Notes

A NOTE ON TECHNICAL PROGRESS IN THE FRAMEWORK OF FACTOR MARKET IMPERFECTIONS AND INTERINDUSTRY FLOWS*

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I. INTRODUCTION

The interindustry flows model was developed by Samuelson (1953) and refined by Vanek (1963) and Rader (1970), among others. Batra and Pattaniak (1971) considered the welfare effects of technical progress in a framework incorporating interindustry flows. The implications of technical progress in a distortionary wage differential model have been analyzed, however, by Batra and Scully (1971), Bhagwati (1968), Hazari (1975), and Hazari and Sgro (1976). Recently, Yu (1980) examined the resource allocation effect of technical progress in a model integrating both interindustry flows and a distortionary wage differential.

As aptly pointed out by Batra and Pattaniak (1971), it is fruitful to distinguish between two types of technical improvement in the presence of intermediate products. The first type deals with the enhancement of the productivities of primary factors of production; the second type involves the reduction in the input requirements of intermediate products. The cost reducing effect of only the first type of technical progress was examined by Yu (1980). The purpose of this note is, therefore, to consider the effects of the second type of technical improvement in the model integrating interindustry flows and wage differentials.

The integrated model is briefly described in Section II. Section III investigates the effects of technical progress reducing material input requirements. Concluding remarks are contained in Section IV.

II. THE MODEL—INTERINDUSTRY FLOWS AND DISTORTIONARY WAGE DIFFERENTIAL[1]

The economy produces two commodities, X_1 and X_2. Each commodity serves the

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[1] We employ the static model of international trade developed previously by Batra and Pattaniak (1971), Hazari and Sgro (1976) and Yu (1980) with modification of allowing reduction in the input requirements of intermediate products in the presence of interindustry flows and wage distortion. Comparative static results are obtained; the model can be interpreted as two single-period analysis in which gross saving and investment etc. need not be dealt with for our purpose.

dual role of a final product for consumers as well as an intermediate input in the production of the other good. The production functions for the two commodities are assumed to be linearly homogeneous:[2]

$$X_1 = F_1(K_1, L_1, X_{21}) = L_1 f_1(k_1, T_1) \tag{1}$$

$$X_2 = F_2(K_2, L_2, X_{12}) = L_2 f_2(k_2, T_2) \tag{2}$$

where K_i and L_i are respectively, the capital and the labor inputs, X_{ji} is the amount of commodity j used for the production of the ith commodity, and k_i and T_i are (K_i/L_i) and (X_{ji}/L_i) $(i=1,2; j=1,2; i \neq j)$.

The social welfare depends on the final demands for the two commodities (x_1, x_2):

$$U = U(x_1, x_2) \tag{3}$$

where U is quasi-concave and twice differentiable with $U_i(=\partial U/\partial X_i) > 0$ and $U_{ii}(=\partial^2 U/\partial X_i^2) < 0$ $(i=1,2)$.

The final demands can be written as:

$$x_1 = X_1 - a_1 X_2 = L_1 f_1 - a_1 L_2 f_2 \tag{4}$$

$$x_2 = X_2 - a_2 X_1 = L_2 f_2 - a_2 L_1 f_1 \tag{5}$$

where a_1 and a_2 indicate, respectively, the requirement of X_1 per unit of X_2 and of X_2 per unit of X_1, i.e. $a_1 = X_{12}/X_2$ and $a_2 = X_{21}/X_1$.

The interindustry wage differential is expressed as follows:

$$w_1 = \alpha w_2 \qquad \alpha \neq 1 \quad \text{and} \quad \alpha > 0 \tag{6}$$

However, the two industries pay the same rentals on capital:

$$r_1 = r_2 = r \tag{7}$$

With perfect competition in the product market and profit maximization, each factor is paid according to its marginal value-added product as follows:

$$w_1 = (f_1 - k_1 f_1')(P_1 - a_2 P_2) = \alpha w_2 = \alpha(f_2 - k_2 f_2')(P_2 - a_1 P_1) \tag{8}$$

$$r = f_1'(P_1 - a_2 P_2) = f_2'(P_2 - a_1 P_1) \tag{9}$$

Factor prices are assumed to be flexible and, therefore, full employment of factors is maintained:

$$\textstyle\sum_i K_i = \bar{K} \tag{10}$$

$$\textstyle\sum_i L_i = \bar{L} \tag{11}$$

Finally, the national income is expressed by:

