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CONVERGENT NON-TATONNEMENT RESOURCE ALLOCATION PROCESSES FOR NON-CLASSICAL ENVIRONMENTS

Hiroaki OSANA[1]

1. INTRODUCTION

The basic theorems of welfare economics assert that, in "classical economic environments," every equilibrium allocation of the perfectly competitive resource allocation process is a Pareto-optimum and every Pareto-optimum can be attained as an equilibrium allocation of that process with the aid of some redistribution of the initial resource endowments (*cf.* Arrow [1, Theorems 4 and 5] and Debreu [4, Theorem (1) of 6.3 and Theorem (1) of 6.4], for instance). According to the terminology introduced by Hurwicz [6], the perfectly competitive resource allocation process is non-wasteful and unbiased for classical economic environments, where by "classical economic environments" we mean economic environemnts free of externalities, non-convexities, and discontinuities. Since the perfectly competitive resource allocation process is neither non-wasteful nor unbiased for non-classical economic environments, it is natural to ask whether there exists a resource allocation process which is non-wasteful and unbiased for those environments. It is of particular interest to ask whether we can find such a process which is informationally decentralized in some sense.

The "greed process" developed by Hurwicz [6] is an example of such processes. This process is non-wasteful and unibiased for every economic environment free of externalities, but fails to be dynamically stable. By relaxing Hurwicz' definition of informational decentralization, Camacho [3] presented a process, called the "*D* process," which is non-wasteful and unbiased for every economic environment. Nor is this process dynamically stable.

Dynamically stable processes have been proposed by Kanemitsu [8 and 9], Ledyard [10], and Hurwicz, Radner, and Reiter [7]. The "inertia-greed process" of Kanemitsu [8] is of the tatonnement type, while the "*A* process" of Kanemitsu [9], the "*P* process" of Ledyard [10], and the "*B* process" of Hurwicz, Radner, and Reiter [7] are of the non-tatonnement type. The inertia-greed process and the *A* process are designed for economic environments free of externalities and indivisibilities, and the *B* process is designed for economic environments which are free of externalities and contain either indivisible commodities only or divisible

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commodities only. On the other hand, the P process is designed for economic environments which admit externalities but not non-convexities.

In the present paper, we shall construct two classes of non-stochastic, non-tatonnement resource allocation processes, one for economies without externalities and the other for economies with externalities. The basic idea is quite simple and obtained by observing the following obvious fact: *Any resource allocation process is non-wasteful and unbiased if* (1) *it finds a Pareto-superior allocation whenever such an allocation exists,* (2) *it generates a Pareto-superior allocation whenever such an allocation is found, and* (3) *it keeps the predetermined allocation unchanged when no Pareto-superior allocation is found.* Such a process will generate a sequence of allocations, and to each allocation in this sequence there corresponds the set of allocations Pareto-superior to it. By properties (2) and (3), the sequence of allocations is non-decreasing with respect to each agent's preference relation, and hence the corresponding sequence of Pareto-superior sets is non-increasing. This kind of monotonicity is likely to facilitate stability of the process, and, for this purpose, non-tatonnement processes seem to be more suitable than tatonnement ones (*cf.* Uzawa [12], Ledyard [10], Hurwicz, Radner, and Reiter [7], and Kanemitsu [9]; see also Hahn and Negishi [5]). Of course, the monotonicity by itself does not guarantee convergence to Pareto-optimal allocations.

In order to get properties (2) and (3), it suffices to specify an outcome rule which implements a Pareto-superior allocation if and only if such an allocation is found. This automatically guarantees the unbiasedness of the process (*cf.* Theorems 2 and 5). Property (1) is essential for non-wastefulness. Note that if each agent proposes the whole set of allocations which he prefers to the status quo then the intersection of the proposals coincides with the set of Pareto-superior allocations, while if some agent proposes only a small set of allocations which he *much* prefers to the status quo then the intersection of the proposals may be empty. In order to get property (1), therefore, it suffices to specify a response rule which "discourages" the agents to be so unamibitious as to accept making proposals which require them to stay within arbitrarily small neighborhoods of the status quo. Such a response rule, together with an outcome rule satisfying properties (2) and (3), constitutes a non-wasteful resource allocation process (*cf.* Theorems 1 and 4). In the present paper, we shall not give any specific device for guaranteeing convergence to Pareto-optimal allocations, but simply try to avoid convergence to non-optimal allocations (*cf.* Theorems 3 and 6). This can be done by "encouraging" the agents to be so ambitious as to reject making any proposals which require them to stay within arbitrarily small neighborhoods of the status quo. If the preference relations are representable by utility functions, then the monotonicity of the sequence of allocations would imply convergence of the corresponding sequence of utility allocations to some point on the utility frontier (*cf.* Corollaries to Theorems 3 and 6). Thus, the crucial step in constructing our processes will be to combine the "encouragement effect" and "discouragement effect" of the response rule.

The response rules of the inertia-greed process and the A process are weighted averages of the greed response and the stationary response. Hence the instability of the greed response is mitigated by the stabilizing effects of the stationary response, while the static properties of the greed process are preserved. The instability of the greed process seems to come from the way in which the opportunity sets are defined to which the greed response is made. That is, the opportunity sets are highly sensitive to the other agents' messages. The basic idea of Kanemitsu's processes is to modify the greed response, while retaining Hurwicz's way of defining the opportunity sets. In the present paper, we shall modify the definition of the "opportunity sets," while retaining the greed response. Many other intermediate variants of these two approaches might be conceivable. Therefore, it will be of great interest to axiomatize the properties of the response rule which will guarantee static optimality and dynamic stability, rather than to increase the number of examples of such response rules.

Although the intertia-greed process, the A process, the B process, and our processes use set-valued messages, the P process uses point-valued messages only. In the P process, the agents exchange messages concerning the directions of reallocation they prefer, a direction of reallocation is determined on the basis of the messages, and a Pareto-superior reallocation is carried out in this direction. The directions of reallocation can be expressed in terms of points of a finite dimensional Euclidean space. This yields a great gain in the simplicity of message processing. However, unless suitable convexity properties are assumed of the environment, a more complicated message space seems to be required, as is the case for the A process and the B process. The situation suggests a trade-off relation between environmental coverage and informational requirements (*cf.* Mount and Reiter [11]).

The inertia-greed process, the A process, the B process, the P process, and our processes all have similar outcome rules. A reallocation is carried out only if it is toward a Pareto-superior allocation. The outcome rules are of the non-tatonnement type in the sense that transactions are permitted to take place even if a message equilibrium has not yet been attained. It is assumed, however, that no consumptions (nor productions) are carried out until some equilibrium obtains. In the present paper, the term *transaction* can be best interpreted as that of contract notes but not actual movement of commodities. Until a new transaction is carried out, the contract notes issued at the preceding transaction time are valid; the latter becomes void when the new transaction is carried out. In particular, the claims to the initial resource endowments become void once the first transaction takes place. This is the essence of our non-tatonnement outcome rules. Therefore it is natural to interpret our resource allocation processes in the short-run context.

The present paper is organized as follows. In Section 2, the assumptions on the economic environment will be postulated, and some preliminary results on the environmental properties will be stated. In Section 3, a general definition of non-tatonnement resource allocation process will be introduced, and two equilibrium concepts will be discussed; in the remaining part of the section, our resource allocation processes will be specified, and the main results on static and dynamic properties of the processes will be stated. All the proofs of the results are given in the Appendix.

2. ENVIRONMENTS

We shall consider economies with m commodities and n agents. The *set of commodities* and the *set of agents* will be denoted by $H=\{1, \cdots, m\}$ and $I=\{1, \cdots, n\}$, respectively. For each agent i, his *consumption* x_i and *production* y_i are points of R^m. An n-tuple $x=(x_i)_{i\in I}$ of individual consumptions and an n-tuple $y=(y_i)_{i\in I}$ of individual productions are called a *consumption allocation* and a *production allocation*, respectively. The ordered pair $a=(x, y)$ of a consumption allocation x and a production allocation y is called an *allocation.* To each agent i, there corresponds (1) the *set D_i of i-possible allocations* which is a subset of R^{2mn}, (2) his *preference relation* $\succsim_i$ which is a complete, reflexive, and transitive binary relation on D_i, and (3) his *initial resource endowment* ω_i which is a point of R^m. The n-tuple $(D_i, \succsim_i, \omega_i)_{i\in I}$ of the ordered triples $(D_i, \succsim_i, \omega_i)$ is called an *environment.*

The *set of possible allocations* and the *set of attainable allocations* are defined by $D=\bigcap_{i\in I}D_i$ and $A=\{(x, y)\in D\colon \sum_{i\in I}(x_i-y_i-\omega_i)=0\}$, respectively. An attainable allocation a is said to be *Pareto-optimal* if there exists no $a'\in A$ such that $a' \succ_i a$ for every $i\in I$, where $a' \succ_i a$ means that not $a \succsim_i a'$. This definition of Pareto-optimality is slightly broader than the usual one (*cf.* Arrow and Hahn [2, p. 91]).

An environment $(D_i, \succsim_i, \omega_i)_{i\in I}$ is said to be *decomposable* if (1) there exists an n-tuple $(D^i)_{i\in I}$ of subsets of R^{2m} such that $D=\{(x, y)\colon (x_i, y_i)\in D^i \text{ for every } i\in I\}$ and (2) for every $i\in I$ there exists a binary relation $\succsim^i$ on D^i such that $\succsim_i=\{((x, y), (x', y'))\in D\times D\colon (x_i, y_i)\succsim^i(x_i', y_i')\}$. It is easy to see that, for every $i\in I$, $\succsim^i$ is complete, reflexive, and transitive. In case $(D_i, \succsim_i, \omega^i)_{i\in I}$ is a decomposable environment, we shall denote it by $(D^i, \succsim^i, \omega_i)_{i\in I}$. Note that decomposability does not imply that, for each agent, his preference relation is independent of his production activities or his consumption (resp. production) possibility is independent of his production (resp. consumption) activities.

