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ON THE GLOBAL STABILITY OF THE MORISHIMA SYSTEM

KOJI OKUGUCHI*

1. INTRODUCTION

Morishima [7] proved the global stability of the competitive equilibrium in the Morishima system, where all non-numéraire goods are classified into two non-overlapping groups, arbitrary two goods in the same group being gross substitutes and arbitrary two goods belonging to the different groups gross complements.

Let there be $m+1$ goods and let goods 0 be the numéraire. Divide m non-numéraire goods into two groups, R and S. The Morishima system is characterized by the following sign patterns for the excess demand functions.

$$(*) \qquad \left.\begin{array}{l} \operatorname{sgn} E_{ij} = \operatorname{sgn} E_{ji} \\ \operatorname{sgn} E_{ik}E_{kj} = \operatorname{sgn} E_{ij} \end{array}\right\} , i, j, k \in R \cup S$$

where $E_i = E_i(P)$ is the excess demand function for the i-th goods, P is a vector of prices of all non-numéraire goods and $E_{hj} = \partial E_h / \partial p_j$, etc. This means that "substitutes (in the gross sense) of substitutes are substitutes, complements of complements are substitutes and substitutes of complements are complements". It was shown by Morishima [6] that (*) is equivalent to (M) below.

$$(\mathrm{M}) \qquad \begin{array}{l} E_{hj} > 0,\ h \neq j,\ h, j \in R:\ E_{hk} < 0,\ h \in R,\ k \in S \\ E_{ij} < 0,\ i \in S,\ j \in R:\ E_{ik} > 0,\ i \neq k,\ i, k \in S\ , \end{array}$$

Morishima proved that the condition (N) specified below, the Walras law and homogeneity of degree zero of excess demand functions ensure the global stability of the competitive equilibrium.

$$(\mathrm{N}) \qquad \begin{array}{l} E_{h0} + 2 \sum_{k \in S} E_{hk}\, p_k > 0,\ h \in R \\ E_{i0} + 2 \sum_{j \in R} E_{ij} p_j + E_i > 0,\ i \in S \end{array}$$

The system of inequalities (N) lacks in symmetry in that the condition for $h \in R$ and that for $i \in S$ are not symmetric.

It should be noted here that the Morishima rule (M) is assumed to hold only for non-numéraire goods. Indeed, Arrow and Hurwicz [1] proved that the Morishima rule is not valid for all goods including the numéraire. Furthermore, Kennedy [3]

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proved that the application of the Morishima rule to all goods entails the instability of the competitive equilibrium.[1] In order to prove the global stability under a symmetric condition, Morishima introduced additional conditions, i.e., regularity condition and weak symmetry. However, the Walras law was not used in his proof of the global stability under the symmetric condition and his proof was rather intricate. Ohyama [8] gave a simple proof of the local stability, using homogeneity of excess demand functions. He pointed out that a similar proof was possible, based on the Walras law. Ichioka [2] derived generalized sufficient conditions for local stability by simultaneous utilization of the Walras law and homogeneity.

In this paper we shall be concerned with deriving alternative symmetric stability conditions for the Morishima system, having in mind rehabilitation of the role of the Walras law.

2. THE SYMMETRIC CONDITIONS AND STABILITY

Let us first consider the following symmetric condition consisting of (N_1) and (N_2).

$$(N_1) \quad \begin{aligned} &E_{h0} + 2\sum_{k\in S} E_{hk}p_k > 0, \ h\in R \\ &E_{i0} + 2\sum_{j\in R} E_{ij}p_j > 0, \ i\in S \end{aligned}$$

$$(N_2) \quad \begin{aligned} &E_{0j} + 2\sum_{i\in S} E_{ij}p_j + E_j > 0, \ j\in R \\ &E_{0k} + 2\sum_{h\in R} E_{hk}p_h + E_k > 0, \ k\in S \end{aligned}$$

Though the first set of conditions (N_1) is independent of the values of excess demand functions, the second set (N_2) is not so. However, in the case of local stability, the excess demand terms in (N_2) vanish. Other things being equal, (N_2) is more likely to be satisfied if excess demands for all non-numéraire goods are non-negative during the adjustment periods.

In the tâtonnement prices are assumed to be adjusted according to

$$dp_i/dt = E_i(P), \qquad i = 1, 2, \cdots, m .$$

We assume all of the conditions adopted by Morishima for the existence of the unique equilibrium price and that all prices remain strictly positive until the equilibrium is attained.

