Title	DIRECT ESTIMATION OF PRODUCTION FUNCTION BY INDUSTRY
Sub Title	
Author	浜田, 文雅(HAMADA, Fumimasa) Chida, Ryokichi
Publisher	Keio Economic Society, Keio University
Publication year	1983
Jtitle	Keio economic studies Vol.20, No.1 (1983.) ,p.33- 65
JaLC DOI	
Abstract	
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19830001-0033

DIRECT ESTIMATION OF PRODUCTION FUNCTION BY INDUSTRY*

Fumimasa HAMADA and Ryokichi CHIDA

I. INTRODUCTION

Most of studies on firm's behaviour assume that production function is an input-output relation as a simplified technological constraint to the firm's optimizing behavior. Modern economic theory interpretes observed quantity of each input and that of output as those to satisfy both this constraint and the optmizing conditions of firm. The main reason why economic theory has been developed on the basis of such an interpretation as this for more than a half century is that it is preferable and effective for making empirical analysis more systematic and for avoiding difficulties and complexities caused from assuming a more realistic framework.

Recently, observable facts and empirical studies based on the optimization conditions have, however, shown that there is some significant inconsistency among the relevant variables as solutions of the optimizing conditions. Particularly, one of the most important is the fact that the gap between rates of expected return on investment by industry cannot be swept away in the long run, and industrial structure does not change toward the equalization of rates of expected return. These facts are not peculiar to Japan. In the United States, for example, this gap appears to have been existed for these twenty five years (see Fraumeni-Jorgenson (1980)). One of the authors also showed that rates of expected return on investment in postwar Japanese economy were very different among industries and investment did not tend to concentrate on industries having high rates (see Hamada (1980a), (1980b)).

These observation leads to the suspicion that, in specifying the optimizing behaviour of firm, some factor might have been overlooked. Factors on which the "insights" have so far done are, (a) imperfect competition in product and factors

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* This paper is a part of the research project which one of the authors engage in cooperatively with other researchers as a Grant-in-Aid for Scientific Research (Type A 57330002) of Ministry of Education, Government of Japan, "International Comparative Study of Industry Productivity and Resource Allocation." One of the authors, Ryokichi Chida, is not a formal member of the above research project, but took part in it as a cooperator. Chida took mainly on data processing and estimation, and in some other respects made some contribution to this paper.

This research was reported at the Annual Meeting of the Japan Association of Economics and Econometrics held at the Kyoto Industry University (October 1982). The authors are grateful to Yushi Kubo, Tsukuba University, for his useful comments at the meeting, and also thankful to Takao Fukuchi, Tsukuba University, and Soshichi Kinoshita, Nagoya University, for fruitful discussions at the other opportunity. But remaining errors are attributed solely to the authors.

markets, (b) the time lag necessary to achieve the optimum (the literatures about lag distribution of investment and labor demand behaviour, in fact, deal with this point). In imperfectly competitive market, firm reduces the supply of product and marginal productivity of capital remains high, compared with those in perfectly competitive market. If we assume the lag-adjustment process for the optimization, the marginal productivity of investment is greater than the intended one. After all, in both cases the marginal rate of expected return on investment exceeds the intended rate of return.

These two insights seem not to be persuasive in the following points respectively. In imperfect market, as is well known, the absolute value of elasticity of demand for product with respect to price must be greater than unity. But, in reality, that value can never exceed unity, that is, the fact demand is inelastic with respect to price is observed. Moreover, it is not realistic to assume that flexibility of labor and capital price is large enough to equate marginal productivity to its real factor price. The lag is too large to regard the observed value as that in an adjustment process for firm to arrive at its optimum position.

As is shown above, it is quetionable to regard, as a self-evident, the necessary conditions for optimization, that is, the equalization of real factor price and marginal productivity of factor. It should be examined whether the necessary conditions of optimization holds or not, in the light of other factors which have not been considered yet.

In this paper, as a first approach, production function by industry is directly estimated in order to check the above necessary conditions, and in a sequel, marginal productivity is estimated and compared with real factor price. In section II, the condition for justifying direct estimation and the meaning of specification of production function are discussed. In section III, the available data and data processing are explained. In section IV, empirical results are presented and best specifications are decided. Marginal productivity of labor and capital by industry in each year will be estimated and compared with factor price respectively in a sequel.

II. THE CONDITION OF DIRECT ESTIMATION AND THE SPECIFICATION OF PRODUCTION FUNCTION

Since a pioneering study of Douglas (1934), many problems related to estimating production function, such as least squared bias by direct estimation, multicollinearity, the simultaneous nature of production process, heteroschedasticity, have been discussed (see Walters (1963)).

Among others, the problem about simultaneous estimation of production function presented by Marschack-Andrews (1944) was very important. Their paper has revealed the possibility that direct estimation of production function may lead to the estimation of a "mongrel" equation. Marschack-Andrews considered that the observed values of output, labor employment, and capital stock were chosen, intently in firm's opitimizing behavior, as the optimal values for firm. They assumed simultaneous equation system which included, as data generating mechanism, production function and the necessary conditions of optimization.

This way of thinking presented by Marschack-Andrews was, thereafter, extended by Hock (1958), Mundlák-Hock (1965), Zeller-Kmenta-Drèze (1966), etc. CES production function derived by Arrow-Chenery-Minhas-Solow (1961) and translog production function derived by Christensen-Jorgenson-Lau (1973) also based on assuming the same data generating mechanism. As these production functions are very often used in recent empirical studies, this indirect estimation, which makes use of the necessary conditions of firm's optimization, becomes prevailing. Indirect estimation assumes that the observed values of output, labor employment, and capital stock can be interpreted to be the value intended by firm in it's optimizing behaviour. If the gap between the intended value and the observed value is too large to neglect, however, this indirect estimation would bring about a serious estimation bias, and the estimates cannot be regarded as true parameters of production function itself.

The important observations which suggest the possibility of such bias are the long-term differentials of the rate of return on investment among industries and too long time lag observed in firm's behavior. Marginal productivity of labor or capital estimated by direct estimation is also much greater than the corresponding real factor price. But indirect estimation precludes all these possibilities.

The problems mentioned above are related to indirect estimation of production function, so those are not positive reasons for estimating production function directly. The positive reasons for direct estimation of production function are that there exist unobservable and non-traded factors like entrepreneur's ability or learning effect of labor which are pointed out by Nelson (1981) as factors disturbing the necessary conditions of optimization, and that the effect of these non-traded factors should be considered in the technological structure of production process. Apart from these reasons, direct estimation can be admitted as long as R^2 exceeds a certain level (for example, 0.95). Though only the mongrel equation is estimated by direct estimation, the estimation bias is probably not so serious if R^2 is very high. It is also desirable if the values of unobservable factors mentioned above are, to some extent, positively correlated to the quantity of labor input or capital input. These factors can then be derived indirectly on the assumption that inputs of labor and capital service should function as the proxy of these unobservable inputs. This assumption cannot be tested, but does not seem to be unrealistic.[1]

III. DATA

In this section, data sources and data processing are explained, and some problems about data in estimating production function directly are discussed.

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[1] For a further discussion, see Hamada (1984).

(1) *Data sources and data processing*

The observation period is from 1960 to 1979, and the real value is measured at 1975 constant billion yen.

1) *Real output* (X)

Real output for the period 1970–1979 is taken from *Annual Report on National Accounts* published by Economic Planning Agency of the Japanese Government (EPA). Real output of EPA is evaluated by producer's price. Since producer's price doesn't include transportation cost and margin, it is more desirable for analysing production process than at purchaiser's price which includes those cost. For the period 1960–1969, real output is computed by data available thanks to Keio Economic Observatory (KEO). We get, for example, 1969 value, X_{69}, as

$$X_{69}=(X_{70}/X'_{70})\times X'_{69}$$

where X_{70} is EPA's value, X'_{69} and X'_{70} are KEO's values respectively.

2) *Number of workers* (L), *number of employees* (E)

For 1970–1979, we use EPA data, and for 1960–1969, we compute by the same method as in the case of real output.

3) *Real value of gross capital stock* (K), *real value of deterioration from gross capital stock* (R)

These data are taken from *Private Firm Gross Capital Stock* published by EPA. EPA estimates gross capital stock and deterioration by accumulating each year's flows, using the results of *1955's National Wealth Census* as bench mark and adjusting those values to the results of *1970's National Wealth Census.*

4) *Value added* (P^*X), *income of employee* (WE)

For 1970–1979, the data are taken from EPA data, and for 1960–1969, computed by the same method as real output. These are nominal values.

5) *Potential productivity* (S)

During the observation period, the decline of the rate of operation in capital stock is caused by Oil Shock and by other reasons, and X must be adjusted to this decline. Instead of adjusting K by rate of utilization, we introduce the potential productivity (S).[2] Though the technological output-capital ratio is considered to be stable in the short term, the sharp declines are observed in movement of X/K. These declines can be considered as caused by the decline of the rate of utilization in k. Then, the value of X/K must be increased between peak and peak in the graph of X/K. The increased $(X/K)^*$ is computed by interpolation and we get

$$S=(X/K)^*\times K$$

where S is the potential productivity in which the influence of the variation in the rate of utilization is eliminated from variations of X.

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[2] The authors are grateful to Yasuo Maeda, a graduate student of master course in Keio University for computing the data of potential productivity.

6) *Net price of output* (P^*), *wage rate* (w)
P^* and w are computed by

$$P^* = P^*X/X\,, \qquad w = WE/E$$

7) *Deterioration fats* (δ)
δ is computed by

$$\delta = R/K_{-1}$$

8) *Price of capital goods* (q)
The weighted average deflator of capital goods and construction goods from *Economic Statistics Monthly* published by The Bank of Japan is available as "q". But, since the combinations of capital goods are thought to be different among industries, to use the same deflator in all industries means to neglects those differences. We made the new data on capital goods.

At first, the weight of jth capital goods, k_{ij} is calculated, using *Fixed Capital Matrix in Input-Output Tables* published by Government of Japan. Capital goods are classified into following four types; general machinery, electric machinery, transportation machinery, and construction goods. Deflators of each capital goods, q_j, are taken from *Economic Statistics Monthly* published by The Bank of Japan. We get "q" of "i"th industry, as

$$q_i = \sum_{j=1}^{4} k_{ij} q_j \Big/ \sum_{j=1}^{4} k_{ij}$$

For 1960–1974, k_{ij} from *1970's Fixed Capital Matrix* is used, and for 1975–1979, k_{ij} from 1975's is used.

9) *Average interest rate on long-term loans* (i)
"i" is taken from *Economic Statistics Monthly.*

(2) *Some problems of data in estimating production function directly*
There are three disputable problems in estimating production function directly.

Firstly, we used real output (X) or potential productivity (S) as dependent variable of production function. Although value added is usually used and so-called value added production function is estimated, it is quetionable to use value added because production function represents the technological structure of each industry or each firm. If it can be assumed that inputs of row materials are proportional to X or S, X or S becomes a function of K and L, which represents how much of X or S is produced using a certain combination of K and M. This is the original definition of production function, so it is meaningless to introduce value added in this relation. It is not value added but real output that is produced. Moreover, value added depends not only on K and L but on market factors through the prices of output and row materials. This is the reason why value added cannot be used for estimating production function. When we compare marginal productivity with factor price, we use P^* instead of P (price of output) in order to keep the consistency in distribution problem. Marginal productivity is to be calculated by X, not by S. This realized marginal productivity will be compared with factor price.

