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# DIFFERENTIALS OF THE RATE OF RETURN ON INVESTMENT BY INDUSTRY -THE POST WAR JAPANESE CASE- 

Fumimasa Hamada*

This paper attempts to make an econometric study of inter-industry allocation of investment in Japan. First, production functions by industry are estimated, and then, based on the profit maximization principle, the sum of discounted marginal revenues due to an additional unit of funds invested is defined, and its value by industry is estimated to test the equalization of the rates between industries. The inter-industry differentials of the rates are found to be large, and discussed from various view-points.

## I. INTRODUCTION

Economic development of a country is accompanied with changes in its industrial structure; the center of gravity of the whole industry shifts from the traditional to the modern, the light industries to the heavy industries, and the manufacturing industries to the third industries. It can be interpreted that changes of industrial structure take place through those of allocation of fixed investment between industries, caused by the occurrence of differentials of the expected rate of return on investment by industry. So, it may be very important to make an empirical analysis of inter-industry allocation of investment, in order to shed more light on the mechanism of changes of industrial structure in the course of economic development.

Inada (1971) developed a theoretical framework of economic development along this line in terms of dynamic specification, and Inada et al. (1972) attempted to apply it to the pre- and the post-War Japanese economic development. This application, however, was not a systematic test of the theory developed in the former literature, but an interpretation of Japanese economic development, using statistical data on the relevant variables. Solow (1963) estimated the rate of return on investment of the economy as a whole in the United States and in West

[^0]Germany directly, and discussed on its plausibility. To make an inquiry into the possibilities of industrial policies, it may be necessary to analyze systematically the actual inter-industry allocation of investment from the view-point of inter-industry comparison of the over-all marginal internal rate of expected return on investment.

Economic theory states that fixed investment is to be allocated so as to equate the sum of discount marginal revenues due to an additional unit of funds invested by industry to each other. In reality, however, this equalization process would be carried out gradually and dynamically, if it certainly works. To make clear whether this equalization process actually exists or not, we have to make an inter-industry comparison of the sum of discounted marginal revenues due to an additional unit of funds invested by industry first. From now on, let us call the sum of discounted marginal revenues due to an additional unit of funds invested, the over-all marginal internal rate of expected return on investment or simply, the over-all rate of return on investment. This concept should be distinguished from the marginal internal rate of expected return on investment or marginal efficiency of investment. ${ }^{1}$

As well known, it seems that the so-called 'marginal internal rate of return on investment' is very difficult to estimate directly in terms of dynamic specification with finite planning horizon. ${ }^{2}$ Moreover, as will be seen later, the equalization of the over-all rate of return on investment between industries does not necessarily imply the equalization of marginal internal rates of return on investment by industry in the sense of Boulding (1935) or Lutz (1951) and Keynes (1936).

In this paper, as the first approach, the over-all rate of return on investment by industry is estimated, based on the hypothesis of stationary expectation with respect to the prices of output and inputs on the finite planning horizon. Then, we will make an inter-industry comparison of the estimated rates to test whether there exists the law of equalization of the rates. It is assumed that there is prevailing the law of diminishing return for each industry, that would be acceptable for the technological conditions of each industrial total with time fixed. Economies of scale could be considered to work upon the technological conditions of production aggregative within an industry with the lapse of historical time.

For the simplicity's sake, the industry as a whole is divided into five sectors. Needless to say, an extention to a more disaggregated classification is also possible for a further development of this study.

Empirical results have shown that the differentials of the over-all marginal internal rates of expected return on investment by sector is quite remarkable, so that the gaps between the theoretical (optimal in potential) and the observed values of investment by sector appeared to be quite large and significant.

[^1]Section II describes the graphical presentation of the basic theory of the interindustry allocation of investment. Section III shows the derivation of the over-all marginal internal rate of expected return on investment, based on the static theory of production and investment. Section IV presents empirical results on production functions by sector, and on the over-all marginal internal rates of expected return on investment by sector. Finally, Section V discusses on the inter-industry comparison of the rates.

## II. the graphical presentation of the basic theory

The neoclassical theory of investment tells us that the inter-industry allocation of investment is made so as to equate the over-all marginal internal rate of expected return on investment by industry to each other. For any optimal solution by industry to exist, the over-all rate of return on investment should be a decreasing function of investment; that is, there should be the decreasing return to scale of production and investment in the economy.
For a simplified explanation, let us assume that there are two industries, each of which has a curve for the over-all marginal internal rate of expected return on investment with respect to the amount of funds invested, showing its investment opportunities, and therefore, this curve is a kind of investment schedule. Figure 1 shows two curves for two industries ( $a$ and $b$ ). The vertical axis measures the overall marginal internal rate of expected return on investment by industry, $\lambda$, and the horizontal axis, the amount of funds invested by industry, $A_{i}(i=a, b)$.

The curve $a a^{\prime}$ in Fig. 1 is the investment schedule for industry $a$, and the curve $b b^{\prime}$ is that for industry $b$. Both curves are drawn as down-sloping, based on the assumption of the decreasing return to scale in production and investment. The curve $c c^{\prime}$ is the curve for the composite of the two curves $a a^{\prime}$ and $b b^{\prime}$, the configuration of which is specified so as to equate the over-all marginal internal rates for the two industries, and to measure the sum of the amounts of funds invested on the horizontal axis, so that the vertical distance $e A$ is equal to the vertical distances $r_{a} A_{a}, r_{b} A_{b}$, and $\lambda^{*} 0$ respectively, and the horizontal distance $0 A$ is equal to the sum of the horizontal distances $0 A_{a}$ and $0 A_{b}$.
If once the total amount of funds available for investment, say $A$, is given exogeneously, the optimal amount of funds to be invested for each industry can be determined as $0 A_{a}$ for industry $a$ and $0 A_{b}$ for industry $b$, respectively. Since these curves like $a a^{\prime}$ and $b b^{\prime}$ make themselves shift as the time goes by, depending on changes in prices of output and inputs, technological progress, expectations, and so forth, the optimal amount of funds to be invested (and real amount of investment also) by industry changes simultaneously. Needless to say, the total amount of funds to be invested may also change depending on its determinants, such as the marginal internal rate of return, private incomes, and networths, and so on, which are not considered in this paper.


Fig. 1. The optimal allocation of investment.

## III. THE OVER-ALL MARGINAL INTERNAL RATE OF EXPECTED RETURN ON INVESTMENT

The over-all marginal internal rate of expected return on investment can be obtained through maximization of the sum of the present values of expected return of the firm. In order to make an empirical approach into this relation, we have to resort to a heroic assumption on the behavior of the firm. Let us assume that the entrepreneur has the stationary expectation with respect to the time-path of prices of output and inputs during the planning horizon. Furthermore, the entrepreneur is assumed to invest its funds for $n$ industries.
The over-all marginal internal rate of expected return on investment for each period during the planning horizon is, after all, determined, based on marginal productivity of capital or investment in fixed capital for the first period of the planning horizon, if we take the assumption of stationary expectation. If we take more complicated or dynamic expectation, the rate for each period will be very intricate and difficult to obtain analytically and directly. ${ }^{3}$
Assuming that production function is homogeneous and of the $v_{j}$ th degree for the $j$ th industry ( $0<v_{j}<1$ ), and given the total amount of funds available for

[^2]investment of $n$ industries, $A$, maximization of the sum of the present values of expected return on fixed capital stock (new investment plus stock in fixed capital at the beginning-of-period) for $n$ industries leads to the system of ( $n-1$ ) equations below ${ }^{4}$ :
\[

$$
\begin{equation*}
1+\lambda=\zeta_{i_{1}}\left(T_{i}\right) \frac{p_{i} \partial X_{i}(t)}{q_{i} \partial I_{i}(t)}=\zeta_{j_{1}}\left(T_{j}\right) \frac{p_{j} \partial X_{j}(t)}{q_{j} \partial I_{j}(t)} \quad \text { for all } \quad i \neq j \tag{1}
\end{equation*}
$$

