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THE UTILIZATION PERIOD OF FIXED CAPITAL*

Eiji HOSODA

1. INTRODUCTION

Many economists have tried to solve the problem of how to determine the utilization period of fixed capital,[1] that is, the economic lifetime of a machine. Recently, several theorists have done research on this subject, using Piero Sraffa's approach [7]. One of the characteristics of his approach is that "all the elements of the analysis are reduced to 'flow'" in his model.[2]

In sraffa's model, the durable means of production is regarded as an annual input just like the other raw materials, and those means which are older by one year at the end of the production process are treated as a part of the annual product of the industry. Referring to this point, Sraffa says,

> "For example, a knitting-machine enters the means of production at the beginning of the year, along with yarn, the fuel, etc., with which it is employed; and at the end of the year the partly worn-out, older machine which emerges from the process will be regarded as a joint product with the year's output of stockings."[3]

Thus fixed capital can be analyzed as a flow just in the same manner as circulating capital which is used up in the production within a year.

Here let us note another characteristic feature of Sraffa's model: that the same fixed capital, "at different ages, should be treated as so many different products, each with its own price."[4] In Sraffa's example of the knitting-machine, for instance, a new machine which enters the means of production at the beginning of the year is considered to be different from the partly worn-out older machine at the end of the year, and in general, they do have different prices. Therefore, Sraffa's model is bound to be much more complicated than one which includes only circulating capital.

But why is it necessary to make a model more complex? To make it more general?

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[1] The utilization period of fixed capital is defined as its economic lifetime, which is different from its physical lifetime. The former is determined as an endogenous variable in an economy while the latter is determined technically, and so beyond the physical lifetime fixed capital is assumed to perish or to be unusable. We use "the utilization period" and "the economic lifetime", or simply "the lifetime" interchangeably.

[2] See Pasinetti [4], p. xiii.

[3] See Sraffa [7], p. 63.

[4] See Sraffa [7], p. 63.

This alone is not enough reason to justify such complexity. Unless fixed capital can be shown to have an essential effect on economic variables it is unnecessary to include it in a model. But production prices *are* in general affected by the utilization period of fixed capital. In other words, the length of time for which a machine can be used has an effect not only on the price of goods produced by the very machine itself but also on the prices of other goods as well. As will be shown, this matter can be analyzed properly only through use of Sraffa's extended model.

In the present paper, we shall attempt to solve the problem of how to determine the utilization period of fixed capital when the rate of profit is given. Baldone [1], Schefold [5], [6], and Varri [8] have published papers that deal with this subject in a similar way. This paper shall follow their method of constructing a model. Our final purpose is to show the important properties of a competitive economy with fixed capital taken into account and to present several interesting results which have hitherto not been obtained.

In Section 2 of this paper, we will examine the assumptions of our approach and construct a basic model. We will also try to transform equations in order to derive prices of fixed capital goods and a factor-price frontier. In Section 3, some properties of the stationary state and the competitive economy will be explained. In Section 4, main results and proofs will be given. In the final section, we will discuss the implications of the results obtained here and the economic meaning of non-basic commodities.

2. THE MODEL

In this section we will examine the assumptions which underlie our analysis, and develop a basic model.

Let us suppose that there are n kinds of finished goods[5] in an economy and that all the goods except the $(n-1)$th one are non-durable goods, that is, goods which are used up in the production within a year. The $(n-1)$th commodity is durable equipment, or fixed capital, which is not used up in the production in a year, and its physical lifetime is assumed to be $\bar{T}$ years. Furthermore let us suppose this fixed capital enters into the nth production process and not into any other process. Namely, let us suppose that there is only one new fixed capital, which enters into only one production process of non-durable goods. (All the above suppositions are required merely for simplicity, and their generalization does not affect the essence of the following discussion at all.)

In addition, both the rate of profit and the wage rate are assumed to be equalized respectively. As Sraffa's system is not closed, either the rate of profit or the wage rate must be given exogenously. We shall assume the rate of profit to be given.

Considering the above premises, let us now formulate the cost of production of industries which do not use fixed capital. It can be expressed by the following

[5] Finished goods mean all non-durable goods together with new fixed capital.

equations:

$$(1) \quad \begin{aligned} &(1+r)(a_{11}p_1+\cdots+a_{n1}p_n)+wl_1=b_1p_1 \\ &(1+r)(a_{12}p_1+\cdots+a_{n2}p_n)+wl_2=b_2p_2 \\ &\qquad \cdots\cdots \\ &(1+r)(a_{1,n-1}p_1+\cdots+a_{n,n-1}p_n)+wl_{n-1}=b_{n-1}p_{n-1} \end{aligned}$$

In these equations r, w, and p_i $(i=1,2,\cdots,n)$ represent rate of profit (given here), wage rate and price of the ith commodity respectively. The notation a_{ij} $(i=1,2,\cdots,n; j=1,2,\cdots,n-1)$ indicates input of the ith commodity into the jth industry which produces b_j units of output, and l_i $(j=1,2,\cdots,n-1)$ indicates labour input into the jth industry. Since we have assumed that the $(n-1)$th commodity is fixed capital and that it is used only in the nth industry, we note that $a_{n-1,j}$ $(j=1,2,\cdots,n-1)$ is zero.

