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A GEOMETRICAL EXPOSITION OF SOME THEOREMS IN TRADE THEORY WITH AN IMPORTED INTERMEDIATE GOOD*

BHARAT R. HAZARI and D. C. SUH

I. INTRODUCTION

In recent years several interesting papers have been written that analyse the role of imported intermediate goods in the pure theory of international trade.[1] The large bulk of this literature is mathematical and concerned mainly with the concept of effective rate of protection. The object of this paper is to present a model of international trade with intermediate goods that can be represented geometrically. Our model is the same as that used by Ruffin [6] in his well known article. The model consists of two domestically produced goods, one of which uses an imported intermediate good in addition to the two primary factors of production, capital and labour.

On the basis of the above mentioned model, we explore and demonstrate several interesting results. First, in terms of a four quadrant diagram, we show explicitly: (a) the determination of production and consumption equilibrium, (b) the international price ratio between the final goods, (c) the price ratio between the imported intermediate good and the exported commodity and (d) the quantity of the imported intermediate good imported and used in the production of the final good.[2] The second issue that we explore is the consequence of a change in the price of the imported intermediate good on welfare, assuming the existence of a tariff on the final good. It is demonstrated that in such a framework an increase (decrease) in the price of the imported intermediate good *may not* result in a decrease (increase) in welfare. This is a new and important result from the policy point of view because there are many developing countries that import intermediate goods but impose very high tariffs on final goods. Finally, we analyse the implications of factor accumulation on welfare, given the presence of a tariff on the final good. It is then shown that factor accumulation may result in 'immiserizing growth'. This result extends Johnson's [5] famous theorem on 'immiserizing growth' to

* Comments received from an annonymous referee are grateful acknowledged.

[1] See for example Batra [1], Casas [2], Corden [3] and Ruffin [6].

[2] Ruffin [6] also uses a geometrical technique to present his results. However, his diagram fails to show (c) and (d) explicitly. Moreover, how exactly is the consumption equilibrium obtained in the diagram is not shown explicitly by Ruffin.

intermediate goods framework.[3] We now proceed to state the model mathematically and then present it geometrically.[4]

II. THE MODEL

In this section, we formally set up our barter model of international trade which is essentially similar to that of Ruffin [6].

Let the utility function for the country as a whole be given by:

$$U = U(D_1, D_2) \tag{1}$$

where U indicates utility and D_1 and D_2 the domestic consumption of commodities 1 and 2 respectively. As usual, we assume that U is strictly quasi-concave and U_i $(i=1, 2)$ are all positive where $U_i = \partial U/\partial D_i$. Thus, the social utility function possesses both behavioural and welfare significance.

It is assumed that part of domestic consumption of commodity 1 is imported and part of domestic production of commodity 2 is exported, so that:

$$D_1 = X_1 + M_1 \tag{2}$$

$$D_2 = X_2 - E_2 \tag{3}$$

where X_i $(i=1, 2)$ indicates the level of output, M_1 imports of good 1 and E_2 represents exports.

It is assumed that the production functions for the two commodities exhibit constant returns to scale. Commodity X_1 is produced with the help of two primary factors of production, capital and labour, in addition to the imported intermediate good M_3. The production function of commodity 1 is assumed to exhibit diminishing returns only with regard to capital and labour. Commodity X_2 is produced with capital and labour but does not use any of the imported intermediate good. The production function for X_2 exhibits diminishing returns along the isoquants. Hence, the production functions are:

$$X_1 = F_1(L_1, K_1, M_{31}) = L_1 f_1(k_1, m_{31}) \tag{4}$$

$$X_2 = F_2(L_2, K_2) = L_2 f_2(k_2) \tag{5}$$

where L_i, K_i denote the labour and capital allocation to the i^{th} sector.[5] The value of M_3 denotes the allocation of the imported intermediate good 3 to sector 1. This value depends on the inputs L_1 and K_1 which make the use of M_3 effective in the

[3] While this result is not surprising, it is presented to show an application of the geometrical technique developed in the paper.

[4] The mathematical proofs of the results contained in the paper can be obained from the authors.