Secondly, production function is estimated by the number of workers, L, and wage rate is calculated by the number of employees, E. If the non-employed worker's wage rate is not so different from the employee's wage rate, this gap is negligible, and this is the case except such industries as Agriculture-Forestry-Fishery or whole sale and retail trade.

Thirdly, a serious problem about capital stock data exists. The data is taken from *Private Firm Gross Capital Stock*, so the industrial calssification is based on each firm. If one firm spreads over a few industries, the firm is classified to the industry where the sale of the firm is largest. The calssification of *Annual Report on National Accounts* is, on the other hand, based on each factory. This gap may influence the results, but since there is no other data available, we shall use these data.

IV. EMPIRICAL RESULTS

Empirical results are shown and the desirable specifications for calculating marginal productivity are ditermined in this section. We choose two specifications by industry, one of which is Cobb-Douglas type and the other is CES type.

(1) *Cobb-Douglas production function*

We estimated following four specifications about Cobb-Douglas type.[3]

$$X = a_0 L^{a_1} K^{a_2} \tag{1}$$

$$X = a_0 L^{a_1} K^{a_2} e^{a_3 t} \tag{2}$$

$$\frac{X}{L} = b_0 \left(\frac{K}{L}\right)^{b_1} \tag{3}$$

$$\frac{X}{L} = b_0 \left(\frac{K}{L}\right)^{b_1} e^{b_2 t} \tag{4}$$

Constant returns to scale is assumed in Eqs. (3) and (4). Tables 1 and 2 show estimated parameters. Potential productivity, S, is used in Table 2. Results are summarized as follows.

1) Since $\bar{R}^2$ is very high except for agriculture-forestry-fishery, the direct estimation bias is not so large.[4]

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[3] We also estimated for the specification including Oil Shock dummy variable, but the results were not so different from those of Table I and II.

[4] In Mundlak-Hock (1965), direct estimation bias in the case of one input Cobb-Douglas type is represented as,

$$p\lim \hat{\alpha} = \alpha + \frac{\sigma_{00}(1-\alpha)}{\sigma_{00} + \sigma_{11}}$$

2) All parameters are statistically significant in 27 equations in Table 1 and 34 equations in Table 2, so S brought slightly better results.

3) Not a few results show negative productivity of labor and negative rate of Hicks neutral technical progress. This unsatisfactory results seem to be caused by multicollinearity between explanatory variables. In Eq. (4), for example, b_1 tends to be overestimated and greater than unity while b_2 tends to be underestimated and negative.

4) Scale economy is observed in many industries in Eqs. (1) and (2). Above all, in the third industry, the labor coefficient is likely to be very large and (a_1+a_2) is often greater than 2.0.

As is shown above, the results of Table 2 is slightly superior to those of Table 1. S is considered more appropriate than X as independent variable. Moreover, the terms of neutral technical progress should be excluded because this term lowers the statistical performance. Finally, non-constraint equations (1) or (2) should be chosen when the statistical performance is not so different. On the basis of these conclusions, the optimal specification in each industry is decided as follows.

Agriculture-Forestry-Fishery (called AFF for short): (3),
Mining (MIN): (3), Food and processed products (FOOD): (1),
Textile products (TETL): (1), Pulp, paper, and paper products (PLPP): (1),
Chemical and related products (CHMS): (1),
Ferreous and non-ferreout metal products (FRNF): (1),
Fablicated metal products (FMTP): (1), General machinery (GNLM): (1),
Electric machinery (ELTM): (2), Transportation machinery (TRNM): (1),
Other manufacturing industries (OTHM): (1), Construction (CNTR): (2),
Electric power, gas, and water supply (EGWS): (1),
Whole sale and retail trade (WSRT): (1),
Financial and insurance business (FIIN): (1),
Transportation and Communication (TRCM): (2), Services (SRVC): (3)

These results are used to estimate marginal productivity of Cobb-Douglas production function in the sequel.

(2) *CES production function* (1)—*linear approximation approach*

Kmenta (1961) shows that CES production function can be linearly approximated by using Taylor's expansion and estimated by OLS. This approximate

where α, $\hat{\alpha}$, σ_{00}, and σ_{11} are respectively the parameter of production function, estimate of α by direct estimation, variance of disturbance in production function, and variance of disturbance in necessary condition for optimization. The covariance σ_{01} is assumed to be zero. If σ_{00} is relatively smaller than σ_{11}, this bias becomes small. But since the values of σ_{00} and σ_{11} are unknown, the estimates, $\hat{\sigma}_{00}$ and $\hat{\sigma}_{11}$, are used to judge the size of bias. Considering that relative share is not constant in the observation period, σ_{11} is probably not so small. $\bar{R}^2$ in direct estimation is, on the contrary, very high, so direct estimation bias can be asumed to be small. In the case of nonlinear direct estimation of CES production function, similar discussion is almost impossible because it is very difficult to solve normal equations simultaneously so as to present the size of estimation bias.

Table 1. Cobb-Douglas Type (Output is used)

Industry	Const.	ln *L*	ln *K*	ln (*K*/*L*)	*T*	$\bar{R}^2$	*SE*	*DW*
AFF	7.5268	0.0829	0.1362			0.8683	0.0313	1.0031
	(3.394)	(0.404)	(1.688)					
	7.8209	0.1229	0.0643		0.0082	0.8619	0.0320	1.0033
	(3.347)	(0.569)	(0.399)		(0.520)			
	1.4516			0.3557		0.9906	0.0365	0.8786
	(43.585)			(31.540)				
	−0.8980			0.3461	0.0012	0.9900	0.0376	0.8685
	(0.025)			(2.383)	(0.066)			
MIN	1.9345	0.0338	0.7495			0.9032	0.0819	1.0022
	(0.599)	(0.160)	(1.889)					
	1.9805	0.0183	0.7524		−0.0014	0.8968	0.0844	1.0235
	(0.539)	(0.050)	(1.824)		(0.052)			
	0.6952			0.8913		0.9925	0.0799	1.0646
	(7.405)			(35.412)				
	−3.0224			0.8759	0.0019	0.9921	0.0822	1.0613
	(0.065)			(4.476)	(0.080)			
FOOD	5.6781	−0.2552	0.6099			0.9924	0.0451	1.0980
	(4.549)	(0.899)	(24.601)					
	7.3932	0.0719	0.1437		0.0464	0.9964	0.0309	1.7854
	(7.898)	(0.346)	(1.368)		(4.498)			
	2.6553			0.5687		0.9980	0.0509	0.7530
	(39.420)			(27.950)				
	−72.9974			0.1868	0.0391	0.9912	0.0437	1.6048
	(2.621)			(1.137)	(2.717)			
TETL	0.5354	0.4376	0.7187			0.9896	0.0350	1.3063
	(0.822)	(28.894)	(4.090)					
	2.1274	0.5407	0.4390		0.0175	0.9926	0.0295	1.9546
	(2.699)	(5.555)	(4.324)		(2.816)			
	1.3869			0.7003		0.9916	0.0357	1.0745
	(20.889)			(33.362)				
	−31.1220			0.4498	0.0169	0.9946	0.0286	1.9704
	(3.168)			(5.799)	(3.310)			
PLPP	−3.3907	1.5871	0.7675			0.9896	0.0681	0.9641
	(3.435)	(5.497)	(27.164)					
	−1.8537	1.9923	0.3301		0.0416	0.9896	0.0681	0.9916
	(1.023)	(4.003)	(0.761)		(1.011)			
	1.3017			0.7890		0.9746	0.1015	0.4120
	(7.310)			(18.989)				
	−119.9392			0.1252	0.0630	0.9747	0.1013	0.3445
	(1.025)			(0.195)	(1.037			
CHMS	−1.0909	0.2993	0.9981			0.9894	0.0910	0.6154
	(0.659)	(0.646)	(22.660)					
	−1.4917	−1.1165	1.7823		−0.0734	0.9918	0.0799	1.0089
	(1.020)	(1.586)	(5.554)		(2.462)			
	0.0311			1.0151		0.9884	0.0897	0.5573
	(0.186)			(28.429)				
	117.9823			1.6406	−0.0614	0.9911	0.0788	0.9190
	(2.513)			(6.539)	(2.513)			

Table 1. (continued)

Industry	Const.	$\ln L$	$\ln K$	$\ln(K/L)$	T	$\bar{R}^2$	SE	DW
FRNF	−1.0108	0.9402	0.7427			0.9895	0.0834	1.0508
	(0.950)	(2.918)	(17.541)					
	0.6423	2.1708	−0.0880		0.0830	0.9915	0.0753	1.6103
	(0.527)	(3.445)	(0.232)		(2.202)			
	−1.4527			0.8103		0.9831	0.0932	0.7757
	(8.314)			(23.425)				
	−18.9251			0.7092	0.0106	0.9822	0.0957	0.7623
	(0.250)			(1.879)	(0.269)			
FMTP	−1.6515	1.8058	0.2174			0.9917	0.0760	1.0733
	(2.399)	(8.339)	(4.662)					
	−1.7321	1.9458	0.1277		0.0105	0.9912	0.0782	1.0747
	(2.263)	(3.522)	(0.390)		(0.277)			
	2.3716			0.4589		0.9426	0.1286	0.5304
	(21.119)			(12.343)				
	203.3596			1.2232	−0.1032	0.9582	0.1101	0.8649
	(2.780)			(4.368)	(2.747)			
GNLM	2.9096	−0.0485	0.7743			0.9755	0.1401	0.7186
	(1.487)	(0.094)	(9.079)					
	−1.4004	2.6432	−0.4965		0.1244	0.9852	0.1092	0.7766
	(0.711)	(3.023)	(1.332)		(3.463)			
	1.6945			0.7296		0.9650	0.1377	0.6605
	(11.518)			(16.070)				
	−125.2315			0.1838	0.0653	0.9735	0.1202	0.6732
	(2.541)			(0.853)	(2.576)			
ELTM	−1.9676	0.9128	0.8025			0.9805	0.1551	0.4629
	(2.220)	(2.050)	(4.521)					
	2.5207	1.5351	−0.2609		0.0964	0.9938	0.0874	1.1858
	(2.844)	(5.674)	(1.303)		(6.129)			
	0.1844			1.2007		0.9439	0.1772	0.5574
	(0.590)			(12.499)				
	−205.2085			−0.2478	0.1066	0.9855	0.0911	1.1552
	(7.138)			(1.187)	(7.145)			
TRNM	−1.7617	1.1584	0.6361			0.9971	0.0574	0.7491
	(1.554)	(2.406)	(4.482)					
	−2.0453	1.7706	0.2921		0.0220	0.9972	0.0567	0.9373
	(1.794)	(2.579)	(0.936)		(1.233)			
	0.8712			0.9620		0.9927	0.0641	0.5379
	(9.003)			(35.911)				
	8.0977			1.0019	−0.0037	0.9923	0.0659	0.5498
	(0.241)			(5.344)	(0.215)			
OTHM	−1.6193	1.1092	0.5440			0.9872	0.0857	0.5012
	(0.933)	(2.835)	(7.376)					
	−3.9175	0.5939	1.2092		−0.0701	0.9917	0.0690	0.5378
	(2.496)	(1.680)	(5.596)		(3.202)			
	1.8714			0.6767		0.9742	0.0927	0.2248
	(15.971)			(18.825)				
	114.9624			1.2558	−0.0584	0.9786	0.0840	0.3919
	(2.261)			(4.786)	(2.224)			

