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Abstract	In this paper, we have presented a trade model with non-traded goods in terms of a four-quadrant diagram. Using simple geometry a number of well known and important results are presented. These are summarized below: (1) The geometrical technique used in this chapter clearly shows the separation between a production set and consumption set. (2) Given the exogenously given terms of trade the determination of the relative price of non-traded goods is explicitly shown. (3) Two theorems on gains from trade, namely, free trade is better than no trade and an improvement (deterioration) in the terms of trade raises (lowers) welfare are demonstrated in the non-traded goods framework. (4) It is demonstrated that in the first best framework economic expansion always raises welfare. However, the output effects of economic expansion are ambiguous. (5) A result on "immiserizing growth" is presented in the context of monopoly oower in trade.
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A SIMPLE GEOMETRICAL TREATMENT OF NON-TRADED GOODS IN THE PURE THEORY OF INTERNATIONAL TRADE*

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and Dong C. SUH

1. INTRODUCTION

In 1967, Komiya [2] incorporated non-traded goods in the pure theory of international trade. Since the appearance of this pioneering paper a rich and interesting literature has grown which focuses on the role of non-traded goods in the real theory of trade.[1] This literature generally uses a model in which the country produces three commodities (two traded and one non-traded) with the help of two primary factors of production. This represents an extension of the traditional two-factor, two-commodity, two-country static model of trade (popularly known as the Heckscher-Ohlin-Samuelson model) to a three-commodities, two-factor, two-country model of barter trade. A number of interesting results have been obtained on the basis of this extended model. For instance, technical progress in the import competing industry may not result in an increase in its output in the presence of non-traded goods. The intuitive appeal of these highly interesting results is brought out with the help of simple geometry in the present paper.

This paper presents a very simple model of international trade with non-traded goods which can be easily represented in terms of geometry.[2] The geometrical representation of the model not only sheds light on several well known propositions in the non-traded goods literature but also makes the results accessible to a non-mathematical audience. It is assumed that two goods are produced domestically, one non-traded (X_1) and the other totally exported (X_2) with the help of two primary factors of production. On the consumption side two goods are consumed, one non-traded (D_1) and the other imported (D_3). Thus, the exported good is not consumed domestically while the imported good is not produced locally. This model can be represented in terms of a four-quadrant diagram which enables us to discuss several trade results with non-traded goods geometrically. Our main focus of attention will be to demonstrate on the basis of this simple model (a) the separation of consumption and production set; (b) explicit determination of the price of non-traded good; (c) explicit representation

* Useful comments received from an anonymous referee are gratefully acknowledged.

[1] See for example: Hazari and Sgro [6], [7], Jones [9], Komiya [11] and McDougall [12].

[2] This simple model was originally presented by Jones [9]. However he did not use the geometrical technique which is used by us.

of theorems on gains from trade and (c) the impact of economic expansion on output and welfare.

We now proceed to provide a brief verbal exposition of the main assumptions of the model.[3]

(i) The production functions for commodities X_1 and X_2 are assumed to be linearly homogeneous and exhibit diminishing returns to factor proportions. The relative price ratio guiding production is P_2/P_1.

(ii) The aggregate utility function which depends on the consumption of D_1 and D_3 is assumed to be strictly concave. It is also assumed to possess both behavioural and welfare significance. The price ratio P_3/P_1 and income determine the value of consumption.

(iii) The factors of production are fully employed and in equilibrium the factor rewards equal the value of their marginal product.

(iv) The balance of payments is always assumed to be in equilibrium. The terms of trade are given by the price ratio P_2/P_3.

Note that there are three price ratios in this model: P_2/P_1 which guides production; P_3/P_1 supports consumption (along with income); and P_2/P_3 the terms of trade. One way of closing this model is to assume that P_2/P_3 is given exogenously.

2. A FOUR QUADRANT DIAGRAM AND THE DETERMINATION OF EQUILIBRIUM WITH NON-TRADED GOODS

We now proceed to develop a four quadrant diagram for representing the model already presented. On the basis of this diagram we determine all our endogenous variables, given the assumption that the country is small, and hence takes the terms of trade to be given exogenously.

In Fig. 1, in the left hand top quadrant I, the production possibility locus TT' is represented. The curve TT' is drawn concave to the origin, given our restrictions on the production functions coupled with the assumption that the capital intensities in the two production sectors X_1 and X_2 are not identical ($k_1 \neq k_2$).[4] In the bottom left hand quadrant II exports are indicated by the horizontal axis X_2 and imports by the vertical axis M_3. The terms of trade (P_2/P_3) can be represented by any straight line, that joins points between the X_2 and M_3 axis in quadrant II, for example $T'A$ indicates the terms of trade. It shows that OA of commodity M_3 can be imported in exchange for OT' of commodity X_2 in terms of exports. The line OHG in the bottom right hand quadrant III, is a 45° lines. This is used to transfer the amount of imports M_3 measured down the vertical axis to the horizontal axis D_3, thus the value of OA equals the value OM ($OA = OM$). In the right hand top quadrant IV the social indifference curves associated with our utility function in assumption (ii) are represented. These are convex to the origin and

[3] The mathematical statement of this model is available in Jones [9].

[4] This locus is explicitly derived in many books. See for example Hazari [5].