[5] The referee of this paper suggested that equation (4′) be introduced which takes the form $M_3 = F_1(L_1, k_1)$. This equation has not been introduced into the main text of the paper though its verbal explanation is discussed in the text that follows equations (4) and (5).

production of X_1 according to equation (6) which is given below:

$$\bar{a}_{M1}X_1 = M_{31} = M_3 \tag{6}$$

It is obvious from (6) that each unit of X_1 requires M_3 in fixed proportions. The term $\bar{a}_{M1}$ in equation (6) represents the fixed Leontief input coefficients. k_i and m_{31} denote the capital and import intensity in sector i.

From the assumptions of profit maximization and perfect competition the following conditions regarding factor rewards can be derived:

$$W = [P_1 - \bar{a}_{M1}P_M]\frac{\partial F_1}{\partial L_1} = P_2\frac{\partial F_2}{\partial L_2} \tag{7}$$

$$r = [P_1 - \bar{a}_{M1}P_M]\frac{\partial F_1}{\partial K_1} = P_2\frac{\partial F_2}{\partial K_2} \tag{8}$$

where W denotes the wage rate, r the rental on capital, P_M the price of imported intermediate good and P_i $(i=1, 2)$ the prices of final goods.

The balance of payments equilibrium requires that the value of exports must equal the total value of imports, i.e., the sum of value of imports of final goods and the imported intermediate good. Hence,

$$P_2E_2 = P_1M_1 + P_MM_3 \tag{9}$$

It is assumed that resources are fully employed, so that:

$$k_1L_1 + k_2L_2 = \bar{K} \tag{10}$$

$$L_1 + L_2 = \bar{L} \tag{11}$$

Finally, we make one more assumption, i.e., throughout our analysis we shall assume that the country is small, hence, the prices P_1, P_2 and P_M are given exogenously.

In equations (1) to (11) we have spelled out the model that will be used to derive results.

III. RESULTS

(a) *Geometrical Presentation of the Model*

We now proceed to develop the four quadrant diagram for representing the above model and for developing the main results.

In Figure 1, in the right hand top quadrant I, we represent the production possibility schedule TT'. TT' is drawn concave to the origin.[6] This follows from the restrictions on the production functions and the assumption that the capital intensities in the two sectors are not equal. Given our assumptions that P_1, P_2 and

[6] Ruffins' [6] diagram is a subset of our diagram and only consists of our quadrant I.

P_M are given exogenously and that $\bar{a}_{M1}$ is technologically determined it follows that the free trade production point P_0 is determined by the following condition:

$$\frac{dX_1}{dX_2} = -\frac{P_2}{(P_1 - \bar{a}_{M1}P_M)} = -\frac{P_2}{V} \tag{12}$$

This implies that at the point of equilibrium the slope of the transformation curve TT' equals P_2/V and not P_2/P_1 as in the conventional model. This is so because producers in sector 1 must be guided by the value added price which is $[P_1 - \bar{a}_{M1}P_M]$. Thus, the slope of P_2/V represents the budget line facing the producers.

Since X_1 use an intermediate imported good, obviously consumption equilibrium cannot occur at the point P_0. We now proceed to determine the consumption equilibrium. In order to determine the consumption equilibrium, we have to first compute the quantity of the imported intermediate good required for producing output P_0Q of X_1. This quantity is derived in quadrant II where the following relationship is represented:

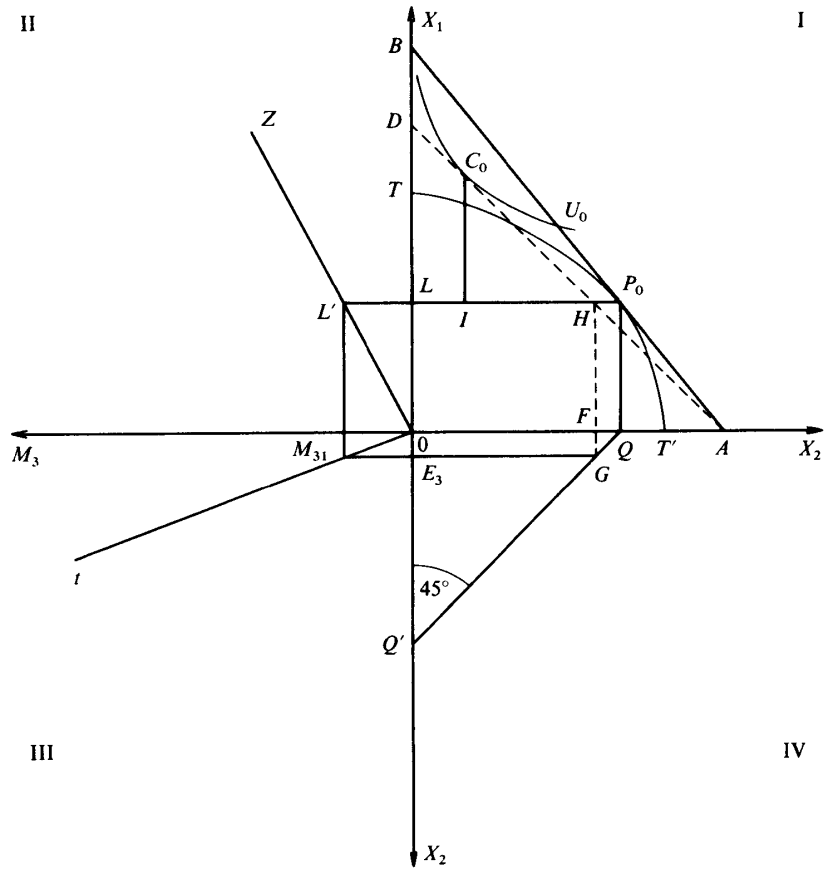

Figure 1

$$\bar{a}_{M1}X_1 = M_3$$

This equation is being represented by the line OZ. Now by drawing the line P_0L', we can determine in quadrant II the amount of intermediate good required for producing X_1. This is indicated by OM_{31}.

Obviously, if we wish to import OM_{31}, we have to pay for it in terms of exports. In quadrant III, we draw the line $0t$ representing the price ratio between commodity M_3 and the exportable good X_2. Given the price ratio $0t$, to import OM_{31}, we need to export OE_3. So from quadrant III, we have determined the amount of exports OE_3 needed for importing OM_{31} of imported intermediate good required for producing QP_0 of X_1.

The amount of X_2 that is needed for acquiring OM_{31} of the intermediate good must be subtracted from the total production of X_2. This can be done by dropping a perpendicular from P_0 onto X_2 the horizontal axis. This perpendicular hits the axis at point Q. From point Q in quadrant IV we draw the 45 degree line QQ'. Now OE_3 can be transferred to the horizontal OX_2 axis. E_3 is joined by a perpendicular with G. And G is joined by a perpendicular to F and H. Now $OE_3 = FG = FQ = HP_0$. Point H represents the output level available to the economy after the country has paid for the intermediate good.

We can now determine the consumption equilibrium. The consumers equate their marginal rate of substitution with the price ratio $P_2/P_1 < P_2/V_1$ (because $V_1 < P_1$). Since X_2 does not use the imported input nothing has to be subtracted from X_2 if no X_1 is produced in the economy. So from A we draw the price ratio P_2/P_1 as shown by line AD which passes through point H. Consumption equilibrium now occurs at C_0 and welfare is indicated by U_0. C_0I of X_1 is imported in exchange for IH of good X_2. P_0H is exported in exchange for OM_{31} of the imported intermediate good. Note that the absolute value of consumption along AHD and AP_0B are always identical to the given value of production at P_0.

(b) *Tariff Equilibrium and the Impact of a Change in the Price of Intermediate Imported Good on Welfare*

In this section of the paper, we first examine the impact of a tariff on the imported final good X_1 on welfare. Though this particular problem has been analysed in the literature earlier by Batra [1] and Ruffin [6], we reproduce their result for two reasons, viz. (a) in order to show the way in which our four quadrant diagram works and (b) to examine the impact of a change in the price of the imported intermediate good on welfare, given that a tariff on the final good already exists.