For a decomposable environment $(D^i, \succsim^i, \omega_i)_{i\in I}$, it will be convenient to introduce some other definitions. The *set of trades* is defined by $F=\{z\in R^{mn}\colon \sum_{i\in I}z_i=0\}$. A point z of R^{mn} is called a *redistribution* if $z-\omega\in F$, where $\omega=(\omega_i)_{i\in I}$. The *set of attainable redistributions* is defined by $B=\{z\in R^{mn}\colon z-\omega\in F$ and there exists a production allocation y such that $(z_i+y_i, y_i)\in D^i$ for every $i\in I\}$. For each agent i, his *production set* is defined by $Y_i=\{y_i\in R^m\colon (x_i, y_i)\in D^i \text{ for some } x_i\in R^m\}$. An attainable redistribution z is said to be *Pareto-optimal* if there exists $y\in \prod_{i\in I}Y_i$ such that (1) $(z_i+y_i, y_i)\in D_i$ for every $i\in I$ and (2) there exists no $(z', y')\in B\times \prod_{i\in I}Y_i$ such that $(z_i'+y_i', y_i')\in D^i$ and $(z_i'+y_i', y_i')\succ^i(z_i+y_i, y_i)$ for every $i\in I$. The following proposition is an immediate consequence of the definitions.

PROPOSITION 1. *If* $(D^i, \succsim^i, \omega_i)_{i \in I}$ *is a decomposable environment, then, for every* $z \in B$, z *is a Pareto-optimal redistribution if and only if there exists* $y \in \Pi_{i \in I} Y_i$ *such that* $(z+y, y)$ *is a Pareto-optimal allocation.*

The class of non-decomposable environments to be considered in this paper is assumed to have the following four properties.

ASSUMPTION 1. *For every* $i \in I$, D_i *is closed in* R^{2mn}.

ASSUMPTION 2. *For every* $i \in I$, *the set* $\{a' \in D_i\colon a' \succsim_i a\}$ *is closed in* D_i *for every* $a \in D_i$.

ASSUMPTION 3. *For every* $i \in I$, *every* $a \in D_i$, *and every positive real number* ε, *there exists* $a' \in D_i$ *such that* $a' \succ_i a$ *and* $d(a, a') < \varepsilon$.[2]

ASSUMPTION 4. $(\omega, 0) \in D$.

The class of decomposable environments to be considered is assumed to have the following four properties. We need two more definitions. For each $i \in I$, his *consumption set* is defined by $X_i = \{x_i \in R^m\colon (x_i, y_i) \in D^i$ for some $y_i \in R^m\}$. For each $i \in I$ and each $b \in R^m$, let $Y_i(b) = \{y_i \in Y_i\colon y_i \geqq b\}$, where the inequality should be understood componentwise.

ASSUMPTION 1*. *For every* $i \in I$, (a) D^i *is closed in* R^{2m}, (b) X_i *is bounded from below, and* (c) $Y_i(b)$ *is bounded for every* $b \in R^m$.

Part (a) of Assumption 1* is equivalent to Assumption 1. The following is equivalent to Assumption 2.

ASSUMPTION 2*. *For every* $i \in I$, *the set* $\{a_i' \in D^i\colon a_i' \succsim^i a_i\}$ *is closed in* D^i *for every* $a_i \in D^i$.

Assumption 3 can be rewritten as follows.

ASSUMPTION 3*. *For every* $i \in I$, *every* $a_i \in D^i$, *and every positive real number* ε, *there exists* $a_i' \in D^i$ *such that* $a_i' \succ^i a_i$ *and* $d(a_i, a_i'\} < \varepsilon$.

Assumption 4 can be weakened so as to admit the possibility that some agents may not survive with their initial resource endowments.

ASSUMPTION 4*. *Either* (a) *there exists* $y \in \Pi_{i \in I} Y_i$ *such that* $(\omega_i + y_i, y_i) \in D^i$ *for every* $i \in I$ *or* (b) *there exists* $(x', y') \in A$, $(x'', y'') \in D$, *and* $\delta \in R^n$ *such that, for every* $i \in I$, $d(x_i'' - y_i'', \omega_i) < \delta_i$ *and* $(x_i', y_i') \succsim^i (x_i, y_i)$ *for every* $(x_i, y_i) \in D^i$ *such that* $d(x_i - y_i, \omega_i) < \delta_i$.

[2] Throughout this paper, d stands for the distance function in some Euclidean space R^k, defined by $d(c, c') = \max\{|c_1 - c_1'|, \cdots, |c_k - c_k'|\}$. The dimension k may be either $2mn$, $2m$, or m; in each case, the same symbol d will be used. The only place in which the reader should be careful is the proof of part (b) of Proposition 3, where the distance in R^m and that in R^{2m} appear in the same series of inequalities.

The meaning of this assumption will be more easily seen by rewriting the assumption in a simplified form (*cf.* Proposition 4).

Under our assumptions, the decomposable environment $(D^i, \succsim^i, \omega_i)_{i \in I}$ can be regarded as a "pure-exchange" economy, by defining, for each agent i, his "consumption set" by

$$Z_i = \{z_i \in R^m: (z_i + y_i, y_i) \in D^i \quad \text{for some} \quad y_i \in R^m\}$$

and his "preference relation" on Z_i by

$$\succsim_i^* = \{(z_i, z_i') \in Z_i \times Z_i: \text{ There exists } y_i \in Y_i \text{ such that } (z_i + y_i, y_i) \in D^i \text{ and } (z_i + y_i, y_i) \succsim^i (z_i' + y_i', y_i') \text{ for every } y_i' \in Y_i \text{ such that } (z_i' + y_i', y_i') \in D^i\}.$$

Then clearly $B = \{z \in \Pi_{i \in I} Z_i: z - \omega \in F\}$. We shall call $(Z_i, \succsim_i^*, \omega_i)_{i \in I}$ the *pure-exchange economy induced by* the decomposable environment $(D^i, \succsim^i, \omega_i)_{i \in I}$. This has the following properties.

PROPOSITION 2. *Under Assumption* 1*, *for every* $i \in I$, Z_i *is closed in* R^m.

PROPOSITION 3. *Under Assumptions* 1*, 2*, *and* 3*, *for every* $i \in I$, $\succsim_i^*$ *is a complete, reflexive, and transitive binary relation on* Z_i *such that* (a) *the set* $\{z_i' \in Z_i: z_i' \succsim_i^* z_i\}$ *is closed in* Z_i *for every* $z_i \in Z_i$ *and* (b) *for every* $z_i \in Z_i$ *and every positive real number* ε *there exists* $z_i' \in Z_i$ *such that* $z_i' \succ_i^* z_i$ *and* $d(z_i, z_i') < \varepsilon$.

PROPOSITION 4. *Under Assumption* 4*, *either* (a) $\omega \in \Pi_{i \in I} Z_i$ *or* (b) *there exists* $z' \in B$, $z'' \in \Pi_{i \in I} Z_i$, *and* $\delta \in R^n$ *such that, for every* $i \in I$, $d(z_i'', \omega_i) < \delta_i$ *and* $z_i' \succsim_i^* z_i$ *for every* $z_i \in Z_i$ *such that* $d(z_i, \omega_i) < \delta_i$.

Proposition 4 is a restatement of Assumption 4* for the induced pure-exchange economy, and part (b) of it asserts that there is an attainable redistribution z' which is Pareto-superior to some sufficiently small neighborhood in $\Pi_{i \in I} Z_i$ of the initial endowment ω (*cf.* Fig. 1). This last property enables our resource allocation processes to realize attainable redistributions even if the initial endowment is not a possible redistribution (*cf.* Lemma 8).

In terms of the notation introduced for the induced pure-exchange economy, we can express the conditions for a Pareto-optimal redistribution in a more familiar way.

PROPOSITION 5. *Under Assumption* 1*, *for every* $z^* \in B$, z^* *is a Pareto-optimal redistribution if and only if there exists no* $z \in B$ *such that* $z_i \succ_i^* z_i^*$ *for every* $i \in I$.

Before concluding this section, we note that, among the assumptions made, the only one which has something to do with divisibility of commodities is that of local non-satiation of preference relations, i.e., Assumption 3 or Assumption 3*. Local non-satiation rules out the purely indivisible case, but it is compatible with the indivisibility of all but one commodity.

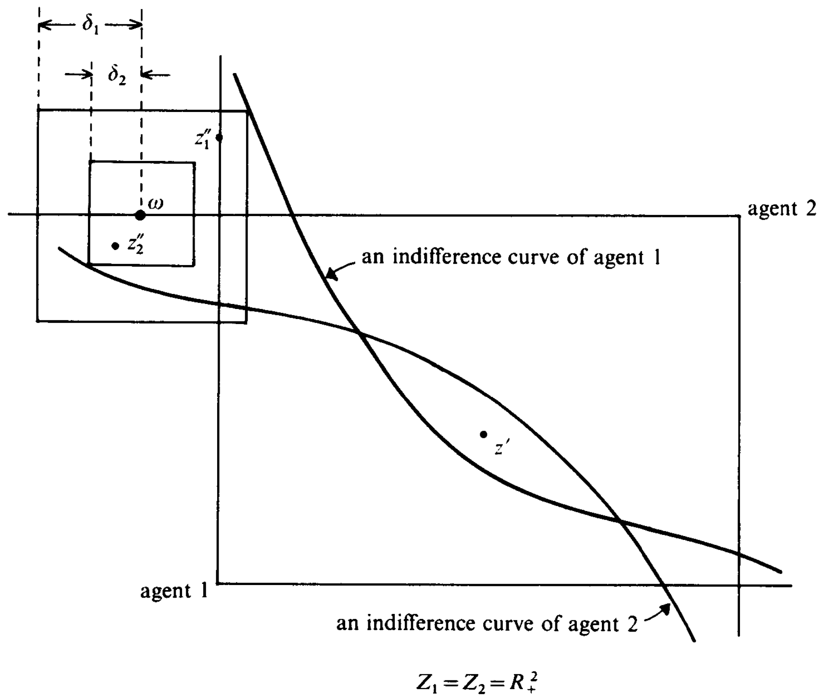

Fig. 1a. An example satisfying Proposition 4.