\]

where $\lambda$ is the over-all marginal internal rate of expected return on investment, $T_{i}$ is the length of planning horizon of the $i$ th industry; $p_{i}, q_{i}, X_{i}, I_{i}$ are the expected value of net price of output, that of investment goods, the volume of output, and the volume of investment of the $i$ th industry respectively, and

$$
\begin{align*}
& \zeta_{i_{1}}\left(T_{i}\right)=\sum_{\tau=1}^{T_{i}}\left\{\left(1-\delta_{i}\right)^{\tau-1}\right\}^{v_{i}}\left(1+r_{i}\right)^{-\tau}  \tag{2}\\
& \zeta_{j_{1}}\left(T_{j}\right)=\sum_{j=1}^{T_{j}}\left\{\left(1-\delta_{j}\right)^{\tau-1}\right\}^{v_{j}}\left(1+r_{j}\right)^{-\tau} \tag{3}
\end{align*}
$$

where $\delta_{i}$ and $r_{i}$ are the physical rate of deterioration of capital stock and discount rate of the $i$ th industry respectively. ${ }^{5}$ Needless to say, the fractions in both sides of eq. (1) are the expected marginal revenues due to an additional unit of funds invested in the $i$ th and the $j$ th industry respectively. So, eq. (1) tells us that the optimization behavior of the entrepreneur leads to the equalization of over-all marginal internal rates of expected return on investment of all industries. Those ( $n-1$ ) equations and an identity showing that the sum of funds to be invested for all industries should be equal to the total amount A, with $n$ equations for marginal productivity of labor determine the values of optimal investment and labor of $n$ industries simultaneously.
To obtain more concrete form of the over-all marginal internal rate of return on investment, production function should be specified. As already mentioned, for the optimal investment to exist, there should be prevailing the law of decreasing return in production and investment for any unit period. We will attempt to adopt two types of production function; the one is the Cobb-Douglas type, the characteristics of which are definite, and the other is somewhat specific type of the so-called "Semi-substitute" or "Limitational" production function. ${ }^{6}$

## III. 1. The case of Cobb-Douglas type

Let be a production function of Cobb-Douglas type of the degree $v$ :

[^3]\[

$$
\begin{equation*}
X(t)=c_{0} L(t)^{c_{1}} K(t)^{c_{2}} e^{c_{3} t} ; \quad 0<c_{1}+c_{2}=v<1 \tag{4}
\end{equation*}
$$

\]

where $L(t)$ is labor input, $K(t)$ is stock in fixed capital in period $t, c_{0}, c_{1}, c_{2}$, and $c_{3}$ are constants. $c_{3}$ is the rate of neutral technical progress. Using eq. (4), eq. (1) can be rewritten as below:

$$
\begin{align*}
1+\lambda & =\zeta_{i_{1}}\left(T_{i}\right) \frac{c_{2 i} p_{i} X_{i}(t)}{q_{i}\left\{I_{i}(t)+\left(1-\delta_{i}\right) K_{i}(t-1)\right\}}  \tag{1}\\
& =\zeta_{j_{1}}\left(T_{j}\right) \frac{c_{2 j} p_{j} X_{j}(t)}{q_{j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}} ; \quad i \neq j .
\end{align*}
$$

Using the equation for the expansion path, and substituting it into production function (4), the equation for the over-all marginal internal rate of expected return on investment of industry $j$ can be written as below ${ }^{7}$ :

$$
\begin{align*}
\lambda_{j}= & \zeta_{j_{1}}\left(T_{j}\right) c_{0 j} c_{2 j}\left(\frac{p_{j}}{q_{j}}\right)\left\{\frac{c_{1 j}}{c_{2 j}} \frac{q_{j}\left(r_{j}+\delta_{j}\right)}{w_{j}}\right\}^{c_{1 j}}  \tag{5}\\
& \cdot\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{c_{1 j}+c_{2 j}-1} e^{c_{3 j} t}-1 .
\end{align*}
$$

The condition for the curve expressed by the equation above to be down-sloping is $0<c_{1 j}+c_{2 j}<1$.
III. 2. The case of semi-substitution type

Let be the technological conditions of industry $j$ as a pair of two relations:

$$
\begin{array}{lll}
X_{j}(t)=a_{0 j} K_{j}(t)^{a_{1 j}} ; & a_{0 j}>0, & 0<a_{1 j}<1, \\
L_{j}(t)=b_{0 j} K_{j}(t)^{b_{1 j} ;} & b_{0 j}>0, & 0<b_{1 j}<1 . \tag{7}
\end{array}
$$

Equation (6) states that the scale of production is technologically related to stock in fixed capital, and eq. (7) states that labor input is substituted by capital services through increasing scale of production; that is, substitution is accompanied with increases in the level of production. ${ }^{8}$

Maximization of the sum of present values of expected return on fixed capital stock (new investment plus stock in fixed capital at the beginning-of-period) with respect to the amount of funds to be invested for $n$ industries leads to the equation for the over-all marginal internal rate of return on investment for industry $j$ as below ${ }^{9}$ :

[^4]\[

$$
\begin{align*}
\lambda_{j}= & \zeta_{j_{3}}\left(T_{j}\right) \frac{p_{j}}{q_{j}} a_{0 j} a_{1 j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{a_{1 j}-1}  \tag{8}\\
& -\zeta_{j_{4}}\left(T_{j}\right) \frac{w_{j}}{q_{j}} b_{0 j} b_{1 j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{b_{1 j}-1}-1
\end{align*}
$$
\]

Equation (8) shows that, given $\zeta_{j}, p_{j}, q_{j}, r_{j}, T_{j}$ and $K_{j}(t-1)$, the over-all marginal internal rate of return on investment is a function of real investment $I_{j}(t)$. The first order conditions of maximization include the $(n-1)$ equalities between those rates of all industries ${ }^{10}$; that is,

$$
\begin{align*}
\zeta_{j_{3}}\left(T_{j}\right) & \frac{p_{j}}{q_{j}} a_{0 j} a_{1 j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{a_{1 j}-1}  \tag{9}\\
& -\zeta_{j_{4}}\left(T_{j}\right) \frac{w_{j}}{q_{j}} b_{0 j} b_{1 j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{b_{1 j}-1} \\
= & \zeta_{i 3}\left(T_{i}\right) \frac{p_{i}}{q_{i}} a_{0 i} a_{1 i}\left\{I_{i}(t)+\left(1-\delta_{i}\right) K_{i}(t-1)\right\}^{a_{1 i}-1} \\
& -\zeta_{i_{4}}\left(T_{i}\right) \frac{w_{i}}{q_{i}} b_{0 i} b_{1 i}\left\{I_{i}(t)+\left(1-\delta_{i}\right) K_{i}(t-1)\right\}^{b_{1 i}-1} ; \quad i=j
\end{align*}
$$

The condition for the curve of $\lambda_{j}$ to be down-sloping is very complicated, which is shown by eq. (B.15) in Mathematical Appendix B.

## IV. EMPIRICAL RESULTS

## IV. 1. The brief outlines of the industrial structure

The industry as a whole was divided into five sectors; that is, Sector I, the primary industry (agriculture, forestry, fishery, and mining), Sector II, the light industry (foods and textiles), Sector III, the heavy industry (chemistry, the ferrous and non-ferrous, metal products, general machines and tools, electric machines and tools, transportation machines and tools, oil-refining, cement-clay-stones, pulps, and other manufacturings), Sector IV, constructions, and Sector V, the service industry (electric power, gas, water, whole sales, retails, bankings and insurance, real estate, transportation and communication, and other services).

Our observation period is from 1960 through 1977 calender year. During this period, real output of all industries has been growing at an annual rate of 9.13 percent. In 1960, the relative share of real output of Sector I, the primary industry, was 14.5 percent of the total industry output, and it declined to 4.2 percent in 1977. Sector II, the light industry, has also shown decreases, during the same period, from 13.8 percent through 7.7 percent. Sector III, the heavy industry, however, has raised its relative share from 31.3 percent in 1960 through 43.3 percent in 1977. Relative share of Sector IV, constructions, has slightly declined from 10.7 percent

[^5]in 1960 through 9.0 percent in 1977. And Sector V, the service industry, has shown a remarkable growth, the relative share of which increased from 29.7 percent in 1960 through 35.7 percent in 1977.

