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# MINIMUM WAGE RATES-OUTPUT RIGIDITY AND PRICE CHANGES* 

Rajindar K. Koshal and R. G. Akrihal


#### Abstract

This paper is an attempt to explain the behavior of a producer who is faced with a fixed wage rate due to either the presence of a union or government minimum wage legislation. Our analysis shows that a monopsonist facing an imperfect market for labor under a minimum wage legislation has built in a profit maximization incentive not to increase production to a point, even though the demand for the product increases. This implies that the increase in price of the product does not give any incentive to increase production, which adds fuel to the problem of inflation. The magnitude of the increase in price is dependent upon the value of the elasticity of the supply of labor and the elasticity of the product. These results also apply to the case of bilateral monopoly.


## I. introduction

Since the passage of the Fair Labor Standards Act of 1938, the United States has had a Federal minimum wage [4, 1034$]$. There have been a number of studies, since then, which deal with the various effects of minimum wages on many aspects of the economy [ $1-3,7,8,10-15,17,19-34]$. A large number of these studies have focused on the impact of minimum wages on the unemployment of teenage workers $[1-3,6,11-14,19,20,23,24,27,31,32]$ and how many of the teenage workers lose opportunities to get training, learn skills and increase productivity [6]. Some authors have made comparisons between white and non-white teenage employment, male and female teenage employment, etc. [32], and some have even estimated elasticities of unemployment with respect to increases in minimum wages [1].

Several studies have shown minimum wages resulting in reduced employment levels while some others suggest the movement of workers from low wage regions to high wage regions or from covered industries to uncovered industries [7, 10, 17, $20,26,30,34]$. The effect of minimum wage legislation on factor substitution and productivity is shown by Kaun [21]. In addition, the work by Brozen [7] draws our attention to the fact that minimum wage hikes might result in increased levels of the structural unemployment rate in the economy (which may very well help explain the increasing structural unemployment rate in the United States in the recent

[^0]past). There is another study by Brozen [6] which explains how thousands of workers lose their jobs and many are forced to migrate; and why many businesses fail and never get revived; and how city slums get deteriorated and overpopulated. These unfavorable events in the economy occur between the time the minimum wages are imposed-or increased-and the productivity of the affected workers and/or inflation catches up with the higher wages.

So far there has been no study that deals with the effects of minimum wages on such an important issue as the behavior of a producer in a monopsonistic market model with regard to his output decisions under demand-pull conditions. The inflationary pressures being as common as they have been in the recent years, producers' reactions to minimum wage rates in terms of their output levels under such conditions are very relevant. The present study is an attempt to explain the behavior of a producer faced with a fixed wage rate due to either the presence of a union or government minimum wage legislation.

## II. PROBLEM AND ANALYSIS

For simplicity, let us consider that a monopsony firm uses two inputs- $L$, labor, and $K$, capital. Let the monopsonist produce according to the following production function with the above two inputs:

$$
\begin{equation*}
Q=F(L, K) \tag{1}
\end{equation*}
$$

where $Q$ is total output, $L$ is the amount of labor, and $K$ is the amount of capital. The firm's total cost $(T C)$ when it uses $L$ units of labor at a price $\left(P_{w}\right)$ and $K$ units of capital at a price $\left(P_{k}\right)$ to produce a specified level of output $Q$ is

$$
\begin{equation*}
T C=A+P_{w} L+P_{k} K \tag{2}
\end{equation*}
$$

where $A$ is fixed cost. It must be pointed out that it is further assumed that markets for factors of production are imperfect, that is: prices $P_{w}$ and $P_{k}$ depend upon the amount of the factors used. It can easily be shown $[9,16]$ that the condition for the most efficient combination of factors to produce an output $Q$ is:

$$
\begin{equation*}
\frac{P_{w}+L \cdot\left(d P_{w} / d L\right)}{d F / d L}=\frac{P_{k}+K \cdot\left(d P_{k} / d K\right)}{d F / d K} \tag{3}
\end{equation*}
$$

Equation (3) can be interpreted as stating that the marginal outlay ( $M O$ ) of a factor divided by the marginal productivity of that factor must be the same for all factors. It can also be shown that marginal cost $(M C)$ is given by

$$
\begin{equation*}
M C=\frac{d T C}{d Q}=\frac{\left(P_{w}+L \cdot\left(d P_{w} / D L\right)\right) d L+\left(P_{k}+K \cdot\left(d P_{k} / d K\right)\right) d k}{(d F / d L) \cdot d L+(d F / d K) \cdot d K} \tag{4}
\end{equation*}
$$