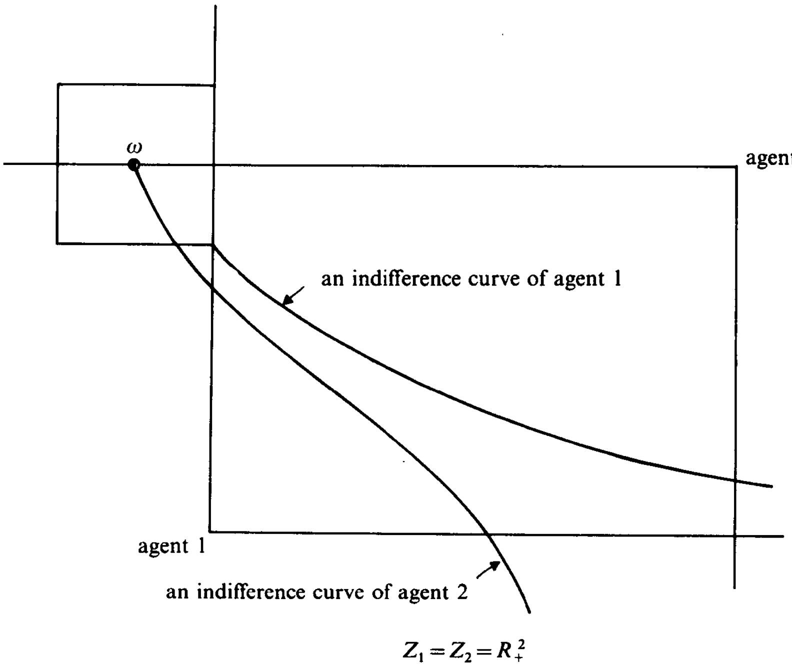

Fig. 1b. An example violating Proposition 4.

3. RESOURCE ALLOCATION PROCESSES

Given two sets L and C, a *non-tatonnement resource allocation process* (or simply, a *process*) with *message space* L and *outcome space* C is the ordered pair (T, h) of a function (called a *response function*) T from $L^n \times C$ to L^n and a function (called an *outcome function*) h from $L^n \times C$ to C. For each $(M, c) \in L^n \times C$, let $P(M, c) = (T(M, c), h(M, c))$. Then P is a function from $L^n \times C$ to itself. A fixed point (M, c) of P is a natural candidate for the equilibrium concepts of the process. For, once the fixed point has been attained, both of the message complex M and the outcome c remain constant. This fixed point (M, c) may be called a *full equilibrium* of the process (T, h), provided c satisfies certain attainability conditions. Furthermore, an attainable outcome c may be called a *full equilibrium outcome* if (M, c) is a full equilibrium for some $M \in L^n$. On the other hand, concentrating our attention on outcomes only, we may also think of the following equilibrium concept. Given an outcome c, we define a function G_c: $L^n \to L^n$ by $G_c(M) = T(M, c)$. Given an outcome c and a positive integer t, we define a function G_c^t: $L^n \to L^n$ by $G_c^t(M) = G_c(G_c^{t-1}(M))$, where $G_c^0(M) = M$. An attainable outcome c is called a *weak equilibrium outcome* if there exists $M \in L^n$ such that $c = h(G_c^{t-1}(M), c)$ for every positive integer t. In this case, if the initial message complex M^0 is chosen equal to M, then $M^t = T(M^{t-1}, c)$ and $c = h(M^{t-1}, c)$ for every positive integer t, so that the outcome c actually remains constant. Clearly, every full equilibrium outcome is a weak equilibrium outcome. The equilibrium concept of Ledyard [10, Definition 5] is similar to that of weak equilibrium, in the sense that messages need not remain constant.

Let

$$L_1 = \{U\colon U \text{ is a non-empty subset of } R^m\},$$

$$C_1 = \{z \in R^{mn}\colon \ z - \omega \in F\}.$$

The elements of L_1 are regarded as sets of individual trades, and the elements of C_1 as redistributions. For the class of decomposable environments satisfying Assumptions 1* through 4*, we shall construct a process with message space L_1 and outcome space C_1.

A message complex $M \in (L_1)^n$ is said to be *consistent* if $F \cap \Pi_{i \in I} M_i$ is non-empty, and M is said to be *inconsistent* if it is not consistent. We shall say that M_i is *ambitious* if $0 \notin M_i$, and that M_i is *unambitious* if $0 \in M_i$. The distance $d(M_i, 0)$ of the message M_i from 0 can be regarded as representing the intensity of agent i's ambition.[3]

For each agent i and each point z_i of R^m, let

$$X_i(z_i) = \{x_i \in R^m\colon \ x_i + z_i \in Z_i\}.$$

[3] Given a subset U of R^m and a point v of R^m, the distance of U from v is defined by $d(U, v) = \inf_{u \in U} d(u, v)$. The same definition will be used for the distance between a subset and a point of R^{2mn}.

This set stands for the individual trades of agent i that are possible for him when his present resource holdings are given by z_i. Given an agent i, a non-negative real number r, and a real number q in the open unit interval]0, 1[, we define a function f_{irq}: $(L_1)^n \times R^m \to R$ and a function T_{irq}: $(L_1)^n \times R^m \to L_1 \cup \{\varnothing\}$ by

$$f_{irq}(M, z_i) = \begin{cases} \max \{d(X_i(z_i), 0), r\} & \text{if } M \text{ is consistent or } M_i \text{ is unambitious,} \\ \max \{(d(X_i(z_i), 0), qd(M_i, 0)\} & \text{otherwise,} \end{cases}$$

$$T_{irq}(M, z_i) = \{x_i^* \in X_i(z_i)\colon \ x_i^* + z_i \succ_i^* x_i + z_i \ \text{ for every } \ x_i \in X_i(z_i) \cap K(0, f_{irq}(M, z_i))\},$$

where $K(0, f_{irq}(M, z_i))$ is the cube with center 0 and edge $2f_{irq}(M, z_i)$, i.e., $K(0, f_{irq}(M, z_i)) = \{x_i \in R^m\colon d(x_i, 0) \leqq f_{irq}(M, z_i)\}$.

Given a non-negative real number r and a number q in]0, 1[, we define a function T_{rq}: $(L_1)^n \times C_1 \to (L_1 \cup \{\varnothing\})^n$ by

$$T_{rq}(M, z) = (T_{irq}(M, z_i))_{i \in I}.$$

Actually, under our assumptions on the environmental properties, T_{rq} will turn out to be a function from $(L_1)^n \times C_1$ to $(L_1)^n$ (*cf.* Lemma 2), so that it can be regarded as the response function of the process being constructed. To simplify the notation, the subscripts r and q will be suppressed in what follows, unless special reference to the numbers r and q is needed.

The set $T_i(M, z_i)$ consists of those individual trades which are preferred to any individual trades in the set $X_i(z_i) \cap K(0, f_i(M, z_i))$. To use the terminology of Hurwicz [6], $T_i(M, z_i)$ may be said to represent the "greed" response to $X_i(z_i) \cap K(0, f_i(M, z_i))$. Note that the latter set is always non-empty and compact since, by Proposition 2, $X_i(z_i)$ is closed in R^m. The function f_i determines how ambitious the agent i may be. He is always permitted to be so ambitious as to exclude from his next proposal $T_i(M, z_i)$ any individual trades which are impossible for him; therefore, the distance $d(T_i(M, z_i), 0)$ of his next proposal from 0 may be at least as large as $d(X_i(z_i), 0)$. On the other hand, if either M is consistent or M_i is unambitious then $d(T_i(M, z_i), 0)$ may be at least as large as the fixed number r; while if M is inconsistent and M_i is ambitious then $d(T_i(M, z_i), 0)$ is required to be a fraction of the distance $d(M_i, 0)$ of the previous proposal M_i from 0 (*cf.* Lemma 2 and Fig. 2). In this way, the function f_i encourages agent i if the previous message complex is consistent or his previous proposal is unamibitious; while it discourages him otherwise. The encouragement effect ($r > 0$) helps the process approach Pareto-optimal redistributions (*cf.* Theorem 3 and its proof (Proof of Theorem 6)), and the discouragement effect ($0 < q < 1$) helps the process find Pareto-superior redistributions (*cf.* Lemma 6).