It is well known that the imposition of a tariff on the final imported good X_1 lowers welfare. We now proceed to establish this result in terms of Figure 2. In Figure 2, TT' is the normal production possibility locus. Free trade production equilibrium occurs at P_0 and consumption equilibrium at C_0. The welfare is indicated by U_0. Imports of intermediate goods are indicated by OM_{31}. Now

suppose a tariff is levied on the imports of commodity 1. This implies that the domestic price of good 1, $P_1'^* = P_1(1+t)$. Since, we have made the small country assumption $P_1'^* > P_1$, which implies that the value added price ratio P_2/V moves in favour of commodity 1. Hence, output of X_1 rises and that of X_2 falls. Suppose that the new output point is given by P'. Now in order to sustain the production of X_1 at P', we need to import the intermediate good. This is indicated by OM'_{31}. Since output of X_1 rises, given fixed coefficients the level of imports of intermediate good must also increase as is evident from $OM'_{31} > OM_{31}$. By following the procedure outlined in section (a) we know that $P'H'$ of X_2 has to be exported to import OM'_{31}. Through H' we pass the international price ratio P_2/P_1 (dashed line) and through P'_0 the value added price ratio P_2/V. Consumption equilibrium now occurs at C'. Welfare is shown by U'. Now $U' < U_0$, hence, a tariff lowers welfare. Note that with the imposition of the tarift D.R.S. $\neq P_2/P_1$ and D.R.T. $\neq P_2/V$. The mechanism through which we arrive at the new equilibrium position is indicated by the dashed lines.

We now proceed to establish the result that in the presence of a tariff on the final

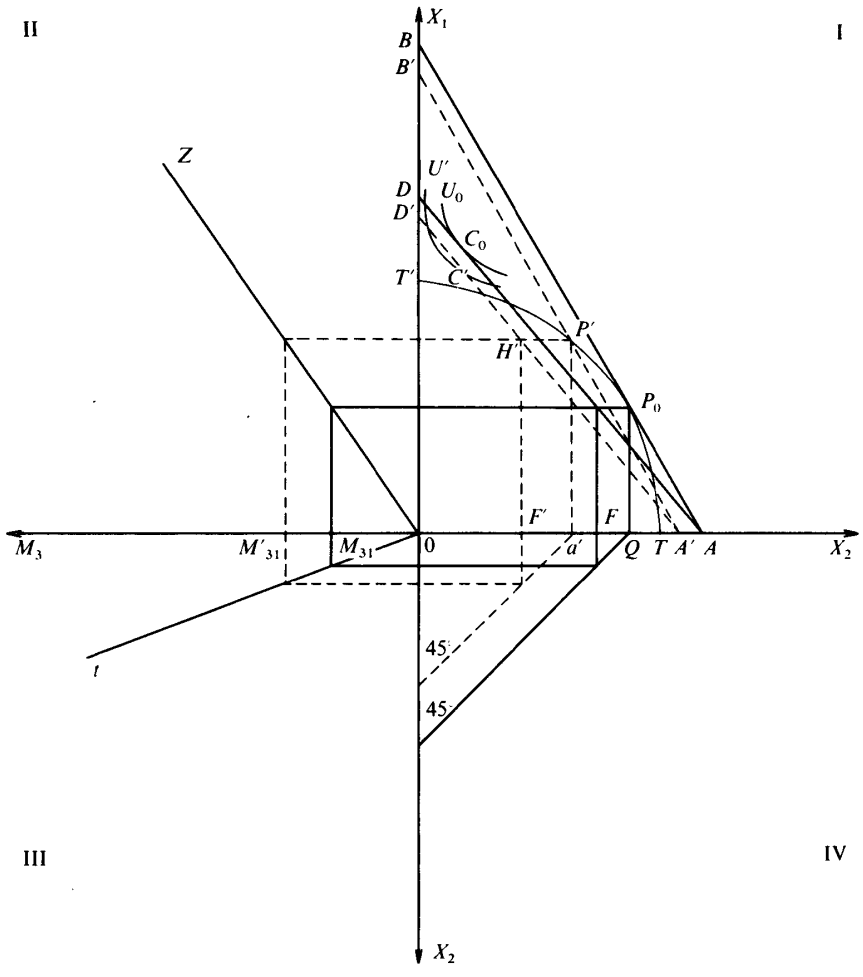

Figure 2

good an increase in the price of the imported intermediate good may increase welfare. This is done in Figure 3. Suppose that the price of the intermediate good increases. This implies that the price line shifts from $0t$ to $0t'$ in quadrant III. Given this shift, we now have to determine the new equilibrium. The original tariff equilibrium is indicated by production point P_0, consumption point C_0 and welfare level U_0. An increase in the price of the intermediate good shifts the value added domestic price in favour of commodity 2. Hence, output of commodity 2 increases and that of commodity 1 falls. New production equilibrium occurs at P'. The dashed lines again indicate the mechanism through which we determine point H through which the international price that consumers face passes, consumption equilibrium occurs at point C'. Welfare level is indicated by U'. Now $U' > U_0$, hence, welfare has increased despite the increase in the price of the imported intermediate good.