[2] The production can be perhaps more clearly described as follows: $X_1 = \min[F_1(K_1, L_1), X_{21}/a_2]$ and $X_2 = \min[F_2(K_2, L_2), X_{12}/a_1]$ where $F_i(K_i, L_i)$ is homogenous of degree one in K_i and L_i, $(i=1,2)$. Accordingly, f_i is defined as $f_i(k_i) = (1/L_i)F_i(K_i, L_i)$, where $k_i = K_i/L_i$, $(i=1,2)$. See Vanek (1963), pp. 130–132.

$$Y=(P_1-a_2P_2)X_1+(P_2-a_1P_1)X_2=P_1x_1+P_2x_2 \tag{12}$$

The above model will be utilized to examine the implications of technical progress concerning reduction in the requirement of material input for the production of X_1.

III. TECHNICAL IMPROVEMENT CONCERNING INTERMEDIATE INPUT

In this section we derive the implications of technical progress which results in a decline in the requirement for intermediate inputs for factor intensities, labor allocation, outputs and national income. For purpose of analysis, but without loss of generality, let technical progress occur in the X_1 industry only and the technical improvement causes a decline in the per unit requirement for X_2, i.e. a_2 decreases.

Factor Intensities

Differentiating (8) and (9) with respect to a_2, keeping P_i constant[3] and remembering that a_1 is not affected, we obtain:

$$\frac{dk_1}{da_2}=-\frac{V\alpha P_2 f_2}{f_1'' V_1(k_1-\alpha k_2)} \tag{13}$$

$$\frac{dk_2}{da_2}=-\frac{P_2 f_1}{f_2'' V_2(k_1-\alpha k_2)} \tag{14}$$

where

$$V_1=P_1-a_2P_2\,, \qquad V_2=P_2-a_1P_1 \quad \text{and} \quad V=(P_2-a_1P_1)/(P_1-a_2P_2)\,.$$

Remembering that $\alpha \neq 1$ but $\alpha>0$, $dk_1/da_2 \gtrless 0$ and $dk_2/da_2 \gtrless 0$ if $k_1 \lessgtr \alpha k_2$. It is also noteworthy that with $\alpha<1$, $k_1>k_2$ implies that $k_1>\alpha k_2$. However, for $\alpha>1$, $k_1>\alpha k_2$ may not necessarily follow from $k_1>k_2$. Thus the following proposition is immediate:

PROPOSITION 1. *Technical improvement concerning reduction in intermediate input in the first industry results in a rise (fall) in the factor intensities in both industries if the first good is labor (capital) intensive in the value sense.*

This proposition along with the proposition deduced in my earlier paper (Yu, 1980) highlights the fact that the two types of technical progress in connection with the presence of interindustry flows lead to similar changes in the capital-labor ratios given that the first industry is always capital or labor intensive in the value sense.

This proposition is illustrated in Figure 1, adapted from the graph used by Bhagwati and Sainivasan (1971) and Hazari (1975). $\bar{K}/\bar{L}$ denotes the total (fixed) factor endowment and the interval (a,b) the feasible set of w/r ratios. k_1 and αk_2

[3] Note that we are dealing with a small country, the conditions of which cannot affect the terms of trade determined in the international market. Hence, P_1 and P_2 are given and not affected by technical progress.