Given a non-negative real number r and a number q in]0, 1[, we define a correspondence H_{rq}: $(L_1)^n \times C_1 \to C_1$ by

$$H_{rq}(M, z) = \begin{cases} (F \cap \Pi_{i \in I} T_{irq}(M, z_i)) + \{z\} & \text{if } T_{rq}(M, z) \text{ is consistent,} \\ \{z\} & \text{otherwise.} \end{cases}$$

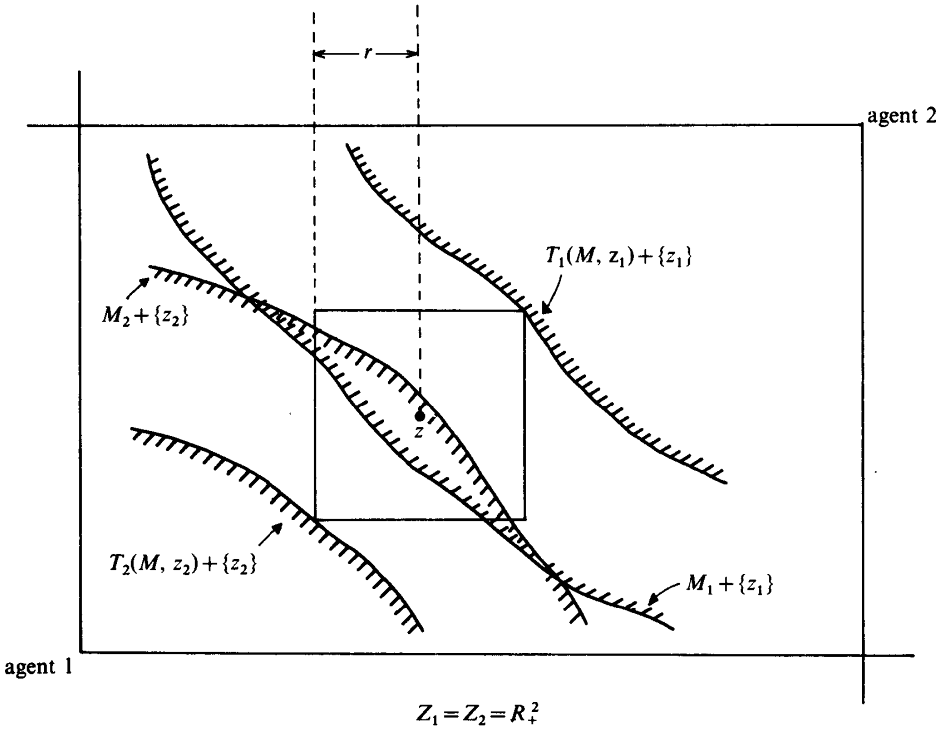

Fig. 2a. M is consistent and $z=h(M, z)$.

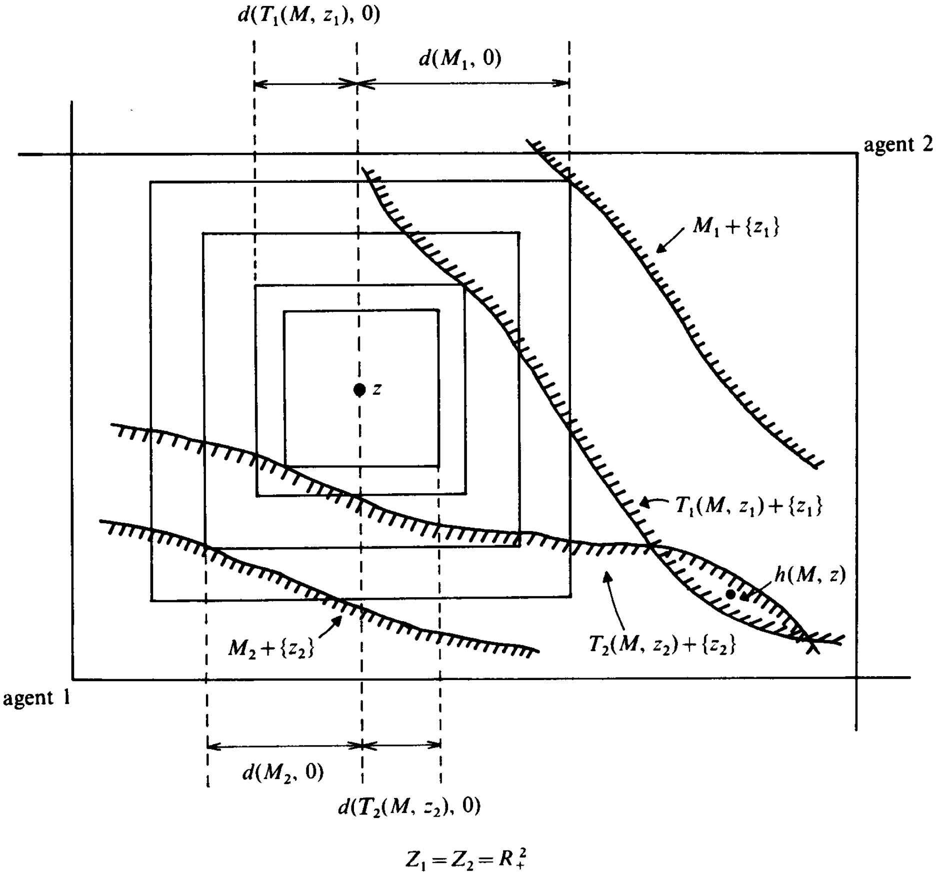

Fig. 2b. M is inconsistent and $T(M, z)$ is consistent.

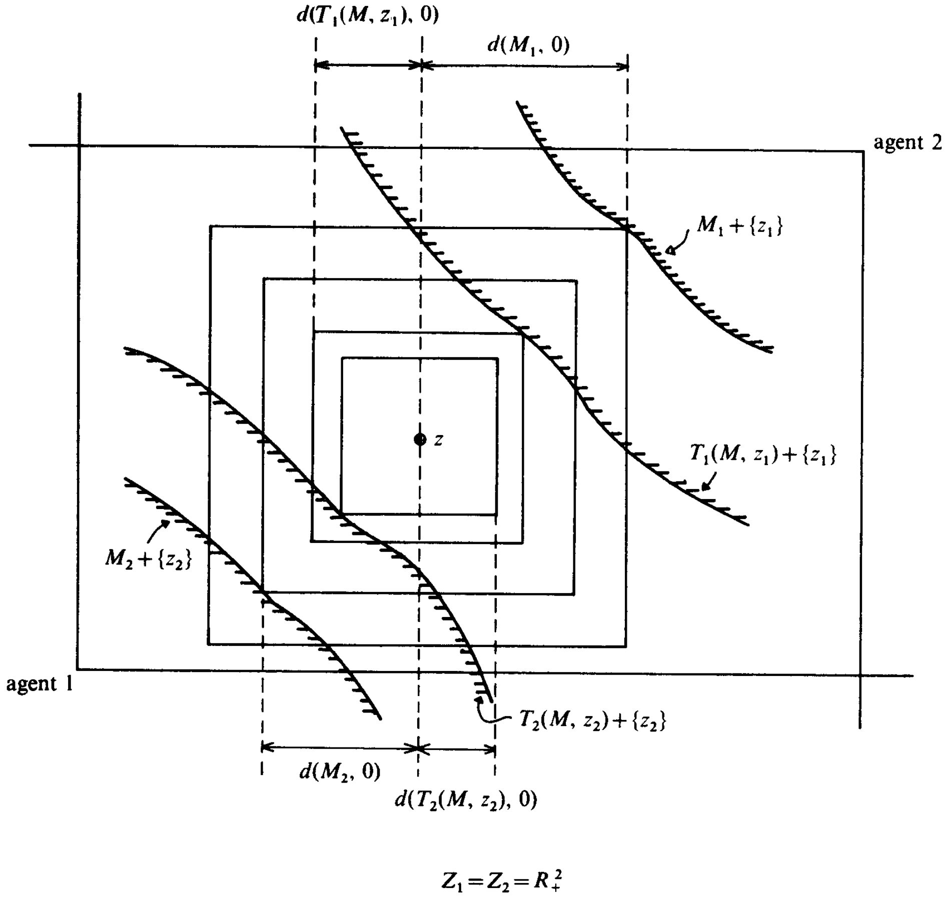

Fig. 2c. M and $T(M, z)$ are inconsistent.

Since any (not necessarily continuous) selection h of H_{rq} can be regarded as an outcome function of the process being constructed, we may call H_{rq} the *outcome correspondence*. Take any selection h of H_{rq}. According to this outcome function, no transactions are performed when the message complex $T(M, z)$ is inconsistent, while some Pareto-superior redistribution is selected from the set $(F \cap \Pi_{i \in I} T_i(M, z_i)) + \{z\}$ when $T(M, z)$ is consistent. Hence the sequences of redistributions generated by the process are non-decreasing with respect to the preference relation of each agent.

Let

$P_1(L_1, C_1) = \{(T, h)$: There exists a positive real number r and a number q in $]0, 1[$ such that $T = T_{rq}$ and $h(M, z) \in H_{rq}(M, z)$ for every $(M, z) \in (L_1)^n \times C_1\}$,

$P_1^*(L_1, C_1) = \{(T, h)$: There exists a non-negative real number r and a number q in $]0, 1[$ such that $T = T_{rq}$ and $h(M, z) \in H_{rq}(M, z)$ for every $(M, z) \in (L_1)^n \times C_1\}$.

Then $P_1(L_1, C_1) \subset P_1^*(L_1, C_1)$, and any process (T, h) in $P_1^*(L_1, C_1)$ is a non-tatonnement resource allocation process. Since, for every agent i, his response function T_i is independent of the environmental characteristics $(D^j, \succsim^j, \omega_j)_{j \in I-\{i\}}$ of the other agents, the response function T is privacy-preserving. T is also anonymous in the sense that, for every agent i, T_i is invariant under any permutation of the other agents.

The *set of attainable outcomes* of any process in $P_1^*(L_1, C_1)$ will always be chosen equal to B, the set of attainable redistributions.

THEOREM 1 (Non-wastefulness). *Under Assumptions* 1*, 2*, *and* 3*, *for every process* (T, h) *in* $P_1^*(L_1, C_1)$, *every weak equilibrium outcome of* (T, h) *is a Pareto-optimal redistribution.*

THEOREM 2 (Unbiasedness). *Under Assumptions* 1*, 2*, *and* 3*, *for every process* (T, h) *in* $P_1^*(L_1, C_1)$, *every Pareto-optimal redistribution is a full equilibrium outcome of* (T, h).