The intuitive explanation of the above result is the following: An increase in the price of the imported intermediate good changes the price ratio that is relevant to

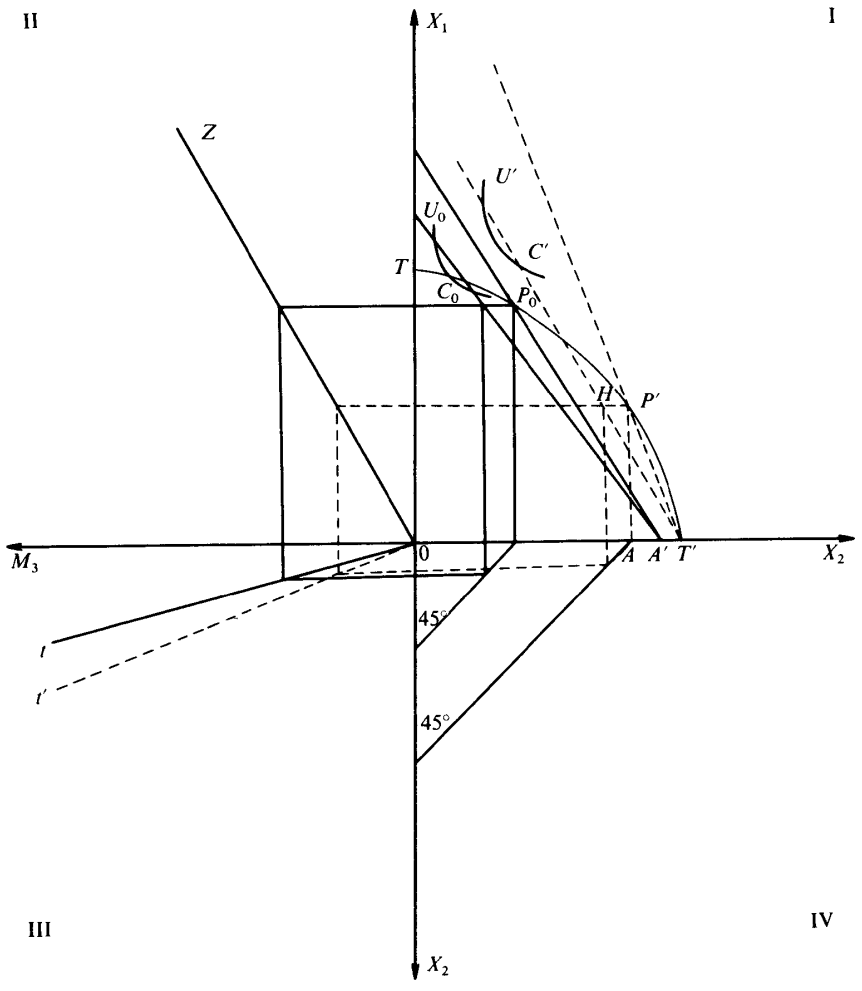

Figure 3

the producers. Hence, it triggers substitution in production along the transformation curve. This substitution is in favour of commodity X_2. The imposition of the tariff in the original situation has resulted in an increase in output or X_1 compared with X_2. Hence in the new situation, the movement in production, i.e., an increase in the output of X_2 is towards the free trade point. This movement eliminates some of the production loss associated from the original imposition of the tariff. The consumption loss due to the imposition of the tariff is constant. In Figure 3 we have shown a case in which the reduction in production loss outweighs the consumption loss resulting from the deterioration in the terms of trade, hence, an increase in the price of the intermediate good leads to an increase in welfare.[7]

This is a highly significant result because many developing countries impose high tariffs on final goods and import intermediate goods that are used in fixed proportions. The above result demonstrates that in the presence of tariffs a rise in the price of the intermediate good may be a good thing from the welfare point of view of the less developed country. In fact, such an increase (given the tariff) leads them at least to produce at more efficient points.

