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TAX DISTORTIONS AND THE RATE OF ECONOMIC GROWTH: A NUMERICAL GENERAL EQUILIBRIUM ANALYSIS

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While economists have long realised that various tax distortions may reduce the benefits to an economy of increases in its production capabilities which come about from a growth of factor supplies or improvements in its technology, little has been done to ascertain the magnitude of these losses or to determine the degree to which they depend on the values of key structural parameters of an economy. Those who have constructed growth models of economies have tended to be concerned with such problems as the definition of capital and the asymptotic behaviour of their models and to omit taxes and the government sectors from their models.[1] On the other hand, those economists who have attempted to analyse or measure in general equilibrium models the consequences of government taxes have almost invariably confined their attention to one period situations in which there can be no change in the production capabilities of the economy. (See, for example, the recent works of Shoven and Whalley (1972 and 1973) and Whalley (1975).) This paper seeks to explore the relationship between rates of taxes on outputs and inputs on the one hand and rates of growth of output and consumption on the other by means of numerical analysis of a multi-commodity Neoclassical model.

The model is a generalisation of that used by Bhagwati (1958 and 1968) and others to examine the phenomenon of "immiserising growth". In particular, it follows in the spirit of Johnson (1967) who was the first to demonstrate that a tax, in his case a tariff on imports, rather than an adverse movement of the commodity terms of trade, may be the cause of immiserising growth. Unfortunately, this literature still suffers from a number of deficiencies. With the exception of the work of Ohyama (1972), the models have not progressed beyond two or at most three commodities and they have used calculus as the method of anlaysis. The first restriction limits the interrelationships among commodities and the taxes which can be considered and the second proves results which hold only locally. These models have assumed there is only one consumer or, what amounts to the same thing in terms of the model, they have employed a social utility function. Finally, the concentration on the rather extreme case of negative growth has tended to divert attention from the more general possibility that taxes and/or terms of trade

* I am greatly indebted to John Whalley for providing the algorithm used to compute competitive equilibria and to referees for suggesting improvements.

[1] For example, see the selection of models in Burmeister and Dobell (1970).
There have been some analyses of alternative tax regimes and their effects on the growth of an economy in macro-planning models of individual economies.

effects may reduce the benefits of increases in the production capabilities of an economy without necessarily negating them completely.

We consider growth in the production capabilities of one country which trades final commodities with the Rest of the World. The model used is capable of handling a large finite number of commodities, taxes which are differentiated among input and output commodities and among buyers of individual commodities and discrete variations in these tax levels. It can also examine the question of how growth affects different individual classes of households in a growing economy. This question is ignored in virtually all formal growth models but it is a major concern of policy-makers.

The method of solution is to obtain a competitive equilibrium solution of the world economy at each period of time. Numerical algorithms developed since the path-breaking work of Scarf (1967 and 1973) enable us to go beyond the two or three commodity range. Thus one can obtain, for any specification of the economy of the home country and its trading partner, the actual solution values of all endogenous variables and thereby assess the effects of variations in the tax set of the home country.

Our comparisons are still, however, limited to the comparative static technique of comparing two points in time, the "pre-growth" and the "post-growth" situations, and to a growth in factor supplies or improvements in technology which are given exogenously to the model. These are serious limitations. In particular, we cannot consider the links between tax regimes and consumption growth which come about because the higher average incomes, different relative prices, or new distribution of incomes under one regime compared to another may affect the rate of capital formation or population growth. In principle, the method of computing competitive equilibria could be used to compute a moving equilibrium timepath for a continuously growing economy and even an economy in which capital formation and labour force growth are determined endogenously. But operational algorithms which can compute such equilibria do not yet exist.

Section I specifies the model and presents the parameter sets. Section II examines an illustrative example of growth and growth indices. It also presents the Ohyama decomposition of the Hicks Index. This decomposition provides a useful method of following the interactions between an increase in aggregate real output, the movement of the terms of trade, and tax revenues on the one hand and, on the other, aggregate real consumption and the consumption of individual households in the economy. In Section III the results of some simulations of different tax regimes and their effects on the growth performance of the economy are presented. Section IV uses the same model to compare the gains from increases in production capabilities with the gains which would accrue from tax reforms in the home country or from growth in the Rest of the World or from increase in access to the markets of the Rest of the World.