Further, it is easy to see that when one of the factors is fixed, the marginal outlay by a firm for a variable factor must be equal to the marginal cost of the firm's output multiplied by the marginal product of the factor. Assuming $K$ is fixed, we get
from (4):

$$
\begin{equation*}
M O_{w}=P_{w}+L \cdot \frac{d P_{w}}{d L}=M C \frac{d F}{d L} \tag{5}
\end{equation*}
$$

where $M O_{w}$ is the marginal outlay of labor.
Equation (5) may be rewritten as

$$
\begin{equation*}
M C=\frac{M O_{w}}{d F / d L}=\frac{P_{w}+L\left(d P_{w} / d L\right)}{d F / d L} \tag{6}
\end{equation*}
$$

Equation (6) shows that marginal cost of the product not only depends on the price of labor but also the rate of change in this price and the marginal productivity of labor. It is well known $[4,8,9,16,18]$ that in the absence of union and/or government interference, the producer would employ $L_{1}$ amount of labor and pay a wage rate equal to $P_{w 1}$ (Fig. 1). However when government fixes the minimum wage rates, say $\bar{P}_{w}$, equal to the wage rate which would prevail if the producer as well as the labor would be a price taker. In this case, the function. (6) reduces to

$$
\begin{equation*}
\bar{M} C=\frac{\bar{P}_{w}}{d F / d L} \quad \text { for } \quad L \leq \bar{L} \tag{7}
\end{equation*}
$$

The imperfect labor market and minimum wage act imply that the firm could produce, say, up to $\bar{Q}$ units of output by hiring labor up to $\bar{L}$ units at the $\bar{P}_{w}$ wage rate (Figures $1 \& 2$ ). However, to produce more than $\bar{Q}$ units, the firm must hire more than $\bar{L}$ units of labor at more than the $\bar{P}_{w}$ wage rate. These facts produce discontinuity both in the marginal outlay function of labor and the marginal cost function (Figures $1 \& 2$ ) of output. It can be easily proved that the marginal


Fig. 1. Labor market.


Fig. 2. Product market.
revenue product (or the marginal revenue) function intersects the marginal outlay (the marginal cost) function at some lower point of this discontinuity. For simplicity let us assume that $M R P(M R)$ intersects the $M O(M C)$ at the lowest point of discontinuity.

The producer faced with rising demand would find that from a profit maximization point, it is optimum to employ the $\bar{L}$ amount of labor and to produce output $\bar{Q}$ until the demand curve $D D$ has risen to $D^{\prime} D^{\prime}$ so that the new marginal revenue function intersects just beyond the highest point of the discontinuity in the marginal cost curve or equivalently the price of the product has risen sufficiently so that the marginal revenue product curve intersects the marginal outlay ( $M O$ ) curve just beyond the highest point of discontinuity of marginal outlay of labor. These situations are also graphically shown in Figs. 1 and 2. The obvious result of this discontinuity is that in spite of the increase in price (within the above limits), the producer has no incentive to increase the output. This adds fuel to the problem of inflation.

Now let the two demand curves $D D$ and $D^{\prime} D^{\prime}$ for the product be represented by the following functions:

$$
\begin{align*}
& P_{q}=\alpha_{0}+\beta Q  \tag{8}\\
& P_{q}=\alpha_{1}+\beta Q \tag{9}
\end{align*}
$$

where $P_{q}$ is the price of the product and $Q$ is the quantity demanded. $\alpha_{0}, \alpha_{1}$, and $\beta$ are constants.
It is easy to see that $\Delta P_{q}$-the minimum increase in the price, $P_{q}$ needed to increase the production is given by:

$$
\begin{equation*}
\Delta P_{q}=\alpha_{1}-\alpha_{0}=F J \tag{10}
\end{equation*}
$$

Now assume that the supply function of labor is given by:

$$
\begin{equation*}
P_{w}=\gamma_{0}+\gamma_{1} L \tag{11}
\end{equation*}
$$

It is easy to see that marginal labor outlay at point $Y$ is given by

$$
\begin{equation*}
\bar{L} Y=\gamma_{0}+2 \gamma_{1} \bar{L} \tag{12}
\end{equation*}
$$