(c) *Factor Accumulation and Welfare in the Presence of a Tariff*

In this final section, we examine the impact of factor accumulation on welfare

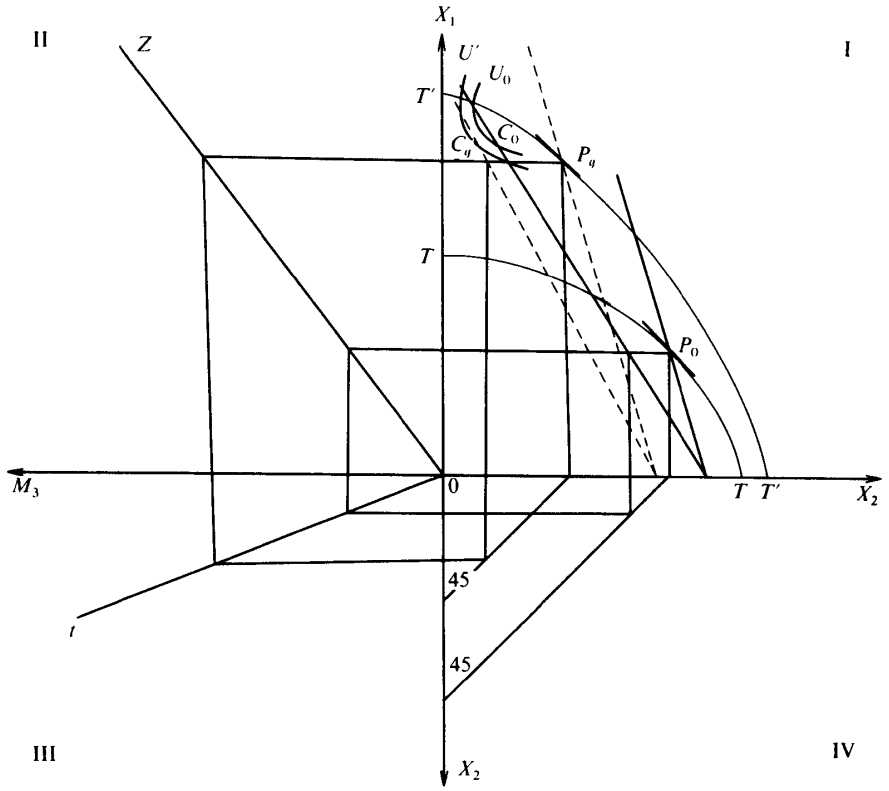

Figure 4

[7] This is a new result and is analogous to some results on terms of trade movement and welfare in the wage differential framework in the standard 2×2 model. On this see, for instance, Hazari [4].

assuming that there exists a tariff on the imported final good. Since there are two primary factors of production capital and labour whose supply is exogenously given, factor accumulation can occur either through an increase in both of them or in one of them. To keep matters simple, we shall assume that the supply of capital increases.

In Figure 4 we assume that the pregrowth transformation curve is given by TT. The pregrowth production equilibrium occurs at point P_0 and consumption equilibrium at point C_0. Due to the presence of the tariff on X_1, D.R.S. $\neq P_2/P_1$ and D.R.T. $\neq P_2/V$. Welfare is indicated by U_0. Let us now suppose that the stock of capital increases. This results in the new production possibility curve $T'T'$. Let us assume that X_1 is capital-intensive compared with X_2. From Rybczynski [7] theorem, we know that the output of X_1 increases and that of X_2 falls. Hence, production equilibrium on the post growth production possibility curve occurs at point P_g. By following the procedure outlined in section (a) of the paper, consumption equilibrium is obtained. This consumption equilibrium occurs at C_g. The welfare level associated with C_g is indicated by U'. Now $U' < U_0$, hence, welfare falls as a result of capital accumulation and 'immiserizing growth' occurs.

The intuitive explanation of this result is quite simple. Given X_1 is capital-intensive an increase in capital leads to an increase in the output of commodity X_1. However, in the pregrowth situation X_1 is already being overproduced compared with the optimal production point due to the presence of a tariff. Hence, capital accumulation further accentuates the production loss present in the model due to the tariff. This loss is the source of 'immiserizing growth' in Figure 4.

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