I

There are two countries, country 1 and 2, which produce four final commodities each, commodities (1, 2, 3, 4) and (5, 6, 7, 8) respectively. At any time period each country has stocks of two primary inputs, labour and capital, which are owned by the households of that country. Both the set of commodities produced and the set of primary input commodities in one country are distinct from the sets of the other country. That is, there is a total of 12 commodities, all of which are distinguished by their country of origin (the country in which they are produced for final commodities and the country in which they are located for inputs). It is assumed that all eight final commodities are consumable and internationally-tradable at zero transport costs. Hence, the set of consumable commodities in each country is the set of consumables in the world economy. Whereas final commodities are assumed to be only available from current production, primary inputs are only available from the country's own stocks because they are not tradable. These assumptions on the supply side of the model are almost universal in the multisectoral models of growth and in the static models of competitive equilibria. On the demand side there are four consumers, consumers (1, 2) and (3, 4) living in countries 1 and 2 respectively. Each country has a government.

Further aspects of the world in which growth takes place are described most simply in terms of the behaviour of the three types of agents, the producers and households and government, in both countries. These descriptions of the economies of the two countries relate to one period of time and, therefore, time scripts for all variables are omitted. It is assumed below that the growth of production capabilities and the variations in tax sets occur in only one country, country 1, and the paper is concerned almost exclusively with the effects of these changes on *this country* alone.

The production of each of the eight commodities is given by a CES production function, using only inputs of the two primary inputs, capital (K) and labour (L), with which the country is endowed, viz.

$$y_j^k = A_j[\delta_j (L_j^k)^{-\rho_j} + (1-\delta_j)(K_j^k)^{-\rho_j}]^{-1/\rho_j} \qquad (1)$$

$$j = 1, \cdots, 8$$
$$k = 1, 2$$

where A_j is the normalising constant which defines units of measurement for outputs and δ_j and ρ_j are the distribution and elasticity parameters. Units of capital and labour are assumed to be intranationally mobile, malleable and without occupational preference.

Given the production functions for one country and this country's endowments of primary inputs, the country's *attainable production set* is defined as the set of all outputs that can be produced given the stock of primary inputs of the country, and no other constraints. *Growth* in the sense of a growth of production capabilities can be understood more precisely as an expansion of this attainable production set.

Producers of each commodity seek to maximise the industry profits for any set of final commodity prices ($\tilde{p}$) and input prices ($\tilde{w}$). This profit-maximising behaviour determines general equilibrium *commodity supply functions*, $y_j(\tilde{p},\tilde{w}) = y_j(\tilde{p})$ for each commodity, which are homogeneous of degree zero in $\tilde{p}$. Given the definition of commodities, these are world supply functions.

There are two consumers in each country. This is the minimum number to permit growth to affect individual households differently and to allow for redistribution among households via government taxation. Households play a dual role. They are consumers and they are owners of the stocks of primary inputs.

Each consumer has a Sato (1968) two-level CES utility function in which the eight commodities are grouped into four groups of two commodities, each group consisting of one commodity produced in country 1 and another produced in country 2:

$$(2) \qquad U_h = \left[\sum_{s=1}^{4} \alpha_{hs} \bar{x}_{hs}^{\ -\rho_h} \right]^{-1/\rho_h} \qquad h = 1, 2, 3, 4$$

where $\bar{x}_{hs}$ is a CES index of the quantity of the s group consumed by consumer h of the form

$$\bar{x}_{hs} = [a_{hd_s} x_{hd_s}^{\ -\rho_{hs}} + (1 - a_{hd_s}) x_{he_s}^{\ -\rho_{hs}}]^{-1/\rho_{hs}} \qquad h = 1, 2, 3, 4 \quad s = 1, 2, 3, 4$$

The subscripts d_s and e_s denote the domestically-produced and imported commodities of group s at the bottom level. The commodity groups are the same for all four consumers but the utility function distribution and elasticity parameters differ among individuals at both the top and bottom levels.

Each consumer maximises his utility subject to his budget constraint. He obtains his income from his ownership of capital and labour and from his share of the distribution of the aggregate receipts from all forms of taxation in his home country. This maximisation yields a set of demand functions $x_h(p,I_h)$ which are continuous, single-valued and homogeneous of degree zero. The *commodity demand functions* for the world markets, $x = \Sigma_h x_h(p,I)$ where $I = I_1, \cdots, I_4)$, are, therefore, continuous and homogeneous of degree zero.