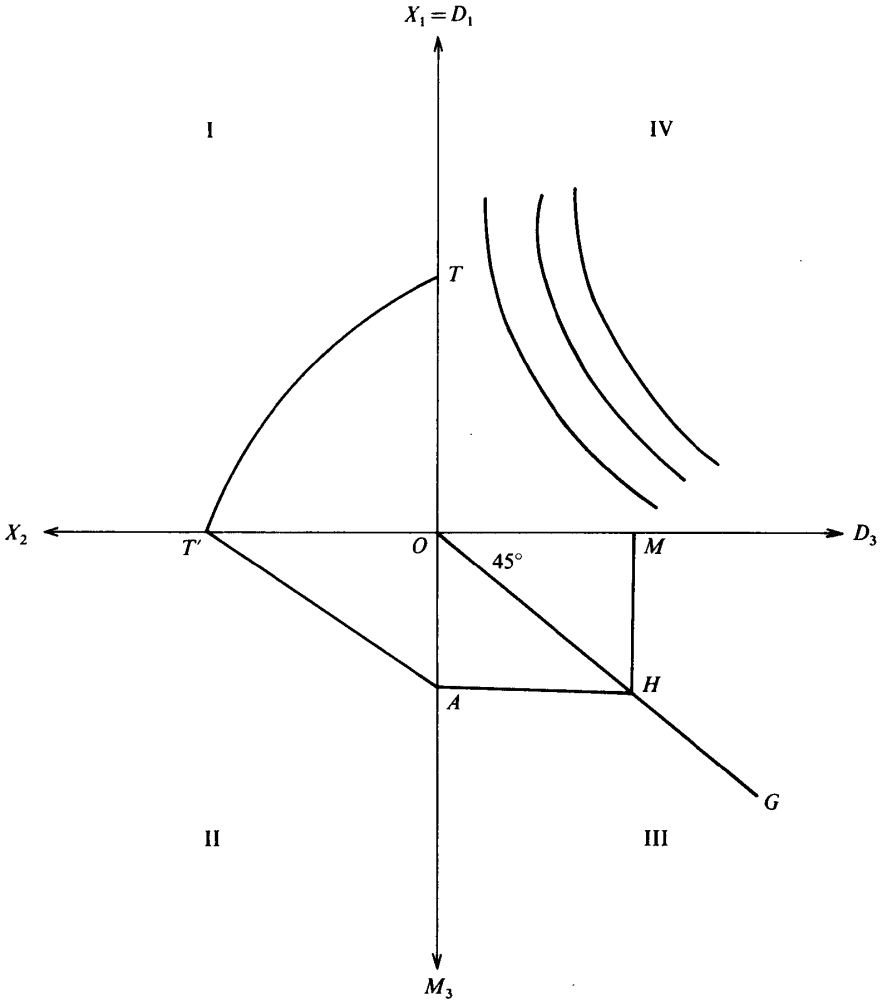

Fig. 1.

non-intersecting given our restrictions that the utility function is strictly concave. Equilibrium cannot be determined on the basis of information that has been so far described. For this purpose we have to introduce a consumption possibility schedule.

Using quadrants I–III a consumption possibility locus is derived. This is done in Fig. 2. The procedure is as follows. Let the terms of trade be given by the slope of the line $T'A$. If production occurs at point T', then these terms of trade allow us to import OA of commodity M_3 which in quadrant IV is represented by point M. This is one point on the consumption possibility locus. Now choose an arbitrary production point on the production transformation locus, let this point be P in quadrant I. At point P the economy produces OD of comodity X_2 and DP of commodity X_1. Given the same terms of trade (slope of line $T'A$), the line DF has the same slope as $T'A$, the economy can import $OF = OJ$ of commodity M_3 in exchange for OD exports of good X_2. The amounts OJ of good M_3 and DP of good

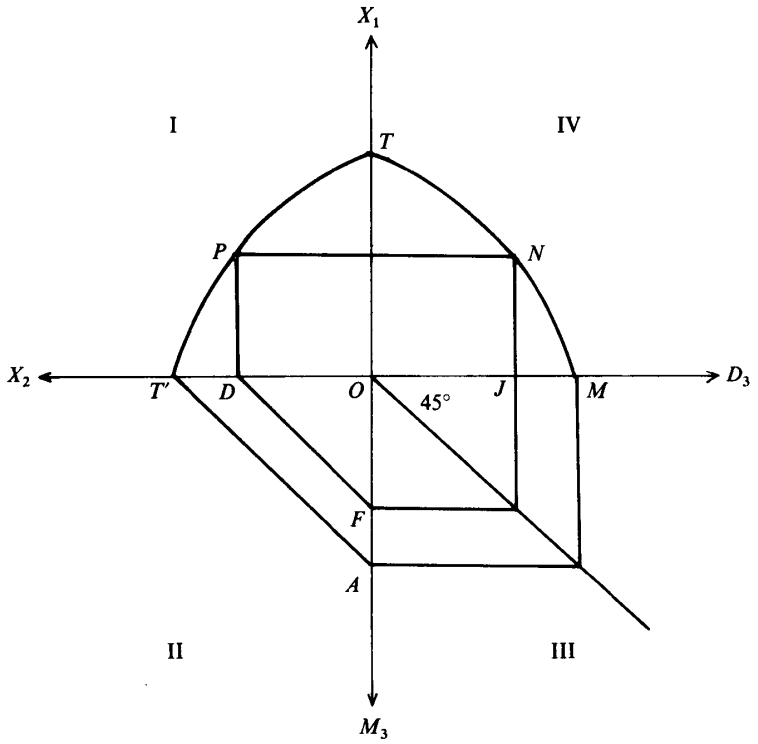

Fig. 2.

X_1, the two commodities consumed determine another point, N, on the consumption possibility frontier. Using this procedure for all the feasible production points along the transformation curve TT' enables us to trace out the concave to the origin consumption possibility locus TNM in quadrant IV. The separation between the production and consumption possibilities set is clearly shown by this diagram. This separation arises because the commodities that are produced are not the same as the commodities that are consumed.