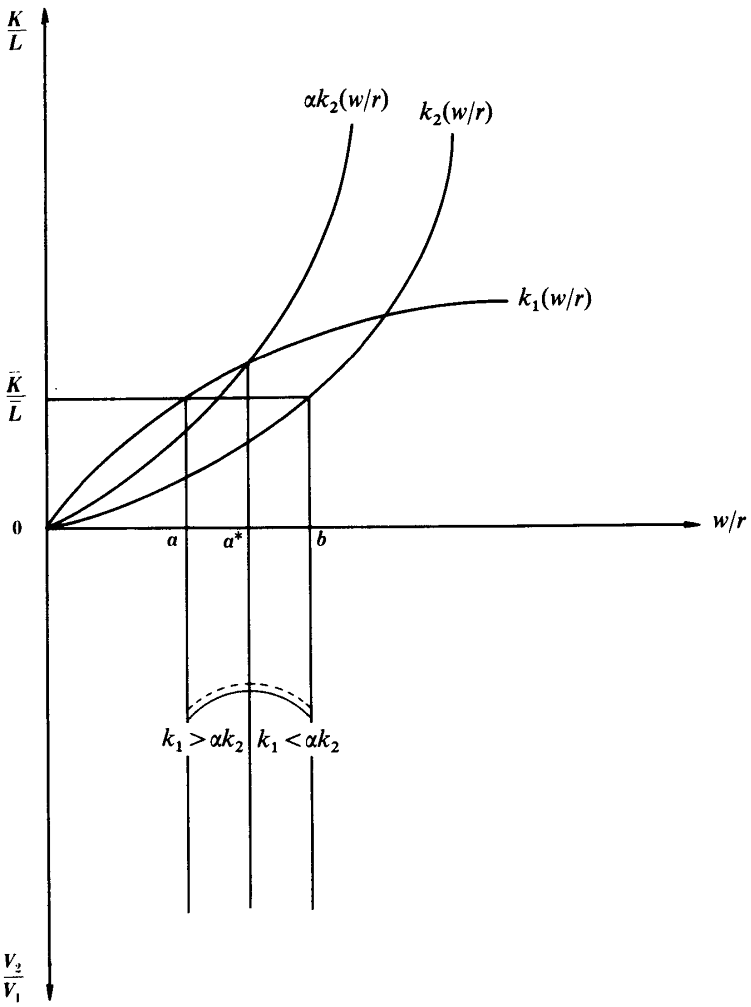

Fig. 1.

indicate the value capital intensities in the two industries. Note that if $\alpha = 1$, the structural relation between k_1 and w/r is represented by $k_1(w/r)$. If $\alpha \gtrless 1$, the curve shifts leftwards (rightwards). For purpose of analysis, assume $\alpha > 1$ such that factor intensity reversal is possible. The relation between αk_2 and w/r is given by the curve $\alpha k_2(w/r)$. Note that at any w/r in the half open interval (a^*, b) factor intensities get reversed, i.e. $k_1 < \alpha k_2$. The value-added (net price) ratio V_2/V_1 (instead of the gross price ratio P_2/P_1) is shown along the vertical axis in the lower half of the diagram. In the interval (a, a^*), industry X_1 is capital intensive in both the physical and the value sense and the w/r increases as V_2/V_1 decreases. For the interval (a^*, b) in which value intensity reversal occurs, w/r rises as V_2/V_1 increases.

Referring to Fig. 2, the isoquants corresponding to fixed intermediate-input levels represent amount of output in the ratio V_2/V_1, instead of P_2/P_1, as often interpreted in this conventional Lerner technique. The value-added ratio involves, say, an exchange of 1 for 2 units of the two commodities. While commodity X_2 is

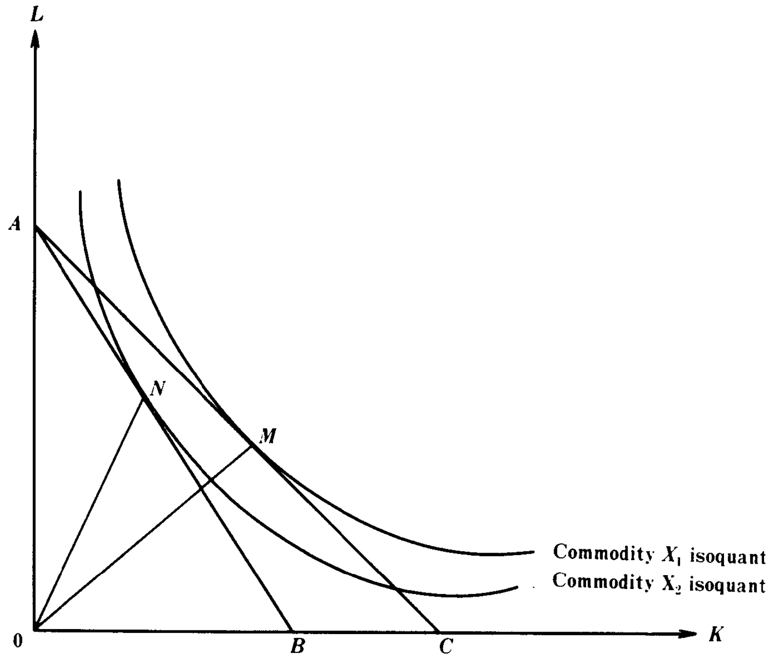

Fig. 2.

labor-intensive, the corresponding factor price ratios is given by the slopes of (AC, AB).[4]

In the event of the second type of the technical progress, a_2 decreases along with constant P_1, P_2, a_1 and α. As a result, V_2/V_1 falls. Hence, along the *same* curve in the lower-half portion in Figure 1, both k_1 and k_2 increases (decreases) as $k_1 \gtrless \alpha k_2$.