Next let us proceed to the formulation of production processes of the nth industry which uses fixed capital. In this industry, at the beginning of the year, $a_{n-1,n}$ units of new fixed capital whose physical lifetime is $\bar{T}$ enter into the production process, and at the end of the year b_n^0 units of the nth commodity and M^1 units of the fixed capital which is one year older are produced. Input of other factors and labour which enter into the same process along with the new fixed capital are a_{in}^0 $(i=1,2,\cdots,n$ & $i\neq n-1)$ and l_n^0 respectively. At the beginning of the next year, M^1 units of this one year older fixed capital, a_{in}^1 $(i=1,2,\cdots,n$ & $i\neq n-1)$ units of the other commodities and l_n^1 units of labour are thrown into the process, and M^2 units of the fixed capital which is older by two years and b_n^1 units of the nth commodity are produced at the end of the year. It is assumed here that new fixed capital *is not* used in the same process along with the older one and that more than two kinds of the older capital *are not* used in the same process. Therefore $a_{n-1,n}^t$ $(t=1,2,\cdots,\bar{T})$ is assumed to be zero. If the fixed capital is used successively till the Tth year $(T\leqq\bar{T})$, then the production process are expressed schematically as follows.

$$(2) \quad \begin{array}{lcl} \textit{input} & & \textit{output} \\ (a_{1n}^0,\cdots,a_{n-2,n}^0,a_{n-1,n}^0,a_{nn}^0,l_n^0) & \rightarrow & (b_n^0,M^1) \\ (a_{1n}^1,\cdots,a_{n-2,n}^1,M^1,a_{nn}^1,l_n^1) & \rightarrow & (b_n^1,M^2) \\ \qquad\cdots\cdots & & \\ (a_{1n}^T,\cdots,a_{n-2,n}^T,M^T,a_{nn}^T,l_n^T) & \rightarrow & (b_n^T,M^{T+1}). \end{array}$$

In regard to these production processes, we explicitly make the following assumptions:

*A*1 (free disposal). Fixed capital can be scrapped free at any time within its physical lifetime.

A2. Production of the same scale is repeated every year, and therefore the same amount of the new fixed capital is thrown into the process and the same amount of the old capital is scrapped after the same amount of time.

Two comments need to be given about these assumptions. First, under *A1* the scrap value of the fixed capital is zero. Second, under *A2* the schema expressed in (2) is seen not only sequentially for $(T+1)$ years but also simultaneously every year.

Now that we have clarified assumptions, we can formulate the equations which express the costs of production in this industry:

$$
(3) \quad
\begin{aligned}
&(1+r)(a_{1n}^{0}+\cdots+a_{nn}^{0}p_n)+wl_n^0=b_n^0p_n+M^1p_m^1\\
&(1+r)(a_{1n}^{1}p_1+\cdots+a_{nn}^{1}p_n+M^1p_m^1)+wl_n^1=b_n^1p_n+M^2p_m^2\\
&\qquad\cdots\cdots\\
&(1+r)(a_{1n}^{T}p_1+\cdots+a_{nn}^{T}p_n+M^Tp_m^T)+wl_n^T=b_n^Tp_n\,.
\end{aligned}
$$

In (3), p_m^t represents the price of the fixed capital whose age is t, and $p_m^{T+1}=0$ under Assumption *A1*.

Before proceeding to our analysis, we need three more assumptions about the economy as a whole.

A3 (viability). For any $T\,(T=0,1,2,\cdots,\bar{T})$, the following inequalities holds.

$$
(4) \quad
\begin{aligned}
&\sum_{j=1}^{n-1} a_{ij}+\sum_{t=0}^{T} a_{in}^t \leqq b_i \quad (i=1,2,\cdots,n-1)\\
&\sum_{j=1}^{n-1} a_{nj}+\sum_{t=0}^{T} a_{nn}^t \leqq \sum_{t=0}^{T} b_n^t\,.
\end{aligned}
$$

Furthermore, at least one of the n inequalities contained in (4) holds with strict inequality.