The diagram is now complete with production being shown in quadrant I, the terms of trade in quadrant II and the consumption possibilities in quadrant IV. The equilibrium values of all our endogenous variables can be determined. We start from quadrant IV in Fig. 3. The tangency between the indifference curve U_0 and the consumption possibility locus TM at point C indicates the consumption equilibrium. The consumers are maximizing utility by consuming CJ of commodity X_1 and OJ of commodity D_3. This consumption equilibrium at C in turn determines the production equilibrium at point P in quadrant I. On the production side PD of good X_1 is produced along with OD of good X_2. Note that the production of commodity X_1 the non-traded good, exactly equals the consumption of non-traded good ($CJ=PD$). The output of OD of commodity X_2 is transformed into imports OD' of commodity M_3 by using the terms of trade given by the slope of the line DD'. The price ratio facing consumers (P_3/P_1) at point C equals the slope of the straight line QC, while the price ratio facing

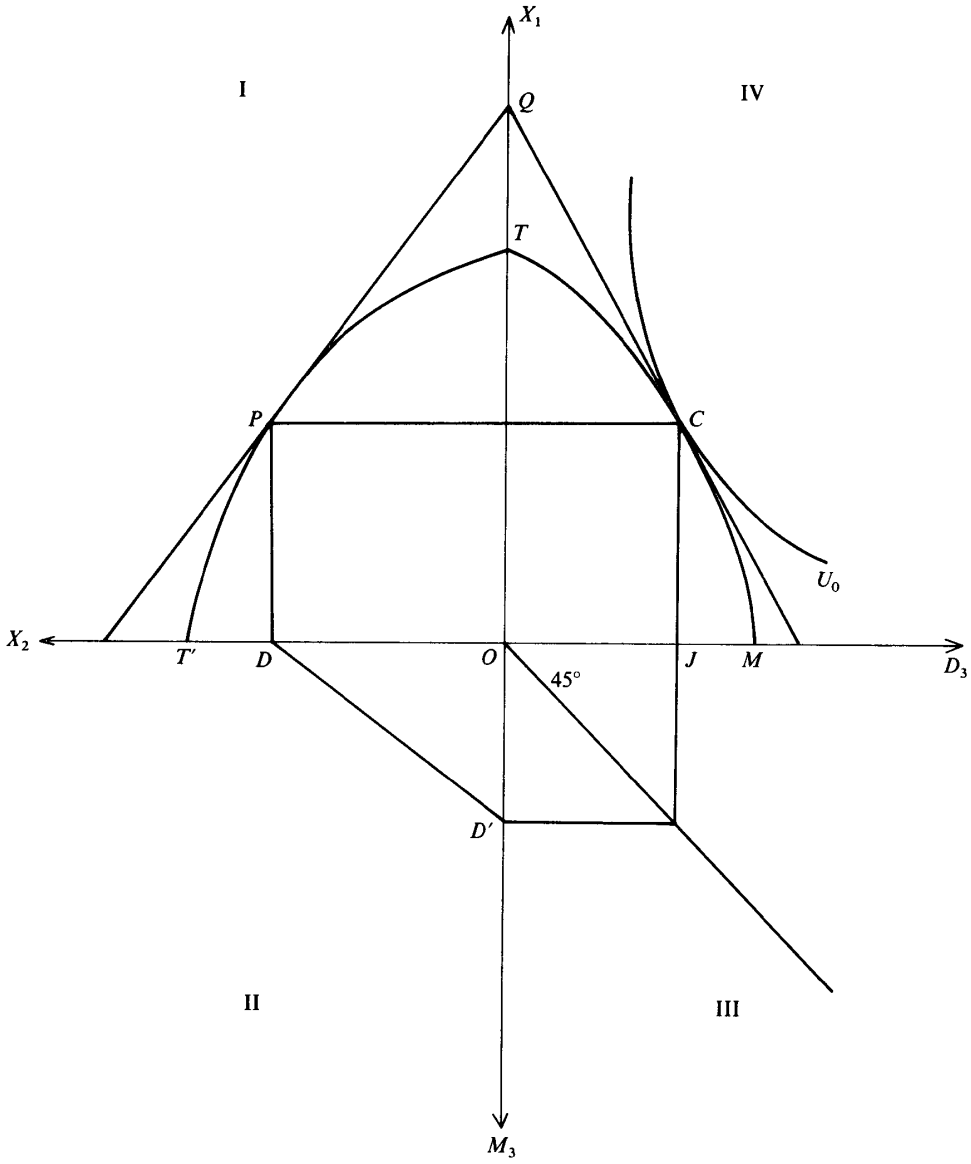

Fig. 3.

producers (P_2/P_1) at P equals the slope of the straight line QP. Note that the straight lines whose slopes represent the two price ratios, facing producers and consumers, commence from the same point Q on the X_1 axis. This is because national income from the production side must equal national income from the consumption side. Given the terms of trade in Fig. 3 we have determined all the endogenous variables. We will now use this simple, but elegant diagram to present some results.

3. FREE TRADE VERSUS NO TRADE

This section examines the welfare ranking of free trade vis-a-vis no trade.

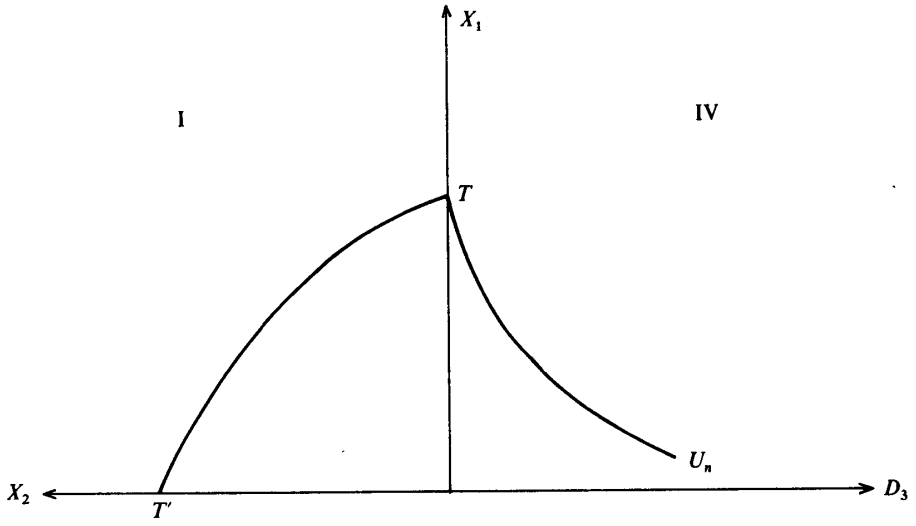

Fig. 4a.