The use of CES functions has the great advantage of allowing us to observe directly and to vary if desired the elasticities of substitution in production and consumption simply by observing or varying the appropriate substitution parameters. The elasticities of substitution in production in particular have, since the time of Hicks' (1932) seminal work, played a major role in growth theory discussion of the effects of increases in factor supplies or of technology on factor incomes. In production the Allen partial elasticities of substitution between any two input commodities produced is $\sigma_j = 1/(1 + \rho_j)$. As a special case, as $\sigma_j \rightarrow 0$ for all j we get the Leontief technology. The use of the two-level CES function in demand gives greater flexibility in that the Allen partial elasticities of substitution are not constrained to be equal for all pairs of commodities (see Sato (1968)). These utility functions also allow us to group domestically-produced and imported commodities. Thus every domestically-produced commodity has an imperfect substitute. The degree of substitutability (σ_{ij}) may vary among groups; that for "automobiles" differing from that for the "clothing" group. As a special case, as $\rho_{hs} \to -1$ for all h and s, we get the standard Neoclassical assumption that the two countries produce identical sets of commodities.

The production function parameters have been chosen so that two commodities which are in the same consumption group have similar production parameters, σ_j and δ_j. These groups may be called "industries". Each industry in one country competes with the product of the industry in the other country, both products being close substitutes in consumption and produced by a similar technology. These assumptions have the important consequence that the dichotomy between import and export industries in traditional trade-and-growth models no longer exists. Each industry is both an export and an import industry.

The government of the country enters only as an imposer of taxes on the consumption of commodities and the use of primary inputs and as a disburser of the net revenue collected. It does not produce or purchase commodities. Thus the only purpose of government activity is to redistribute incomes and purchasing power among individual households. A particular distribution of taxes in country k is called a *distribution policy*.

Each country imposes a vector of *ad valorem consumption taxes* $t^k = (t_1{}^k, \cdots, t_8{}^k)$ and a vector of *ad valorem input taxes* $\tau^k = (\tau_{ij})$ whose element τ_{ij} is the *ad valorem* rate of tax on the use of input i in the production of commodity j within each country. There are no inter-consumer differentials in consumption taxes but there are inter-commodity differentials in input taxes. Since the set of home-produced commodities and the set of imported commodities in a country are disjoint, each country is imposing a set of four excise taxes and four tariffs on the commodities which it produces and imports respectively. Negative taxes are perimitted provided they do not exceed 100 per cent of the world price.

The consumption taxes distort both producer and consumer choice and cause the familiar losses of welfare to the economy. Input taxes also distort prices to producers. Another consequence of the input market distortions is that the *realisable production set* of profit-maximising outputs lies in the interior of the attainable production set, except for the points of complete attainable production set (see Magee (1973)). This is an additional source of welfare loss. We are concerned with the effect of all of these losses in reducing the gains from growth of the attainable production set in country 1.

In order to carry out simulations one must specify the parameters. The production and consumption function parameters are given in Table 1. In consumption the elasticity of substitution is taken to be greater between paired commodities at the bottom level (1.5 in all groups) than between groups at the top-level (0.75). Moreover all top-level parameters are identical for all four consumers.

This implies there is a uniform pattern of tastes in broad commodity groups throughout the world. At the bottom level there are intra-national and international differences in tastes. This has been done so that in the first two groups ("industries"), each consumer attaches a greater weight to the commodity which is domestically-produced; that is, there is a systematic bias towards the domestically-produced commodity in some industries.

The factor endowments of Table 1 indicate that country 1 is a small part of the world economy.[2]

The initial tax set is given in Table 2. As with the non-tax parameters, the numbers chosen are intended to be illustrative but realistic. This set represents a tax-ridden world economy. There are high barriers to imports in both countries and high differentials in input taxes, both between industries for the tax on one input and between the tax rates on inputs in one industry. These differentials represent the fact that taxes on value added or wage or capital incomes are differentiated by industry. Inter-commodity and inter-factor differentials in tax rates are common to developed countries.[3] High rates here have been chosen deliberately so as to encompass situations with more differentiated rate structures. Other cases with less differentiation are covered when all taxes and sub-sets of taxes are reduced uniformly in the simulations reported in Sections III and IV.

The method used in this paper is to make binary comparisons, for any given set of tax (and other) parameters, between the pre-growth situation and the post-growth situation. Then the rates of growth between these terminal situations of various indices of output, consumption and welfare are computed. Finally the various growth indices under different tax regimes are compared. These comparisons isolate the effects of these tax variations on the rate of growth of output and other variables, holding constant the growth of the economy's attainable production set and preferences and all other parameters.

