov}(p, f_K)$ and the risk-aversion effect, measured by $R'(K)$ in Eq. (3b). It is shown in Fig. 1 that if the former outweighs the latter, the risk-averse firm will hire more capital ($K_u$) than the firm facing a certain price ($K_c$). It is also quite interesting to note that if the firm is risk-neutral, it will always hire more capital than the firm facing a certain price. In the case where factors are substitutes ($f_{LK}<0$) the factor substitutability effect in terms of negative $\text{cov}(p, f_K)$ strengthens the risk-aversion effect. In this case the analysis runs along traditional lines.

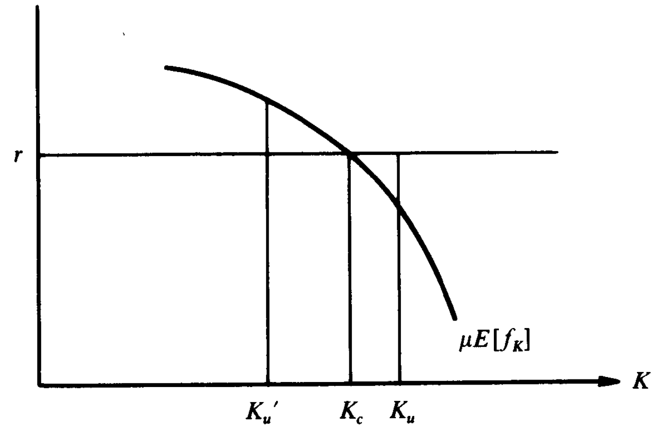


Fig. 1.

If $f_{LK}$ is positive and the factor complementarity effect outweighs the risk-aversion effect, (5c) clearly shows that the risk-averse firm would not only hire more capital but would also hire more labour and produce more output.[9] For the sake of comparison we have assumed that the risk-averse firm finds $\mu$ to be the market price in the *ex post* situation, whereas the firm operating under certainty takes $\mu$ as the given market price. In the case of the factor complementarity effect and risk-aversion effect offsetting each other the risk-averse firm would produce the certainty output. The analysis clearly shows that the conventional theory connecting price uncertainty and risk-aversion with production loss may not be universally applicable.[10]

The effect of uncertainty on the choice of production technique can be deduced from the analysis presented above. Mayer dealt exclusively with this problem without finding a satisfactory answer. Dividing Eq. (3b) by Eq. (2) we get

$$\frac{r}{w}=\frac{\mu E(f_K)}{pf_L}+\frac{\text{cov}(p, f_K)}{pf_L}+\frac{\text{cov}(U', pf_K)}{pf_L E(U')} \tag{6}$$

[9] This will definitely be true if the actual market price is revealed to be greater than $\mu$. See footnote 8 for further clarification.

[10] Similar conclusions can be derived from Oi (1961) and Epstein (1978).

In Eq. (6) $p$ is the *ex post* price which is taken to be equal to $\mu$.[11] The firm's capital-labour ratio will be higher than the cost-minimising one if the factor complementarity effect is stronger than the risk-aversion effect. With $f_{LK}<0$, the firm's capital-labour ratio will be lower than the cost-minimising capital-labour ratio. Again, the analysis does not exclude the possibility of the firm choosing the cost-minimising technique of production even though a portion of risk is transferred to the labour market.[12]

The case of a competitive firm choosing both capital and labour *ex ante* has been elaborately discussed by Batra and Ullah. The comparison between the Batra-Ullah firm and firm which is allowed to choose labour *ex post* rests primarily on the sign of $f_{LK}$. The Batra-Ullah firm would be paying risk-premium on both labour and capital, whereas the firm choosing labour *ex post* would pay risk-premium on capital only. Thus we get

$$r=\mu f_K-\frac{\partial R^*(L,K)}{\partial K}$$

for the firm choosing both $L$ and $K$ *ex ante*, and

$$r=\mu E(f_K)+\operatorname{cov}(p,f_K)-R'(K)$$

for the firm choosing only $K$ *ex ante*.