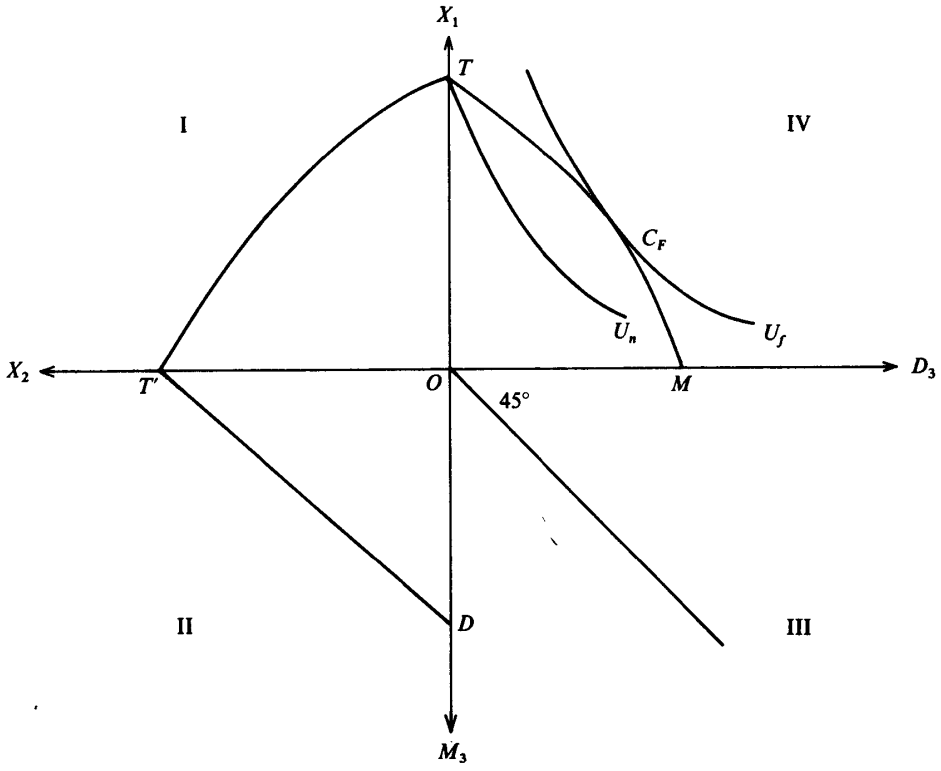

Fig. 4b.

Interest in the welfare ranking of these two commercial policies goes back to David Ricardo [13]. He was probably the first economist to present a highly simplified model demonstrating the superiority of free trade over no trade in welfare terms. Samuelson [14] developed the first rigorous proof of the gains from trade theorem that compensated free trade is better or as good as no trade. This result was

extended to cover non-traded goods by Kemp [10].

It should be emphasized that in all statements of the above theorem the terms "compensated" is used. This is because introduction of free trade into a situation of no trade results in changes in the distribution of income. Some individuals become better off by the introduction of free trade while others become worse off. Hence, the usage of the term compensated free trade. We side track this problem by assuming a well behaved aggregate utility function which incorporates the welfare judgement about income distribution.

In Fig. 4 we present the result that compensated free trade is better or no worse than no trade in terms of welfare. Consider the no trade situation shown in Fig. 4a where in quandrant I TT' is the production possibility locus. The no trade situation corresponds to a corner position at T where consumption consists solely of good X_1, the non-traded good. The production point is also at point T where production occurs at the point of complete specialization, corresponding to this no trade point, the welfare level is represented by the indifference curve U_n.

Let us now introduce trade in this setting. Assuming the country to be small the terms of trade, (P_2/P_3), are represented by the slope of the line $T'D$ in Fig. 4b. Given the terms of trade and the locus the consumption possibility frontier TM is traced out in quadrant IV. The free trade consumption point is now at C_F. The welfare level is indicated by the indifference curve U_f. Since $U_f > U_n$, it follows that free trade is better than no trade in welfare terms.

4. TERMS OF TRADE AND WELFARE

In this section, we present the result that for a small country an improvement (deterioration) in the terms of trade raises (lowers) welfare. An improvement in the terms of trade obtains when for the same amount of exports more imports can be purchased.[5] This result is presented diagramatically in Figs. 5a and 5b.

A favourable movement in the terms of trade is indicated in Figs. 5a and b by a rotation of the line $T'A$ to $T'A'$. The change in the slope of this line indicates movements in the terms of trade. Before the improvement OT' of exports could be exchanged for OA of imports. After the favourable movement in terms of trade, OT' of exports can be exchanged for OA' of imports. As a consequence of improvement in the terms of trade a new consumption possibility locus emerges—this is traced out in quadrant IV as TM'. In Fig. 5a consumption equilibrium occurs at C_t and production equilibrium at P_t in quadrant I. Consumption C_t occurs on indifference curve U_t, and $U_t > U_f$, therefore welfare unambiguously increases due to the favourable movement in the terms of trade.