Table 1. (continued)

Industry	Const.	$\ln L$	$\ln K$	$\ln(K/L)$	T	$\bar{R}^2$	SE	DW
CNTR	7.7316	−0.5516	0.6918			0.9909	0.0652	0.7895
	(2.772)	(0.807)	(4.093)					
	1.9984	0.3106	0.8166		−0.0496	0.9928	0.0581	0.7877
	(0.570)	(0.435)	(5.106)		(2.323)			
	3.0801			0.4126		0.9691	0.0683	0.4639
	(59.578)			(17.166)				
	94.1405			0.8314	−0.0461	0.9791	0.0565	0.8152
	(3.167)			(6.017)	(3.063)			
EGWS	−1.6184	−0.4735	1.2244			0.9837	0.1028	0.2110
	(1.246)	(0.458)	(4.945)					
	−14.1764	−1.1212	2.9638		−0.1361	0.9857	0.0965	0.4681
	(2.022)	(1.086)	(3.012)		(1.819)			
	−2.0115			1.1488		0.9756	0.1002	0.1802
	(5.722)			(19.393)				
	175.7539			2.5877	−0.0946	0.9777	0.0958	0.5432
	(1.618)			(2.937)	(1.636)			
WSRT	−9.6038	2.5098	0.2826			0.9848	0.0965	0.5579
	(7.346)	(7.690)	(2.532)					
	−11.0001	2.6357	0.3517		−0.0091	0.9838	0.0994	0.5759
	(0.806)	(2.076)	(0.516)		(0.103)			
	0.5389			0.9941		0.8587	0.2012	0.1475
	(1.490)			(7.371)				
	−157.1929			−0.3294	0.0819	0.9672	0.0997	0.4243
	(7.481)			(7.507)	(1.732)			
FIIN	−2.4517	0.5498	1.0585			0.9761	0.1527	0.3661
	(1.167)	(0.360)	(1.562)					
	1.1084	3.5387	−1.3332		0.0987	0.9926	0.0857	1.3021
	(0.845)	(3.592)	(2.455)		(6.168)			
	−0.9843			1.5322		0.9489	0.1506	0.3817
	(2.683)			(13.141)				
	−174.5695			−0.2427	0.0910	0.9789	0.0075	0.6176
	(5.119)			(0.680)	(5.090)			
TRCM	−3.9400	1.0431	0.7890			0.9858	0.0898	0.1728
	(1.287)	(1.498)	(7.132)					
	−18.2488	2.5426	1.5325		−0.0922	0.9936	0.0607	0.6750
	(4.899)	(4.445)	(8.618)		(4.608)			
	0.3501			0.9318		0.9790	0.0922	0.1805
	(2.368)			(20.939)				
	32.8817			1.1381	−0.0169	0.9787	0.0928	0.2131
	(0.876)			(1.697)	(0.867)			
SRVC	−9.337	3.3314	−0.2819			0.9821	0.1045	0.4382
	(2.137)	(2.980)	(0.834)					
	−17.9377	4.0802	0.2716		−0.0989	0.9828	0.1024	0.6217
	(2.279)	(3.298)	(0.504)		(1.303)			
	2.1139			0.5961		0.9329	0.1203	0.1810
	(18.379)			(11.337)				
	−117.4742			−0.0942	0.0615	0.9341	0.1193	0.2061
	(1.121)			(0.155)	(1.141)			

Note: Figures in parentheses are t-values for the estimates.

Table 2. Cobb-Douglas Type (Potential output is used)

Industry	Const.	$\ln L$	$\ln K$	$\ln(K/L)$	T	$\bar{R}^2$	SE	DW
AFF	10.9850	−0.2249	0.0062			0.6659	0.0349	0.4155
	(3.5612)	(0.7861)	(0.0561)					
	10.7040	−0.2222	0.0357		−0.0026	0.6454	0.0360	0.4120
	(2.7698)	(0.7520)	(0.1387)		(0.1278)			
	1.5080			0.3466		0.9741	0.0423	0.3115
	(39.2496)			(26.7278)				
	1.1579			0.5743	−0.0289	0.9756	0.0411	0.3025
	(4.7855)			(3.6809)	(1.4643)			
MIN	−1.7122	0.1985	1.2013			0.9472	0.0500	0.7130
	(0.8094)	(1.5406)	(4.9567)					
	−0.7231	−0.1346	1.2629		−0.0297	0.9556	0.0459	0.8494
	(0.3621)	(0.6724)	(5.6355)		(2.0602)			
	0.5745			0.9398		0.9946	0.0503	0.6164
	(9.7267)			(59.3403)				
	−0.0243			1.1938	−0.0316	0.9958	0.0447	0.8789
	(0.0957)			(11.2290)	(2.4107)			
FOOD	3.6214	0.2431	0.5674			0.9958	0.0232	0.5883
	(5.6500)	(1.6689)	(44.5610)					
	4.5350	0.4173	0.3190		0.0247	0.9982	0.0151	1.7547
	(9.9225)	(4.1199)	(6.2211)		(4.9076)			
	2.7335			0.5552		0.9945	0.0238	0.6318
	(86.8918)			(58.4277)				
	3.2433			0.3274	0.0225	0.9968	0.0181	1.6240
	(23.3781)			(5.3231)	(3.7310)			
TETL	−0.7829	0.5577	0.8097			0.9935	0.0218	0.7533
	(1.9259)	(8.3558)	(52.1856)					
	−1.8786	0.4867	1.0022		−0.0120	0.9959	0.0174	0.9194
	(4.0445)	(8.4863)	(16.7544)		(3.2890)			
	1.2195			0.7666		0.9879	0.0331	0.2121
	(19.7753)			(39.3174)				
	1.2763			0.7432	0.0016	0.9872	0.0340	0.2246
	(5.6026)			(8.0614)	(0.2596)			
PLPP	−2.9987	1.3036	0.8259			0.9962	0.0307	1.3222
	(6.7435)	(10.4852)	(64.8893)					
	−4.3702	1.0021	1.2162		−0.0371	0.9969	0.0277	1.1218
	(5.9353)	(4.9941)	(6.9032)		(2.2203)			
	1.1218			0.8447		0.9762	0.0738	0.1341
	(8.6596)			(27.9467)				
	0.5268			1.0275	−0.0173	0.9750	0.0756	0.1310
	(0.3373)			(2.1439)	(0.3822)			
CHMS	−2.8780	0.7287	1.0107			0.9968	0.0366	0.4467
	(4.3217)	(3.9146)	(57.0661)					
	−3.0724	0.0418	1.3911		−0.0356	0.9980	0.0292	0.4243
	(5.7422)	(0.1625)	(11.8533)		(3.2655)			
	−0.0883			1.0530		0.9930	0.0509	0.1336
	(0.9108)			(51.9982)				
	−2.0294			1.5746	−0.0512	0.9969	0.0337	0.5720
	(5.0566)			(14.6759)	(4.8998)			

Table 2. (continued)

Industry	Const.	$\ln L$	$\ln K$	$\ln (K/L)$	T	$\bar{R}^2$	SE	DW
FRNF	−0.4465	0.7280	0.7540			0.9991	0.0168	1.5243
	(0.6851)	(11.2441)	(88.6175)					
	−0.7943	0.2457	1.0795		−0.0325	0.9999	0.0059	1.4043
	(8.3445)	(4.9958)	(36.4717)		(11.0539)			
	1.5922			0.8017		0.9947	0.0363	0.2154
	(23.3794)			(59.4678)				
	−0.4321			1.3189	−0.0543	0.9985	0.0194	0.5306
	(1.4373)			(17.2194)	(6.7826)			
FMTP	−1.1728	1.5764	0.3074			0.9986	0.0673	0.6209
	(1.9256)	(8.2293)	(7.4523)					
	−0.2476	−0.0304	1.3373		−0.1206	0.9976	0.0308	0.7497
	(0.8209)	(0.1396)	(10.3599)		(8.0651)			
	2.3022			0.5160		0.9302	0.1119	0.2028
	(23.5456)			(15.9420)				
	0.5973			1.6506	−0.1532	0.9921	0.0376	0.7941
	(4.0770)			(17.2576)	(11.9398)			
GNLM	1.8566	0.3586	0.6805			0.9950	0.0423	0.5939
	(3.1400)	(2.3091)	(26.4090)					
	3.1932	−0.4761	1.0746		−0.0386	0.9971	0.0323	1.0115
	(5.4803)	(1.8394)	(9.7345)		(3.6282)			
	2.0299			0.6868		0.9926	0.0413	0.5915
	(46.0704)			(50.5067)				
	1.7441			0.8363	−0.0179	0.9940	0.0371	0.5356
	(13.2844)			(12.5561)	(2.2833)			
ELTM	0.1260	0.0019	1.1043			0.9986	0.0281	0.3450
	(0.7859)	(0.0238)	(34.3885)					
	0.7457	0.0878	0.9575		0.0133	0.9991	0.0222	0.4038
	(3.3044)	(1.2750)	(18.7799)		(3.3235)			
	0.4452			1.1634		0.9962	0.0306	0.3690
	(8.3517)			(70.1853)				
	0.9374			0.9596	0.0150	0.9980	0.0223	0.4038
	(7.4469)			(18.8130)	(4.1111)			
TRNM	−0.3888	0.6348	0.7796			0.9993	0.0203	0.6066
	(0.9705)	(3.7301)	(15.5414)					
	−0.1159	0.0458	1.1105		−0.0211	0.9997	0.0125	0.4850
	(0.4603)	(0.3020)	(16.1104)		(5.3700)			
	0.9843			0.9496		0.9976	0.0257	0.1853
	(25.3922)			(88.4789)				
	0.2979			1.2149	−0.0249	0.994	0.0133	0.5882
	(3.0217)			(32.2035)	(7.1106)			
OTHM	−0.9270	0.8907	0.6150			0.9846	0.0691	0.4770
	(0.6630)	(2.8249)	(10.3466)					
	−3.4472	0.3257	1.3445		−0.0769	0.9955	0.0375	0.3904
	(4.0400)	(1.6947)	(11.4462)		(6.4591)			
	1.7756			0.7177		0.9704	0.0742	0.1338
	(18.9419)			(24.9593)				
	0.3442			1.3834	−0.0671	0.9832	0.0558	0.2646
	(0.9088)			(7.9317)	(3.8464)			

Table 2. (continued)