The global stability of the equilibrium under (N_1) and (N_2) is established as

[1] It is reported that Iritani proved that if the Morishima rule is applied to all goods and if, in addition, substitutes (in net sense) of substitutes are always substitutes, there exists no pair of complementary goods.

follows. From the homogeneity of excess demand functions,

(1) $$\sum_{j\in R} E_{hj}p_j+\sum_{k\in S} E_{hk}p_k+E_{h0}=0,\ h\in R$$

(2) $$\sum_{j\in R} E_{ij}p_j+\sum_{k\in S} E_{ik}p_k+E_{i0}=0,\ i\in S$$

Differentiating the Walras law with respect to p_j and p_k,

(3) $$\sum_{h\in R} E_{hj}p_h+\sum_{i\in S} E_{ij}p_i+E_{0j}+E_j=0,\ j\in R$$

(4) $$\sum_{h\in R} E_{hk}p_h+\sum_{i\in S} E_{ik}p_i+E_{0k}+E_k=0,\ k\in S$$

The condition (N_1) together with (1) and (2) yields

(5) $$\sum_{j\in R} E_{hj}p_j-\sum_{k\in S} E_{hk}p_k<0,\ h\in R$$

(6) $$-\sum_{j\in R} E_{ij}p_j+\sum_{k\in S} E_{ik}p_k<0,\ i\in S$$

From (N_2), (3) and (4) we get

(7) $$\sum_{h\in R} E_{hj}p_h-\sum_{i\in S} E_{ij}p_i<0,\ j\in R$$

(8) $$-\sum_{h\in R} E_{hk}p_h+\sum_{i\in S} E_{ik}p_i<0,\ k\in S$$

Inequalities (5) and (6) show that $A(P)=[E_{ij}(P)]$, the Jacobian matrix of the excess demand functions, is a matrix with negative dominant diagonals everywhere, and (7) and (8) that $A'(P)$ is a matrix with negative dominant diagonals everywhere.[2] Hence a symmetric matrix $A(P)+A'(P)$ is shown to be a matrix with negative dominant diagonals, its characteristic roots all being negative.[3] It follows that $A(P)$ is quasi-negative definite, leading to the global stability of the competitive equilibrium[4] under the symmetric conditions (N_1) and (N_2). Incidentally, it should be noted that quasi-negative definiteness of $A(P)$ implies that the matrix is totally stable, hence that it is Hicksian. See Metzler [5] and Quirk and Saposnik [9, pp. 166–7].

Let us next introduce the following conditions. The corresponding local version of these condition was used by Ichioka [2].

(N_1^*) $$2\Big(\sum_{k\in S} E_{hk}p_k+\sum_{k\in S} E_{kh}p_k\Big)+E_{h0}+E_{0h}+E_h>0,\ h\in R$$

[2] Note that both P and $A(P)$ depend on t.

[3] McKenzie [3].

[4] Arrow and Hurwicz [1].

$$(N_2^*) \qquad 2\Big(\sum_{j\in R} E_{ij}p_j + \sum_{j\in R} E_{ji}p_j\Big) + E_{i0} + E_{0i} + E_i > 0, \; i\in S$$

It is noted that (N_1) and (N_2) together imply (N_1*) and (N_2*), but the reverse implication is not necessarily true. To prove the global stability under the set of conditions (N_1*) and (N_2*), rewrite (3) and (4) as

$$(3') \qquad \sum_{j\in R} E_{jh}p_j + \sum_{k\in S} E_{kh}p_k + E_{0h} + E_h = 0, \; h\in R\,,$$

$$(4') \qquad \sum_{j\in R} E_{ji}p_j + \sum_{k\in S} E_{ki}p_k + E_{0i} + E_i = 0, \; i\in S\,.$$

We then have from (1), (3′) and (N_1*)

$$(9) \qquad \sum_{j\in R} (E_{hj} + E_{jh})p_j - \sum_{k\in S} (E_{hk} + E_{kh})p_k < 0, \; h\in R$$

On the other hand, (2), (4′) and (N_2*) yield

$$(10) \qquad -\sum_{j\in R} (E_{ij} + E_{ji})p_j + \sum_{k\in S} (E_{ik} + E_{ki})p_k < 0, \; i\in S$$

From (9) and (10) the symmetric matrix $A(P) + A'(P)$ is seen to have negative dominant diagonals, hence the global stability.

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