An interesting feature of this model is that output response cannot be predicted from the movement in the terms of trade. This is illustrated in Figs. 5a and b. In Fig. 5a production of X_2 falls as a consequence of improvement in terms of trade,

[5] This result was first proved by Kemp [10], in the non-traded goods framework.

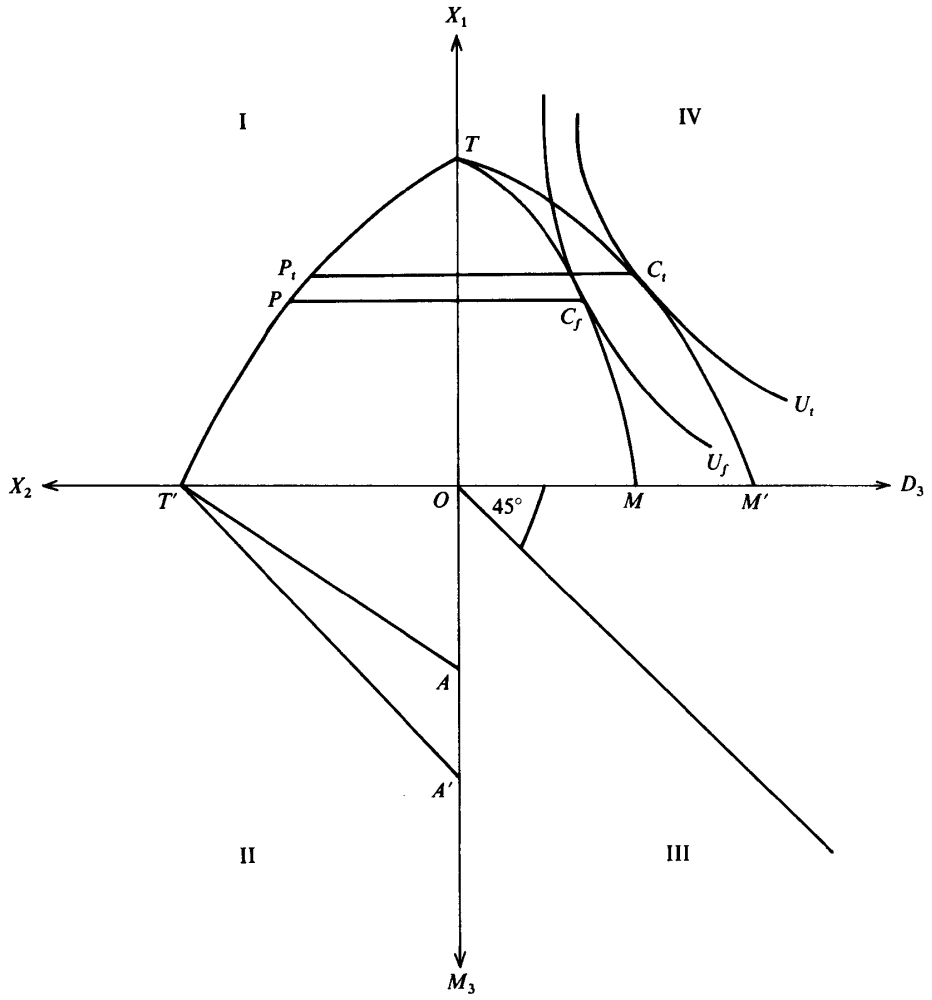

Fig. 5a.

while in Fig. 5b the production of X_2 increases as a consequence of favourable movement in the terms of trade. This is so because output movements are governed by the price ratio (P_2/P_1) and the terms of trade by (P_3/P_2). These price movements are linked by (P_3/P_1) the price ratio that guides consumption. It is also clear from Fig. 5a and b that the price movement (P_3/P_1) is ambiguous; therefore the movement in (P_2/P_1) is not of a definite sign and hence, the ambiguity of output response to a change in terms of grade.

5. ECONOMIC EXPANSION, OUTPUT LEVELS AND WELFARE

In the model presented in Section 1 there are two alternative but not mutually exclusive ways of introducing economic expansion. Economic growth can either be introduced by allowing technical progress in the production sectors of the economy and/or by introducing growth in capital and labour the two primary