Assumption A3 tells us that the whole economy is always viable however long a lifetime of fixed capital may be within $\bar{T}$ years.

A4. There is at least one basis commodity[6] in an economy.

A5. All industries produce under constant returns to scale.

We note that non-negativity of the prices of all finished goods is assured when the rate of profit is given within a certain range,[7] and now will proceed to our own discussion.

Let us rewrite Eqs. (3) and (4) for analysis in our next section. First, let us set

[6] As for the definition of a basic commodity, see Sraffa [7], p. 7–8.

[7] Non-negativity of the prices of the finished goods does not depend upon the lifetime. See Baldone [1], and Schefold [5], [6].

matrices and vectors as

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1,n-1} \\ a_{21} & a_{22} & \cdots & a_{2,n-1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n-1,1} & a_{n-1,2} & \cdots & a_{n-1,n-1} \end{pmatrix}$$

$$a_n=(a_{n1}, a_{n2}, \cdots, a_{n,n-1})$$

$$L=(l_1, l_2, \cdots, l_{n-1}) .$$

$$B=\begin{pmatrix} b_1 & 0 & \cdots & & 0 \\ 0 & b_2 & \cdots & & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & b_{n-2} & 0 \\ 0 & 0 & \cdots & 0 & b_{n-1} \end{pmatrix}$$

$$P=(p_1, p_2, \cdots, p_{n-1})$$

Using these, we can transform (1) into

$$(1)' \qquad (1+r)(PA+p_n a_n)+wL=PB .$$

Next, by setting a vector as

$$a_n^t=(a_{1n}^t, a_{2n}^t, \cdots, a_{n-1,n}^t)'^{8} \quad (t=0, 1, \cdots, T) ,$$

we can rewrite (3) as

$$(3)' \quad \begin{aligned} &(1+r)(Pa_n^0+p_n a_{nn}^0) + wl_n^0=p_n b_n^0+p_m^1 M^1 \\ &(1+r)(Pa_n^1+p_n a_{nn}^1+p_m^1 M^1)+wl_n^1=p_n b_n^1+p_m^2 M^2 \\ &\qquad \cdots\cdots \\ &(1+r)(Pa_n^T+p_n a_{nn}^T+p_m^T M^T)+wl_n^T=p_n b_n^T . \end{aligned}$$

In (3)′ (or (3)), we can eliminate the prices of the old fixed capital goods. Multiplying both sides of the tth equation ($t=0, 1, \cdots, T$) of (3)′ by $1/(1+r)^t$ and adding up both sides of all such $(T+1)$ equations, we can obtain

[8] If a is a row vector, a' is its transposed vector.

(5)
$$(1+r)\left\{P\sum_{t=0}^{T}\frac{1}{(1+r)^{t+1}}a_n^t+p_n\sum_{t=0}^{T}\frac{1}{(1+r)^{t+1}}a_{nn}^t\right\}+w\sum_{t=0}^{T}\frac{1}{(1+r)^{t+1}}l_n^t=p_n\sum_{t=0}^{T}\frac{1}{(1+r)^{t+1}}b_n^t\,.$$

By redefining $\hat{a}_n(T)$, $\hat{a}_{nn}(T)$, $\hat{l}_n(T)$, and $\hat{b}_n(T)$ in this way

$$\hat{a}_n(T)=\sum_{t=0}^{T}\frac{1}{(1+r)^{t+1}}a_n^t$$

$$\hat{a}_{nn}(T)=\sum_{t=0}^{T}\frac{1}{(1+r)^{t+1}}a_{nn}^t$$

$$\hat{l}_n(T)=\sum_{t=0}^{T}\frac{1}{(1+r)^{t+1}}l_n^t$$

$$\hat{b}_n(T)=\sum_{t=0}^{T}\frac{1}{(1+r)^{t+1}}b_n^t\,,$$

we can transform (5) into

(5)′
$$(1+r)\{P\hat{a}_n(T)+p_n\hat{a}_{nn}(T)\}+w\hat{l}_n(T)=p_n\hat{b}_n(T)\,.$$

Thus all the prices which appear in (5)′ are those of the finished goods alone. Furthermore, by defining

$$\tilde{A}=\begin{pmatrix}A & \hat{a}_n(T)\\ a_n & \hat{a}_{nn}(T)\end{pmatrix}\qquad \tilde{B}=\begin{pmatrix}B & 0\\ 0 & \hat{b}_n(T)\end{pmatrix}$$

$$\tilde{L}=(L,\hat{l}_n(T))\qquad \tilde{P}=(P,p_n)\,,$$

finally we can obtain

(6)
$$(1+r)\tilde{P}\tilde{A}+w\tilde{L}=\tilde{P}\tilde{B}$$

which embodies both (1) and (3). We regard (6) to be an *integrated system*, following Pasinetti [3]. As noted before, it is assured that $P\geq 0$[9] when the rate of profit is given within a certain range.