These computations require the solution values of all variables, given a tax set, in the pre-growth and the post-growth situations. This is done by computing for each situation a *competitive equilibrium for the world economy* in which world demand equals world supply for each economy. (A full definition of this equilibrium is given by Shoven and Whalley (1974).) In this equilibrium there is a vector of prices for the 12 commodities in the world economy including the 6 commodities of country 2 as well as those of country 1: $\tilde{p}=(\tilde{p}, \tilde{w})=(\tilde{p}_1,\cdots,\tilde{p}_8;$

[2] It is not possible to measure national size in terms of either some index of productive capacity or some index of the quantity of primary input endowments since the sets of input and output commodities are disjoint. However, since the pairs of commodities produced in the two commodities are highly substitutable and the technologies are similar for the production of commodities within the same group, we may say that country 2 is very roughly 100 times the size of country 1.

The assumption that a country be small in the sense that it has no influence on the market price of any commodity it buys or sells on the world markets, is a logical impossibility in a competitive model of a world economy with a finite number of countries.

[3] Some estimates of factor taxes and consumption taxes may be found for the UK (Whalley (1975) and the EC, Whalley (1975)). Shoven and Whalley (1973) gives estimates of factor taxes in the US.

TABLE 1. BASIC NON-TAX PARAMETER SET

PRODUCTION

Commodity	A	ρ_j	δ_j
1	0.80	1.30	0.60
2	1.50	0.75	0.50
3	1.25	1.75	0.70
4	1.50	1.30	0.45
5	0.80	1.30	0.60
6	1.20	0.75	0.50
7	1.60	2.00	0.70
8	2.00	1.25	0.45

CONSUMPTION

Top-Level of utility function

Consumer	ρ_h	α_1	α_2	α_3	α_4
$h=1, 2, 3, 4$	0.75	0.30	0.30	0.20	0.20

Bottom-Level of utility function

Consumer (h)	ρ_{hs}	a_{hd_1}	a_{he_1}	a_{hd_2}	a_{he_2}	a_{hd_3}	a_{he_3}	a_{hd4}	a_{he4}
1	1.5	0.20	0.05	0.20	0.05	0.20	0.20	0.05	0.05
2	1.5	0.15	0.10	0.15	0.10	0.15	0.15	0.10	0.10
3	1.5	0.05	0.20	0.05	0.20	0.05	0.05	0.20	0.20
4	1.5	0.10	0.15	0.10	0.15	0.10	0.10	0.15	0.15

Consumer	Initial Input Endowments: Capital	Initial Input Endowments: Labour	Shares in Tax Revenue
1	50	100	0.5
2	150	100	0.5
3	5,000	10,000	0.5
4	15,000	10,000	0.5

$\tilde{w}_1,\cdots,\tilde{w}_4$). Both $\tilde{p}$ and $\tilde{w}$ are net of taxes. That is, they are the prices received by producers for the sales to consumers of the final commodities and the prices received by consumers for sales of the primary inputs to producers. The vector of market or consumer prices of the final commodities in country k is obtained from the prices p by the relation $p^k=\tilde{p}(1+t^k)$ whose j- th element is $p_j^k=\tilde{p}_j^k(1+t_j^k)$. The vector of input prices in one country, including input taxes, in one industry j is obtained from the equilibrium world prices by the relation $w_j=\tilde{w}(1+\tau_j)$ whose i-th element is $w_{ij}=\tilde{w}_i(1+\tau_{ij})$.

The assumptions regarding the structure of the two economies and the restrictions on the functions, and in particular the property of homegeneity of

TABLE 2. BASIC TAX PARAMETER SET

Country 1	1	2	3	4	5	6	7	8
Consumption tax rate	0.2	0.6	−0.3	0.5	0.5	0.2	0.2	0.3
Capital tax rate	0.0	0.0	0.3	0.6				
Labour tax rate	0.6	0.3	0.0	0.0				
Country 2								
Consumption tax rate	0.5	0.2	0.2	0.3	0.1	0.2	0.0	−0.1
Capital tax rate					0.0	0.0	0.3	0.6
Labour tax rate					0.6	0.3	0.0	0.0

degree zero of the commodity demand and supply functions, are such that the competitive equilibrium may be computed by an algorithm which is derived from Shoven and Whalley (1974). Because of the homogeneity of degree zero of both the demand and supply functions, one need consider only normalised price vectors which lie in the unit simplex.

$$S=\left\{P\colon \sum_{j=1}^{8} \tilde{p}_j+\sum_{i=1}^{4} \tilde{w}_j=1;\tilde{p}_j,\ \tilde{w}_i\geq 0\right\} \tag{3}$$

The algorithm employed is essentially a search procedure on the unit simplex of these prices augmented by the vector of revenues, (P, R). Augmentation is required because $x_j=x_j(P, R)$ and in each trial a value of R must be taken for demand to be calculated.