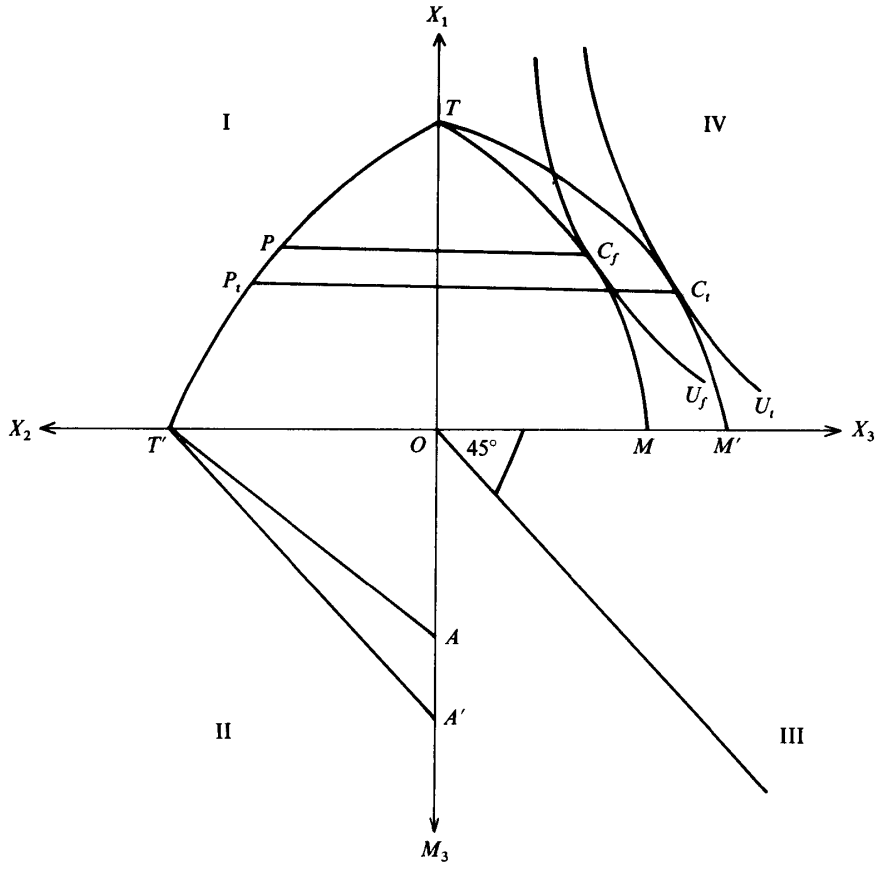

Fig. 5b.

factors of production. We shall only analyse the consequences of technical progress.

It is assumed that Hicks—neutral technical progress occurs in the export goods sector X_2.[6] We are interested in analysing the consequences of this type of technical progress for output levels and welfare. Technical progress in sector X_2 shifts the production possibility curve from TT' to TT'' in quadrant I of Fig. 6a. Given the new production possibility frontier and the exogenously given terms of trade, the post-growth consumption possibility locus TM' is traced out in quadrant IV of Fig. 6a. The pre-growth consumption equilibrium is at C_0 at the corresponding equilibrium at P_0.

It is clear from quandrant IV of Fig. 6a that new consumption possibility frontier lies uniformly outside the old frontier (except at the point of complete specialization in X_1). Consumption equilibrium now occurs at a point of tangency between the post-growth locus and an indifference curve. Given that the new

[6] Other types of technical progress, and factor accumulation can be easily handled in terms of our geometrical technique. The basic principles underlying the analysis of the impact of factor accumulation on output and welfare are identical with those of technical progress, and hence an explicit discussion of factor accumulation is omitted. The interested reader can attempt this as an exercise.

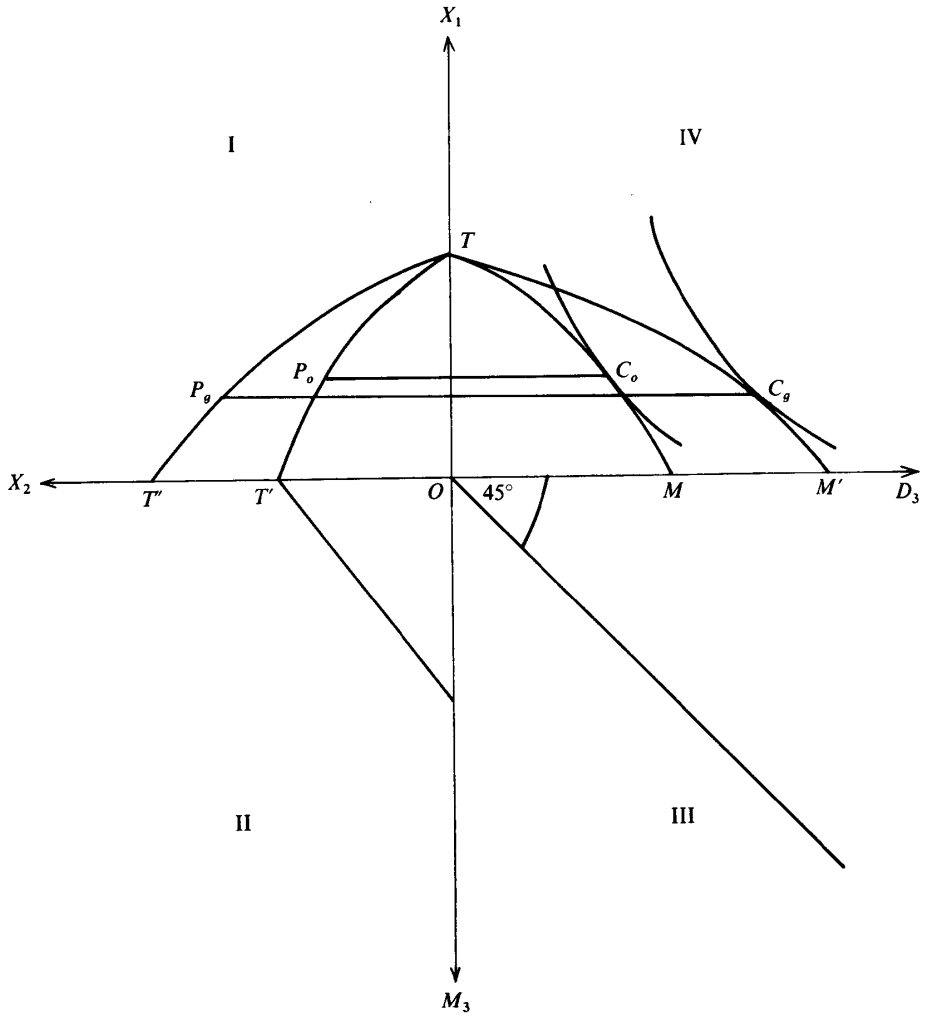

Fig. 6a.

consumption possibility frontier dominates the old frontier, welfare unambiguously increases as a consequence of technical progress. The output effect of technical progress depends on consumption equilibrium which in turn depends on the indifference map. Suppose that in Fig. 6a consumption equilibrium occurs at C_g. Associated with C_g is the production point P_g in quadrant I. Equilibrium at P_g indicates that as a result of technical progress in X_2 its output has increased and that of X_1 has decreased. This of course is a standard result. In Fig. 6b it is shown that output of X_2 decreases in spite of the fact that it is the sector that experiences technical progress. The intuitive reason for the occurrence of this result can be easily seen by a closer examination of quadrant IV of Fig. 6b. Output of X_2 decreases because with the shift in the consumption possibility locus the consumers decide to decrease the absolute amount of consumption of the imported good M_3, therefore given fixed terms of trade the output of X_2 needed to buy M_3 must decline. This is precisely what is happening in Fig. 6b. In other words if D_3 is an inferior good then there exists the possibility that technical progress in commodity