It follows that, for any process in $P_1^*(L_1, C_1)$, the weak equilibrium outocmes and the full equilibrium outcomes happen to coincide with each other. Note that the "encouragement" effect of the process is irrelevant to these two static properties.

THEOREM 3. *Suppose that Assumptions* 1*, 2*, 3*, *and* 4* *hold. For every process* (T, h) *in* $P_1(L_1, C_1)$ *and every sequence* $\{z^t\}_{t=0}^{\infty}$ *in* R^{mn}, *if* $z^0 = \omega$ *and there exists a sequence* $\{M^t\}_{t=0}^{\infty}$ *in* $(L_1)^n$ *such that* $M^t = T(M^{t-1}, z^{t-1})$ *and* $z^t = h(M^{t-1}, z^{t-1})$ *for every positive integer* t, *then every cluster point of* $\{z^t\}_{t=0}^{\infty}$ *is a Pareto-optimal redistribution.*

This theorem asserts that if the sequence of redistributions converges to some redistribution then the limit redistribution is necessarily Pareto-optimal. However, the sequence is not necessarily convergent. Nor is it obvious, under our assumptions on the environmental properties, that the sequence has a cluster point, though this will be guaranteed if the set B of attainable redistributions is assumed to be compact.

COROLLARY TO THEOREM 3 (Convergence to a Pareto-optimal utility allocation). *Suppose, in addition to Assumptions* 1*, 2*, 3*, *and* 4*, *that* B *is compact and, for each* $i \in I$, *the preference relation* $\succsim_i^*$ *can be represented by a utility function* u_i^*. *Let* $\{z^t\}_{t=0}^{\infty}$ *be as in Theorem* 3. *For each non-negative integer* t, *let* $u^t = (u_i^*(z_i^t))_{i \in I}$. *Then the sequence* $\{u^t\}_{t=0}^{\infty}$ *in* R^n *is convergent. If, furthermore,* u_i^* *is lower semi-continuous for every* $i \in I$, *then* $\lim_{t \to \infty} u^t = (u_i^*(z_i^*))_{i \in I}$ *for some Pareto-optimal redistribution* z^*.

The B process of Hurwicz, Radner, and Reiter [7] has this kind of stability properties (*cf.* [7, Theorem 5.2]). Theorem 3 is also related to the "global value stability" of Ledyard [10]. Kanemitsu [9] obtains convergence in the allocation space, by specifying an appropriate transaction rule.

We can construct similar processes for the class of non-decomposable environments satisfying Assumptions 1, 2, 3, and 4. Let

$$L_2=\{U: \quad U \text{ is a non-empty subset of } R^{2mn}\},$$
$$C_2=R^{2mn}.$$

The elements of the message space L_2 are regarded as sets of allocations, and the elements of the outcome space C_2 as allocations. Non-decomposability requires the message space L_2 much larger than L_1. A message complex M is said to be *consistent* if $A \cap (\bigcap_{i \in I} M_i)$ is non-empty, and M is said to be *inconsistent* if it is not consistent. We shall say that M_i is *ambitious at a* if $a \notin M_i$ and that M_i is *unambitious at a* if $a \in M_i$.

Given an agent i, a non-negative real number r, and a number q in $]0, 1[$, we define a function f^*_{irq}: $(L_2)^n \times C_2 \to R$ and a function T^*_{irq}: $(L_2)^n \times C_2 \to L_2 \cup \{\emptyset\}$ by

$$f^*_{irq}(M, a)=\begin{cases}\max\{d(D_i, a), r\} & \text{if } M \text{ is consistent or } M_i \text{ is unambitious at } a,\\ \max\{d(D_i, a), qd(M_i, a)\} & \text{otherwise,}\end{cases}$$

$$T^*_{irq}(M, a)=\{a^* \in D_i: \quad a^* \succ_i a' \quad \text{for every} \quad a' \in D_i \cap K^*(a, f^*_{irq}(M, a))\},$$

where $K^*(a, f^*_{irq}(M, a))$ is the cube with center a and edge $2f^*_{irq}(M, a)$, i.e., $K^*(a, f^*_{irq}(M, a))=\{a' \in R^{2mn}: d(a', a) \leqq f^*_{irq}(M, a)\}$.

Given a non-negative real number r and a number q in $]0, 1[$, we define a function T^*_{rq}: $(L_2)^n \times C_2 \to (L_2 \cup \{\emptyset\})^n$ by

$$T^*_{rq}(M, a)=(T^*_{irq}(M, a))_{i \in I}.$$

Actually, T^*_{rq} is a function from $(L_2)^n \times C_2$ to $(L_2)^n$ (*cf.* Lemma 3), so that it may be regarded as a response function of the process being constructed. This response function can be interpreted in the same way as before. Given a non-negative real number r and a number q in $]0, 1[$, we define an outcome correspondence H^*_{rq}: $(L_2)^n \times C_2 \to C_2$ by

$$H^*_{rq}(M, a)=\begin{cases}A \cap (\bigcap_{i \in I} T^*_{irq}(M, a)) & \text{if } T^*_{rq}(M, a) \text{ is consistent,}\\ \{a\} & \text{otherwise.}\end{cases}$$

Let

$P_2(L_2, C_2)=\{(T, h)$: There exists a positive real number r and a number q in $]0, 1[$ such that $T=T^*_{rq}$ and $h(M, a) \in H^*_{rq}(M, a)$ for every $(M, a) \in (L_2)^n \times C_2\}$,

$P^*_2(L_2, C_2)=\{(T, h)$: There exists a non-negative real number r and a number q in $]0, 1[$ such that $T=T^*_{rq}$ and $h(M, a) \in H^*_{rq}(M, a)$ for every $(M, a) \in (L_2)^n \times C_2\}$.

Then $P_2(L_2, C_2) \subset P^*_2(L_2, C_2)$, and any process (T, h) in $P^*_2(L_2, C_2)$ is a non-tatonnement resource allocation process. For every agent i, T_i is independent of $(D_j, \succsim_j)_{j \in I-\{i\}}$ but depends on consumptions and productions of other agents. At each stage of the process, all agents are assumed to be informed of the current allocation. In particular, they have to know the initial allocation at the first stage of the process, which usually implies that each agent has to know the initial resource endowments of other agents. In this sense, the response function T is not privacy-preserving. It may be emphasized again, however, that the preference fields (D_j, $\succsim_j$) need not be communicated. These are much more difficult to communicate than the initial resource endowments. The response function T is anonymous, so that the process (T, h) may still be qualified as an informationally decentralized process in a very weak sense.

The *set of attainable outcomes* of any process in $P_2^*(L_2, C_2)$ will always be chosen equal to A, the set of attainable allocations.

THEOREM 4 (Non-wastefulness). *Under Assumptions* 1, 2, *and* 3, *for every process* (T, h) *in* $P_2^*(L_2, C_2)$, *every weak equilibrium outcome of* (T, h) *is a Pareto-optimal allocation.*

THEOREM 5 (Unbiasedness). *Under Assumptions* 1, 2, *and* 3, *for every process* (T, h) *in* $P_2^*(L_2, C_2)$, *every Pareto-optimal allocation is a full equilibrium outcome of* (T, h).

THEOREM 6. *Suppose that Assumptions* 1, 2, 3, *and* 4 *hold. For every process* (T, h) *in* $P_2(L_2, C_2)$ *and every sequence* $\{a^t\}_{t=0}^{\infty}$ *in* R^{2mn}, *if* $a^0=(\omega, 0)$ *and there exists a sequence* $\{M^t\}_{t=0}^{\infty}$ *in* $(L_2)^n$ *such that* $M^t=T(M^{t-1}, a^{t-1})$ *and* $a^t=h(M^{t-1}, a^{t-1})$ *for every positive integer* t, *then every cluster point of* $\{a^t\}_{t=0}^{\infty}$ *is a Pareto-optimal allocation.*

COROLLARY TO THEOREM 6 (Convergence to a Pareto-optimal utility allocation). *Suppose, in addition to Assumptions* 1, 2, 3, *and* 4, *that* A *is compact and, for each* $i\in I$, *the preference relation* $\succsim_i$ *can be represented by a utility function* u_i. *Let* $\{a^t\}_{t=0}^{\infty}$ *be as in Theorem* 6. *For each non-negative integer* t, *let* $u^t=(u_i(a^t))_{i\in I}$. *Then the sequence* $\{u^t\}_{t=0}^{\infty}$ *in* R^n *is convergent. If, furthermore,* u_i *is lower semi-continuous for every* $i\in I$, *then* $\lim_{t\to\infty}u^t=(u_i(a^*))_{i\in I}$ *for some Pareto-optimal allocation* a^*.

APPENDIX: PROOF OF THE RESULTS

Proof of Proposition 1. Straightforward.

Proof of Proposition 2. Let $\{z_i^\nu\}_{\nu=1}^{\infty}$ be any sequence in Z_i converging to some $z_i\in R^m$. For each positive integer ν there is $y_i^\nu\in R^m$ such that $(z_i^\nu+y_i^\nu, y_i^\nu)\in D^i$. Since, by Assumption 1(b), X_i is bounded from below, we may assume, without loss of generality, that there is $b\in R^m$ such that $y_i^\nu\geqq b$, i.e., $y_i^\nu\in Y_i(b)$ for every ν. By Assumption 1(c), $Y_i(b)$ is bounded, so that we may assume, without loss of generality, that $\{y_i^\nu\}_{\nu=1}^{\infty}$ converges to some $y_i\in R^m$. Since, by Assumption 1(a), D^i is closed in R^{2m}, it follows that $(z_i+y_i, y_i)\in D^i$ so that $z_i\in Z_i$.