Factor Rewards

Differentiating (8) and (9) with respect to a_2 yields:

$$\frac{dr}{da_2} = -\frac{P_2 f_1}{k_1 - \alpha k_2} \tag{15}$$

$$\frac{dw}{da_2} = \frac{\alpha k_2 P_2 f_1}{k_1 - \alpha k_2} \tag{16}$$

From (15) and (16), $dr/da_2 \gtrless 0$ and $dw/da_2 \gtrless 0$ if $k_1 \gtrless \alpha k_2$. Noting that $da_2 < 0$, we state the following proposition:

PROPOSITION 2. *Technical improvement regarding reduction in intermediate input in the first industry leads to a rise (fall) in the capital rental coupled with a fall (rise) in the wage rate if the first industry is capital (labor) intensive in the value sense.*

Both Propositions 1 and 2 highlight the importance of value intensity in

[4] A more complicated Lerner diagram is utilized by Bhagwati and Srinivasan (1971) to illustrate the possible multiple equilibrium situation in connection with wage differential.

determining the effect of a reduction in intermediate input on factor intensities and factor rewards. The intuitive rationale for these results is that technical progress in the first industry results in a decline in a_2P_2. Hence, at constant P_1, V_1 is increased implying that the value added ratio $V\,(=V_2/V_1)$ decreases. The price of the factor used intensively in the first industry should rise and that of the other factor should fall in order to maintain previous gross or net commodity prices. It follows that in the absence of factor intensity reversal, capital rental should rise and wage rate should decrease. However, with factor intensity reversal, the reward of the now intensively used factor, i.e. wage rate, should rise and that of the other factor, i.e. capital rental, should decline.

Labor Allocation

Differentiating (10) and (11) with respect to a_2, we obtain:

$$\frac{dL_2}{da_2}=-\frac{dL_1}{da_2}=-\frac{P_2}{k_1-k_2}\left[\frac{VV_2f_2''f_2L_1\alpha+V_1f_1f_1''L_2}{f_1''f_2''V_1V_2(k_1-\alpha k_2)}\right] \tag{17}$$

In view of (17), $dL_1/da_2 \lesseqgtr 0$ and $dL_2/da_2 \gtreqless 0$ if (k_1-k_2) and $(k_1-\alpha k_2)$ are of the same (opposite) signs. Thus, the following proposition is immediate:

PROPOSITION 3. *Technical progress reducing intermediate input requirement in the first industry results in an allocation of labor input from the first (second) industry to the second (first) industry if the factor intensities rankings get reversed (unchanged) in the presence of the wage differential.*

The implication of this result can be seen from the well-known full employment identity, $\bar{K}/\bar{L}=k_1(L_1/\bar{L})+k_2(L_2/\bar{L})$. In the absence of factor intensity reversal, the first industry gains (loses) both capital and labor as k_1 and k_2 fall (rise). However, with factor intensity reversal, the direction of allocating the resources including labor between the two industries is reversed.

Output Response

Differentiating (4) and (5) with respect to a_2 yields:

$$\frac{dx_1}{da_2}=\frac{\alpha L_1f_2P_2V[\alpha(V+a_1)f_2+(1-\alpha)k_2f_2'V]}{V_1(k_1-k_2)(k_1-\alpha k_2)f_1''} +\frac{L_2f_1P_2[(a_1/\alpha+V)f_1+(1-1/\alpha)a_1k_1f_1']}{VV_2(k_1-k_2)(k_1-\alpha k_2)f_1''} \tag{18}$$

$$\frac{dx_2}{da_2}=-L_1f_1-\frac{\alpha L_1f_2P_2V[(1+\alpha Va_2)f_2+(1-\alpha)k_2f_2'Va_2]}{V_1(k_1-k_2)(k_1-\alpha k_2)f_1''} -\frac{L_2f_1P_2[(1/\alpha+Va_2)f_1+(1-1/\alpha)k_1f_1']}{VV_2(k_1-k_2)(k_1-\alpha k_2)f_2''} \tag{19}$$