Growth in fixed capital stock was about 10 percent at an annual rate, and real gross investment has grown at an annual rate of 12.7 percent in all industries during the same period. The most interesting in those observations is that the tendency of changes in the relative share of real output of each sector does not necessarily correspond with those of real gross investments. Though the relative share of real investment has increased in Sector I, conspicuous declines of relative share of real investment have been observed in Sector III (the heavy industry), particularly during the period from 1968 through 1977. Sector V has shown striking increases in shares during the same period. The relative shares of Sector II (the light industry) and Sector IV (constructions) remained almost unchanged.

Labor productivity has strikingly increased in Sector III, and in 1977, it was about twice as large as that in Sector II. In Sector II, IV, and V, increases in labor productivity were moderate (annual increases of about 5.5, 5.9, and 6.7 percent respectively), while in Sector I, it was 4.2 percent during the period from 1960 through 1977.

All in all, changes in the structure of demand for industrial product are complicatedly related to those in inter-industry allocation of investment, and also to those in labor productivity by industry or by sector.

## IV. 2. Estimation of production functions

Estimation of production function by sector was made, using two types of production function; Cobb-Douglas type and the semi-substitute type (or the factor limitational type). Empirical results are shown in Tables 1 and 2. In Table 1, the estimated results of production functions by sector are shown. It seems clear, from the table, that this type of production function is not suitable to all the sectors except for Sector II (the light industry).

The reason why the results were not satisfactory may be considered to be relating to the estimation method, the ordinary least squares, on which I will not discuss here, because this is the first approach to the problem of this sort. But, neglecting the discussion on the estimation method, there may be some problems concerning the empirical results. Firstly, there certainly exists the multicollinearity among the relevant variables, which seems innevitable in estimating parameters of Cobb-Douglas production functions, using time series data.

Secondly, some estimates show that the sum of coefficients of elasticity of production with respect to labor and capital is greater than unity for Sector III, IV, and $V$. It could be interpreted that the influences of technological progress overtime were not removed from the variations of production, ${ }^{11}$ or the vintage model

[^6]TABLE 1. Estimation of Production Functions of Cobb-Douglas Type

| Sector | $\ln L$ | $\ln K$ | $\ln X_{-1}$ | $t$ | $\bar{R}^{2}$ | SE | DW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | $\begin{gathered} -0.12297 \\ (0.497) \end{gathered}$ | $\begin{gathered} 0.11114 \\ (1.090) \end{gathered}$ | - | - | 0.911 | 0.0253 | 1.073 |
|  | $\begin{aligned} & 0.01020 \\ & (0.042) \end{aligned}$ | $\begin{gathered} -0.08585 \\ (0.586) \end{gathered}$ | - | $\begin{aligned} & 0.02378 \\ & (1.773) \end{aligned}$ | 0.922 | 0.0237 | 1.261 |
|  | $\begin{gathered} -0.06944 \\ (0.283) \end{gathered}$ | $\begin{aligned} & 0.05368 \\ & (0.466) \end{aligned}$ | $\begin{aligned} & 0.44607 \\ & (1.727) \end{aligned}$ | (1.73) | 0.915 | 0.0229 | 1.648 |
| II | $\begin{aligned} & 0.56543 \\ & (5.145) \end{aligned}$ | $\begin{gathered} 0.61980 \\ (54.617) \end{gathered}$ | - | - | 0.995 | 0.0218 | 2.218 |
|  | $\begin{aligned} & 0.51852 \\ & (5.444) \end{aligned}$ | $\begin{aligned} & 0.44981 \\ & (3.818) \end{aligned}$ | - | $\begin{aligned} & 0.01509 \\ & (1.449) \end{aligned}$ | 0.996 | 0.0211 | 2.396 |
|  | $\begin{aligned} & 0.66721 \\ & (3.780) \end{aligned}$ | $\begin{aligned} & 0.81079 \\ & (3.529) \end{aligned}$ | $\begin{gathered} -0.28955 \\ (0.816) \end{gathered}$ | (1.44) | 0.994 | 0.0223 | 1.906 |
| III | $\begin{aligned} & 1.70212 \\ & (4.896) \end{aligned}$ | $\begin{aligned} & 0.51906 \\ & (7.256) \end{aligned}$ | - | - | 0.993 | 0.0533 | 1.021 |
|  | $\begin{aligned} & 2.62426 \\ & (4.687) \end{aligned}$ | $-0.14323$ | - | $\begin{aligned} & 0.06409 \\ & (1.999) \end{aligned}$ | 0.994 | 0.0487 | 1.184 |
|  | $\begin{aligned} & 1.79731 \\ & (3.578) \end{aligned}$ | $\begin{aligned} & 0.43088 \\ & (1.998) \end{aligned}$ | $\begin{aligned} & 0.09030 \\ & (0.309) \end{aligned}$ | (1.9) | 0.991 | 0.0524 | 1.326 |
| IV | $\begin{aligned} & 0.59526 \\ & (0.059) \end{aligned}$ | $\begin{aligned} & 0.50316 \\ & (2.205) \end{aligned}$ | - | - | 0.965 | 0.0862 | 0.561 |
|  | $\begin{gathered} 0.62103 \\ (0.704) \end{gathered}$ | $\begin{aligned} & 1.01423 \\ & (3.666) \end{aligned}$ | - | $\underset{(2.593)}{-0.10477}$ | 0.974 | 0.0733 | 0.815 |
|  | $\begin{aligned} & 0.54665 \\ & (0.665) \end{aligned}$ | $\begin{gathered} -0.13056 \\ (0.485) \end{gathered}$ | $\begin{aligned} & 0.93865 \\ & (3.292) \end{aligned}$ | - | 0.975 | 0.0669 | 1.696 |
| V | $\begin{aligned} & 1.88702 \\ & (4.279) \end{aligned}$ | $\begin{aligned} & 0.42277 \\ & (2.861) \end{aligned}$ | - | - | 0.986 | 0.0668 | 0.432 |
|  | $\begin{aligned} & 2.76944 \\ & (4.181) \end{aligned}$ | $\begin{aligned} & 1.60539 \\ & (2.276) \end{aligned}$ | - | $\underset{(1.710)}{-0.14733}$ | 0.988 | 0.0629 | 0.663 |
|  | $\begin{aligned} & 0.52228 \\ & (1.798) \end{aligned}$ | $\begin{gathered} -0.21611 \\ (1.840) \end{gathered}$ | $\begin{aligned} & 0.99968 \\ & (7.204) \end{aligned}$ | - | 0.996 | 0.0320 | 1.742 |

[^7]might have been preferable in estimating production functions for these sectors. But, those things will be taken up in a further extension of this study, After all, Cobb-Douglas type was adopted for Sector II.

Table 2 shows the estimates for the parameters of production functions of semisubstitution or factor limitational type. In Table 2, all the estimates appear to be fairly plausible from the view-point of the conditions for the curve of the over-all marginal internal rate of return on investment to be down-sloping, except for those of Sector II and the estimate of the coefficient of elasticity of labor input with respect to capital stock in Sector I.