However, we know that the value of the marginal labor outlay at point $X$ is

$$
\begin{equation*}
\bar{L} X=\bar{P}_{w} \tag{13}
\end{equation*}
$$

Using (12) and (13) we get the value of the gap $X Y$ as follows

$$
\begin{equation*}
X Y=\gamma_{0}+2 \gamma_{1} \bar{L}-\bar{P}_{w} \tag{14}
\end{equation*}
$$

Dividing (14) by $d F / d L$ gives the value of the gap ( $F J$ ) in marginal cost

$$
\begin{align*}
F J & =\frac{X Y}{(d F / d L)}=\Delta P_{q}=\frac{\gamma_{0}+2 \gamma_{1} \bar{L}_{1}-P_{w}}{d F / d L}  \tag{15}\\
\Delta P_{q} & =\frac{\left(\gamma_{0}+\gamma_{1} \bar{L}\right)-\bar{P}_{w}+\gamma_{1} \bar{L}}{d F / d L} \\
& =\frac{\gamma_{1} \bar{L}}{d F / d L}=\frac{\left(d P_{w} / d L\right) \cdot \bar{L}}{d F / d L} \\
& =\frac{1}{\bar{E}_{1 s}} \cdot \frac{\bar{P}_{w}}{d F / d L} ; \quad \text { where } \quad \bar{E}_{1 s}=\frac{d L}{d P_{w}} \cdot \frac{\bar{P}_{w}}{\bar{L}}
\end{align*}
$$

(Supply elasticity of labor)

$$
\begin{equation*}
\Delta P_{q}=\frac{1}{E_{1 s}} \cdot \overline{M C} \tag{16}
\end{equation*}
$$

Thus (16) suggests that the increase in price of the product needed to induce the producer to hire more labor is indirectly related to the elasticity of supply of labor at $\left(\bar{L}, \bar{P}_{w}\right)$ and directly related to the value of the marginal cost of the product at ( $\bar{Q}, \bar{P}_{q}$ ). It is easy to see that if the supply of labor is perfectly elastic, $\Delta P_{q}$ would be zero. On the other hand, if the supply of labor is perfectly inelastic, $\Delta P_{q}$ would approach infinity.

It is also interesting to examine the relative change in price of the product, from (16), we get

$$
\begin{equation*}
\frac{\Delta P_{q}}{\bar{P}_{q}}=\frac{1}{\bar{E}_{1 s}} \cdot \frac{\overline{M C}}{\bar{P}_{q}} \tag{17}
\end{equation*}
$$

But

$$
\bar{P}_{q}=\frac{\overline{M R}}{\left(1-1 / \bar{E}_{q}\right)} \quad \text { where } \quad \bar{E}_{q}=\frac{d Q}{d P_{q}} \cdot \frac{P_{q}}{Q}
$$

Therefore

$$
\frac{\Delta P_{q}}{\bar{P}_{q}}=\frac{1}{\bar{E}_{1 s}} \cdot \frac{\overline{M C}}{\overline{M R}} \cdot\left(1-1 / \bar{E}_{q}\right)
$$

or

$$
\begin{equation*}
\frac{\Delta P_{q}}{\bar{P}_{q}}=\frac{1}{\bar{E}_{1 s}} \cdot\left(1-1 / \bar{E}_{q}\right) \quad \text { since } \quad \overline{M C}=\overline{M R} \tag{18}
\end{equation*}
$$

Relation (18) suggests that the relative increase in price of the product decreases as demand elasticity of the product increases, and the relative change in price approaches $1 / \bar{E}_{1 s}$ (the reciprocal of the supply elasticity of labor) as demand elasticity approaches infinity.

## SUMMARY

We have shown that a monopsonist facing an imperfect market for labor under a minimum wage legislation has built-in profit maximization incentive not to increase production to a point, even though the demand for the product increases. This implies that the increase in price of the product does not give any incentive to increase production, which adds fuel to the problem of inflation. However, only when the price of the product increases further and inflation catches up with minimum wages, rendering them ineffective, will additional output come forth! Of course the magnitude of the needed increase in price is dependent upon the value of the elasticity of the supply of labor and the elasticity of demand for the product. But this increased price may very well justify the next round of minumum wage hikes leading to inflationary pressures and perhaps needing more of the same for bringing about higher input. The minimum relative increase in price of the product needed to bring forth increased output levels will be smaller (larger) the lower (higher) the demand elasticity of the product, given the elasticity of supply of labor. These results also apply to the case of a bilateral monopoly model.

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