Let us continue to make some transformations of equations for further analysis. In the same way as we obtained (5), by multiplying both sides of the tth equation ($t=1,2,\cdots,T$) by $1/(1+r)^t$ and adding up both sides of those multiplied equations over T except for the last one, we get

[9] Suppose p is a vector. When all the elements of p are zero, we denote $p=0$. When they are not smaller than zero, $p\geqq 0$. When $p\geqq 0$ and $p\neq 0$, we denote $p\geq 0$. We use the same notation for inequalities as applied to matrices.

$$(1+r)\left\{P\sum_{t=0}^{T-1}\frac{1}{(1+r)^{t+1}}a_n^t+p_n\sum_{t=0}^{T-1}\frac{1}{(1+r)^{t+1}}a_{nn}^t\right\}$$

$$+w\sum_{t=0}^{T-1}\frac{1}{(1+r)^{t+1}}l_n^t=p_n\sum_{t=0}^{T-1}\frac{1}{(1+r)^{t+1}}b_n^t+\frac{1}{(1+r)^T}M^Tp_m^T$$

Using our previous notations, we can express the above equation as

$$(1+r)\{P\hat{a}_n(T-1)+p_n\hat{a}_{nn}(T-1)\}+w\hat{l}_n(T-1)=p_n\hat{b}_n(T-1)+\frac{1}{(1+r)^T}M^Tp_m^T.$$

In just the same way, by adding up both sides of those $(T-1)$ multiplied equations, and so on successively, we can obtain

$$(7)\quad \begin{aligned}&(1+r)\{P\hat{a}_n(T-1)+p_n\hat{a}_{nn}(T-1)\}+w\hat{l}_n(T-1)=p_n\hat{b}_n(T-1)+\frac{1}{(1+r)^T}M^Tp_m^T.\\ &\qquad\cdots\cdots\\ &(1+r)\{P\hat{a}_n(0)+p_n\hat{a}_{nn}(0)\}+w\hat{l}_n(0)=p_n\hat{b}_n(0)+\frac{1}{1+r}M^1p_m^1\end{aligned}$$

Now (1)′ can be transformed into

$$(8)\qquad P=\{(1+r)p_na_n+wl\}[B-(1+r)A]^{-1}.$$

From (7) and (8), the following equations can be obtained.

$$(9)\quad \begin{aligned}&w\{(1+r)L[B-(1+r)A]^{-1}\hat{a}_n(T-1)+\hat{l}_n(T-1)\}\\ &\quad=p_n\{\hat{b}_n(T-1)-(1+r)\hat{a}_{nn}(T-1)-(1+r)^2a_n[B-(1+r)A]^{-1}\hat{a}_n(T-1)\}\\ &\quad+\frac{1}{(1+r)^T}M^Tp_m^T\\ &\qquad\cdots\cdots\\ &w\{(1+r)L[B-(1+r)A]^{-1}\hat{a}_n(0)+\hat{l}_n(0)\}\\ &\quad=p_n\{\hat{b}_n(0)-(1+r)\hat{a}_{nn}(0)-(1+r)^2a_n[B-(1+r)A]^{-1}\hat{a}_n(0)\}\\ &\quad+\frac{1}{1+r}M^1p_m^1.\end{aligned}$$

Following Baldone [1], we define

$$D(t)=(1+r)L[B-(1+r)A]^{-1}\hat{a}_n(t)+\hat{l}_n(t)$$

$$N(t)=\hat{b}_n(t)-(1+r)\hat{a}_{nn}(t)-(1+r)^2a_n[B-(1+r)A]^{-1}\hat{a}_n(t)$$

$$(t=0,1,\cdots,T-1).$$

Using these notations, we can transform (9) into

$$(10) \qquad wD(t)=p_nN(t)+M^{t+1}p_m^{t+1}\frac{1}{(1+r)^{t+1}} \quad (t=0,1,\cdots,T-1).$$

We note that the right-hand side of (10) contains p_n, the price of the nth commodity. Incidentally, we also remark that $D(t)$ is positive[10] since $[B-(1+r)A]^{-1}\geq 0$ under Assumption $A3$.[11] (We omit the proof of its positivity, since it is simply a mathematical calculation which has been published in other articles (e.g., Baldone [1])).