Weak conditions for decreasing return to scale of production are satisfied of the estimates for parameters of Sector I, III, and IV, but not of Sector V; that is, the estimate of coefficient of elasticity of production with respect to capital stock is greater than unity. Needless to say, the strong conditions of decreasing return to

TABLE 2. Estimation of Production Functions of Semi-substitution Type

| Sector | Dpd. Var. | $\ln K$ | Const. | $\bar{R}^{2}$ | SE | DW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | $\ln X$ | $\begin{gathered} 0.16143 \\ (13.545) \end{gathered}$ | $\begin{aligned} & 7.52504 \\ & (67.726) \end{aligned}$ | 0.915 | 0.0247 | 1.028 |
|  | $\ln L$ | $\begin{array}{r} -0.40889 \\ (33.163) \end{array}$ | $\begin{aligned} & 10.83804 \\ & (94.284) \end{aligned}$ | 0.986 | 0.0256 | 0.509 |
| II | $\ln X$ | $\begin{gathered} 0.62932 \\ (34.911) \end{gathered}$ | $\begin{aligned} & 4.06504 \\ & (26.916) \end{aligned}$ | 0.986 | 0.0351 | 0.862 |
|  | $\ln L$ | $\begin{aligned} & 0.18834 \\ & (0.661) \end{aligned}$ | $\begin{gathered} 5.53205 \\ (23.172) \end{gathered}$ | 0.010 | 0.5550 | 0.343 |
| III | $\ln X$ | $\begin{array}{r} 0.85776 \\ (30.183) \end{array}$ | $\begin{aligned} & 2.10181 \\ & (7.329) \end{aligned}$ | 0.982 | 0.0831 | 0.697 |
|  | $\ln L$ | $\begin{gathered} 0.19898 \\ (15.197) \end{gathered}$ | $\begin{gathered} 4.93408 \\ (37.330) \end{gathered}$ | 0.931 | 0.0383 | 0.434 |
| IV | $\ln X$ | $\begin{gathered} 0.51660 \\ (22.306) \end{gathered}$ | $\begin{gathered} 5.46857 \\ (30.946) \end{gathered}$ | 0.967 | 0.0834 | 0.574 |
|  | $\ln L$ | $\begin{gathered} 0.22563 \\ (37.915) \end{gathered}$ | $\begin{gathered} 4.33197 \\ (95.491) \end{gathered}$ | 0.988 | 0.0214 | 1.662 |
| V | $\ln X$ | $\begin{gathered} 1.04162 \\ (23.757) \end{gathered}$ | $\begin{aligned} & 0.01276 \\ & (0.029) \end{aligned}$ | 0.970 | 0.0965 | 0.199 |
|  | $\ln L$ | $\begin{gathered} 0.32795 \\ (19.038) \end{gathered}$ | $\begin{gathered} 4.24225 \\ (24.332) \end{gathered}$ | 0.955 | 0.0379 | 0.180 |

Note: Figures in parentheses are $t$-values for the estimates. $\bar{R}^{2}$, SE, and DW are the coefficient of determination adjusted to the degree of freedom, standard error of regression estimates, and Durbin-Watson statistic, respectively.
scale should be checked by computation, using eqs. (B.12), (B.14), and (B.15) in Mathematical Appendix B.
Reviewing the possibility of existence of serial correlation in disturbances in the regression models, values for Durbin-Watson statistic appear to warn us the positive correlations for almost all sectors. However, the coefficients of determination $\bar{R}^{2}$ appear to encourage us, and suggest us to rely on the estimates. Finally, I determined to adopt the estimates of parameters for production function of semi-substitute type for Sector I, III, IV, and V in Table 2, and those for CobbDouglas type for Sector II in Table 1.
IV. 3. Estimation of the over-all marginal internal rate of return on investment Estimation of the over-all marginal internal rate of return on investment is made, using eq. (5) for Sector II and eq. (8) for Sector I, III, IV, and V. The estimated value for each period can be obtained by substituting into eqs. (5) or (8) the values of prices of output and inputs, $p, q, w$, the value of discount rate, $r$, the value of rate of deterioration of capital stock, $\delta$, values of real investment and

TABLE 3. Estimates for the Over-all Rate of Return on Investment by Sector

| Period | Sector |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | I | II | III | IV | V |
| 1961 | $\begin{aligned} & 2.79342 \\ & 0.31038 \end{aligned}$ | $\begin{aligned} & 2.46526 \\ & 0.35218 \end{aligned}$ | $\begin{aligned} & 2.26856 \\ & 0.32408 \end{aligned}$ | $\begin{aligned} & 3.51274 \\ & 0.25091 \end{aligned}$ | $\begin{aligned} & 3.82130 \\ & 0.54590 \end{aligned}$ |
| 1962 | $\begin{aligned} & 2.83779 \\ & 0.31531 \end{aligned}$ | $\begin{aligned} & 1.58802 \\ & 0.22686 \end{aligned}$ | $\begin{aligned} & 2.55493 \\ & 0.36499 \end{aligned}$ | $\begin{aligned} & 5.85032 \\ & 0.41788 \end{aligned}$ | $\begin{aligned} & 4.00799 \\ & 0.57257 \end{aligned}$ |
| 1963 | $\begin{aligned} & 3.19590 \\ & 0.35510 \end{aligned}$ | $\begin{aligned} & 1.85248 \\ & 0.26464 \end{aligned}$ | $\begin{aligned} & 2.74400 \\ & 0.39200 \end{aligned}$ | $\begin{aligned} & 5.93502 \\ & 0.42393 \end{aligned}$ | $\begin{aligned} & 4.06287 \\ & 0.58041 \end{aligned}$ |
| 1964 | $\begin{aligned} & 3.32883 \\ & 0.36987 \end{aligned}$ | $\begin{aligned} & 1.92703 \\ & 0.27529 \end{aligned}$ | $\begin{aligned} & 2.38413 \\ & 0.34059 \end{aligned}$ | $\begin{aligned} & 5.38692 \\ & 0.38478 \end{aligned}$ | $\begin{aligned} & 4.39747 \\ & 0.62821 \end{aligned}$ |
| 1965 | $\begin{aligned} & 3.25476 \\ & 0.36164 \end{aligned}$ | $\begin{aligned} & 1.75252 \\ & 0.25036 \end{aligned}$ | $\begin{aligned} & 2.63032 \\ & 0.37576 \end{aligned}$ | $\begin{aligned} & 6.58420 \\ & 0.47030 \end{aligned}$ | $\begin{aligned} & 4.51185 \\ & 0.64455 \end{aligned}$ |
| 1966 | $\begin{aligned} & 3.27006 \\ & 0.36334 \end{aligned}$ | $\begin{aligned} & 1.63058 \\ & 0.23294 \end{aligned}$ | $\begin{aligned} & 2.42263 \\ & 0.34609 \end{aligned}$ | $\begin{aligned} & 6.71608 \\ & 0.47972 \end{aligned}$ | $\begin{aligned} & 4.51486 \\ & 0.64498 \end{aligned}$ |
| 1967 | $\begin{aligned} & 3.27168 \\ & 0.36352 \end{aligned}$ | $\begin{aligned} & 1.82917 \\ & 0.26131 \end{aligned}$ | $\begin{aligned} & 2.34416 \\ & 0.33488 \end{aligned}$ | $\begin{aligned} & 7.18018 \\ & 0.51287 \end{aligned}$ | $\begin{aligned} & 4.58297 \\ & 0.65471 \end{aligned}$ |
| 1968 | $\begin{aligned} & 3.43512 \\ & 0.38168 \end{aligned}$ | $\begin{aligned} & 1.71563 \\ & 0.24509 \end{aligned}$ | $\begin{aligned} & 2.35851 \\ & 0.33693 \end{aligned}$ | $\begin{aligned} & 6.22426 \\ & 0.44459 \end{aligned}$ | $\begin{aligned} & 4.60355 \\ & 0.65765 \end{aligned}$ |
| 1969 | $\begin{aligned} & 3.35646 \\ & 0.37294 \end{aligned}$ | $\begin{aligned} & 1.36038 \\ & 0.19434 \end{aligned}$ | $\begin{aligned} & 2.12100 \\ & 0.30300 \end{aligned}$ | $\begin{aligned} & 4.97644 \\ & 0.35546 \end{aligned}$ | $\begin{aligned} & 4.23906 \\ & 0.60558 \end{aligned}$ |
| 1970 | $\begin{aligned} & 3.10842 \\ & 0.34538 \end{aligned}$ | $\begin{aligned} & 1.19266 \\ & 0.17038 \end{aligned}$ | $\begin{aligned} & 1.91898 \\ & 0.27414 \end{aligned}$ | $\begin{aligned} & 4.31018 \\ & 0.30787 \end{aligned}$ | $\begin{aligned} & 4.25544 \\ & 0.60792 \end{aligned}$ |
| 1971 | $\begin{aligned} & 2.89701 \\ & 0.32189 \end{aligned}$ | $\begin{aligned} & 1.14037 \\ & 0.16291 \end{aligned}$ | $\begin{aligned} & 2.11190 \\ & 0.30170 \end{aligned}$ | $\begin{aligned} & 4.76924 \\ & 0.34066 \end{aligned}$ | $\begin{aligned} & 4.53544 \\ & 0.64792 \end{aligned}$ |
| 1972 | $\begin{aligned} & 2.52135 \\ & 0.28015 \end{aligned}$ | $\begin{aligned} & 0.78288 \\ & 0.11184 \end{aligned}$ | $\begin{aligned} & 2.13143 \\ & 0.30449 \end{aligned}$ | $\begin{aligned} & 4.68482 \\ & 0.33463 \end{aligned}$ | $\begin{aligned} & 4.81418 \\ & 0.68774 \end{aligned}$ |
| 1973 | $\begin{aligned} & 1.99458 \\ & 0.22162 \end{aligned}$ | $\begin{aligned} & 0.88634 \\ & 0.12662 \end{aligned}$ | $\begin{aligned} & 1.73096 \\ & 0.24728 \end{aligned}$ | $\begin{aligned} & 3.44120 \\ & 0.24580 \end{aligned}$ | $\begin{aligned} & 4.37920 \\ & 0.62560 \end{aligned}$ |
| 1974 | $\begin{aligned} & 1.54575 \\ & 0.17175 \end{aligned}$ | $\begin{aligned} & 0.69261 \\ & 0.09894 \end{aligned}$ | $\begin{aligned} & 1.42072 \\ & 0.20296 \end{aligned}$ | $\begin{aligned} & 4.58416 \\ & 0.32744 \end{aligned}$ | $\begin{aligned} & 4.09913 \\ & 0.58599 \end{aligned}$ |
| 1975 | $\begin{aligned} & 1.69848 \\ & 0.18872 \end{aligned}$ | $\begin{aligned} & 0.57378 \\ & 0.08197 \end{aligned}$ | $\begin{aligned} & 1.55540 \\ & 0.22220 \end{aligned}$ | $\begin{aligned} & 4.52368 \\ & 0.32312 \end{aligned}$ | $\begin{aligned} & 4.91323 \\ & 0.70189 \end{aligned}$ |
| 1976 | $\begin{aligned} & 1.60659 \\ & 0.17851 \end{aligned}$ | $\begin{aligned} & 0.79898 \\ & 0.11414 \end{aligned}$ | $\begin{aligned} & 1.46090 \\ & 0.20870 \end{aligned}$ | $\begin{aligned} & 4.12454 \\ & 0.29461 \end{aligned}$ | $\begin{aligned} & 5.18420 \\ & 0.74060 \end{aligned}$ |
| 1977 | $\begin{aligned} & 1.60713 \\ & 0.17857 \end{aligned}$ | $\begin{aligned} & 0.55513 \\ & 0.07930 \end{aligned}$ | $\begin{aligned} & 1.52355 \\ & 0.21765 \end{aligned}$ | $\begin{aligned} & 4.09976 \\ & 0.29284 \end{aligned}$ | $\begin{aligned} & 5.58432 \\ & 0.79776 \end{aligned}$ |