Under competitive equilibria all the endogenous variables are, *ceteris parabus*, a function of the tax sets (t^k, τ^k). Varying the tax sets of country 1 generates new equilibria for the pre-growth and post-growth situations, enabling us to isolate the effects of the tax variations on rates of growth of endogenous variables. It is assumed that the country 1 is free to change its taxes with no retaliation from country 2.

II

In order to gain some appreciation of the model and of the effects of growth in production capabilities in this model, Table 3 gives the solution values of the principal endogenous macro-variables for one pre-growth and one post-growth situation. The pre-growth structure of the two national economies including the stocks of capital and labour are as described above in Tables 1 and 2. The post-

TABLE 3. COMPARISON OF THE PRE-GROWTH AND POST-GROWTH COMPETITIVE EQUILIBRIA

	Commodities							
	1	2	3	4	5	6	7	8
Price	3.766	1.744	1.782	1.938	0.164	0.095	0.061	0.063
	(3.743	1.726	1.810	1.904	0.170	0.098	0.063	0.065)
Production	36.87	143.66	52.00	26.34	5670.73	6063.73	6855.95	7195.31
	(39.39	152.68	54.03	28.62	5669.53	6044.84	6880.10	7200.23)
Aggregate Consumption	8.30	5.66	37.98	7.72	654.71	1385.89	1123.33	1643.04
in Country 1	(8.99	6.22	39.54	8.47	664.81	1410.25	1141.45	1662.98)
Aggregate Consumption	28.57	138.00	14.02	18.62	5016.01	4667.84	5732.61	5552.27
in Country 2	(30.40	146.47	14.49	20.14	5004.72	4364.60	5738.65	5537.25)

Note: The prices have to be divided by 12 to obtain the prices in the unit simplex.

growth situation is identical in all respects except that the aggregate stock of capital has been assumed to have grown by 10 per cent and there has been an exogenous 2 per cent Hicks-neutral increase in technology in all four industries of country 1.[4] The increase in the capital stock is all held by individual 1 who is the "capitalist", owning 75 per cent of the capital stock in the pre-growth situation. The 10 per cent increase in its capital stock and the rate of technological change are of the order of the annual increases experienced in many countries.

Growth in country 1 induces changes in the equilibrium world prices (net of taxes) of all eight commodities and consequential changes in the production and consumption of all commodities in both countries. Country 1 is small in relation to the Rest of the World (country 2) but its growth does have a small effect on prices on world markets. Looking at the prices of the individual final commodities in country 1, we find that the price of commodity 3 has risen whereas the prices of the commodities 1, 2 and 4 have fallen. This is an illustration of the duality of the factor prices and final commodity prices within each country which follows under the assumptions of perfect competition and constant returns to scale. For commodity 3 is the commodity in country 1 which is most labour-intensive[5] and labour is the primary input in country 1 whose relative price has risen. The solution shows that the wage rate/rental rate ratio rises from 1.220 to 1.334. Further, the

[4] This technological improvement is represented by a uniform 2 per cent increase in the normalising or technology parameters A_j, for $j=1, \cdots, 4$ in Table 1. This shifts the boundary of the attainable production set uniformly outwards.

[5] With CES production functions competitive input and output markets, and neutral technological change, the capital-labour ratio in each industry is given by the equation

$$\log (K_j/L_j)=\sigma_j \log [(1-\delta_j)/\delta_j]+\sigma_j \log (p_k/p_L)$$

wage rate in country 1 has risen whereas the rental rate on capital has fallen. This illustrates immediately one source of potential conflict between households owning different proportions of labour and capital. In country 2 the wage and rental rate both rise and the wage rate/rental rate ratio also rises, from 1.298 to 1.299. Thus the pre-growth factor price ratio of country 1 is less than that of country 2 but the post-growth ratio is greater. The price of commodity 7, which uses labour most intensively of the four commodities produced in country 2, rises.

Looking at the production of commodities, we find that in country 2, the production of commodity 7 (whose price has risen relative to the prices of the other commodities produced in this country) has risen and that the production of commodity 8 has also increased marginally. In country 1 the effect of an increase in the aggregate stock of capital is to increase the production of all four commodities. But how does one evaluate the welfare significance of these production changes? This question requires some careful preliminary examination.

An advantage of a solvable general equilibrium model is that one can obtain the values of the utilities of all individual consumers directly from the solutions corresponding to the different situations. This permits one to dispense with any measure of economic growth and the need to consider its welfare significance. However, there remain two basic problems. What significance can be attached to the magnitude of the change in an individual's utility? And how does one weight the changes of the two individuals?