It follows from the Batra-Ullah model[13] that

$$\frac{\partial R^*}{\partial K}=\frac{-\operatorname{cov}(U'(\pi^*),pf_K)}{E(U'(\pi^*))},$$

where $\pi^*$ is the firm's profit when both $L$ and $K$ are chosen *ex ante*. Let us suppose that marginal risk-premium at the optimum level of capital input hired is the same in two cases, i.e., $R'(K)=R_K{}^*$. It then follows that the Batra-Ullah firm would hire less capital if $f_{LK}>0$ and more capital if $f_{LK}<0$. The case of positive $f_{LK}$ is shown in Fig. 2 where $K_u$ is capital hired by the firm choosing labour *ex post* and $K_u{}'$ is capital hired by the Batra-Ullah firm.

To compare labour employment and output let us assume that $\mu$ turns out to be the actual price in the *ex post* situation. With $f_{LK}>0$, the Batra-Ullah firm would hire less labour and produce less output. The reason is simple. The amount of capital hired by the firm choosing labour *ex post* being higher, its *ex post* marginal-revenue-product-of-labour curve is raised above the expected marginal revenue-product-of-labour curve of the Batra-Ullah firm. On top of this the Batra-Ullah firm pays risk-premium on labour. In the case of negative $f_{LK}$ the comparison does

[11] Here again we are using the ground rules for comparison.

[12] Batra, Ullah and others who have developed models input choices under uncertainty have shown that product price uncertainty *does not* force a competitive firm to choose a non-optimum technique of production. Our results clearly show that this claim may not be valid.

[13] For necessary details see the paper by Batra and Ullah from which a risk-premium function $R^*(L,K)$ can be derived.

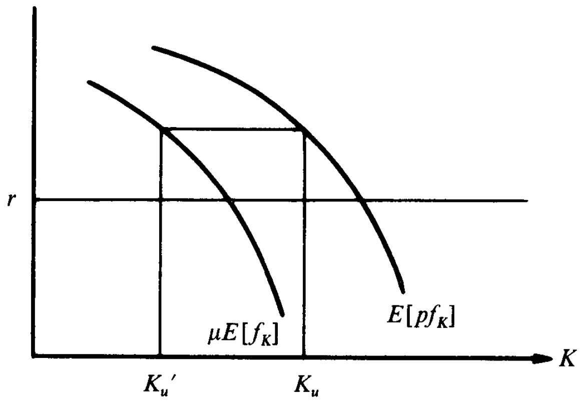


Fig. 2.

not yield any determinate result. It is true that the Batra-Ullah firm hires more capital in this case which raises its expected marginal-revenue-product-of-labour curve above the *ex post* marginal-revenue-product-of-labour curve of its counterpart. However, the Batra-Ullah firm pays a risk-premium on labour which the firm choosing labour *ex post* does not have to pay. Hence the indeterminacy arises.

## IV. INPUT DEMAND FUNCTIONS

An interesting aspect of our model is the effect of a change in wage rate on the *ex ante* demand for capital. It cannot be denied that this effect is somewhat novel in the uncertainty literature. In certainty models we are concerned with the effect of a change in the price of a factor chosen *ex post* on the demand for another factor chosen *ex post*. In the Batra-Ullah model both these factors are chosen *ex ante*, whereas in this model we seek to analyse the effect of a change in the price of an *ex post* factor on the demand for an *ex ante* factor.