capital stock at the beginning-of-period, $I(t), K(t-1)$ for each period. This computation was done for the period from 1961 through 1977. The computed results are shown in Table 3. At a glance, it may easily be understood that there seems to exist considerable differentials between the estimates for the over-all
marginal internal rate of expected return on investment by sector.
These estimates are considered to be what the entrepreneurs themselves estimated and used at the time of decisions of investment, that were actually observed in each period. To make an inter-industry comparison easier, a sort of average annual rate of return were also calculated. They are shown under the corresponding figures. These annual rates were obtained by dividing the corresponding figures with the length of planning horizon by sector; that is, they are 9 years for Sector I, 7 years for Sector II and III, 14 years for Sector IV, and 7 years for Sector V. The next section reviews the empirical results and discusses on the implications of them.

## V. REVIEW AND IMPLICATION OF THE EMPIRICAL RESULTS

Estimation of the over-all marginal internal rate of return on investment seem to be quite plausible for all sectors. Figures divided by the length of planning horizon are roughly comparable with those obtained, by R. M. Solow, for the United States economy and for West German economy as a whole. ${ }^{12}$ For instance, his marginal rate of return on capital in the United States and in West Germany were 0.40 and 0.39 with an annual rate of technological progress of 2 percent respectively in 1954, while those of mine were 0.31 for Sector I (the primary industry), 0.35 for Sector II (the light industry), 0.32 for Sector III (the heavy industry), 0.25 for Sector IV (construction), and 0.55 for Sector V (the third industry) respectively in Japan in 1961. The similarity between Solow's and mine is rather striking.


Fig. 2. Time configuration of the over-all marginal rate of return on investment.

[^8]In Table 3, or in Fig. 2, the over-all marginal rate of return on investment is declining, in trend, in Sector, I, II, III, while Sector V shows rather an increasing trend. Sector IV is sustaining a high level of rate of return after a dramatic fluctuations from 1962 through 1968. It should be noted that Sector I, II, and III produce the traded goods, while Sector IV and V supply the so-called "nontraded" goods.

Industries producing the traded goods are constantly affected by the international economic and political movements from abroad. The recent big movements, such as the Nixon Shock, the collapse of IMF system and the shift to flexible exchange rate, Oil-Crises, Islamic revolutions, world inflation, and Yenappreciation and Yen-depreciation, have brought about uncertainty in expectations to the future of economic activities, and the growth rate of world economy seems to be lower and lower.

In contrast to these, sectors producing the non-traded goods, particularly the services industry, such as whole sales, retails, real estate, and services, appear to be enjoying some prosperity for these years. Since 1975, the over-all marginal rate of return is dramatically increasing, and really arrived at a high rate of about 5.58 , an annual rate of about 0.8 in 1977.

The reason why the over-all marginal rate of return in Sector IV, construction, is considerably high could be interpreted in various ways. The main causes, however, may be the following three points; firstly, competition in market for construction may be quite imperfect, and the new entries, hardly possible, so that the prices of output may have a downward rigidity. This might be brought about through the old-fashioned organization of this industry, such as traditional unions of workers concerning the natures of work specific to construction, the systems of order (bidding). Moreover, a considerable part of demand comes from the public sector, oftenly affected by the political forces.

Secondly, demand for construction does not seem to be elastic with respect to price of output. It may be almost given from out-side of the market. This means that the demand curve is kinked vertically at the point of the observed output. Thirdly, errors in expectation should be taken into consideration. It may be considered that the industry of high marginal rate of return should have invested more than the observed, but the entrepreneur had made expectation biased systematically lower in pricing of its product. This is why the estimates showed rather higher rate of return. Any way, the differentials of the over-all marginal internal rate of expected return by sector appear to be significantly remarkable. Needless to say, it could also restated that there certainly exist the significant differentials of discount rate between these sectors.
Finally, the curves for $\lambda_{j} / T_{j}(j=1, \cdots, 5)$ are shown in Figs. 3-19 for the period from 1961 through 1977. For the derivation of these curves, see Mathematical Appendix A and B, particularly eqs. (A.23) and (B.12). As easily seen, all the curves are down-sloping, except for Sector V. This is just what was observed and discussed in Section IV. 2.

## MATHEMATICAL APPENDIX A

This appendix is to present the derivation process of the over-all marginal internal rate of expected return on investment by industry. First, we proceed for a general case of labor-capital substitution as the technological conditions of production. Production function is assumed to be homogeneous and of $v_{j}$ th degree. The sum of present values of the firm is written as below:

$$
\Pi=\sum_{j=1}^{n}\left[\sum_{j=1}^{T_{j}}\left\{p_{j} X_{j}(t, \tau)-w_{j} L_{j}(t, \tau)\right\}\left(1+r_{j}\right)^{-\tau}-q_{j} I_{j}(t)\right]-\lambda\left\{\sum_{j=1}^{n} q_{j} I_{j}(t)-A\right\}
$$

where $p_{j}, X_{j}, w_{j}, L_{j}, q_{j}, I_{j}, r_{j}$ are price of output, the volume of output, wage rate, labor input, prices of investment goods, the volume of investment in real terms, discount rate of industry $j$ respectively. The value within the parenthesis $\}$ is the non-wage incomes in the period of the planning horizon expected at period $t$, So the value within the parenthesis [ ] is the sum of the present values of non-wage incomes expected during the planning horizon, less of the amount of funds invested at the period $t$ for industry $j$. The last term in the equation above is a constraint for maximization of the gain-function, so that $\lambda$ is the lagrange's multiplier, which is to be inter in this case, as the over-all marginal internal rate of expected return on investment.