The comparison of the utility changes of an individual h is facilitated by expressing the values of the utilities in two situations as an index, viz. U_h^c/U_h^b where b and c denote the pre-growth (base) and post-growth (current) situations respectively. But, for the individual, the values of this index are quite arbitrary since U_h is an ordinal scale representing preference-orderings. Any other increasing transformation of this function gives the same demand functions but will give a different index. However, the linearly homogeneous representation has one distinct advantage. This choice means that the movements of the utility index can also be interpreted as an index of the quantity consumed by the consumer. The Malmquist quantity index of consumption is the index γ such that $U_h(\gamma x_h^b) = U_h(x_h^c)$ for each current situation, c.[6] Since U_h is linearly homogeneous $\gamma = U_h(x_h^b)/U_h(x_h^c)$. γ itself is free of the influence of the arbitrary choice of transformation since it is a measure in commodity space. Yet, this index retains the feature that an increase (decrease) in the index indicates whether the consumer concerned is better or worse off than in the base situation to which his current situation is compared. These Malmquist indices will be employed extensively.

Concerning the problem of weighting the gains to individual consumers in an economy, there is really no escape, as Chipman and Moore (1978) have shown, from the use of some Bergson-type social welfare function. Let $U^k = (U_m, U_n)$. This

[6] See Malmquist (1953) and Diewert (1976, p. 148). This index is in fact the ratio of two values of the distance function (see Shephard (1970, p. 76)). For non-homothetic functions it is not independent of the consumption bundles of the base situation but for homothetic functions it depends only on U^b.

is the vector of utilities of the two households in country k. A *social welfare function* is a function, $W^k(U^k)$ which may be used to rank alternative utility vectors. It is assumed here that the social welfare function of country k is Benthamite, viz. $W^k(U^k) = U_m + U_n$. There is no overwhelming reason to choose the Bentham form. As with any positively-weighted average of the two utilities, it is intermediate in value between the limits of the values of the utilities of individuals 1 and 2. Alternatively, this index may be regarded as a Malmquist index of the percentage change in aggregate consumption for the economy. It is an average of the two individual consumers' Malmquist indices, using each consumer's share of aggregate utility in the base situation as weights. This *Bentham-Malmquist index of social consumption* and social welfare is employed below.

A second index of social welfare, the *Hicks Index*, is also employed. This is

$$Z = p^c x^c / p^c x^b \gtreqless 1 \tag{4}$$

The country superscripts have been omitted for convenience. If $Z > 1$ social welfare is said to increase.[7] The main advantage of the Hicks measure which concerns the theory of growth is that it can be decomposed to reveal the relationship between real output and real consumption expenditure and the manner in which these magnitudes in turn are related to distortionary taxes. From the definitions of prices we have

$$p^c x^t = \tilde{p}^c (1 + T^c) x^t \tag{5}$$

T is a diagonal matrix with the *ad valorem* consumption taxes along the diagonal. Substituting Equation (5) in (4) and using the zero balance of payments constraint obtained from Walras Law, the Hicks Index in one country is decomposed into the following components[8]

$$Z = (\tilde{p}^c y^c / \tilde{p}^c y^b) w_1 + (\tilde{p}^c T^c x^c / \tilde{p}^c T^c x^b) w_2 \tag{6}$$

$$w_1 = \tilde{p}^c y^b / p^c x^b, \; w_2 = \tilde{p}^c T^c x^b / p^c x^b,$$

$$(w_1 + w_2) \gtreqless 1, \qquad w_2 < 1$$

In our model there are only eight consumables but these relationships hold for any dimension.

In Equation (6) the rate of growth of real consumption expenditure is the weighted sum of the two components, the weights being the respective shares of

[7] This is the familiar Hicks (1940) criterion put in index form. Chipman and Moore (1971, 1973b) have given a precise interpretation of this criterion as it is applied to competitive equilibrium situations. This criterion assumes, unrealistically, that lump-sum non-distortionary transfers of income are possible without limit and without changing aggregate consumption. As an ordering of competitive equilibria it has the severe disadvantages that it is partial and not asymmetric (see Chipman and Moore (1973b)).

[8] This is the decomposition of the Hicks criterion proposed by Ohyama (1972) and put into index form.

base consumption valued at current prices. The first component is the index of real product at factor cost in constant prices. Note the appropriate index is a Paasche index using current period price weights rather than the base period weights usually employed in measuring economic growth. This term is positive under profit-maximising behaviour.[9] The commodity terms of trade enter the expression through the weights: if the Laspeyres index of the terms of trade deteriorates the sum of the weights is less than unity.