Industry	Const.	$\ln L$	$\ln K$	$\ln(K/L)$	T	$\bar{R}^2$	SE	DW
CNTR	11.3768	−1.4925	0.9551			0.9862	0.0603	0.5964
	(4.4077)	(2.3600)	(6.1082)					
	1.6681	−0.0323	1.1665		−0.0840	0.9977	0.0245	0.4810
	(1.1293)	(0.1074)	(17.2984)		(9.3296)			
	3.0614			0.4559		0.9409	0.0744	0.2667
	(54.3858)			(17.4263)				
	2.4184			1.1821	−0.0809	0.9939	0.0240	0.5146
	(44.3813)			(20.1559)	(12.5117)			
EGWS	−2.3905	0.4381	0.9810			0.9704	0.0959	0.1258
	(1.9733)	(0.4550)	(4.2495)					
	−21.3046	−0.5374	3.6008		−0.2050	0.9826	0.0735	0.4661
	(3.9911)	(0.6835)	(4.8057)		(3.5982)			
	−1.7291			1.1082		0.9543	0.0940	0.1327
	(5.2407)			(19.9355)				
	−9.9529			2.6799	−0.1033	0.9605	0.0874	0.5511
	(2.3658)			(3.3352)	(1.9601)			
WSRT	−9.3833	2.3950	0.3455			0.9724	0.0938	0.4002
	(7.3810)	(7.5464)	(3.1843)					
	−17.2210	3.1001	0.7326		−0.0508	0.9713	0.0956	0.3217
	(1.3111)	(2.5396)	(1.1184)		(0.5996)			
	0.4655			1.0365		0.7642	0.1954	0.1218
	(1.3248)			(7.9115)				
	3.0300			−0.2384	0.0789	0.9390	0.0994	0.2742
	(7.6420)			(1.2672)	(7.2466)			
FIIN	−3.0019	1.7131	0.4429			0.9966	0.0373	1.0388
	(5.8439)	(4.5820)	(2.6722)					
	−3.0107	1.7057	0.4488		−0.0002	0.9963	0.0385	1.0320
	(5.1065)	(3.8535)	(1.8392)		(0.0340)			
	−0.2131			1.3432		0.9773	0.0606	0.2145
	(1.4430)			(28.6217)				
	−0.6025			1.4932	−0.0077	0.9766	0.0615	0.2393
	(1.0205)			(6.6337)	(0.6819)			
TRCM	−2.6828	0.7321	0.8188			0.9696	0.0945	0.1403
	(0.8323)	(0.9987)	(7.2857)					
	−19.5699	2.5018	1.7263		−0.1088	0.9908	0.0519	0.9022
	(6.1298)	(5.1115)	(11.3446)		(6.3555)			
	0.3124			0.9485		0.9581	0.0942	0.1381
	(2.0676)			(20.8554)				
	−0.5723			1.3123	−0.0297	0.9612	0.0906	0.1823
	(0.9804)			(5.5470)	(1.5648)			
SRVC	−8.2938	3.0104	−0.1546			0.9690	0.1014	0.3648
	(1.9555)	(2.7748)	(0.4713)					
	−18.2082	3.8737	0.4834		−0.1141	0.9715	0.0972	0.5441
	(2.4374)	(3.2984)	(0.9451)		(1.5820)			
	2.0748			0.6404		0.8953	0.1145	0.1337
	(18.9427)			(12.7883)				
	2.6985			0.1170	0.0466	0.8942	0.1152	0.1655
	(3.8297)			(0.1997)	(0.8962)			

function coincides with one-output and two-input translog production function assumed constant returns to scale and group additivity between factors. The functional form to be estimated is

$$\ln\left(\frac{K}{L}\right)=a_0+a_1\ln\left(\frac{K}{L}\right)-\frac{1}{2}a_2\left\{\ln\left(\frac{K}{L}\right)\right\}^2 \tag{5}$$

marginal productivity of labor and capital are

$$\frac{\partial X}{\partial L}=\frac{X}{L}\left\{(1-a_1)-a_2\ln\frac{K}{L}\right\} \tag{6}$$

Table 3. Kmenta Type (Linear approximation of CES type)

Industry	Const.	$\ln(K/L)$	$\{\ln(K/L)\}^2$	$\bar{R}^2$	SE	DW
AFF	0.8919	0.8068	−0.0804	0.9904	0.0250	0.8701
	(8.3147)	(10.2533)	(5.8769)			
MIN	−0.3085	1.4541	−0.0719	0.9966	0.0402	0.8045
	(1.1507)	(9.4269)	(3.3457)			
FOOD	3.3598	0.1545	0.0622	0.9966	0.0187	1.0187
	(18.5343)	(1.3414)	(3.4876)			
TETL	−1.5599	2.5438	−0.2800	0.9948	0.0216	0.6076
	(2.8194)	(7.2048)	(5.0369)			
PLPP	−3.9347	.32641	−0.2846	0.9992	0.0139	1.1019
	(17.1537)	(29.8693)	(22.1696)			
CHMS	−3.6565	2.5990	−0.1650	0.9983	0.0249	0.3863
	(7.7560)	(12.7741)	(7.6075)			
FRNF	−0.7237	1.7489	−0.0954	0.9988	0.0172	0.6419
	(2.4562)	(14.5806)	(7.9081)			
FMTP	1.1028	1.4385	−0.1637	0.9765	0.0650	0.3487
	(5.3375)	(9.3445)	(6.0375)			
GNLM	1.6748	0.9359	−0.0413	0.9938	0.0378	0.5730
	(9.6528)	(7.8653)	(2.1046)			
ELTM	0.9517	0.8301	0.0579	0.9965	0.0291	0.3679
	(3.1240)	(4.1870)	(1.6863)			
TRNM	0.8962	1.0597	−0.0225	0.9822	0.0668	2.6508
	(1.2542)	(2.5363)	(0.3713)			
OTHM	−0.8221	2.4172	−0.2687	0.9973	0.0225	0.5685
	(4.1837)	(18.9575)	(13.3598)			
CNTR	2.7241	0.8367	−0.0962	0.9499	0.0685	0.2821
	(15.8295)	(4.4790)	(2.0554)			
EGWS	−23.4082	8.4789	−0.6239	0.9923	0.0386	0.3897
	(10.2164)	(10.8988)	(9.4783)			
WRST	−12.4460	10.4460	−1.7146	0.8464	0.1577	0.1797
	(3.1328)	(3.6172)	(3.2605)			
FIIN	−3.3142	3.3916	−0.3350	0.9833	0.0520	0.3790
	(2.9054)	(4.5225)	(2.7354)			
TRCM	−6.0974	4.9916	−0.6244	0.9977	0.0219	0.7510
	(16.8317)	(21.9246)	(17.7778)			
SRVC	−0.2225	2.9081	−0.5279	0.9952	0.0245	0.4297
	(1.8445)	(24.7894)	(19.4119)			

$$\frac{\partial X}{\partial K}=\frac{X}{K}\left(a_1-a_2\ln\frac{K}{L}\right), \tag{7}$$

and, elasticity of substitution σ is

$$\sigma=\frac{\dfrac{\partial\ln X}{\partial\ln L}\cdot\dfrac{\partial\ln X}{\partial\ln K}}{\dfrac{\partial\ln X}{\partial\ln L}\cdot\dfrac{\partial\ln X}{\partial\ln K}+a_2}, \tag{8}$$

then it is not constant. If $a_2=0$, it reduced to Cobb-Douglas type whose elasticity of substitution is unity.

Table 3 shows the estimation results of Eq. (5). Since $\bar{R}^2$ is very high and most of the parameters are statistically significant in all industries, the direct estimation bias is small. a_1 is positive in all industries and a_2 is negative in 16 out of 18 industries. Sign of marginal productivity is determined not only by sign of a_1 and a_2 but also by the value of K/L. When a_1 is very large, marginal productivity of labor can be negative although a_2 is positive. We calculated the value of marginal productivity and found the fact that marginal productivity of labor was negative in 11 industries in some years. We don't, therefore, use linearly approximated CES function in following analysis though it is very general type production function.

(3) *Estimation results of CES production function* (2)—*nonlinear estimation*

Since Arrow-Chenery-Minhas-Solow (1961), the first necessary conditions of profit maximization of the firm has been mainly used in order to estimate CES production function. Brown-De Cani (1963), Fergreson (1965), Sheshinski (1967), Tsujimura-Kuroda (1974) estimated CES parameters indirectly using this profit maximization conditions. The reason lies not only in the difficulty of nonlinear estimation but in the assumption that the first condition of profit maximization is always satisfied. On the other hand, nonlinear estimation of CES production function were tried in Bodkin-Klein (1967) and Kumer-Capinski (1974) but this kind of estimation is much fewer than indirect linear estimation. We will make use of Gauss-Newton method to estimate nonlinear CES function directly in this section.[5]

[5] According to Maddala (1977), Gauss-Newton method can be summarized as follows.
We have a set of n observations, $y_1, y_2, \cdots, y_n$ where

$$y_i=f_i(\theta)+u_i \qquad \theta=(\theta_1,\cdots,\theta_{12})$$

f_i are nonlinear functions respect to parameter θ. Then, parameter $\theta_1,\cdots,\theta_n$ are chosen so as to minimize

$$S(\theta)=\sum_{i=1}^{n}[y_i-f_i(\theta)]^2$$

Taking a linear expansion of $f(\theta)$ around θ_0 and using ordinary least squares, we get the estimates $\hat{\theta}$. If $|(\hat{\theta}-\theta_0)/\theta_0|$ is smaller than a prescribed tolerance (in our estimation 0.01), computation process is over. Otherwise, the same process is repeated setting $\hat{\theta}$ as the new θ_0. If disturbances u_i are assumed to be independently and identically distributed with mean 0 and variance σ^2, the final estimator of θ is approximatedly normally distributed with mean 0 and covariance matrix $\hat{\sigma}^2[F(\hat{\theta})'F(\hat{\theta})]^{-1}$, where $\hat{\sigma}^2=(1/n)SSE(\hat{\theta})$, F is $n\times m$ matrix of partial derivatives $\partial f_i/\partial\theta_j$ $i=1,2,\cdots,n; j=1,2,\cdots,n$.

Table 4. CES Type (Homogeneous degree of v, Output is used)

Industry	γ	δ	ρ	v	λ_L	λ_K	*SE/DW*	*IN*
AFF	4.0211	0.4220	0.2905	0.9615			881.899	50
	(0.1641)	(1.6144)	(0.8177)	(1.2467)			0.2578	
MIN	2.2136	−0.0001	1.7066	0.8611			540.614	50
	(0.0624)	(0.0109)	(0.0984)	(0.3577)			0.0539	
FOOD	11.5787	0.9828	−1.1610	−.2842			520.173	43
	(0.5666)	(53.9751)	(3.6941)	(3.4749)			1.4769	
	21.6750	0.9873	−1.2826	1.1583	0.0078	0.0016	546.436	50
	(0.2425)	(6.6442)	(0.3152)	(1.4929)	(0.0847)	(0.0315)	1.4729	
TETL	3.3505	0.0795	0.4907	0.9698			235.636	32
	(0.7795)	(0.0709)	(0.1116)	(1.8275)			1.2126	
	19.0737	0.0020	1.2861	0.6912	−0.0020	0.0218	236.45	50
	(0.1738)	(0.0314)	(0.1668)	(0.8272)	(0.2115)	(0.4684)	1.1818	
PLPP	0.0300	0.0241	0.8967	1.6871			323.34	50
	(0.2144)	(0.1099)	(0.5030)	(1.4251)			0.8533	
CHMS								
FRNF								
FMTP								
GNLM	0.0001	1.0000	−5.9182	3.4692				50
	(0.02476)	(0.74×10^{-8})	(0.1036)	(0.4737)				
ELTM								
TRNM								
OTHM	0.2067	0.0004	2.4578	1.3197			1158.78	50
	(0.4018)	(0.2372)	(2.5277)	(3.8844)			0.6509	
CNTR	68.8915	0.0001	4.6727	0.7219			1277.39	50
	(1.5193)	(0.1184)	(1.6452)	(8.0742)			0.9597	
EGWS								
WSRT	−0.2184	0.0211	4.7663	1.2141			30215.3	4
	(0.29×10^{-5})	(0.49×10^{-6})	(0.57×10^{-5})	(0.27×10^{-1})			0.0051	
FIIN								
TRCM	0.0068	0.0547	6.9144	2.1801			7838.88	4
	(0.13×10^{-6})	(0.40×10^{-7})	(0.0)	(0.2881)			0.0086	
SRVC								

Note: IN is iteration number. Maximum value of IN is 50.
The estimates are not gained in the blank.