For each $i\in I$ and each $z_i\in Z_i$, define $Y_i(z_i)=\{y_i\in R^m\colon (z_i+y_i, y_i)\in D^i\}$.

LEMMA 1. *Under Assumptions* 1* *and* 2*, *for every* $i \in I$ *and every* $z_i \in Z_i$, *there is* $y_i \in Y_i'(z_i)$ *such that* $(z_i+y_i, y_i) \succsim^i (z_i+y_i', y_i')$ *for every* $y_i' \in Y_i'(z_i)$.

Proof. By the definition of Z_i, $Y_i'(z_i)$ is non-empty. By Assumption 1(b), X_i is bounded from below, so that $Y_i'(z_i)$ is also bounded from below. Hence there is $b \in R^m$ such that $y_i \geqq b$ for every $y_i \in Y_i'(z_i)$, i.e., $Y_i'(z_i) \subset Y_i(b)$ so that, by Assumption 1(c), $Y_i'(z_i)$ is bounded. It is clearly closed in R^m since, by Assumption 1(a), D^i is closed in R^{2m}. Thus $Y_i'(z_i)$ is non-empty and compact.

Define a binary relation Q_i on $Y_i'(z_i)$ by $Q_i = \{(y_i, y_i') \in Y_i'(z_i) \times Y_i'(z_i)$: $(z_i+y_i, y_i) \succsim^i (z_i+y_i', y_i')\}$. Clearly, Q_i is complete, reflexive, and transitive, and the set $\{y_i' \in Y_i'(z_i)$: $y_i' Q_i y_i\}$ is closed in $Y_i'(z_i)$ for every $y_i \in Y_i'(z_i)$. Since $Y_i'(z_i)$ is non-empty and compact, it follows that there is $y_i \in Y_i'(z_i)$ such that $y_i Q_i y_i'$, i.e., $(z_i+y_i, y_i) \succsim^i (z_i+y_i', y_i')$ for every $y_i' \in Y_i'(z_i)$.[4]

Proof of Proposition 3. Suppose that $z_i, z_i' \in Z_i$ but not $z_i \succsim_i^* z_i'$. Let $y_i \in Y_i'(z_i)$. Then $(z_i'+y_i', y_i') \succ^i (z_i+y_i, y_i)$ for some $y_i' \in Y_i'(z_i')$. By Lemma 1, $(z_i'+y_i'', y_i'') \succsim^i (z_i'+y_i', y_i')$ for some $y_i'' \in Y_i'(z_i')$, so that $(z_i'+y_i'', y_i'') \succsim^i (z_i+y_i, y_i)$. Since y_i is arbitrary in $Y_i'(z_i)$, this implies that $z_i' \succsim_i^* z_i$. Thus $\succsim_i^*$ is complete and reflexive. Transitivity is obvious.

Let $z_i \in Z_i$ and let $\{z_i^\nu\}_{\nu=1}^\infty$ be any sequence in the set $\{z_i' \in Z_i$: $z_i' \succsim_i^* z_i\}$ converging to some $z_i^0 \in Z_i$. For each positive integer ν there is $y_i^\nu \in Y_i'(z_i^\nu)$ such that $(z_i^\nu+y_i^\nu, y_i^\nu) \succsim^i (z_i+y_i, y_i)$ for every $y_i \in Y_i'(z_i)$. Since $\{z_i^\nu\}_{\nu=1}^\infty$ converges to z_i^0 and $(z_i^\nu+y_i^\nu, y_i^\nu) \in D^i$ for every ν, we may assume, as in the proof of Proposition 2, that the sequence $\{y_i^\nu\}_{\nu=1}^\infty$ converges to some $y_i^0 \in R^m$. By Assumption 1*, $y_i^0 \in Y_i'(z_i^0)$. Let $y_i \in Y_i'(z_i)$. Then $(z_i^\nu+y_i^\nu, y_i^\nu) \succsim^i (z_i+y_i\ y_i)$ for every ν, so that, by Assumption 2*, $(z_i^0+y_i^0, y_i^0) \succsim^i (z_i+y_i, y_i)$. Since y_i is arbitrary in $Y_i'(z_i)$, this implies that $z_i^0 \succsim_i^* z_i$, i.e., $z_i^0 \in \{z_i' \in Z_i$: $z_i' \succsim_i^* z_i\}$. Thus, part (a) of the proposition is proved.

Let $z_i \in Z_i$ and $\varepsilon > 0$. By Lemma 1, there is $y_i \in Y_i'(z_i)$ such that

$$(1) \qquad (z_i+y_i, y_i) \succsim^i (z_1+y_i', y_i') \qquad \text{for every} \quad y_i' \in Y_i'(z_i)\,.$$

By Assumption 3*, for each positive integer ν there is $(x_i^\nu, y_i^\nu) \in D^i$ such that $(x_i^\nu, y_i^\nu) \succ^i (z_i+y_i, y_i)$ and $d((x_i^\nu, y_i^\nu), (z_i+y_i, y_i)) < 1/\nu$. There is a positive integer ν such that $1/\nu < \varepsilon/2$. Let $z_i' = x_i^\nu - y_i^\nu$. Then $(z_i'+y_i^\nu, y_i^\nu) \in D^i$ so that $y_i^\nu \in Y_i'(z_i')$ and $z_i' \in Z_i$. Furthermore, $d(z_i', z_i) \leqq d(x_i^\nu - y_i^\nu, z_i+y_i-y_i^\nu) + d(z_i+y_i-y_i^\nu, z_i) = d(x_i^\nu, z_i+y_i) + d(y_i^\nu, y_i) \leqq d((x_i^\nu, y_i^\nu), (z_i+y_i, y_i)) + d((x_i^\nu, y_i^\nu), (z_i+y_i, y_i)) < \varepsilon$. Suppose $z_i \succsim_i^* z_i'$. Then there would be $y_i' \in Y_i'(z_i)$ such that $(z_i+y_i', y_i') \succsim^i (z_i'+y_i'', y_i'')$ for every $y_i'' \in Y_i'(z_i')$, so that $(z_i+y_i', y_i') \succsim^i (z_i'+y_i^\nu, y_i^\nu)$. Since $(x_i^\nu, y_i^\nu) \succ^i (z_i+y_i, y_i)$ and $x_i^\nu = z_i'+y_i^\nu$, it follows that $(z_i+y_i', y_i') \succ^i (z_i+y_i, y_i)$, contradicting (1). Thus $z_i' \succ_i^* z_i$ and part (b) of the proposition is proved.

[4] The assertion follows from the proposition that a non-empty compact space endowed with a reflexive, transitive binary relation has a maximal element if the upper contour sets are closed. In the present context, the relation is complete, so that the maximal element is a greatest element. See Ward [13, Theorem 1].

Proof of Propostion 4. Straightforward.

Proof of Proposition 5. Necessity is obvious. To prove sufficiency, note that, by Lemma 1, for each $i \in I$ there is $y_i^* \in Y_i'(z_i^*)$ such that $(z_i^* + y_i^*, y_i^*) \gtrsim^i (z_i^* + y_i', y_i')$ for every $y_i' \in Y_i'(z_i^*)$. If z^* were not a Pareto-optimal redistribution, then there would be $(z, y) \in B \times \Pi_{i \in I} Y_i$ such that $y_i \in Y_i'(z_i)$ and $(z_i + y_i, y_i) \succ^i (z_i^* + y_i^*, y_i^*)$ for every $i \in I$, so that, for every $i \in I$, $(z_i + y_i, y_i) \succ^i (z_i^* + y_i', y_i')$ for every $y_i' \in Y_i'(z_i^*)$, i.e., $z_i \succ_i^* z_i^*$, a contradiction.

LEMMA 2. *Under Assumptions* 1*, 2*, *and* 3*, *if* (T, h) *is a process in* $P_2^*(L_1, C_1)$ *then, for every* $i \in I$, *every* $M \in (L_1)^n$, *and every* $z_i \in R^m$, $T_i(M, z_i)$ *is non-empty and* $d(T_i(M, z_i), 0) = f_i(M, z_i)$.

Proof. Similar to the proof of Lemma 3 below.

LEMMA 3. *Under Assumptions* 1, 2, *and* 3, *if* (T, h) *is a process in* $P_2^*(L_2, C_2)$ *then, for every* $i \in I$, *every* $M \in (L_2)^n$, *and every* $a \in R^{2mn}$, $T_i(M, a)$ *is non-empty and* $d(T_i(M, a), a) = f_i^*(M, a)$.

Proof. Since, by Assumption 1, D_i is closed in R^{2mn}, $D_i \cap K^*(a, f_i^*(M, a))$ is non-empty and compact, so that, by Assumption 2, there is $a^* \in D_i \cap K^*(a, f_i^*(M, a))$ such that $a^* \gtrsim_i a'$ for every $a' \in D_i \cap K^*(a, f_i^*(M, a))$. By Assumption 3, $a^{**} \succ_i a^*$ for some $a^{**} \in D_i$; therefore $a^{**} \in T_i(M, a)$. Thus $T_i(M, a)$ is non-empty. Note that $T_i(M, a) = \{a' \in D_i\colon a' \succ_i a^*\}$. Let $\varepsilon > 0$. By Assumption 3, there is $a' \in D_i$ such that $a' \succ_i a^*$ and $d(a', a^*) < \varepsilon$. Hence $d(T_i(M, a), a) \leqq d(a', a) \leqq d(a', a^*) + d(a^*, a) < \varepsilon + f_i^*(M, a)$. Since ε can be arbitrarily small, $d(T_i(M, a), a) \leqq f_i^*(M, a)$. Suppose $d(T_i(M, a), a) < f_i^*(M, a)$. Then $d(a', a) < f_i^*(M, a)$ for some $a' \in T_i(M, a)$. By Assumption 3, there is $a'' \in D_i$ such that $a'' \succ_i a'$ and $d(a', a'') < f_i^*(M, a) - d(a', a)$. Hence $d(a'', a) \leqq d(a'', a') + d(a', a) < f_i^*(M, a)$ so that $a'' \in D_i \cap K^*(a, f_i^*(M, a))$, implying that $a' \succ_i a''$, a contradiction.