Production function of industry $j$ is written as

$$
\begin{equation*}
X_{j}(t, \tau)=F_{j}\left[L_{j}(t, \tau), K_{j}(t, \tau)\right] \tag{A.2}
\end{equation*}
$$

where,

$$
\begin{equation*}
L_{j}\left(t, \tau=\left(1-\delta_{j}\right)^{\tau-1} L_{j}(t),\right. \tag{A.3}
\end{equation*}
$$

and

$$
\begin{align*}
K_{j}(t, \tau) & =\left(1-\delta_{j}\right)^{\tau-1} K_{j}(t)  \tag{A.4}\\
& =\left(1-\delta_{j}\right)^{\tau-1}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\} .
\end{align*}
$$

Substituting (A.3) and (A.4) into (A.2), production function can be rewritten as below:

$$
\begin{align*}
X_{j}(t, \tau) & =\left\{\left(1-\delta_{j}\right)^{\tau-1}\right\}^{v_{j}} F_{j}\left[L_{j}(t), K_{j}(t)\right] ; \quad 0<v_{j}<1 .  \tag{A.5}\\
& =\left\{\left(1-\delta_{j}\right)^{\tau-1}\right\}^{v_{j}} X_{j}(t)
\end{align*}
$$

Now, taking account of (A.3), (A.4), and (A.5), eq. (A.1) can be rewritten as below:

$$
\begin{align*}
\Pi=\sum_{j=1}^{n} & \left.\sum_{\tau=1}^{T_{j}}\left[p_{j}\left(1-\delta_{j}\right)^{\tau-1}\right\}^{v_{j}} X_{j}(t)-w_{j}\left(1-\delta_{j}\right)^{\tau-1} L_{j}(t)\right]\left(1+r_{j}\right)^{-\tau}  \tag{A.6}\\
& -\sum_{j=1}^{n} q_{j} I_{j}(t)-\lambda\left\{\sum_{j=1}^{n} q_{j} I_{j}(t)-A\right\}
\end{align*}
$$

$$
=\sum_{j=1}^{n}\left[\zeta_{j_{1}}\left(T_{j}\right) p_{j} X_{j}(t)-\zeta_{j_{2}} w_{j} L_{j}(t)-q_{j} I_{j}(t)\right]-\lambda\left\{\sum_{j=1}^{n} q_{j} I_{j}(t)-A\right\} .
$$

where,

$$
\begin{align*}
& \zeta_{j_{1}}\left(T_{j}\right)=\sum_{\tau=1}^{T_{j}}\left\{\left(1-\delta_{j}\right)^{\tau-1}\right\}^{v_{j}}\left(1+r_{j}\right)^{-\tau}  \tag{A.7}\\
& \zeta_{j_{2}}\left(T_{j}\right)=\sum_{\tau=1}^{T_{j}}\left(1-\delta_{j}\right)^{\tau-1}\left(1+r_{j}\right)^{\tau}
\end{align*}
$$

The first order conditions for maximization of the right hand side of eq. (A.1) are:

$$
\begin{equation*}
\frac{\partial \Pi}{\partial I_{j}(t)}=\zeta_{j_{1}}\left(T_{j}\right) p_{j} \frac{\partial X_{j}(t)}{\partial I_{j}(t)}-(1+\lambda) q_{j}=0, \quad j=1, \cdots, n \tag{A.9}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial \Pi}{\partial L_{j}(t)}=\zeta_{j_{1}}\left(T_{j}\right) p_{j} \frac{\partial X_{j}(t)}{\partial L_{j}(t)}-\zeta_{j_{2}}\left(T_{j}\right) w_{j}=0, \quad j=1, \cdots, n \tag{A.10}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial \Pi}{\partial \lambda}=\sum_{j=1}^{n} q_{j} I_{j}(t)-A=0 . \tag{A.11}
\end{equation*}
$$

From eq. (A.9), the law of equalization of the over-all marginal internal rate of expected return on investment between industries can be obtained:

$$
\begin{equation*}
\zeta_{i_{1}}\left(T_{i}\right) \frac{p_{i} \partial X_{i}(t)}{q_{i} \partial I_{i}(t)}=\zeta_{j_{1}}\left(T_{j}\right) \frac{p_{j} \partial X_{j}(t)}{q_{j} \partial I_{j}(t)} \quad \text { for all } \quad i \neq j \tag{A.12}
\end{equation*}
$$

and also from eq. (A.10), the equalities of marginal productivity of labor and real wage rate are obtained:

$$
\begin{equation*}
\frac{\zeta_{j_{1}}\left(T_{j}\right)}{\zeta_{j_{2}}\left(T_{j}\right)} \frac{p_{j} \partial X_{j}(t)}{w_{j} \partial L_{j}(t)}=1, \quad j=1, \cdots, n . \tag{A.13}
\end{equation*}
$$

Consequently, an identity (A.11), ( $n-1$ ) eq. (A.12), and $n$ eq. (A.13) determine $2 n$ solutions for labor input and investment of $n$ industries simultaneously, and therefore, substituting these $2 n$ solutions into the right hand side of eq. (A.5), the optimal volumes of production of $n$ industries are also determined. Equations (A.2), (A.3), and (A.4) determine the solutions for quantities of output, labor and capital stock during the planning horizon.

From eq. (A.9), the over-all marginal internal rate of expected return on investment for industry $j, \lambda_{j}$ can be written as below:

$$
\begin{equation*}
\lambda_{j}=\zeta_{j_{1}}\left(T_{j}\right) \frac{p_{j} \partial X_{j}(t)}{q_{j} \partial I_{j}(t)}-1, \quad j=1, \cdots, n \tag{A.14}
\end{equation*}
$$

Moreover, marginal efficiency of investment or marginal internal rate of expected
return on investment can be obtained by putting $\lambda_{j}=0$, and using eq. (A.7) as below:

$$
\begin{equation*}
\left[\sum_{\tau=1}^{T_{j}}\left\{\left(1-\delta_{j}\right)^{\tau-1}\right\}^{v_{j}}\left(1+r_{j}^{*}\right)^{-\tau}\right]^{-1}=\frac{p_{j} \partial X_{j}(t)}{q_{j} \partial I_{j}(t)}, \quad j=1, \cdots, n, \tag{A.15}
\end{equation*}
$$

where $r_{j}^{*}$ on the right hand side is marginal efficiency of investment. Taking the length of the planning horizon for industry $j$ to infinity, marginal efficiency of investment can be shown more clearly,

$$
\begin{equation*}
r_{j}^{*}=\frac{p_{j} \partial X_{j}(t)}{q_{j} \partial I_{j}(t)}+\left(1-\delta_{j}\right)^{v_{j}}-1, \quad j=1, \cdots, n \tag{A.16}
\end{equation*}
$$

In eq. (A.15), putting $r_{j}^{*}=r$, where $r$ is discount rate, the solution for investment $I_{j}(t),(j=1, \cdots, n)$, can be obtained for the case where there is no constraint on the budget; that is,

$$
\begin{equation*}
I_{j}(t)=G_{j}\left[p_{j}, q_{j}, r(t), \delta_{j}(t), v_{j}\right], \quad j=1, \cdots, n . \tag{A.17}
\end{equation*}
$$

The equality of marginal efficiency of investment between industries certainly holds:

$$
\begin{equation*}
r_{i}^{*}=r_{j}^{*}=r \quad \text { for all } \quad i \neq j, \tag{A.18}
\end{equation*}
$$

where $r$ is the market discount rate, that may be determined through general interdependence between all markets.