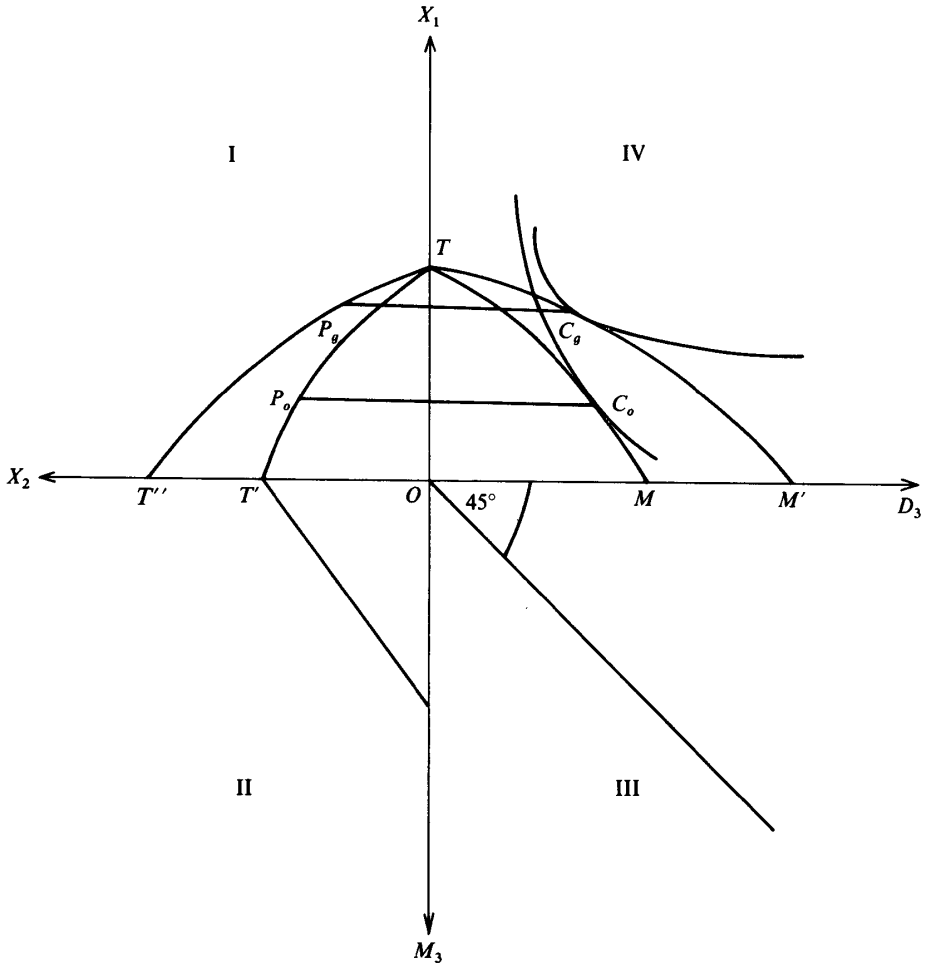

Fig. 6b.

X_2 may lead to fall in its output and an increase in the output of the non-progressive sector X_1. Thus, *Hicks—neutral technical progress in sector 2 does not necessarily result in an increase in the output of* X_2.[7] *Welfare always increases as a consequence of technical progress.*

6. A RESULT ON IMMISERIZING GROWTH

The object of this section is to present a result on immiserizing growth in the non-traded goods framework. Over the last ten years several immiserizing growth theorems have appeared in the literature.[8] A necessary condition for "immiserizing growth" is the presence of a distortion, for example, factor market imperfections, externalities and monopoly in trade. This section is devoted to presenting an

[7] This type of ambiguity was also noted by Komiya [11].

[8] See for instance: Bhagwati [1], [3], Hazari [4], [5], Hazari and Sgro [6], [7], and Johnson [8].