3. THE STATIONARY STATE AND THE COMPETITIVE EQUILIBRIUM

Since the rate of profit is given exogenously, the relative values of w, p_i $(i=1,2,\cdots,n)$, and p_m^t $(t=1,2,\cdots,T)$ can be determined in the system expressed by (1) and (3) (or (3)′) if T is determined. There are $(n-1)+T+1=n+T$ equations and $1+n+T$ unknowns, so the relative prices and wage rate can be determined if a commodity is arbitrarily adopted as *numeraire*. In the following, the wage rate will be adopted as the main unit of value, i.e., $w=1$.[12]

In order to explain how T is determined, it seems to be convenient to clarify the concepts of the stationary state and the competitive equilibrium, since they are important concepts in the present paper.[13]

We have already implicitly assumed the stationary state. Namely, in Assumption $A2$ we have supposed that exactly the same scale of input and output is repeated in every industry when the utilization period of fixed capital is determined.[14] As Joan Robinson [10] says, in the stationary state, "tradition rules and the cycle of production and distribution repeats itself from year to year, from generation to generation, without changes in population, technical innovations, or concentration of wealth." Therefore, once the utilization period of fixed capital is determined, it never changes.

Next, let us refer to the concept of the competitive equilibrium. We shall provisionally call any technique chosen under the competitive equilibrium the α-technique, and any other technique the β-technique. If the rate of profit is uniform and given exogenously, the economy which corresponds to each technique has its own prices. Since the α-technique is chosen under the competitive equilibrium, there can be no extra profit if the economy of the β-technique is evaluated using the prices of the economy of the α-technique. For example, let us suppose the utilization period of fixed capital chosen under the competitive equilibrium is T, and any other utilization period is $\tau\ (\neq T)$. In addition, let us suppose the price vector $\tilde{P}=(P,p_n)$ of

[10] It is implicitly assumed here that labour is an indispensable input in all the industries.

[11] See, for example, Baldone [1] Corollary 1.

[12] We will change the selection of *numeraire* if necessary.

[13] We are grateful to Professor Kamiya for helpful suggestions about the properties of the competitive equilibrium.

[14] In the stationary state, $a_{n-1,n}=b_{n-1}=M^1=M^2=\cdots=M^T$.

finished goods corresponds to the economy of the utilization period T, and $\tilde{P}'=(P',p_n')$ corresponds to the economy of the utilization period τ. As we have just explained, there can be no extra profit if the economy of the utilization period τ is evaluated using the prices of the economy of the period T. Therefore, the following inequality must hold.

$$(11) \qquad (1+r)\tilde{P}\tilde{A}'+\tilde{L}\geqq \tilde{P}\tilde{B}' .$$

Owing to A3, we can rewrite (11) as

$$(12) \qquad \tilde{P}\leqq \tilde{L}'[\tilde{B}'-(1+r)\tilde{A}']^{-1} .$$

Notice that the right-hand side is equal to $\tilde{P}'=(P',p_n')$. Therefore, the inequality (12) implies the following as a property of the competitive equilibrium:

$N1$. When the rate of profit is uniform and given exogenously, the price vector of the finished goods which corresponds to the utilization period chosen under the competitive equilibrium is not larger in terms of the wage rate than that which corresponds to any other utilization period.

Thus, as for finished goods, the same property can be obtained as in an economy without fixed capital goods. (But we must notice that the prices of old fixed capital goods are not referred to in $N1$.) In addition to the property $N1$, we can deduce other important properties of the competitive economy. We will now take up three such properties, and prove that they can be deduced from $N1$.

First, let us consider the charge for fixed capital. As Sraffa [7] says, "the difference between the values of the asset at two consecutive ages gives the allowance to be made for depreciation for that year." Namely, $(M^t p_m^t - M^{t+1} p_m^{t+1})$ expresses the depreciation allowance for the fixed capital whose age is t. This amount added to the profit at the general rate on the value of the asset at the beginning of the year is called the *annual charge for that year*. When we define $C_m(t)$ as

$$C_m(t)=(M^t p_m^t - M^{t+1} p_m^{t+1})+rM^t p_m^t ,$$

we can interpret

$$\frac{C_m(0)}{1+r}+\frac{C_m(1)}{(1+r)^2}+\cdots+\frac{C_m(T)}{(1+r)^{T+1}}$$

to be the sum of the present value of the annual charge when the fixed capital is used up till the year T.

Next, let us suppose that the utilization period of fixed capital must be determined so that this sum is the minimum. Thus, we can obtain the following property.:

$N2$. When the rate of profit is uniform and given exogenously, the sum of the present value of the annual charge on fixed capital which corresponds to the utilization period chosen under the competitive equilibrium is not larger than that which corresponds to any other utilization period.