LEMMA 4. *Under Assumptions* 1*, 2*, *and* 3*, *for every* $(T, h) \in P_i^*(L_1, C_1)$ *and every sequence* $\{M^t\}_{t=1}^{\infty}$ *in* $(L_1)^n$, *if there exists a sequence* $\{z^t\}_{t=1}^{\infty}$ *in* $\Pi_{i \in I} Z_i$ *such that* $M^{t+1} = T(M^t, z^t)$ *and* $z^{t+1} = h(M^t, z^t)$ *for every positive integer* t, *and if there exists a positive integer* t^* *such that* M^{t^*+t} *is inconsistent for every positive integer* t, *then* $\lim_{t \to \infty} d(M_i^t, 0) = 0$ *for every* $i \in I$.

Proof. Similar to the proof of Lemma 5 below.

LEMMA 5. *Under Assumptions* 1, 2, *and* 3, *for every* $(T, h) \in P_2^*(L_2, C_2)$, *every sequence* $\{(M^t, a^t)\}_{t=1}^{\infty}$ *in* $(L_2)^n \times D$, *and every positive integer* t^*, *if* $M^{t+1} = T(M^t, a^t)$, $a^{t+1} = h(M^t, a^t)$, *and* M^{t^*+t} *is inconsistent for every positive integer* t, *then* $\lim_{t \to \infty} d(M_i^t, a^{t^*}) = 0$ *for every* $i \in I$.

Proof. Note that $a^{t^*+t} = a^{t^*}$ for every positive integer t. If $a^{t^*} \notin M_i^{t^*+t}$ for every positive integer t, then, by Lemma 3, $d(M_i^{t^*+t+1}, a^{t^*}) = d(T_i(M^{t^*+t}, a^{t^*}), a^{t^*}) = f_i^*(M^{t^*+t}, a^{t^*}) = qd(M_i^{t^*+t}, a^{t^*}) = \cdots = q^{t+1} d(M_i^{t^*}, a^{t^*})$ for every positive integer

t, so that $\lim_{t\to\infty} d(M_i^t, a^{t^*}) = \lim_{t\to\infty} d(M_i^{t^*+t+1}, a^{t^*}) = \lim_{t\to\infty} q^{t+1} d(M_i^{t^*}, a^{t^*}) = 0$. It remains to consider the case in which $a^{t^*} \in M_i^{t^*+t'}$ for some positive integer t'. If $r > 0$ then $a^{t^*} \notin M_i^{t^*+t'+t}$ for every positive integer t so that the argument for the previous case applies; while if $r = 0$ then $d(M_i^{t+1}, a^{t^*}) = f_i^*(M^t, a^{t^*}) = 0$ for every $t \geqq t^* + t'$ so that $\lim_{t\to\infty} d(M_i^t, a^{t^*}) = 0$.

LEMMA 6. *Under Assumptions* 1*, 2*, *and* 3*, *for every* $(T, h) \in P_1^*(L_1, C_1)$ *and every sequence* $\{M^t\}_{t=1}^{\infty}$ *in* $(L_1)^n$, *if there exists a sequence* $\{z^t\}_{t=1}^{\infty}$ *in* B *such that* $M^{t+1} = T(M^t, z^t)$ *and* $z^{t+1} = h(M^t, z^t)$ *for every positive integer* t, *then, for every positive integer* t^* *such that* z^{t^*} *is not a Pareto-optimal redistribution, there exists a positive integer* t' *such that* $t' \geqq t^*$ *and* $M^{t'}$ *is consistent.*

Proof. Similar to the proof of Lemma 7 below.

LEMMA 7. *Under Assumptions* 1, 2, *and* 3, *for every* $(T, h) \in P_2^*(L_2, C_2)$ *and every sequence* $\{M^t\}_{t=1}^{\infty}$ *in* $(L_2)^n$, *if there exists a sequence* $\{a^t\}_{t=1}^{\infty}$ *in* A *such that* $M^{t+1} = T(M^t, a^t)$ *and* $a^{t+1} = h(M^t, a^t)$ *for every positive integer* t, *then, for every positive integer* t^* *such that* a^{t^*} *is not a Pareto-optimal allocation, there exists a positive integer* t' *such that* $t' \geqq t^*$ *and* $M^{t'}$ *is consistent.*

Proof. Suppose M^t is inconsistent for every $t \geqq t^*$. Then $a^t = a^{t^*}$ for every $t \geqq t^*$. Since a^{t^*} is not Pareto-optimal, there is $a \in A$ such that $a \succ_i a^{t^*}$ for every $i \in I$. By Assumption 2, there is $\varepsilon > 0$ such that, for every $i \in I$, $a \succ_i a'$ for every $a' \in D_i$ such that $d(a', a^{t^*}) < \varepsilon$ (the number ε can be chosen so that $\varepsilon < r$ if $r > 0$). By Lemma 5, there is $t' > t^*$ such that $d(M_i^t, a^{t^*}) < \varepsilon$ for every $i \in I$ and every $t \geqq t' - 1$. Suppose $a \notin T_i(M^{t'-1}, a^{t^*})$ for some $i \in I$. Then $a' \succsim_i a$ for some $a' \in D_i \cap K^*(a^{t^*}, f_i^*(M^{t'-1}, a^{t^*}))$. If $a^{t^*} \notin M_i^{t'-1}$ then $d(a', a^{t^*}) \leqq f_i^*(M^{t'-1}, a^{t^*}) = qd(M_i^{t'-1}, a^{t^*}) \leqq d(M_i^{t'-1}, a^{t^*}) < \varepsilon$ so that $a \succ_i a'$, a contradiction. Hence $a^{t^*} \in M_i^{t'-1}$. If $r > 0$ then $\varepsilon < r = f_i^*(M^{t'-1}, a^{t^*}) = d(M_i^{t'}, a^{t^*}) < \varepsilon$, a contradiction; while if $r = 0$ then $d(a', a^{t^*}) \leqq f_i^*(M^{t'-1}, a^{t^*}) = r = 0$, i.e., $a' = a^{t^*}$ so that $a^{t^*} \succsim_i a$, a contradiction. Thus $a \in T_i(M^{t'-1}, a^{t^*}) = M_i^{t'}$ for every $i \in I$, so that $M^{t'}$ is consistent, a contradiction.

Proof of Theorem 1. Similar to the proof of Theorem 4 below.

Proof of Theorem 4. Let a be a weak equilibrium outcome. Then there is $M \in (L_2)^n$ such that $a = h(G_a^{t-1}(M), a)$ for every positive integert t. For each t, let $M^t = T(M^{t-1}, a)$ with $M^0 = M$. Then $a = h(M^{t-1}, a)$ for every t, so that M^t is inconsistent for every t; therefore, by Lemma 5, a is a Pareto-optimal allocation.

Proof of Theorem 2. Similar to the proof of Theorem 5 below.

Proof of Theorem 5. Let a be a Pareto-optimal allocation. For each $i \in I$, let $M_i = \{a' \in D_i : a' \succ_i a\}$. Let $i \in I$. Then M_i is ambitious at a and, by Assumption 3, $d(M_i, a) = 0$, so that $f_i^*(M, a) = qd(M_i, a) = 0$. Let $a' \in M_i$. If $a'' \in D_i \cap K^*(a, f_i^*(M, a))$ then $d(a'', a) \leqq f_i^*(M, a) = 0$, i.e., $a'' = a$ so that $a' \succ_i a = a''$; therefore $a' \in T_i(M, a)$. Hence $M_i \subset T_i(M, a)$. If $a' \in T_i(M, a)$ then $a' \in D_i$ and

$a' \succ_i a''$ for every $a'' \in D_i \cap K^*(a, f_i^*(M, a))$ so that $a' \succ_i a$; therefore $a' \in M_i$. Hence $T_i(M, a) \subset M_i$. Thus $M_i = T_i(M, a)$. Since i is arbitrary in I, $M = T(M, a)$. Hence $T(M, a)$ is inconsistent, so that $a = h(M, a)$.

LEMMA 8. *Under Asssumptions* 1*, 2*, 3*, *and* 4*, *for every* $(T, h) \in P_1(L_1, C_1)$ *and every sequence* $\{z^t\}_{t=0}^{\infty}$ *in* R^{mn}, *if* $z^0 = \omega$ *and there exists a sequence* $\{M^t\}_{t=0}^{\infty}$ *in* $(L_1)^n$ *such that* $M^t = T(M^{t-1}, z^{t-1})$ *and* $z^t = h(M^{t-1}, z^{t-1})$ *for every positive integer t, then there exists a positive integer t such that* $z^t \in B$.