At first, initial values for iteration process must be given. These values are taken by indirect estimation as follows.[6] The condition that marginal productivity of labour equals to wage rate is

$$\ln\left(\frac{X}{L}\right)=\alpha_0+\alpha_1\ln\left(\frac{w}{P*}\right) \tag{9}$$

$\alpha_1=\sigma=1/(1+\rho)$ can be estimated by ordinary least square method, and using ρ, new data $X^{-\hat{\rho}}$, $K^{-\hat{\rho}}$, $L^{-\hat{\rho}}$ are calculated. As Eq. (10) is linear in γ and δ,

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[6] This initial values mere estimated by members of Hamada seminar for the resport in 1981 Mita Festival at Keio University.

$$X^{-\rho}=\gamma\delta L^{-\rho}+\gamma(1-\delta)K^{-\rho} \tag{10}$$

γ and δ can be estimated by OLS. These estimates, $\hat{\rho}$, $\hat{\delta}$, $\hat{\gamma}$ are used as initial values, and the initial value of scale parameter, "v", which is necessary in the case not assuming constant returns to scale, is 1.5, considering the results of Cobb-Douglas type.

(a) The case of homogeneous degree of "v"

Table 4 shows the results of estimating following two equations

$$X=\gamma[\delta L^{-\rho}+(1-\delta)K^{-\rho}]^{-v/\rho} \tag{11}$$

$$X=\gamma[\delta(e^{\lambda_L t}L)^{-\rho}+(1-\delta)(e^{\lambda_K t}K)^{-\rho}]^{-v/\rho} \tag{12}$$

Factor augmenting technical progress is introduced in Eq. (12). The results can be summarized as follows.

1) There are many cases that iteration is stopped by numerical error, especially in (12), and estimates could be gained only in FOOD and TXTL.[7] Moreover, the convergence could not be achieved within the maximum number of iteration (50) in almost all cases.

2) Standard Error of Regression is considerably large and Durbin-Watson Statistics is very low.

3) Most of the estimates are not statistically significant, and every industry has at least one insignificant estimates.

These unsatisfactory results are considered to be caused by the arbitrary choice of initial value of "v", multicollinearity between K and L, and the complexity of function itself. It is necessary, therefore, to sacrifice the generality to get better results.

(b) The case of homogeneous degree of one

Table 5 shows the results of estimating following two equations.

$$\frac{X}{L}=\gamma\left\{\delta+(1-\delta)\left(\frac{K}{L}\right)^{-\rho}\right\}^{-1/\rho} \tag{13}$$

$$\frac{X}{L}=\gamma\left\{\delta e^{-\rho\lambda_L t}+(1-\delta)\left(\frac{Ke^{\lambda_K}}{L}\right)^{-\rho}\right\}^{-1/\rho} \tag{14}$$

Compared with the results in Table 4, the number of cases that iteration process is stopped by numerical error decreases. But almost all estimates of parameters are not significant. Estimates are all significant and the convergence is achieved within 50 iterations in only few industries. As for estimated values, the values of ρ are much smaller than those in Table 4. Table 6 uses potential productivity instead of real output to estimate following equations.

$$\frac{S}{L}=\gamma\left\{\delta+(1-\delta)\left(\frac{K}{L}\right)^{-\rho}\right\}^{-1/\rho} \tag{15}$$

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[7] Numerical errors are caused by anti-logarithms in linear expansion being negative.

Table 5. CES Type (Linear homogeneous, Output is used)

Industry	γ	δ	ρ	λ_L	λ_K	*SE*	*DW*	*IN*
AFF	2.8809	0.3509	0.4237			0.4328	1.1103	3
	(5.4738)	(2.9017)	(2.3523)					
	2.0057	0.0875	1.2453	0.0169	−0.0319	0.4590	1.0742	49
	(0.6122)	(0.0954)	(0.2281)	(0.3707)	(0.2780)			
MIN	4.8851	0.6607	−0.7347			6.1482	1.1201	33
	(0.4555)	(0.5640)	(0.5149)					
FOOD	32.7260	0.9049	−0.7898			4.3504	1.0551	50
	(2.8463)	(7.4206)	(1.8981)					
	13.5312	0.1851	0.9948	−0.0715	−0.0601	5.3975	0.7688	50
	(0.1947)	(0.0513)	(0.6070)	(0.1014)	(0.3714)			
TETL	2.8175	0.1015	0.4159			1.4265	1.2201	50
	(1.5861)	(0.3609)	(0.4268)					
	11.4672	0.2110	0.5420	0.0467	−0.0297	5.5922	0.7339	50
	(0.0758)	(0.0231)	(0.0236)	(0.1555)	(0.0937)			
PLPP	2.0938	0.0139	0.6707			12.9530	0.4941	50
	(1.5622)	(0.1455)	(0.4503)					
	2.5912	0.0715	0.5329	−0.0062	−0.0046	1.5194	1.2309	50
	(0.6877)	(0.1150)	(0.1732)	(0.0510)	(0.0754)			
CHMS	0.8985	−0.0834	−0.3505			16.1952	0.5495	50
	(0.0987)	(0.0166)	(0.0211)					
FRNF	2.5803	0.0282	0.4093			24.9126	0.8743	50
	(1.1685)	(0.1769)	(0.3815)					
	2.9234	0.0296	0.6978	0.0228	0.0698	22.8859	1.1307	50
	(0.2118)	(0.0442)	(0.1583)	(0.1567)	(0.1901)			
FMTP	3.5234	0.0010	2.4163			5.1005	0.9417	50
	(6.7672)	(0.2842)	(2.3157)					
GNLM	11.1841	0.7813	−0.7853			7.0518	0.9645	17
	(0.9708)	(1.5169)	(0.8309)					
	10.6597	0.6676	0.0910	−0.0178	0.3495	7.6178	0.7360	50
ELTM	1.4223	−0.0617	0.4234			8.4495	0.9629	50
	(1.0242)	(0.2095)	(0.3235)					
TRNM	2.1880	0.0015	0.8967			5.2234	0.4397	50
	(4.3604)	(0.0510)	(0.1851)					
OTHM	3.0093	0.0013	1.7634			5.8985	0.2093	50
	(6.5890)	(0.2315)	(1.5253)					
	3.6514	0.0349	0.7999	0.0123	−0.0056	6.7841	0.2197	50
	(0.3482)	(0.0315)	(0.0698)	(0.0346)	(0.0275)			
CNTR	11.9090	0.0474	1.6725			2.9216	0.7838	19
	(8.6366)	(1.0528)	(3.3529)					
	12.8419	0.0330	1.7289	0.0011	−0.0355	3.3703	0.6818	50
	(2.8413)	(0.1039)	(0.2705)	(0.0190)	(0.5050)			
EGWS	0.1526	−0.0733	0.1322			18.9333	0.1524	50
	(0.1787)	(0.0564)	(0.0480)					
WRST	1.0644	−0.0670	0.1570			7.4229	0.0569	50
	(0.1760)	(0.0196)	(0.0090)					
FIIN	0.4837	−0.3908	0.0591			6.9886	0.4606	50
	(0.2551)	(0.1822)	(0.0477)					
TRCM	1.1948	0.0014	1.0859			2.9143	0.1463	50
	(3.4042)	(0.0384)	(0.1518)					
	1.0218	0.0004	1.9642	0.0008	0.0490	0.8775	0.6497	50
	(19.0556)	(0.1898)	(1.3601)	(0.0469)	(2.0747)			
SRVC	4.4218	0.0007	2.9898			3.6700	0.1509	50
	(9.6800)	(0.2057)	(1.6083)					

Table 6. CES Type (Linear homogeneous, Potential Productivity is used)

Industry	γ	δ	ρ	λ_L	SE	DW	IN
AFF	2.2313	0.1031	0.7992		0.3738	0.9167	10
	(7.4888)	(2.4771)	(4.8039)				
	2.3832	0.2378	0.3888	−0.0327	0.3574	0.9729	5
	(10.3222)	(4.2725)	(1.9387)	(1.2333)			
MIN	1.5439	0.0014	1.0682		1.4157	0.5552	50
	(22.2859)	(0.4362)	(1.9860)				
FOOD	33.8688	0.8871	−0.7075		1.9506	1.0485	37
	(6.4620)	(15.3877)	(4.0638)				
	38.9038	0.9343	−0.7465	0.0267	1.8807	1.5287	50
	(5.1306)	(12.9750)	(2.1140)	(3.3049)			
TETL	1.9849	0.0029	1.4616		1.2935	0.3158	50
	(7.9966)	(0.2924)	(1.4248)				
	1.8239	0.0012	1.0359	−0.0936	0.4587	0.6352	28
	(70.8233)	(1.0965)	(3.1591)	(3.0220)			
PLPP	1.9181	0.0023	1.0254		8.5235	0.1509	50
	(4.7017)	(0.1853)	(0.9114)				
	1.9400	0.0043	0.8514	−0.0040	9.2734	0.1458	50
	(3.0078)	(0.0926)	(0.2635)	(0.0192)			
CHMS	0.7843	−0.0896	−0.0999		24.7978	0.0179	50
	(0.1437)	(0.0264)	(0.0139)				
	0.7335	−0.0894	−0.0319	0.0125	17.3620	0.0390	50
	(0.1829)	(0.0339)	(0.0047)	(0.0235)			
FRNF	2.2389	0.0016	0.9514		15.3816	0.1602	50
	(6.0577)	(0.2502)	(1.3562)				
	2.1837	0.0048	0.0791	−1.0541	1.2860	0.6922	33
	(75.7347)	(1.9358)	(1.0964)	(1.0702)			
FMTP	3.5753	0.0006	2.4889		3.7810	0.3861	50
	(11.4675)	(0.4611)	(3.3512)				
	4.9204	0.0706	0.0381	−0.4964	5.6546	0.1542	50
	(2.0152)	(0.1941)	(0.0966)	(0.1277)			
GNLM	8.2311	0.3509	−0.0374		1.8804	0.6561	5
	(4.9280)	(2.7583)	(0.2199)				
	5.5811	0.1188	−0.1018	−0.5301	2.1526	0.2766	50
ELTM	0.3119	−2.3027	−0.4832		2.1374	0.3837	42
	(0.2033)	(0.2067)	(1.0872)				
TRNM	2.3165	0.1126×10^{-6}	3.5827		0.5482	0.8978	36
	(236.300)	(0.6767)	(9.8882)				
OTHM	2.9960	0.0012	1.7524		3.8436	0.1448	50
	(10.5456)	(0.3412)	(2.2603)				
	2.9008	0.0005	1.6470	−0.0436	0.7018	1.4305	46
	(69.8533)	(1.0233)	(4.7572)	(3.0370)			
CNTR	12.0633	0.0456	1.5765		4.3051	0.2817	17
	(6.5023)	(0.7698)	(2.3676)				
	14.1128	0.0959	−0.0007	−0.4744	4.3333	0.0993	50
	(5.5996)	(0.4200)	(0.3073)	(0.0102)			
EGWS	0.1610	−0.0625	0.1598		20.5697	0.1201	50
	(0.1876)	(0.0542)	(0.0553)				

Table 6. (continued).