The second component shows the direct effects of consumption taxes on real national expenditure. It is zero if there are no consumption tax distortions. If there are non-zero tax distortions, this index may be greater or less than unity, that is, the consumption tax structure *per se* may increase or decrease the benefits of an expansion in the attainable production set.

The formal theory of immiserising growth has emphasised effects which are adverse and large and there has often been a presumption in the growth and development literature that distortionary taxes reduce the rate of growth of real expenditures (for example, Ranis (1973)). However, this presumption is unwarranted. In any situation, if a commodity is subject to a high consumption tax relative to other commodities, and especially those for which it is more substitutable, this commodity will be underconsumed relative to the situation with no such distortions. If the effect of growth on incomes and relative prices, given the tax structure is to increase the consumption of this commodity, the static welfare loss due to the distortion in the pre-growth situation is reduced. Equation (6) shows that this component is greater (less) than unity if the consumption tax revenue increases (decreases), given the changes in the quantities consumed of the taxed commodities and the tax rates held constant at the current level. This is a generalisation of the tariff revenue effect noted by Johnson (1967).[10] Because of the differentiation in our model between imported and domestically-produced commodities, this tax revenue effect in turn can be decomposed into the changes due to tariffs and that due to excise duties.

Input tax distortions affect the rate of growth of real consumption expenditure principally through the index of real output.

These components are all *mutatis mutandis* adjustments which reflect the total adjustments of all variables to two different situations. Therefore, if one compares growth of consumption between the pre-growth and post-growth situations under one tax regime with that under a different regime, one finds that the differences in taxes affect the value of the components due to real output and terms of trade changes as well as the direct tax component.

[9] There is a slight difficulty with the sign of this component. While economic growth has been defined as an expansion of the attainable production set, it has not been shown that, in the presence of non-zero distortions, the realisable production set also necessarily expands.

[10] The possibility that distortions may increase the rate of growth of consumption is implicit in Johnson (1967) and it is recognised by Corden (1974, p. 326). Corden suggests one compare the rate of growth of real expenditure under a given set of distortions with that under zero distortions.

In the example above, the index of real national expenditure, Z, increases by 2.54 per cent. Thus according to this criterion the economy of country 1 is better off after the growth of the attainable production set. This is verified by the average Bentham-Malmquist index which increases by 2.70 per cent.[11] The gain in real national expenditure may be decomposed using equation (6). We find that the Paasche index of national product increases by 6.22 per cent which overstates substantially the increase in national expenditure because the increase in production and incomes in country 1 induces a significant deterioration in the Laspeyres index of the terms of trade. There is a slight increase in the revenue index. That is, the changes in consumption and imports in the general equilibrium adjustment to the growth in country 1 reduce the distortionary consequences of these taxes and thereby serve to *increase* the rate of growth of real national expenditure and social welfare. (This is not to deny that in each of the pre-growth and the post-growth situations there are losses from distortionary taxes.)

III

This Section reports some simulations showing the effects of differences in the tax regimes on the rates of growth of the Bentham-Malmquist index and the utility-Malmquist index for individuals 1 and 2 in country 1. The pre-growth and post-growth situations are those described in Section II, except for the tax sets. The tax sets are first fixed at the base levels reported in Table 1. This produces the case examined in Section II. Then taxes in country 1 are varied. The variations are all cases of proportional reductions in the tax set or in sub-sets of the taxes, that is, the tax rates or sub-sets of the tax rate, such as the taxes on inputs only, are lowered uniformly to 70, then 50, 30 and 0 per cent of their base levels in Table 2. This is the kind of tax reduction considered by Foster and Sonnenschein (1970) for all taxes in their simpler model of a closed economy. Four sets or sub-sets are considered: all inputs taxes only, all tariffs only, all excise taxes only and all taxes together. Together these variations produce quite different tax structures. Comparing the pre-growth and post-growth situations for each tax structure isolates the effects of

[11] The Hicks and Malmquist indices tend to move closely together. This is because the Hicks index, like the aggregate Malmquist index, is a weighted average of individual consumption indices. Since in Equation (11) $x=\sum_h x_h$, we have

$$Z=\sum_h [(p^c x_h{}^c)/(p^c x_h{}^b)\cdot w_h],\ w_h=p^c x_h{}^b/p^c x^b$$

Moreover, these individual indices always predict correctly the sign of the individual's utility change in any current situation compared to a base situation because

$$(p^c x_h{}^c/p^c x_h{}^b)\geq 1 \rightarrow U_h(x_h{}^c)/U(x_h{}^b)>1$$

provided $U(x_h)$ is strictly quasi-concave, which the 2-level CES function is.