We shall prove that this property is obtained from $N1$, but before proceeding to the proof, let us introduce another property by changing the *numeraire* and adopting arbitrarily one of finished goods as a unit of value. Needless to say, wage rate is also measured in terms of that commodity. Generally speaking, when a uniform rate of profit is given, the choice of technique has an effect on the wage rate. So, the following property is plausible in the competitive economy.

$N3$. When the rate of profit is uniform and given exogenously, the wage rate which corresponds to the utilization period chosen under the competitive equilibrium is not smaller in terms of an arbitrary commodity (except old machines) than that which corresponds to any other utilization period.

When some technique is chosen and fixed, the wage rate changes as the rate of profit changes. We call the curve which expresses this relationship a factor-price curve. There are as many curves as the number of techniques available. We call the outermost envelope of those curves a *factor-price frontier*. Thus $N3$ implies that this frontier consists of the factor-price curves which correspond to the utilization period chosen under the competitive equilibrium.

In the next section, we shall prove rigorously that the above properties are compatible with one another.

4. THE UTILIZATION PERIOD AND PRICES OF FIXED CAPITAL GOODS

In this section, we will present the main propositions for the above model. It is usually assumed in the literature that the technical coefficient matrix $\tilde{A}^*$ is indecomposable. We do not, however, assume the indecomposability of $\tilde{A}$ in this paper precisely because whether or not $\tilde{A}$ is indecomposable will make a great difference in the utilization period, and because it is our main aim to elucidate this difference. In order to clarify this point, let us first consider the case in which the commodity produced by means of fixed capital is a basic commodity.

Incidentally, we assume in the following that all the finished goods except the $(n-1)$th and the nth commodities are basic, since this assumption has no essential effect.

4-a. *When the Commodity Produced by Means of Fixed Capital Is a Basic Commodity*

A basic commodity is, in short, one which enters all the production processes directly or indirectly. If the nth commodity produced by means of fixed capital is a basic commodity, the production condition of the industry has a crucial effect on the determination of all the prices and the wage rate ($1/p_i$ in this case). Therefore, all the prices and the wage rate depend upon the lifetime of fixed capital.

In the preceding section, we showed three properties of the competitive equilibrium, from which we now obtain the following proposition.

PROPOSITION 1. *If a commodity produced by means of fixed capital is a basic commodity, N1 is equivalent to N2 and N3.*

Proof. We shall show first that $N1$ is equivalent to $N2$. We suppose T is the utilization period chosen under the competitive equilibrium, and τ is any other period. Denoting by $C(\tau)$ the sum of the present value of the annual charge when the utilization period is τ, we can obtain

$$C(\tau)=\sum_{t=0}^{\tau}\frac{C_m(t)}{(1+r)^{t+1}}\,.$$

By definition,

$$C_m(t)=(1+r)M^t p_m^t-M^{t+1}p_m^{t+1}$$

and so

$$\frac{C_m(t)}{(1+r)^{t+1}}=\frac{1}{(1+r)^t}M^t p_m^t-\frac{1}{(1+r)^{t+1}}M^{t+1}p_m^{t+1}\,.$$

Putting this equation into (13), we obtain

$$C(\tau)=M^0 p_m^0=a_{n-1,n}p_{n-1}\,.$$

We denote by $(p_1^*,p_2^*,\cdots,p_n^*)$ the price vector corresponding to T. Since $C(T)=a_{n-1,n}p_{n-1}^*$, then

$$C(T)\leqq C(\tau)\quad\Leftrightarrow\quad p_{n-1}^*\leqq p_{n-1}$$

holds.

Now we define e_i $(i=1,2,\cdots,n-1)$ to be a column vector whose ith element is unity with the other element being all zero. Then from (8),

$$p_{n-1}=Pe_{n-1}=\{(1+r)p_n a_n+L\}[B-(1+r)A]^{-1}e_{n-1}\,.$$

Thus we arrive at the following equation:

$$(8)'\qquad p_{n-1}^*-p_{n-1}=(1+r)(p_n^*-p_n)a_n[B-(1+r)A]^{-1}e_{n-1}\leqq 0\,.$$

Since the nth commodity is basic according to the hypothesis of the proposition and thus $a_n[B-(1+r)A]^{-1}>0$,[15] the following holds:

$$p_{n-1}^*\leqq p_{n-1}\quad\Leftrightarrow\quad p_n^*\leqq p_n\,.$$

From (8) again, for any $i=1,2,\cdots,n-1$

$$(8)\qquad p_i^*-p_i=(1+r)(p_n^*-p_n)a_n[B-(1+r)A]^{-1}e_i\leqq 0$$

must hold. Therefore,

$$C(T)\leqq C(\tau)\quad\Leftrightarrow\quad\tilde{P}^*\leqq\tilde{P}$$

[15] See Baldone [1], p. 105–106.

holds.