The properties of the *ex-post* demand function of labour have been outlined in (5a), (5b) and (5c). It should be mentioned that $L^*$ is not a function of $r$ in (2a), though $K^*$ is affected by a change in $r$ or $w$. To find the properties of the *ex ante* demand function of capital let us totally differentiate equation (3) with respect to $K$, $w$ and $r$.

$$\begin{aligned} DdK + E[U''(\pi)(pf_K - r)\{(pf_{L^*} - w)(\partial L^*/\partial w) - L^*\} \\ + U'(\pi)pf_{KL^*}(\partial L^*/\partial w)]dw \\ - E[U''(\pi)(pf_K - r) + U'(\pi)]dr = 0 \end{aligned}$$

Using Eq. (2) we get

$$\frac{\partial K}{\partial w} = \frac{1}{D} E[U''(\pi)(pf_K - r)L^* - U'(\pi)pf_{KL^*}(\partial L^*/\partial w)] \tag{7}$$

$$\frac{\partial K}{\partial r}=\frac{1}{D}E[U''(\pi)(pf_K-r)+U'(\pi)] \tag{8}$$

Let $a=E[U''(\pi)(pf_K-r)\mid L^*]$ which is clearly non-negative with the assumption of non-increasing absolute risk-aversion.[14] Thus $\partial K/\partial r$ is negative. Also,

$$\begin{aligned}E[U''(\pi)(pf_K-r)L^*]&=E[E\{U''(\pi)(pf_K-r)L^*\mid L^*\}]\\&=E[aL^*]>0\,.\end{aligned}$$

If $f_{LK}$ is positive, the sign of $\partial K/\partial w$ is unambiguously negative[15] under the assumption of non-increasing absolute risk-aversion. This is what we should expect in view of the earlier analysis. The probable gain in the form of the factor complementarity effect which raises the *ex ante* value of capital's marginal product is reduced if the price of the complementary factor is increased. If $f_{LK}$ is negative, $\partial K/\partial w$ has an indeterminate sign as in the Batra-Ullah model.[16]

## V. CONCLUDING REMARKS

In conclusion it is necessary to highlight a decision-making problem which precedes the decision-making problems discussed in this paper as well as in the Batra-Ullah paper. It is the problem of choosing the industry in which a new investment is to be made. The nature of the industry and the length of the production lag are likely to be the most important factors at this initial stage. The greater is the number of factors chosen *ex ante*, the greater is level of uncertainty involved in production decisions. But it should be noted that even if all factors are chosen *ex post*, there is uncertainty at the initial stage of decision-making. The firm knows the price with certainty at the stage of choosing labour and capital, but not at the initial stage at which it has to choose the industry. Let us suppose that the choice is between three industries, $A$, $B$ and $C$. In $A$ capital is chosen *ex ante* and labour *ex post*. In $B$ both factors are chosen *ex ante* and in $C$ both are chosen *ex post*. At the initial stage price is random. Therefore, the firm would be comparing three expected utilities, $V_A$, $V_B$ and $V_C$, which are defined as

$$\begin{aligned}V_A&=E[U(pf(L_p,K_a)-wL_p-rK_a)]\\V_B&=E[U(pf(L_a,K_a)-wL_a-rK_a)]\\V_C&=E[U(pf(L_p,K_p)-wL_p-rK_p)]\,,\end{aligned}$$

where $L_p$ and $K_p$ are optimum *ex post* values, whereas $L_a$ and $K_a$ are optimum *ex*

[14] A proof of this is quite easily available in the literature. See, for example, Batra and Ullah (1974). The idea of an absolute risk-aversion function used in this connection is due to Pratt (1964).

[15] $\partial L^*/\partial w<0$, by (5b).

[16] In the Batra-Ullah model the sign of $\partial K/\partial w$ is indeterminate even if $f_{LK}$ has a positive sign. In the certainty case $\partial K/\partial w$ has its sign determined by that of $f_{LK}$. In this sense our model lies between the certainty model and the Batra-Ullah model.

*ante* values of labour and capital respectively.

We can conceive of a long run competitive equilibrium in which $V_A = V_B = V_C = 0$. From the viewpoint of our analysis it interesting to note that an increase in the wage rate affects choices between $A$ and $B$ and between $B$ and $C$ but not between $A$ and $C$. On the other hand, an increase in capital's rental alters choices between $A$ and $C$ and between $B$ and $C$ but not between $A$ and $B$.

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