Industry	γ	δ	ρ	λ_L	SE	DW	IN
	0.1800	−0.0353	0.2807	−0.0149	20.3400	0.1295	50
	(0.1957)	(0.0415)	(0.0641)	(0.0465)			
WSRT							
FIIN	0.4298	−0.4979	−0.0086		10.6201	0.0446	50
	(0.1329)	(0.1053)	(0.0043)				
	0.4736	−0.4670	−0.0183	0.0061	14.4199	0.0220	50
	(0.0715)	(0.0462)	(0.0032)	(0.0237)			
TRCM	1.1867	0.0004	1.3886		3.0160	0.0982	50
	(4.7395)	(0.0325)	(0.1093)				
	1.1896	0.0006	1.3289	0.0070	3.2223	0.0966	50
	(2.5576)	(0.0145)	(0.0592)	(0.0201)			
SRVC	4.4359	0.0001	3.5546		3.4966	0.0864	50
	(13.5322)	(0.1799)	(1.7433)				
	4.4582	0.0002	3.2958	−0.0010	4.0129	0.0779	50
	(8.9510)	(0.0728)	(0.4410)	(0.0114)			

$$\frac{S}{L}=\gamma\left\{\delta(e^{\lambda_L t})^{-\rho}+(1-\delta)\left(\frac{K}{L}\right)^{-\rho}\right\}^{-1/\rho} \tag{16}$$

The results are not so different from the results in Table 5 except following points. Firstly, the number of cases that iteration process is stopped by numerical errors decreases from 9 to 5, and secondly the number of significant estimates increases from 13 to 20. But most of the estimates are not significant. All estimates are significant only in AFF and FOOD.[8]

As mentioned above, since good estimates could not be obtained except in two industries, AFF and FOOD, the results cannot be used to estimate marginal productivity. It is not effective to introduce the term of technical progress. Since the problem of multicollinearity or initial value of v is removed in Eqs. (13) or (15), the cause of such unsatisfactory results seems to be the specification of CES function itself or the method of nonlinear estimation. Indeed, Gauss-Newton method tends to be largely influenced by initial values. But another non-linear estimation method, pattern-search method brings the similar results.[9] Therefore, the relation between the functional form and the data should be examined in order to obtain better results if the CES specification is kept unchanged. In AFF and FOOD in which the results were better than in any other industry, labor

[8] Except these specification, $S/L=\gamma\cdot e^{\lambda t}[\delta+(1-\delta)(K/L)^{-\rho}]^{-1/\rho}$ which includes Hicks neutral technical progress was estimated. But the results were unsatisfactory.

[9] Nonlinear direct estimation of CES production function by pattern search method was attempted for the report of Hamada seminar at 1981 Mita Festival at Keio University. Pattern search method approaches the optimum value more slowly but more surely than Gauss-Newton method. But the results are very similar to the results of Gauss-Newton method, Bodkin-Klein (1967) estimates macro CES production functions directly and indirectly. Those direct estimation results are not so different from ours in respect to very small δ and insignificant estimates.

employment is decreasing or nearly constant. This fact seems to suggest that CES function fits well if capital input increases much more rapidly than labor input. As it is unrealistic to decrease labor employment, we attempts to increase capital stock exponentially, which means to assume the constant-rate capital-augmenting technical progress. Considering the increasing performance of machinery used as capital stock in the observation period, this assumption does not seem to be unrealistic. Though vintage model is more suitable for this assumption, as the first approach, the value of the rate of technical progress, λ_K, is given a priori. The new capital stock data is $e^{\lambda_K t}K$, and the other parameters are estimated by

$$\frac{S}{L}=\gamma\left\{\delta+(1-\delta)\left(\frac{e^{\lambda_K t}K}{L}\right)^{-\rho}\right\}^{-1/\rho} \tag{17}$$

Table 7 shows the estimation results of Eq. (17) which are obtained by moving the value of λ_K from 0.0 to 0.2. The results are summarized as follows.

1) R^2 is very high.[10]

2) The larger λ_K becomes, the faster the convergence is achieved.

3) The estimated value of γ is not influenced by the value of λ_K.

4) As λ_K becomes larger, not only the estimate of δ but also it's t-value increases, on the other hand, the estimate of ρ decreases.

It is natural that the estimate of δ becomes larger because the increase of λ_K makes $e^{\lambda_K t}K/L$ larger. But the statistical performance is much improved by the increase in t-values and the fast convergence. Looking at each industry, there are many numerical error and small t-values in the third industries, in which L increase conspicuously. If λ_K is greater than 0.2, the better results may be obtained. But it is unrealistic to set the value of λ_K too high.

Since the estimation of Eq. (17) brought good results, we tried with the specification of homogeneous degree of v and the specification of estimating λ_K instead of setting the value a priori, but neither of them was successful. Therefore, marginal productivity is to be estimated by using the results in Table 7. As there is little difference in R^2, we choose the result the sum of whose t-values is maximum by industry. The optimal values of λ_K are as follows.

AFF: 0.20, MIN: 0.08, FOOD: 0.04, TXTL: 0.12, PLPP: 0.06, CHMS: 0.12, FRNF: 0.08, FMTP: 0.20, CNLM: 0.20, LETM: 0.20, TRNM: 0.02, OTHM: 0.10, CNTR: 0.20, EGWS: 0.12, WSRT: 0.16, FIIN: 0.10, TRCM: 0.14, SRVC: 0.14

These results are used to estimate marginal productivity of CES production function in the sequel.

Looking at the estimated chosen above, the estimate of ρ is positive except in FOOD and fairly large in some industries so that the elasticity of substitution, $\sigma=1/(1+\rho)$, is smaller than unity. Considering this fact, it is quetionable to use

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[10] Nonlinear program does not compute R^2. R^2 has a different meaning in the case of nonlinear regression, but we computed R^2 as the ratio of sum of squared residuals to total variation in dependent variables in order to show the degree of fitness.

Table 7-1. AFF

λ_K	γ	δ	ρ	*SE*	*DW*	R^2	*IN*
0.00	2.2313	0.1631	0.7992	0.3738	0.9167	0.9875	10
	(7.4888)	(2.4771)	(4.8039)				
0.02	2.5361	0.2498	0.6421	0.3937	0.8647	0.9859	3
	(7.0072)	(3.1558)	(4.7157)				
0.04	2.8287	0.3302	0.5364	0.4104	0.8257	0.9850	3
	6.9109)	(3.9079)	(4.6336)				
0.06	3.0994	0.4007	0.4607	0.4243	0.7961	0.9838	3
	(6.9579)	(4.6986)	(4.5586)				
0.08	3.3487	0.4618	0.4030	0.4360	0.7730	0.9830	4
	(7.0847)	(5.5310)	(4.4522)				
0.10	3.5714	0.5133	0.3586	0.4460	0.7547	0.9823	4
	(7.3004)	(6.4327)	(4.3958)				
0.12	3.7709	0.5572	0.3229	0.4545	0.7399	0.9816	4
	(7.5427)	(7.3771)	(4.3493)				
0.14	3.9501	0.5946	0.2937	0.4618	0.7277	0.9810	4
	(7.7974)	(8.3602)	(4.3113)				
0.16	4.1111	0.6268	0.2694	0.4683	0.7174	0.9804	4
	(8.0560)	(9.3786)	(4.2805)				
0.18	4.2562	0.6547	0.2488	0.4739	0.7088	0.9800	4
	(8.3134)	(10.4283)	(4.2552)				
0.20	4.3874	0.6789	0.2311	0.4789	0.7013	0.9795	4
	(8.5660)	(11.5052)	(4.2337)				

Table 7-2. MIN

λ_K	γ	δ	ρ	*SE*	*DW*	R^2	*IN*
0.00	1.5439	0.0014	1.0682	1.4157	0.5552	0.9989	50
	(22.2859)	(0.4362)	(1.9860)				
0.02	1.5108	0.0076	0.8763	1.7167	1.0848	0.9983	11
	(15.1958)	(1.1565)	(4.3573)				
0.04	1.5766	0.0258	0.6651	2.1226	1.1950	0.9974	5
	(9.5147)	(1.5760)	(4.6591)				
0.06	1.6768	0.0516	0.5488	2.4448	1.2464	0.9966	9
	(7.2053)	(1.9245)	(4.7974)				
0.08	1.7976	0.0820	0.4716	2.7033	1.2735	0.9959	5
	(6.0110)	(2.2384)	(4.8912)				
0.10	1.9366	0.1157	0.4143	2.9142	1.2889	0.9952	10
	(5.2985)	(2.5670)	(4.9433)				
0.12	2.0872	0.1509	0.3700	3.0895	1.2985	0.9946	13
	(4.8166)	(2.8756)	(4.9740)				
0.14	2.2435	0.1860	0.3348	3.2373	1.3047	0.9941	12
	(4.5227)	(3.1985)	(4.9962)				
0.16	6.2439	0.9915	−0.4585	76.3112	0.0127	0.0	3
0.18							
0.20							

Table 7-3. FOOD

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	33.8688	0.8871	−0.7075	1.9506	1.0485	0.9965	37
	(6.4620)	(15.3877)	(4.0638)				
0.02	29.8761	0.7955	−0.3491	1.6503	1.6992	0.9975	6
	(8.5703)	(14.5117)	(3.6971)				
0.04	28.8282	0.7539	−0.1888	1.6253	2.0198	0.9975	5
	(10.0629)	(15.6230)	(2.7761)				
0.06	28.9699	0.7421	−0.1069	1.6908	2.0887	0.9973	5
	(11.0108)	(17.1174)	(1.9194)				
0.08	29.5914	0.7442	−0.0608	1.7784	2.0598	0.9971	5
	(11.6944)	(18.8133)	(1.2605)				
0.10	30.4014	0.7524	−0.0331	1.8657	2.0045	0.9968	5
	(12.2606)	(20.6693)	(0.7695)				
0.12	31.2707	0.7632	−0.0156	1.9461	1.9471	0.9965	5
	(12.7698)	(22.6526)	(0.3395)				
0.14	32.1393	0.7747	−0.0041	2.0178	1.8950	0.9962	5
	(13.2446)	(24.7358)	(0.1146)				
0.16	32.9791	0.7861	0.0361	2.0182	1.8495	0.9960	5
	(13.6919)	(26.8952)	(0.1095)				
0.18	33.7776	0.7970	0.0089	2.1377	1.8103	0.9957	5
	(14.1164)	(29.1151)	(0.2891)				
0.20	34.5231	0.8072	0.0126	2.1877	1.7765	0.9955	6
	(14.5334)	(31.5137)	(0.4390)				