Next, we will show that $N1$ is equivalent to $N3$.[16] When we adopt labour as *numeraire* and set $w=1$, then $1/p_i$ $(i=1,2,\cdots,n)$ means wage rate in terms of the ith commodity. Then, from (8)′ and (8)″,

$$P^* \leqq P \quad \Leftrightarrow \quad \frac{1}{p_i^*} \geqq \frac{1}{p_i} \qquad \text{for any } i \quad (i=1,2,\cdots,n).$$

The latter inequality means that the wage rate which corresponds to the utilization period chosen under the competitive equilibrium is not smaller (in terms of finished goods) than that wage rate which corresponds to any other utilization period. Therefore $N1$ is equivalent to $N3$. Q.E.D.

Owing to this proposition, we can conclude that the above properties are equivalent *when the commodity produced by means of fixed capital is basic.* We will find, however, this proposition does not hold in a more general case.

4-b. When the Commodity Produced by Means of Fixed Capital Is a Non-basic Commodity

We shall now explain the reason why our above proposition does not hold in this case.

When the commodity produced by means of fixed capital is non-basic, its production condition has an effect on its own price and on some other non-basic commodities, but does not have an effect on prices of basic commodities. Furthermore, when the rate of profit is given, the wage rate is determined independent of the production conditions of non-basic commodities, and so the industries producing non-basic commodities have no effect on the wage rate determined by the production conditions of basic commodities. In other words, the production conditions of non-basic commodities do not essentially affect the formation of a factor-price curve measured by a basic commodity at all. Since we have assumed only that the nth industry uses fixed capital, we can draw only one factor-price curve irrespective of the lifetime of this fixed capital. In this case, the curve itself is the factor-price frontier. This fact means that the lifetime corresponding to the curve which constitutes the frontier can be any value from 0 to $\bar{T}$. Consequently, $N3$ is not equivalent to $N1$. Thus, $N3$ is ineligable as a criterion to determine the utilization period. But, as we will show later, if $N3$ is modified a little, it will become equivalent to $N1$ and still be valid as such a criterion.

Next, let us have a look at $N2$. We must point out that fixed capital goods must necessarily be non-basic in our system when the commodity produced by them is non-basic. Since only the nth industry is assumed to use the fixed capital goods and since the nth commodity is non-basic, those fixed capital goods which enter into only the production processes of the nth commodity are also non-basic.

[16] See Baldone [1].

Let us consider the case in which the commodity produced by means of fixed capital goods (the nth commodity) *do not* enter into the production of the new fixed capital (the $n-1$th commodity) either directly or indirectly. Though in this situation both the nth commodity and the $n-1$th are non-basic, there is a slight difference between them. We can see that the $n-1$th commodity is used in the production of the nth commodity, but that the latter *is not* used in the production of the former either directly or indirectly. This means that the price of the $n-1$th commodity is independent of the production condition of the nth commodity while the price of the nth commodity is dependent on the production condition of the $n-1$th commodity. The lifetime has, therefore, no effect on the price formation of the new fixed capital or on the $n-1$th commodity at all. But we know that the sum of the present value of the annual charge for fixed capital goods $C(\tau)$ is equal to the value of the new fixed capital $M^0 p_m^0$, i.e.,

$$C(\tau) = M^0 p_m^0 = a_{n-1,n} p_{n-1} ,$$

the lifetime being τ. Since p_{n-1} is independent of τ, we can see

$$C(\tau) = a_{n-1,n} \bar{p}_{n-1} = constant$$

for any τ ($\tau = 0, 1, \cdots, \bar{T}$). That is, the sum of the present value of the annual charge is constant irrespective of the lifetime. Consequently, $N2$ is not equivalent to $N1$.

But when the nth commodity enters into the production of the $n-1$th commodity, the lifetime affects the price of the $n-1$th commodity (the new fixed capital) and so the value of $C(\tau)$ depends upon the lifetime. Only in this limited case, therefore, $N2$ is valid.

Now let us return to $N3$, and revise it in the following way:

$N3'$. When the rate of profit is uniform and given exogenously, the wage rate in terms of the nth commodity which corresponds to the utilization period chosen under the competitive equilibrium is not smaller than that which corresponds to any other utilization period.