Proof. If $\omega \in \Pi_{i \in I} Z_i$ then the lemma is trivial. We shall, therefore, assume that $\omega \notin \Pi_{i \in I} Z_i$. Suppose $z^t = z^0 = \omega$ for every positive integer t. Then M^t is inconsistent for every t. Let $i \in I$. If $0 \notin M_i^t$ for every t then

$$\begin{aligned} d(M_i^t, 0) = f_i(M^{t-1}, \omega_i) &= \max\{d(X_i(\omega_i), 0), qd(M_i^{t-1}, 0)\} \\ &= \max\{d(X_i(\omega_i), 0), q \max\{d(X_i(\omega_i), 0), qd(M_i^{t-2}, 0)\}\} \\ &= \max\{d(X_i(\omega_i), 0), q^2 d(M_i^{t-2}, 0)\} = \cdots \\ &= \max\{d(X_i(\omega_i), 0), q^t d(M_i^0, 0)\} \end{aligned}$$

for every $t > 0$, so that

$$\lim_{t \to \infty} d(M_i^t, 0) = d(X_i(\omega_i), 0);$$

while if $0 \in M_i^{t'}$ for some $t' \geqq 0$ then

$$d(M_i^{t'+1}, 0) = f_i(M^{t'}, \omega_i) = \max\{d(X_i(\omega_i), 0), r\} > 0$$

so that

$$\begin{aligned} d(M_i^{t'+t}, 0) = f_i(M^{t'+t-1}, \omega_i) &= \max\{d(X_i(\omega_i), 0), qd(M_i^{t'+t-1}, 0)\} \\ &= \max\{d(X_i(\omega_i), 0), q^{t-1} d(M_i^{t'+1}, 0)\} \end{aligned}$$

for every $t > 0$; therefore $\lim_{t \to \infty} d(M_i^t, 0) = d(X_i(\omega_i), 0)$. Thus $\lim_{t \to \infty} d(M_i^t, 0) = d(X_i(\omega_i), 0)$ for every $i \in I$. By Proposition 4, there is $z' \in B$ and $\delta \in R^n$ such that, for every $i \in I$, $d(X_i(\omega_i), 0) < \delta_i$ and $z_i' \succsim_i^* z_i$ for every $z_i \in Z_i$ such that $d(z_i, \omega_i) < \delta_i$. Hence there is a positive integer t such that $d(M_i^t, 0) < \delta_i$ for every $i \in I$. For every $i \in I$ there is $x_i \in M_i^t = T_i(M^{t-1}, \omega_i)$ such that $d(x_i, 0) < \delta_i$. Hence, for every $i \in I$, $x_i + \omega_i \in Z_i$ and $d(x_i + \omega_i, \omega_i) < \delta_i$, implying that $z_i' \succsim_i^* x_i + \omega_i$; therefore $z_i' - \omega_i \in M_i^t$. Thus $z' - \omega \in F \cap \Pi_{i \in I} M_i^t$, contradicting the fact that M^t is inconsistent. Hence $z^t \neq z^0$ for some $t > 0$. Without loss of generality, we may assume that $z^{t'} = z^0$ for every $t' < t$. Then $z^t \neq z^{t-1}$ so that $z^t \in (F \cap \Pi_{i \in I} M_i^t) + \{z^{t-1}\}$; therefore $z^t = x + z^{t-1}$ for some $x \in F \cap \Pi_{i \in I} M_i^t$. For every $i \in I$, $x_i \in M_i^t = T_i(M^{t-1}, z_i^{t-1}) \subset X_i(z_i^{t-1})$ so that $z_i^t = x_i + z_i^{t-1} \in Z_i$. Thus $z^t \in B$.

Proof of Theorem 3. Similar to the proof of Theorem 6 below, in view of Lemma 8.

Proof of Theorem 6. Let a be a cluster point of $\{a^t\}_{t=0}^{\infty}$. Then there is a subsequence $\{a^{t^\nu}\}_{\nu=1}^{\infty}$ converging to a. Suppose a is not a Pareto-optimal allocation. Then there is $a' \in A$ such that $a' \succ_i a$ for every $i \in I$. By Assumption 2, there is $\varepsilon > 0$ such that, for every $i \in I$, $a' \succ_i a''$ for every $a'' \in D_i$ such that $d(a'', a) < \varepsilon$. There is a positive integer t' such that $3rq^{t'} < \varepsilon$. Without loss of generality, we may assume that $t^{\nu-1} + t' + 1 < t^\nu$ for every ν. Since $\{a^{t^\nu}\}_{\nu=1}^{\infty}$ is convergent and hence Cauchy-convergent, there is a positive integer μ such that

(2) $$d(a^{t^\nu}, a) < rq^{t'} \quad \text{and} \quad d(a^{t^\nu}, a^{t^{\nu+1}}) < rq^{t'} \qquad \text{for every} \quad \nu \geqq \mu .$$

Without loss of generality, we may assume that $a^{t^\mu} \neq a^{t^\mu - 1}$ and hence that M^{t^μ} is consistent, which, by Lemma 3, implies that $d(M_i^{t^\mu+1}, a^{t^\mu}) = r$ for every $i \in I$. Since for every positive integer t if $M^{t^\mu+s}$ is inconsistent for every $s \in \{1, \cdots, t\}$ then $a^{t^\mu} = a^{t^\mu+s}$ for every $s \in \{1, \cdots, t\}$, it follows from Lemma 3 that

(3) for every positive integer t, if $M^{t^\mu+s}$ is inconsistent for every $s \in \{1, \cdots, t\}$ then $d(M_i^{t^\mu+s}, a^{t^\mu}) = rq^{s-1}$ for every $s \in \{1, \cdots, t+1\}$ and every $i \in I$.

Suppose $M^{t^\mu+t}$ is consistent for some $t \in \{1, \cdots, t'+1\}$. Without loss of generality, we may assume that $M^{t^\mu+s}$ is inconsistent for every $s \in \{1, \cdots, t-1\}$. By (3), $d(M_i^{t^\mu+t}, a^{t^\mu}) = rq^{t-1} \geqq rq^{t'}$ for every $i \in I$, so that, by (2), $a^{t^{\mu+1}} \notin M_i^{t^\mu+t}$ for every $i \in I$. Since $M^{t^\mu+t}$ is consistent, $a^{t^\mu+t} \in M_i^{t^\mu+t}$ for every $i \in I$. If $a^{t^{\mu+1}} \succsim_i a^{t^\mu+t}$ for some $i \in I$ then $a^{t^{\mu+1}} \in M_i^{t^\mu+t}$, a contradiction. Hence $a^{t^\mu+t} \succ_i a^{t^{\mu+1}}$ for every $i \in I$. Since $t^\mu + t \leqq t^\mu + t' + 1 \leqq t^{\mu+1}$, this contradicts the fact that, for every $i \in I$, the sequence $\{a^t\}_{t=1}^{\infty}$ is non-decreasing with respect to $\succsim_i$. Then $M^{t^\mu+t}$ is inconsistent for every $t \in \{1, \cdots, t'+1\}$.

Therefore, by (3), $d(M_i^{t^\mu+t'+1}, a^{t^\mu}) = rq^{t'}$ for every $i \in I$. Let $i \in I$. Then $d(a'', a^{t^\mu}) < 2rq^{t'}$ for some $a'' \in M_i^{t^\mu+t'+1}$, so that, by (2), $d(a'', a) \leqq d(a'', a^{t^\mu}) + d(a^{t^\mu}, a) < 3rq^{t'} < \varepsilon$, implying that $a' \succ_i a''$. Since $a'' \in M_i^{t^\mu+t'+1}$, it follows that $a' \in M_i^{t^\mu+t'+1}$. Since i is arbitrary in I, $a' \in A \cap (\bigcap_{i \in I} M_i^{t^\mu+t'+1})$, which contradicts the fact that $M^{t^\mu+t}$ is inconsistent for every $t \in \{1, \cdots, t'+1\}$.

Proof of the Corollary to Theorem 3. Similar to the proof of the corollary to Theorem 6.

Proof of the Corollary to Theorem 6. Since A is compact, the sequence $\{a^t\}_{t=0}^{\infty}$ has a cluster point a^*, which is, by Theorem 6, a Pareto-optimal allocation. Hence there is a subsequence $\{a^{t^\nu}\}_{\nu=1}^{\infty}$ converging to a^*. Since the sequence $\{a^t\}_{t=1}^{\infty}$ is non-decreasing with respect to $\succsim_i$ for every $i \in I$, it follows that, for every $i \in I$, $a^{t^{\nu'}} \succsim_i a^{t^\nu}$ whenever $\nu' \geqq \nu$. By Assumption 2, $a^* \succsim_i a^{t^\nu}$ for every $i \in I$ and every positive integer ν. If $u_i(a^t) > u_i(a^*)$ for some $i \in I$ and some positive integer t, then $u_i(a^{t^\nu}) \geqq u_i(a^t) > u_i(a^*)$, i.e., $a^{t^\nu} \succ_i a^*$ for some positive integer ν such that $t^\nu > t$, a contradiction. Hence $(u_i(a^*))_{i \in I} \geqq u^t$ for every positive integer t. Thus, for every $i \in I$, the sequence $\{u_i^t\}_{t=1}^{\infty}$ is non-decreasing and bounded from above, so that $\lim_{t \to \infty} u_i^t$ exists. Therefore, the sequence $\{u^t\}_{t=0}^{\infty}$ is convergent. Now assume that, for every $i \in I$, u_i is lower semi-continuous. Suppose $u_i(a^*) > \lim_{t \to \infty} u_i^t$ for some $i \in I$. Since u_i is lower semi-continuous and $\{a^{t^\nu}\}_{\nu=1}^{\infty}$ converges to a^*, $u_i(a^{t^\nu}) > \lim_{t \to \infty} u_i^t$

for some positive integer v, so that $u_i(a^{t'}) \geqq u_i(a^{t^v}) > \lim_{t\to\infty} u_i^t$ for some positive integer t' such that $t' \geqq t^v$, a contradiction. Thus $u_i(a^*) = \lim_{t\to\infty} u_i^t$ for evry $i \in I$, i.e., $\lim_{t\to\infty} u^t = (u_i(a^*))_{i \in I}$.

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