Table 7-4. TETL

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	1.9849	0.0029	1.4616	1.2935	0.3158	0.9889	50
	(7.9966)	(0.2924)	(1.4248)				
0.02	1.8681	0.0033	1.5614	1.4534	0.5297	0.9860	9
	(10.7073)	(0.6449)	(3.0410)				
0.04	2.2847	0.0355	0.9356	1.7691	0.4872	0.9793	6
	(4.2458)	(0.7971)	(2.6054)				
0.06	2.9008	0.1038	0.6669	2.0051	0.4615	0.9734	5
	(2.9730)	(1.0311)	(2.3766)				
0.08	3.6558	0.1908	0.5151	2.1805	0.4470	0.9685	6
	(2.5313)	(1.2994)	(2.2215)				
0.10	4.4749	0.2788	0.4185	2.3152	0.4378	0.9645	7
	(2.3991)	(1.6203)	(2.1119)				
0.12	5.2988	0.3585	0.3523	2.4215	0.4314	0.9612	7
	(2.4176)	(1.9932)	(2.0366)				
0.14	6.0995	0.4277	0.3039	2.5075	0.4267	0.9584	8
	(2.4724)	(2.3989)	(1.9796)				
0.16	6.8536	0.4864	0.2673	2.5784	0.4232	0.9571	8
	(2.5706)	(2.8433)	(1.9365)				
0.18	7.5571	0.5361	0.2385	2.6379	0.4203	0.9540	8
	(2.6888)	(3.3212)	(1.9019)				
0.20	8.2108	0.5784	0.2151	2.6884	0.4180	0.9522	8
	(2.8211)	(3.8210)	(1.8690)				

Table 7-5. PLPP

λ_K	γ	δ	ρ	*SE*	*DW*	R^2	*IN*
0.01	1.9181	0.0023	1.0254	8.5235	0.1509	0.9768	50
	(4.7017)	(0.1853)	(0.9114)				
0.02	1.7963	0.0014	1.1801	23.8824	0.0338	0.8178	50
	(2.5989)	(0.1110)	(0.7402)				
0.04	1.5841	0.0006	1.4776	2.1820	1.6809	0.9985	22
	(33.2732)	(1.8355)	(14.2838)				
0.06	1.6684	0.0045	1.0799	2.6999	1.3173	0.9977	14
	(18.3186)	(2.2366)	(12.2728)				
0.08	1.8272	0.0149	0.8512	3.0916	1.1505	0.9969	13
	(12.3892)	(2.5536)	11.3840)				
0.10	2.0621	0.0336	0.6996	3.3912	1.0615	0.9963	11
	(9.2095)	(2.8290)	(10.5646)				
0.12	2.3603	0.0601	0.5935	3.6277	1.0068	0.9958	13
	(7.5067)	(3.1493)	(10.0122)				
0.14	2.7179	0.0929	0.5148	3.8189	0.9705	0.9953	15
	(6.4027)	(3.4377)	(9.5799)				
0.16	3.1195	0.1294	0.4551	3.9770	0.9446	0.9949	14
	((5.8252)	(3.8164)	(9.3008)				
0.18	3.5643	0.1682	0.4076	4.1099	0.9253	0.9946	14
	(5.3656)	(4.1625)	(9.0771)				
0.20	4.0431	0.2076	0.3691	4.2233	0.9105	0.9943	15
	(5.1300)	(4.5819)	(8.8880)				

Table 7-6. CHMS

λ_K	γ	δ	ρ	*SE*	*DW*	R^2	*IN*
0.00	0.7343	−0.0896	−0.0999	24.7978	0.0179	0.9196	50
	(0.1437)	(0.0264)	(0.0139)				
0.02	0.9814	0.0003	0.8842	23.0717	0.0777	0.9304	50
	(2.4857)	(0.0187)	(0.1053)				
0.04	0.9915	0.00087	1.6058	3.6642	1.0766	0.9982	19
	(32.2418)	(1.0115)	(9.3641)				
0.06	0.9788	0.0010	1.1650	4.1591	1.0585	0.9977	35
	(19.2059)	(1.3720)	(9.6571)				
0.08							
0.10	1.0680	0.0111	0.7585	5.0551	0.9726	0.9967	20
	(9.5507)	(1.9847)	(9.3039)				
0.12	1.1562	0.0224	0.6477	5.4012	0.9430	0.9962	21
	(7.6139)	(2.2373)	(9.2175)				
0.14							
0.16							
0.18							
0.20							

Table 7-7. FRNF

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	2.2389 (6.0577)	0.0016 (0.2502)	0.9514 (1.3562)	15.3816	0.1602	0.9887	50
0.02	2.2356 (16.0746)	0.0021 (1.2484)	1.0347 (7.0990)	6.7722	1.2740	0.9978	28
0.04	2.5618 (8.9776)	0.0123 (1.5996)	0.7439 (6.6929)	8.0442	1.2201	0.9970	11
0.06	3.0461 (6.3838)	0.0334 (1.9288)	0.5887 (6.4804)	8.9666	1.1970	0.9962	23
0.08	3.6676 (5.1794)	0.0637 (2.2471)	0.4910 (6.4138)	9.6718	1.1838	0.9955	8
0.10	4.4498 (4.4607)	0.1018 (2.5895)	0.4208 (6.2322)	10.2283	1.1759	0.9950	15
0.12	5.3475 (4.0702)	0.1434 (2.9452)	0.3695 (6.1623)	10.6808	1.1701	0.9946	15
0.14							
0.16							
0.18							
0.20							

Table 7-8. FMTP

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	3.5753 (11.4675)	0.0006 (0.4611)	2.4889 (3.3512)	3.7310	0.3861	0.9605	50
0.02	3.5885 (8.0678)	0.0050 (0.7145)	1.7679 (3.7077)	3.9163	0.3720	0.9565	31
0.04	3.7165 (5.8801)	0.0172 (0.8429)	1.3657 (3.8645)	4.0344	0.3668	0.9538	26
0.06	3.8628 (4.9439)	0.0360 (1.0404)	1.1282 (3.9520)	4.1215	0.3633	0.9518	11
0.08	4.0056 (4.3903)	0.0585 (1.2052)	0.9733 (4.1216)	4.1877	0.3613	0.9503	10
0.10	4.1841 (3.9896)	0.0857 (1.3693)	0.8539 (4.2115)	4.2398	0.3601	0.9490	5
0.12	4.3990 (3.6893)	0.1170 (1.5356)	0.7575 (4.2081)	4.2819	0.3594	0.9480	6
0.14	4.5969 (3.5033)	0.1481 (1.7063)	0.6842 (4.2585)	4.3169	0.3590	0.9472	5
0.16	4.7798 (3.3949)	0.1784 (1.8864)	0.6259 (4.3354)	4.3464	0.3588	0.9464	5
0.18	4.9767 (3.2809)	0.2094 (2.0545)	0.5759 (4.3756)	4.3716	0.3587	0.9458	5
0.20	5.1705 (3.2059)	0.2396 (2.2329)	0.5334 (4.4092)	4.3934	0.3586	0.9453	5

Table 7-9. GNLM

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.01	8.2311	0.3509	−0.0374	1.8804	0.6561	0.9967	5
	(4.9380)	(2.7583)	(0.2199)				
0.02	8.0567	0.3116	0.1486	2.0728	0.7926	0.9960	5
	(5.8428)	(3.3470)	(1.2167)				
0.04	8.3348	0.3249	0.2074	2.2590	0.8601	0.9953	4
	(6.1795)	(3.9787)	(2.1110)				
0.06	8.7603	0.3532	0.2238	2.4138	0.8969	0.9946	4
	(6.3108)	(4.6270)	(2.7034)				
0.08	9.2407	0.3858	0.2242	2.5410	0.9189	0.9940	5
	(6.3332)	(5.2667)	(3.1007)				
0.10	9.7404	0.4191	0.2179	2.6465	0.9330	0.9935	5
	(6.4128)	(5.9556)	(3.3964)				
0.12	10.2372	0.4511	0.2091	2.7352	0.9426	0.9931	5
	(6.4842)	(6.6468)	(3.6141)				
0.14	10.7192	0.4812	0.1996	2.8108	0.9492	0.9927	5
	(6.5256)	(7.3321)	(3.7744)				
0.16	11.1901	0.5094	0.1898	2.8760	0.9541	0.9923	6
	(6.6344)	(8.1471)	(3.9434)				
0.18	11.6379	0.5354	0.1806	2.9328	0.9577	0.9920	6
	(6.7257)	(8.9124)	(4.0591)				
0.20	12.0643	0.5593	0.1720	2.9828	0.9604	0.9918	6
	(6.8171)	(9.6888)	(4.1537)				

Table 7-10. ELTM

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	0.3119	−0.2303	−0.4833	2.1374	0.3837	0.9956	42
	(0.2033)	(0.2067)	(1.0872)				
0.02	1.3845	−0.4186	1.0425	6.6294	0.0500	0.9579	50
	0.0465)	(0.0101)	(0.0501)				
0.04	2.5998	0.0314	0.6347	1.7155	0.4937	0.9972	10
	(9.5126)	(1.3504)	(3.1574)				
0.06							
0.08	4.4573	0.2264	0.2587	2.2016	0.5872	0.9954	9
	(5.2193)	(3.0016)	(2.6323)				
0.10	5.4686	0.3143	0.2026	2.3967	0.6050	0.9945	7
	(4.9529)	(3.8319)	(2.4621)				
0.12	6.4594	0.3883	0.1668	2.5604	0.6153	0.9937	6
	(4.9125)	(4.6989)	(2.3351)				
0.14	7.4075	0.4500	0.1419	2.6981	0.6217	0.9930	11
	(4.9766)	(5.6022)	(2.2353)				
0.16							
0.18							
0.20	9.9146	0.5822	0.0980	3.0015	0.6328	0.9914	8
	(5.3568)	(8.5097)	(2.0372)				

Table 7-11. TRNM

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	2.3165	0.1126×10^{-6}	3.5827	0.5482	0.8978	0.9998	36
	(23.6300)	(0.6767)	(9.8882)				
0.02	2.2303	0.0010	1.4697	0.6812	1.4319	0.9997	17
	(73.4124)	(2.7172)	(16.3099)				
0.04	2.3214	0.0090	1.0069	1.0705	1.0050	0.9993	8
	(30.4531)	(3.2149)	(13.6587)				
0.06	2.5044	0.0268	0.7859	1.3469	0.8804	0.9989	9
	(18.9895)	(3.6441)	(12.7620)				
0.08							
0.10	3.0423	0.0857	0.5569	1.7050	0.7965	0.9982	15
	(11.9959)	(4.8189)	(12.0084)				
0.12	3.3672	0.1221	0.4882	1.8276	0.7787	0.9980	21
	(10.5736)	(5.3897)	(11.8185)				
0.14							
0.16							
0.18							
0.20							

Table 7-12. OTHM

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	2.9960	0.0012	1.7524	3.8436	0.1448	0.9776	50
	(10.5456)	(0.3412)	(2.2603)				
0.02	2.6422	0.0002	2.3690	1.5364	0.8930	0.9964	42
	(40.4926)	(1.3095)	(10.1239)				
0.04	2.6206	0.0022	1.6769	1.8876	0.7273	0.9946	12
	(21.5960)	(1.5312)	(9.3092)				
0.06	2.6885	0.0088	1.2931	2.1671	0.6466	0.9929	8
	(14.0793)	(1.8045)	(8.6741)				
0.08	2.8205	0.0220	1.0541	2.3814	0.6069	0.9914	9
	(10.2485)	(1.9803)	(8.3470)				
0.10	2.9981	0.0415	0.8909	2.5520	0.5833	0.9901	6
	(8.2227)	(2.2039)	(8.1073)				
0.12	3.2124	0.0666	0.7716	2.6907	0.5686	0.9890	8
	(7.0105)	(2.4443)	(7.9164)				
0.14	3.4539	0.0957	0.6808	2.8055	0.5588	0.9880	7
	(6.1988)	(2.6765)	(7.7758)				
0.16	3.7164	0.1277	0.6091	2.9022	0.5519	0.9872	7
	(5.6564)	(2.9249)	(7.6537)				
0.18	3.9921	0.1611	0.5511	2.9849	0.5470	0.9865	7
	(5.2949)	(3.1928)	(7.5559)				
0.20	4.2773	0.1950	0.5033	3.0563	0.5433	0.9858	7
	(5.0377)	(3.4705)	(7.4736)				