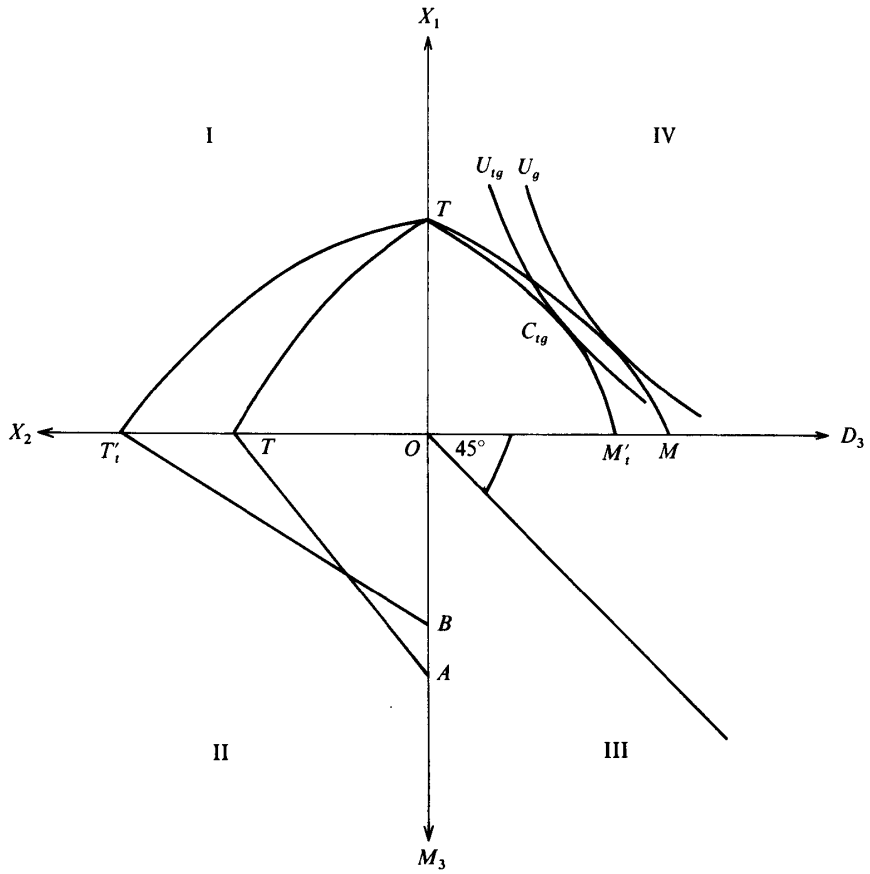

Fig. 7.

immiserizing growth theorem in the presence of monopoly power in trade.[9]

Assume that export biased technical progress occurs in our model. In other words technical progress takes place in the exporting sector of the economy, namely, sector 2. In the previous section this type of expansion was assumed to obtain in the presence of fixed terms of trade. We now assume that the country has monopoly power in trade and therefore its terms of trade deteriorate as a consequence of export biased growth. It can then be shown that the post growth welfare may be below the pre growth level of welfare. This result is explained and demonstrated in Fig. 7.

Technical progress in sector 2 results in an outward shift of the production possibility locus, as shown by TT_t' in Fig. 7. Assume now that the terms of trade deteriorate and are indicated by the slope of line $T_t'B$ instead of $T'A$. This deterioration arises due to export biased technical progress. With the deteriorated terms of trade and the post growth transformation locus is associated a post growth consumption possibility locus as shown by TM_t'. The consumption locus

[9] In the standard model this possibility was originally shown by Bhagwati [1].

TM_t' due to deterioration in the terms of trade does not lie uniformly outside the original consumption possibility locus TM. Suppose on this new locus consumption equilibrium occurs at C_{tg}. The welfare is indicated by the indifference curve U_{tg}. It is clear that $U_{tg} < U_g$, therefore welfare declines as a result of export biased technical progress at non-constant terms of trade. Immiserizing growth arises because the adverse movement in the terms of trade wipes out the gain in productivity from technical progess. Thus in the presence of monopoly power in trade export biased technical progress does not necessarily raise welfare.

7. SUMMARY

In this paper, we have presented a trade model with non-traded goods in terms of a four-quadrant diagram. Using simple geometry a number of well known and important results are presented. These are summarized below:

(1) The geometrical technique used in this chapter clearly shows the separation between a production set and consumption set.

(2) Given the exogenously given terms of trade the determination of the relative price of non-traded goods is explicitly shown.

(3) Two theorems on gains from trade, namely, free trade is better than no trade and an improvement (deterioration) in the terms of trade raises (lowers) welfare are demonstrated in the non-traded goods framework.

(4) It is demonstrated that in the first best framework economic expansion always raises welfare. However, the output effects of economic expansion are ambiguous.

(5) A result on "immiserizing growth" is presented in the context of monopoly power in trade.

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REFERENCES

[1] Bhagwati, J. N., "Immiserizing Growth: A Note," *Review of Economics Studies*, Vol. 25 (June 1958), pp. 201–205.

[2] Bhagwati, J. N., "The Gains from Trade Once Again," *Oxord Economic Papers*, Vol. 20 N.S. (July 1968), pp. 137–148.

[3] Bhagwati, J. N., "Distortions and Immiserizing Growth: A Generalization," *Review of Economic Studies*, Vol. 35 (October 1968), pp. 481–485.

[4] Hazari, B. R., "Factor Market Distortions, Technical Progress and Trade," *Oxford Economics Papers*, Vol. 27 N.S. (March 1975), pp. 47–60.

[5] Hazari, B. R., *The Pure Theory of International Trade and Distortions*, New York: Halsted Press, John Wiley, 1978.

[6] Hazari, B. R. and Sgro, P. M. "Theorems on Immiserizing (Normal Growth) in the Non-Traded Goods and Wage Differentials Framework," *Southern Economic Journal*, Vol. 41 (January 1975), pp. 515–519.

[7] Hazari, B. R. and Sgro, P. M., "Some Notes on Technical Progress in the Framework of Factor Market Imperfections and Non-Traded Goods," *Australian Economic Papers*, Vol. 16 (June 1976), pp. 76–86.

[8] Johnson, H. G., "The Possibilities of Income Losses from Increased Efficiency or Factor Accumulation in the Presence of Tariffs," *Economic Journal*, Vol. 77 (March 1967), pp. 151–154.

[9] Jones, R. W., "Trade with Non-Traded Goods: The Anatomy of Interconnected Markets," *Economics*, Vol. 41 (May 1974).

[10] Kemp, M. C., *The Pure Theory of International Trade and Investment*. Englewood Cliffs, New Jersey: Prentice Hall, 1969.

[11] Komiya, R., "Non-Traded Goods and the Pure Theory of International Trade," *International Economic Review*, Vol. 8 (June 1967), pp. 132–152.

[12] McDougall, I., "Non-Traded Commodities and the Pure Theory of International Trade," in I. McDougall and R. Snape (eds) *Studies in International Economics*, Amsterdam: North-Holland, 1970.

[13] Ricardo, D., *On the Principles of Political Economy and Taxation*, London: John Murray, 1817.

[14] Samuelson, P. A., *"The Gains from International Trade," Canadian Journal of Economics and Political Science*, Vol. 5 (May 1939), pp. 195–205.