Now, let be production function as Cobb-Douglas type of the degree $v_{j}$ for industry $j$; that is,

$$
\begin{align*}
X_{j}(t)= & c_{0 j} L_{j}(t)^{c_{1}} k_{j}(t)^{c_{2 j}} e^{c_{3 j} t}, \quad \text { and } \\
& 0<c_{1 j}+c_{2 j}=v_{j}<1, \quad j=1, \cdots, n . \tag{A.19}
\end{align*}
$$

Using the equation above, and combining eq. (A.9) with eq. (A.10), and also putting $\lambda=0$ and $T_{j}=T_{i}=\infty$ as an approximation, the equation for the expansionpath can be written as below:

$$
\begin{equation*}
\frac{c_{1 j}}{c_{2 j}} \frac{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)}{L_{j}(t)}=\frac{w_{j}}{q_{j}\left(r_{j}+\delta_{j}\right)} \tag{A.20}
\end{equation*}
$$

Substituting eq. (A.20) into eq. (A.19), production function (A.19) can be rewritten as the next:

$$
\begin{align*}
X_{j}(t)= & c_{0 j}\left\{\frac{c_{1 j}}{c_{2 j}} \frac{q_{j}\left(r_{j}+\delta_{j}\right)}{w_{j}}\right\}^{c_{1 j}}  \tag{A.21}\\
& \cdot\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{c_{1 j}+c_{2 j}} e^{c_{3 j} t}
\end{align*}
$$

Differentiating the equation above with respect to $I_{j}(t)$,

$$
\begin{align*}
\frac{\partial X_{j}(t)}{\partial I_{j}(t)}= & c_{0 j}\left(c_{1 j}+c_{2 j}\right)\left\{\frac{c_{1 j}}{c_{2 j}} \frac{q_{j}\left(r_{j}+\delta_{j}\right)}{w_{j}}\right\}^{c_{1 j}}  \tag{A.22}\\
& \cdot\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{c_{1 j}+c_{2 j}-1} e^{c_{3 j} t}
\end{align*}
$$

Substituting the equation above into the right hand side of eq. (A.14), the over-all marginal internal rate of expected return on investment for industry $j$ can be written as below:

$$
\begin{align*}
\lambda_{j}= & \zeta_{j_{1}}\left(T_{j}\right) c_{0 j} c_{2 j}\left(\frac{p_{j}}{q_{j}}\right)\left\{\frac{c_{1 j}}{c_{2 j}} \frac{q_{j}\left(r_{j}+\delta_{j}\right)}{w_{j}}\right\}^{c_{1 j}}  \tag{A.23}\\
& \cdot\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{c_{1 j}+c_{2 j}-1} e^{c_{3 j} t}-1
\end{align*}
$$

The condition for the curve shown with eq. (A.23) to be down-sloping is:
(A.24)

$$
\begin{aligned}
& \frac{\mathrm{d} \lambda_{j}}{\mathrm{~d}\left(q_{j} I_{j}(t)\right)}=\frac{1}{q_{j}}\left(T_{j}\right) c_{0 j} c_{2 j}\left(c_{1 j}+c_{2 j}-1\right)\left(\frac{p_{j}}{q_{j}}\right) \\
& \cdot\left\{\frac{c_{1 j}}{c_{2 j}} \frac{q_{j}}{}\left(r_{j}+\delta_{j}\right)\right\}^{w_{j j}}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{c_{1 j}+c_{2 j}-2} e^{c_{3 j} t}<0 \\
& \text { for } 0<c_{1 j} \neq c_{2 j}<1 .
\end{aligned}
$$

## MATHEMATICAL APPENDIX B

If production function is of the semi-substitute type, the over-all marginal internal rate of expected return on investment for industry $j$ is obtained in the following procedures. Let be the technological conditions of production of industry $j$ as a pair of two relations as follows:

$$
\begin{array}{llll}
\text { (B.1) } & X_{j}(t, \tau)=a_{0 j} K_{j}(t, \tau)^{a_{1 j} ;} ; & a_{0 j}>0, & 0<a_{1 j}<1 ;  \tag{B.1}\\
\text { (B.2) } & L_{j}(t, \tau)=b_{0 j} K_{j}(t, \tau)^{b_{1 j} ;} & b_{0 j}>0, & 0<b_{1 j}<1,
\end{array}
$$

where,

$$
\begin{equation*}
K_{j}(t, \tau)=\left(1-\delta_{j}\right)^{\tau-1} K_{j}(t) . \tag{B.3}
\end{equation*}
$$

Substituting eq. (B.3) into eqs. (B.1) and (B.2), (B.1) and (B.2) can be rewritten as follows:

$$
\begin{equation*}
X_{j}(t, \tau)=a_{0 j}\left\{\left(1-\delta_{j}\right)^{\tau-1} K_{j}(t)\right\}^{a_{1 j}}=\left(1-\delta_{j}\right)^{a_{1 j}(\tau-1)} X_{j}(t) \tag{B.4}
\end{equation*}
$$

$$
\begin{equation*}
L_{j}(t, \tau)=b_{0 j}\left\{\left(1-\delta_{j}\right)^{\tau-1} K_{j}(t)\right\}^{b_{1 j}}=\left(1-\delta_{j}\right)^{b_{1 j}(\tau-1)} L_{j}(t) . \tag{B.5}
\end{equation*}
$$

Maximization of the sum of the present values of the firm under the constraint saying that the sum of funds invested should be equal to $A$ is, making use of eq. (A.6) in the appendix A , to maximize the function as below:

$$
\begin{align*}
\Pi= & \sum_{j=1}^{n}\left\{\zeta_{j_{3}}\left(T_{j}\right) p_{j} X_{j}(t)-\zeta_{j_{4}}\left(T_{j}\right) w_{j} L_{j}(t)-q_{j} I_{j}(t)\right\}  \tag{B.6}\\
& -\lambda\left\{\sum_{j=1}^{n} q_{j} I_{j}(t)-A\right\}
\end{align*}
$$

where,

$$
\begin{equation*}
\zeta_{j_{3}}\left(T_{j}\right)=\sum_{j=1}^{T_{j}}\left\{\left(1-\delta_{j}\right)^{\tau-1}\right\}^{a_{1 j}}\left(1+r_{j}\right)^{-\tau}, \text { and } \tag{B.7}
\end{equation*}
$$

$$
\begin{equation*}
\zeta_{j_{4}}\left(T_{j}\right)=\sum_{j=1}^{T_{j}}\left\{\left(1-\delta_{j}\right)^{\tau-1}\right\}^{b_{1 j}}\left(1+r_{j}\right)^{-\tau} \tag{B.8}
\end{equation*}
$$

The first order condition for maximization of eq. (B.6) is written, taking account of eqs. (B.4) and (B.5), as below:

$$
\begin{align*}
\frac{d \Pi}{d\left\{q_{j} I_{j}(t)\right\}} & =\zeta_{j_{3}}\left(T_{j}\right) \frac{p_{j} d X_{j}(t)}{q_{j} d I_{j}(t)}-\zeta_{j_{4}}\left(T_{j}\right) \frac{w_{j} d L_{j}(t)}{q_{j} d I_{j}(t)}  \tag{B.9}\\
& -(1+\lambda)=0, \quad j=1, \cdots, n .
\end{align*}
$$

Differentiating eqs. (B.4) and (B.5) with respect to $I_{j}(t)$, the following equations are obtained:

$$
\begin{align*}
& \frac{d X_{j}(t)}{d I_{j}(t)}=a_{0 j} a_{1 j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{a_{1 j}-1}  \tag{B.10}\\
& \frac{d X_{j}(t)}{d I_{j}(t)}=b_{0 j} b_{1 j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{b_{1 j}-1} \tag{B.11}
\end{align*}
$$

Substituting eqs. (B.10) and (B.11) into the right hand side of eq. (B.9), and solving (B.9) with respect to, the over-all marginal internal rate of expected return on investment for industry $j$ can be written as the next:

$$
\begin{align*}
& \lambda_{j}=\zeta_{j_{3}}\left(T_{j}\right) \frac{p_{j}}{q_{j}} a_{0 j} a_{1 j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{a_{1 j}-1}  \tag{B.12}\\
&-\zeta_{j_{4}}\left(T_{j}\right) \frac{w_{j}}{q_{j}} b_{0 j} b_{1 j}\left(I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{b_{1 j}-1}-1, \\
& \quad j=1, \cdots, n .
\end{align*}
$$