If we revise $N3$ in this way, the revised statement $N3'$ is equivalent to $N1$ even if the nth commodity is a non-basic commodity. That is,

PROPOSITION 2. *N1 is equivalent to N3′.*

Proof. If the nth commodity is basic, Proposition 2 follows clearly from Proposition 1. But suppose it is non-basic. Then, all the prices of finished goods except the $n-1$th and the nth commodity are independent of the utilization period. Therefore,

$$\tilde{P}^* \leqq \tilde{P} \Leftrightarrow p_i^* = p_i \ (i = 1, 2, \cdots, n-2) , \quad p_{n-1}^* \leqq p_{n-1} , \quad p_n^* \leqq p_n .$$

So we obtain $1/p_n^* \geqq 1/p_n$, which implies that the wage rate in terms of the nth commodity is not smaller than that which corresponds to any other utilization period.

Conversely, suppose $1/p_n^* \geqq 1/p_n$. Then, $p_n^* \leqq p_n$, which implies $p_{n-1}^* \leqq p_{n-1}$ as we have seen in our final formulation of Eq. (1). Therefore, $P^* \leqq P$. Q.E.D.

4-c. Another Property of the Competitive Equilibrium

The properties of the competitive equilibrium we have stated so far only refer to prices of finished goods or wage rate in terms of finished goods. Do we have something to say about the prices of old fixed capital goods? The answer is yes. In fact, the following property can be deduced from $N1$.[17]

$N4$. When the rate of profit is uniform and given exogenously, the prices of old fixed capital goods which correspond to the utilization period chosen under the competitive equilibrium are non-negative.

PROPOSITION 3. *N1 is equivalent to N4.*

Proof. Suppose T is the utilization period chosen under the competitive equilibrium, and $\tilde{P}^* = (p_1^*, p_2^*, \cdots, p_n^*)$ is the price vector corresponding to T. And suppose τ is any other utilization period and $\tilde{P} = (p_1, p_2, \cdots, p_n)$ is the price vector corresponding to τ. In addition, we assume wage rate w^* corresponds to T, and w to τ. Then, from (5)′ and (8), we obtain

$$w\{(1+r)L[B-(1+r)A]^{-1}\hat{a}_n(\tau)+\hat{l}_n(\tau)\}$$
$$=p_n\{\hat{b}_n(\tau)-(1+r)\hat{a}_{nn}(\tau)-(1+r)^2 a_n[B-(1+r)A]^{-1}\hat{a}_n(\tau)\}\,.$$

Therefore, the following holds:

$$w = p_n \frac{N(\tau)}{D(\tau)}\,. \tag{14}$$

From (10),

$$w^* - p_n^* \frac{N(t)}{D(t)} = \frac{M^{t+1}}{D(t)} \frac{1}{(1+r)^{t+1}} p_m^{t+1} \quad (t=0,1,\cdots,T-1) \tag{15}$$

is obtained. For any $\tau < T$, putting (14) into (15), we can obtain

$$w^* - \frac{p_n^*}{p_n} w = \frac{M^{\tau+1}}{D(\tau)} \frac{1}{(1+r)^{\tau+1}} p_m^{\tau+1} \quad (\tau=0,1,\cdots,T-1)\,,$$

which implies

$$\frac{w^*}{p_n^*} \geqq \frac{w}{p_n} \quad \Leftrightarrow \quad p_m^{\tau+1} \geqq 0 \quad (\tau=0,1,\cdots,T-1)\,.$$

Therefore, $N3' \Leftrightarrow N4$. So, $N1 \Leftrightarrow N4$. Q.E.D.

[17] We owe the idea for the following proof to Baldone [1].

5. CONCLUDING REMARKS

As we have shown in the preceding section, the equivalence of *N*1, *N*2, *N*3, and *N*4 is guaranteed only in a limited case, i.e., the case in which the commodity produced by means of fixed capital is a basic commodity, but it fails to be so in general. If the *n*th commodity is non-basic, *N*1 is equivalent to only *N*3′ and *N*4, but not to *N*2 and *N*3.

Finally, let us discuss briefly the economic meaning of non-basic commodities. By analogy to the distinction between wage goods and luxury goods in the Ricardian system, Sraffa's basic commodities and non-basic commodities are often regarded as wage goods and luxury goods respectively. But this interpretation is very misleading. To be sure, Sraffa himself sometimes uses the words in the Ricardian sense. He does specify, however, that his non-basic commodities are not equal to Ricardo's luxury goods. "Non-basic" does not always mean "luxury". The distinction between basic commodities and non-basic commodities is only a technical one. If a commodity enters into all the production processes as input directly or indirectly, it is basic. If not, it is non-basic. That's all. Thus, there may be a case in which many goods in the wage basket are non-basic. Though corn is included in wage goods in the Ricardian system, it could be quite possibly a non-basic commodity in Sraffa's system.

Though they are often regarded as unimportant because of their name, non-basic commodities play an important role in an economy since they are in the wage basket. Thus, the subject we have considered in this paper has important implications.

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