Table 7-13. CNTR

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	12.0633	0.0456	1.5765	4.3051	0.2817	0.9379	17
	(6.5023)	(0.7698)	(2.3676)				
0.02	12.6348	0.0973	1.2128	4.5568	0.2825	0.9304	11
	(5.2402)	(1.0024)	(2.5128)				
0.04	13.2218	0.1541	0.9954	4.7364	0.2836	0.9248	8
	(4.6970)	(1.2537)	(2.6227)				
0.06	13.7764	0.2100	0.8490	4.8713	0.2848	0.9205	4
	(4.4776)	(1.5364)	(2.7201)				
0.08	14.4561	0.2702	0.7292	4.9760	0.2861	0.9170	5
	(4.2755)	(1.8079)	(2.6939)				
0.10	14.9862	0.3205	0.6468	5.0601	0.2872	0.9142	5
	(4.1767)	(2.0765)	(2.7401)				
0.12	15.4673	0.3659	0.5820	5.1291	0.2882	0.9119	5
	(4.1212)	(2.3533)	(2.7818)				
0.14	15.9647	0.4094	0.5259	5.1866	0.2891	0.9099	6
	(4.1329)	(2.6608)	(2.7819)				
0.16	16.3903	0.4472	0.4814	5.2354	0.2899	0.9082	6
	(4.1407)	(2.9611)	(2.7995)				
0.18	16.7748	0.4808	0.4442	5.2772	0.2906	0.9067	6
	(4.0372)	(0.1472)	(0.1577)				
0.20	17.0902	0.5096	0.4139	5.3135	0.2911	0.9054	5
	(4.1752)	(3.5698)	(2.8501)				

Table 7-14. EGWS

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	0.1610	−0.0625	0.1598	20.5697	0.1201	0.8635	50
	(0.1876)	(0.0542)	(0.0553)				
0.02							
0.04							
0.06							
0.08	0.2430	0.00001	1.7085	4.3512	0.6406	0.9989	24
(20.9820)	(0.8782)	(9.7901)					
0.10	0.2513	0.0001	1.3005	4.7709	0.6044	0.9927	31
	(13.7638)	(1.0639)	(9.6350)				
0.12	0.2694	0.0009	1.0461	5.0643	0.5864	0.9917	21
	(9.8028)	(1.2827)	(9.2799)				
0.14							
0.16							
0.18							
0.20							

Table 7-15. WSRT

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00							
0.02							
0.04	1.3588	0.0000001	4.4476	4.2633	0.0618	0.8342	50
	(13.1662)	(0.0637)	(1.3388)				
0.06	1.2068	0.0000001	4.8446	0.6749	0.6214	0.9958	30
	(70.8128)	(0.4343)	(8.4590)				
0.08	1.1036	0.00002	3.0689	0.5837	0.7031	0.9969	46
	(58.1720)	(1.2275)	(12.8872)				
0.10	1.0555	0.0004	2.1464	0.7599	0.4815	0.9947	44
	(30.5093)	(1.4122)	(11.5383)				
0.12	1.0526	0.0028	1.6135	0.9948	0.3826	0.9919	19
	(17.7791)	(1.5959)	(10.0008)				
0.14	1.0847	0.0096	1.2801	1.0951	0.3395	0.9891	31
	(0.0906)	(0.0054)	(0.1404)				
0.16	1.1518	0.0234	1.0489	1.2136	0.3184	0.9866	14
	(8.6037)	(1.8840)	(8.3255)				
0.18							
0.20							

Table 7-16. FIIN

λ_K	γ	δ	ρ	SE	DW	R^2	IN
0.00	0.4298	−0.4979	−0.0086	10.6201	0.0446	0.7538	50
	(0.1329)	(0.1053)	(0.0043)				
0.02	0.3083	−0.9731	−0.2994	12.5702	0.0487	0.6550	50
	(0.0521)	(0.4443)	(0.8935)				
0.04	1.7516	0.00000003	4.4609	21.2510	2.0657	0.9973	48
	(80.3553)	(0.5414)	(8.9044)				
0.06	1.7060	0.0001	2.2108	1.3411	1.5941	0.9961	37
	(34.2336)	(1.1733)	(9.9298)				
0.08	1.7743	0.0027	1.4819	1.4241	1.5183	0.9956	11
	(20.2374)	(1.7419)	(10.5461)				
0.10	1.9153	0.0118	1.1276	1.4577	1.5178	0.9954	27
	(14.6129)	(2.3155)	(10.9883)				
0.12	2.1113	0.0294	0.9157	1.4741	1.5340	0.9953	16
	(11.7408)	(2.8557)	(11.3435)				
0.14	2.3492	0.0547	0.7740	1.4837	1.5524	0.9952	14
	(10.1713)	(3.4070)	(11.6659)				
0.16							
0.18							
0.20	3.2424	0.1593	0.5326	1.4995	1.5976	0.9951	14
	(8.2312)	(5.1523)	(12.1820)				

Table 7-17. TRCM

λ_K	γ	δ	ρ	*SE*	*DW*	R^2	*IN*
0.00	1.1867	0.0004	1.3886	3.0161	0.0982	0.9533	50
	(4.7395)	(0.0325)	(0.1693)				
0.02	1.1262	0.000001	3.3090	5.6967	0.0251	0.8333	50
	(8.7554)	(0.0580)	(0.9210)				
0.04							
0.06	0.9375	0.00009	2.3495	0.7382	0.9394	0.9972	44
	(41.6718)	(1.3875)	(12.8330)				
0.08							
0.10	0.8969	0.0026	1.4702	1.0119	0.6020	0.9947	19
	(18.9560)	(1.8730)	(11.6235)				
0.12	0.9091	0.0070	1.2273	1.1257	0.5307	0.9935	40
	(13.7345)	(2.0096)	(10.7833)				
0.14	0.9244	0.0138	1.0644	1.2232	0.4853	0.9923	17
	(11.2942)	(2.2047)	(10.9519)				
0.16							
0.18							
0.20	1.0659	0.0554	0.7416	1.4317	0.4194	0.9894	12
	(6.7418)	(2.6388)	(9.5509)				

Table 7-18. SRVC

λ_K	γ	δ	ρ	*SE*	*DW*	R^2	*IN*
0.00	4.4359	0.0001	3.5546	3.4966	0.0864	0.9006	50
	(13.5322)	(0.1799)	(1.7433)				
0.02	4.1569	0.0009	2.7058	4.5544	0.0458	0.8287	50
	(8.0953)	(0.2138)	(1.8692)				
0.04	3.6991	0.0007	2.9029	1.2768	0.3785	0.9865	35
	(26.1923)	(0.9184)	(6.8304)				
0.06	3.6635	0.0059	2.0384	1.4818	0.3458	0.9819	14
	(16.0576)	(1.2407)	(6.7253)				
0.08	3.7386	0.0120	1.5593	1.6228	0.3328	0.9783	10
	(11.2917)	(1.4984)	(6.5869)				
0.10	3.8835	0.0439	1.2594	1.7250	0.3261	0.9754	10
	(8.7925)	(1.7326)	(6.4500)				
0.12	4.0698	0.0757	1.0553	1.8023	0.3219	0.9732	6
	(7.4291)	(1.9914)	(6.3202)				
0.14	4.2859	0.1133	0.9068	1.8628	0.3192	0.9713	7
	(6.4990)	(2.2143)	(6.2038)				
0.16	4.5104	0.1534	0.7953	1.9116	0.3173	0.9698	7
	(5.9660)	(2.4801)	(6.1137)				
0.18	4.7393	0.1944	0.7080	1.9517	0.3159	0.9685	7
	(5.6124)	(2.7555)	(6.0405)				
0.20	4.9668	0.2347	0.6380	1.9854	0.3148	0.9675	7
	(5.3735)	(3.0412)	(5.9806)				

Table 8. Cobb-Douglas Type with λ_K (Linear homogeneous)

Industry	λ_K	Const.	$\ln(K/L)$	$\bar{R}^2$	SE	DW
AFF	0.20	1.8371	0.1340	0.9662	0.0421	0.3140
		(60.1402)	(23.3118)			
MIN	0.08	1.4542	0.5690	0.9878	0.1033	0.5353
		(21.5774)	(39.2639)			
FOOD	0.04	3.0938	0.3941	0.9967	0.0060	1.3259
		(158.967)	(76.2691)			
TETL	0.12	2.4357	0.2707	0.9691	0.0502	0.3739
		(48.5489)	(24.4250)			
PLPP	0.06	2.1890	0.5170	0.9748	0.1037	0.1662
		(23.1727)	(27.1549)			
CHMS	0.12	2.0933	0.4695	0.9790	0.1401	0.1587
		(21.6150)	(29.7711)			
FRNF	0.08	2.9608	0.4530	0.0877	0.0547	0.3425
		(42.8850)	(39.0449)			
FMTP	0.20	2.7878	0.2034	0.8922	0.3484	0.2169
		(32.0661)	(12.5784)			
GNLM	0.20	2.8760	0.2525	0.9686	0.1299	0.4450
		(49.5389)	(24.2440)			
ELTM	0.10	2.0923	0.4867	0.9882	0.0515	0.4292
		(39.4730)	(39.9525)			
TRNM	0.02	1.4211	0.7813	0.9946	0.0266	0.1920
		(28.0065)	(59.0928)			
OTHM	0.10	2.5612	0.3563	0.9517	0.1613	0.2053
		(31.5721)	(19.3727)			
CNTR	0.20	3.3410	0.1582	0.8902	0.1852	0.2819
		(58.1343)	(12.4523)			
EGWS	0.12	2.0332	0.3900	0.9449	0.1919	0.1368
		(12.9717)	(18.0720)			
WRST	0.16	1.9076	0.3032	0.9140	0.2508	0.1817
		(19.8518)	(14.2438)			
FIIN	0.10	2.1368	0.4444	0.9496	0.1470	0.2215
		(21.3169)	(18.9386)			
TRCM	0.14	1.8061	0.3418	0.9269	0.2784	0.1242
		(16.6854)	(15.5501)			
SRVC	0.14	2.5404	0.2492	0.8998	0.2262	0.1556
		(34.8631)	(13.0977)			

Cobb-Douglas function where σ is set to be unity. In six industries whose σ is less than 0.5, three industries have negative marginal productivity in the case of constant returns Cobb-Douglas function. Therefore, the cause of unsatisfactory results in Cobb-Douglas case may be the constraint that σ is set to be unity a priori though the true value is considerably smaller than unity. This small value of σ is not caused by the existence of λ_K. If λ_K is not considered, σ will be much smaller than unity because the increases in λ_K leads to the increase in σ.

Furthermore, the fact that the existence of capital augmenting technical progress improves the results is true of Cobb-Douglas case. Table 8 shows the estimation results of Eq. (18).

$$\ln\left(\frac{S}{L}\right)=b_0+b_1\ln\left(\frac{e^{\lambda_\kappa t}K}{L}\right) \tag{18}$$

Compared with the results in Table 2, marignal productivity of labor and capital is positive in all industries and the estimates are all significant though $\bar{R}^2$ is slightly lower.

Keio University

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