Making use of eq. (B.12), the eq. (B.9) showing the equalities between the over-all marginal rates can be rewritten as below:

$$
\begin{align*}
\zeta_{j_{3}}\left(T_{j}\right) \frac{p_{j}}{q_{j}} & a_{0 j} a_{1 j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{a_{1 j}-1}  \tag{B.13}\\
& \quad-\zeta_{j_{4}}\left(T_{j}\right) \frac{w_{j}}{q_{j}} b_{0 j} b_{1 j}\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{b_{1 j}-1} \\
= & \zeta_{i_{3}}\left(T_{i}\right) \frac{p_{i}}{q_{i}} a_{0 i} a_{1 i}\left\{I_{i}(t)+\left(1-\delta_{i}\right) K_{i}(t-1)\right\}^{a_{1 i}-1} \\
& \quad-\zeta_{i_{4}}\left(T_{i}\right) \frac{w_{i}}{q_{i}} b_{0 i} b_{1 i}\left\{I_{i}(t)+\left(1-\delta_{i}\right) K_{i}(t-1)\right\}^{b_{1 i}-1} ; \quad i \neq j
\end{align*}
$$

Similarly as before, putting $\lambda=0$, marginal efficiency of investment or marginal internal rate of return on investment by industry can be obtained as a solution of eq. (B.12), which seems to be very complicated.

The conditions for the curve of the over-all marginal internal rate of return on investment to be down-sloping is obtained by differentiating eq. (B.12) with respect to the amount of funds to be invested for industry $j, q_{j} I_{j}(t)$; that is,

$$
\begin{align*}
\frac{d \lambda_{j}}{d\left\{q_{j} I_{j}(t)\right\}}= & \frac{1}{q_{j}} \frac{d \lambda_{j}}{d I_{j}(t)}  \tag{B.14}\\
= & \frac{1}{q_{j}}\left[\zeta_{j_{3}}\left(T_{j}\right) \frac{p_{j}}{q_{j}} a_{0 j} a_{1 j}\left(a_{1 j}-1\right)\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}\right]^{a_{1 j}-2} \\
& -\zeta_{j_{4}}\left(T_{j}\right) \frac{w_{j}}{q_{j}} b_{0 j} b_{1 j}\left(b_{1 j}-1\right)\left\{I_{j}(t)+\left(1-\delta_{j}\right) K_{j}(t-1)\right\}^{b_{1 j}-2}<0 .
\end{align*}
$$

Consequently, if $a_{0 j}, b_{0 j}>0$ and $0<a_{1 j}<1, b_{1 j}<1$, the condition above is rewritten as below:

$$
\begin{equation*}
\frac{\zeta_{j_{3}}\left(T_{j}\right)}{\zeta_{j_{4}}\left(T_{j}\right)} a_{0 j 0} a_{1 j} \frac{1-a_{1 j}}{b_{0 j} b_{1 j}} \frac{p_{j}}{1-b_{1 j}} \frac{w_{j}}{w_{j}}\left\{I_{j}+(1) K_{j}(t-1){ }^{a_{1 j}-b_{1 j}}<1\right. \tag{B.15}
\end{equation*}
$$

If $a_{1 j}$ is greater than unity, the curve will turn out to be ascendant. This is the case for Sector V .

## APPENDIX C DATA

The observation period is from 1960 through 1977. Data for the period from 1970 through 1977 were taken from Annual Report on National Accounts published by Economic Planning Agency of the Japanese Government, while data for the period from 1960 through 1969 were computed making use of the backward indices for the relevant variables, the data on which were available thanks to Keio Economic Observatory. Data on gross capital stock were available for the period from 1965 through 1977 from EPA's Annual Report on National Accounts. Data
processing are as the followings:
$K(t) \quad$ EPA data for the periods 1965-1977. $K$ (1965) times the backward index for the period 1960-1964
$R(t) \quad$ Real value of deterioration of gross capital stock taken from EPA unpublished data, deflated by the index for investment goods prices reported by the Bank of Japan, for the periods 1965-1977. $R(t)=I(t)-(K(t)-K(t-1))$ for the periods 1960-1969.
$I(t) \quad=R(t)+(K(t)-K(t-1))$ for the periods 1965-1977. $I$ (1965) times the backward index for the periods 1960-1969.
$\delta(t) \quad=R(t) / K(t-1)$ for the periods 1960-1977.
$X(t) \quad$ Real output, from EPA and KEO, 1970 const. bln Yen.
$L(t) \quad$ Number of workers, from EPA and KEO, 10000 persons.
$p(t) \quad$ The net price of output, $1970=1,(=p(t) X(t) / X(t))$.
$q(t) \quad$ Prices of investment goods, $1970=1$, from the Bank of Japan.
$P(t) X(t) \quad$ The value added, from EPA and KEO, bln Yen.
$E(t) \quad$ Number of employee, 10000 persons. from EPA and KEO.
$w(t) E(t) \quad$ Incomes of employee, bln Yen, from EPA and KEO.
$w(t) \quad=w(t) E(t) / E(t)$.
$r(t) \quad$ Average interest rate on long-term loans, from the Bank of Japan.

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[^9]Fig. 3. 1961


Fig. 4. 1962


Fig. 5.


Fig. 6. 1964


Fig. 7.
1965


Fig. 8. 1966


Fig. 9.
1967


Fig. 10.
1968


Fig. 11.


Fig. 12.


Fig. 13.


Fig. 14


Fig. 15.
1973


Fig. 16. 1974


Fig. 17


Fig. 18.
1976


Fig. 19.


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[^1]:    ${ }^{1}$ The over-all rate of return on investment is defined for the whole length of the planning horizon, and the marginal internal rate of return on investment is defined for a unit period during the planning horizon.
    ${ }^{2}$ For instance, Solow (1963) discussed on this problem, and pointed out that the most productive is to obtain marginal productivity of capital.

[^2]:    ${ }^{3}$ See, for instance, Solow, op. cit.

[^3]:    ${ }^{4}$ The derivation process in detail is shown in Mathematical Appendix A.
    ${ }^{5}$ Putting $\lambda=0$, the discount rate $r_{\mathrm{i}}$ turns out to be the marginal internal rate of return on investment or marginal efficiency of investment in the sense of Keynes on which Lutz (1951) discussed. It should be noted that marginal rates equalization certainly holds for the over-all marginal internal rates of return on investment by industry as shown with eq. (1), but it does not hold for marginal efficiency of investment. See the Mathematical Appendix A, particularly eq. (A.18).
    ${ }^{6}$ See Komiya (1962) and also Ozaki (1970) and (1974).

[^4]:    ${ }^{7}$ The equation for the expansion path is approximated by that for the case of the infinite planning horizon. See Mathematical Appendix A.
    ${ }^{8}$ See Ozaki (1970) and (1974). Particularly, Ozaki (1974) presented the estimates for time series aggregates by industry in Japan. Komiya (1962) pointed out, first, that the estimates for the parameters of this type of production functions showed a considerable stability.
    ${ }^{9}$ See Mathematical Appendix B, particularly eq. (B.12).

[^5]:    ${ }^{10}$ See Mathematical Appendix B, particularly eq. (B.13).

[^6]:    ${ }^{11}$ Komiya (1962) discussed almost on the same problem and attempted, first, to use the limitational type in analyzing technological progress and economies of scale in production of steam power industry in the United States, which seemed to be quite satisfactory.

[^7]:    Note: Figures in parentheses are $t$-values for the estimates. $\bar{R}^{2}$, SE, and DW are the coefficient of determination adjusted to the degree of freedom, standard error of regression estimates, and Durbin-Watson statistic, respectively.

[^8]:    ${ }^{12}$ See Table 3 in Solow (1963).

[^9]:    * Gross capital stock is less of the construction work in progress. $K, R$, and $I$ are in 1970 constant billion Yen.