From (18) and (19), $dx_1/da_2>0$ and $dx_2/da_2<0$, if $(k_1-k_2)(k_1-\alpha k_2)<0$. Hence technical progress which results in a decline in the intermediate input requirement is ultra-biased provided that the wage differential results in factor intensity reversal. However, with $(k_1 - k_2)(k_1 - \alpha k_2) > 0$, $dx_1/da_2 < 0$ and dx_2/da_2 may be of any sign since the first term is negative and the last two are positive. It follows that the technical improvement may not be ultra-biased if the wage differential fails to reverse the factor intensity rankings. These observations lead to the following proposition:

PROPOSITION 4. *Technical progress in favor of the first industry regarding a reduction in the non-primary input requirement is (may not be) ultra-biased if the interindustry wage differential causes (fails to cause) a reversal in the intensities.*

This proposition highlights a condition, i.e. factor intensity reversal, for ultra-biased growth; this condition does not emerge in the analysis by Batra and Pattaniak (1971) simply because their model assumes away factor market distortions.

National Income

In the presence of the second type of technical progress, equation (12) should be written as:

$$\frac{dY}{da_2} = P_1 \frac{dx_1}{da_2} + P_2 \frac{dx_2}{da_2} \tag{20}$$

The existence of an interindustry wage differential generally results in the nontangency between the net "input-output" transformation curve and the price line:[5]

$$\frac{dx_1}{dx_2} = -\beta \frac{V_1 a_1 + V_2 \beta}{V_1 + V_2 \beta a_2} \quad \text{and} \quad \beta = \frac{u dL_1 + v dK_1}{(1/\alpha) u dL_1 + v dK_1} \tag{21}$$

where u and v denote, respectively, the marginal productivities of labor and capital in the first industry. Note that β captures the divergence between the marginal rate of transformation and the commodity price ratio. It is well-known that β is related to the wage differential parameter α as follows: $\beta \gtreqless 1$ as $\alpha \gtreqless 1$.

Following Hazari (1975), equations (21) and (19) are substituted into (20) to obtain:

$$\frac{dY}{da_2} = \frac{V_1 V_2}{V_1 + V_2 \beta a_2} (1 - \beta) \frac{dx_2}{da_2} \tag{22}$$

where dx_2/da_2 is given by (19).

It is clear that $dY/da_2 \gtrless 0$ if $\beta \gtrless 1$ and $(k_1 - k_2)(k_1 - \alpha k_2) < 0$; dY/da_2 may have any sign if $\beta \gtrless 1$ and $(k_1 - k_2)(k_1 - \alpha k_2) > 0$. Hence we state the following proposition:

[5] The marginal gross rate of transformation in the presence of an interindustry wage differential can be expressed as: $-dX_1/dX_2 = \beta V_2 / V_1$ where β is defined in (21) in the text.

Proposition 5. *Technical progress lowering the non-primary input requirement in the first industry reduced (enhances) national income if the wage differential is paid by the second (first) industry coupled with a reversal in the factor intensities.*

It is notable that this result indicates only one of the several situations which leads to immiserizing growth. Consider the case in which $\beta < 1$ and the wage differential do not result in reversal in factor intensities, it is possible, from (22), that growth lowers income. The economic rationales underlying the major results presented in this paper run parallel to those provided by Batra and Pattaniak (1971) and the analysis of the implications of a wage differential developed recently by, for example, Hazari (1975) among many others.

IV. CONCLUDING REMARKS

It is noteworthy that a distinction should be made between two types of cost reducing technical progress in the presence of intermediate products. In an earlier paper (Yu, 1980) I have examined the effects of the first type of technical progress concerning the improvement of the productivities of primary inputs. This type of technical progress is identifiable with the idea of technical improvements in the traditional final-good model. Of more analytical interest is to investigate the implications of the second type of technical progress regarding the reduction of the requirement of the intermediate input. This paper represents a modest step in this direction.

In this paper, we have examined the effects of the second-type of technical progress for factor intensities, factor rewards, labor allocation, output levels, and national income in a model integrating wage distortions and interindustry flows. Our results highlight the significance of the value intensity reversal in explaining the ultra-biased growth. We found that the two types of technical progress in the presence of interindustry flows result in similar changes in the factor-intensity ratios, labor allocation, and output levels provided that each of the two industries is always capital or labor intensive in the value sense.

It is worth noting that Hazari–Sgro (1976) has argued, using a non-traded good model, that value intensity reversal may be less important in a more than two commodity model. Our propositions are compatible with their argument because the results in this paper are derived essentially from the two commodity model with allowances for interindustry flows.

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