The Malmquist index is free of the bad properties of non-assymmetry and incomplete ordering which apply to the Hicks index (see footnote 7). The similar values of these two indices suggest that these properties may not vitiate use of the Hicks index.

the tax structure *per se* on growth of utilities and consumption. The changes in the indices for country 1 are reported in Table 4 as percentages of the base levels.

The first clear result is that, for all of the tax structures considered but one, there are positive gains in the consumption and utility of both individuals. Hence efforts to increase capital formation and improve technology are rewarded. The positivity of the gains is worth noting because in this model there are adverse terms of trade effects and a possibility of adverse tax effects. Furthermore, all of the four industries are export industries. The Bhagwati analysis of immiserising growth has suggested that negative gains occur only if the growth in production possibilities is biased towards export industries. On the other hand, the Johnston-type immiserising growth occurs only if growth is biased towards the import-competing industries (Bertrant and Flatters (1971)). But in our model every industry is both export- and import-competing. Hence, these two sources of negative effects oppose each other.

The Hicks and Malmquist indices of aggregate consumption are always close to each other and always change in the same direction in response to changes in input or final commodity taxes. Using the decomposition of the Hicks index, we see that the real output index overstates substantially the gains in real consumption in these examples because the growth in incomes in country 1 induces an adverse movement of the commodity terms of trade. However, that if there were simultaneous growth in country 2 the terms of trade could improve. The real output index would then understate the increase in consumption.

Concerning the effects of the tax variations themselves, higher tariffs raise the rate of economic growth while higher input taxes and higher excise taxes lower it. This holds for both the Hicks and the Bentham-Malmquist indices of aggregate consumption. In these comparisons higher levels of taxation mean equi-proportionate increases in the taxes concerned. This benefit from higher tariffs stem from the fact that higher tariffs reduce the loss of potential income gains when adverse movement of the terms of trade transfer some of the gains to the trading partner. With excise taxes the effect is the opposite in sign since excise taxes are in effect negative tariffs on the import substitutes. Input taxes give rise to a lower growth rate in addition to the one-period loss of aggregate consumption in each of the pre-growth and post-growth situations.

However, the growth rate of aggregate consumption is insensitive to large changes in sub-sets of taxes and even to the elimination of all taxes. This small effect of tax variations on the rates of growth is a result similar to the small effect which tax reforms have on the one-period situations. It is noteworthy because these tax changes do change substantially the vectors of prices and production in the pre-growth and post-growth competitive equilibria. This is not inconsistent because we are here measuring the rates of growth between these terminal points.

The increase in aggregate consumption are unequally divided between the two individuals since the increase in the stock of capital accrues to individual 1 alone. Nevertheless, for all tax sets with the one exception of the zero tax set, the consumption of both individuals 1 and 2 are greater after growth. Individual 1 gains partly because capital accumulation raises the real wage rate and he is the "labouring" household, and partly because the tax system redistributes to him a share of the increase in national income. Individual 1 gains more in all cases the higher all taxes or subsets of taxes because higher taxes lead to higher tax revenues and disbursements.

Tests were carried out to see whether the results above were sensitive to the assumed values of some parameters. Most of the results were robust with respect to large parametric variations. For example, we increased by scalar multiplication the base situation elasticities of substitution in production in all commodities ($\sigma_i = \lambda \sigma_i^b$) and the elasticities of substitution at the bottom level and top-level separately for all households in both countries ($\sigma_{hs} = \lambda \sigma_{hs}^b$ and $\sigma_h = \lambda \sigma_h^b$). Increases up to tenfold in these parameters were simulated. For any given tax set, the rates of growth of the Malmquist index for the economy increases with all three sets of elasticities. However, the rate of growth of aggregate consumption remains insensitive to changes in the tax set and the direction of change remains as in Table 3. In fact when the bottom-level or top-level elasticities of substitution are doubled, so that pairs of domestically-produced and imported goods or groups are more substitutable, tax variations make almost no difference to the growth rates. That is increases in substitutability among commodities reduce the effects of discriminatory taxes on the growth rate.

The effects of tax variations on the growth of the Malmquist index of consumption of the two individuals are more sensitive to variation of the non-tax parameters. When the elasticities of substitution in production are doubled for all commodities, for example, individual 2 gains substantially more than in Table 4 and individual 1 gains substantially less and actually loses when tariffs and all taxes are reduced below their base levels. Greater substitutability of capital for labour gives more of the gains to the "capitalist" whose capital stock increases in the growth situation since the wage rate/rental rate does not increase as much. Lowering taxes denies the other individual a share of the gains through the redistribution of tax revenues. On the other hand, when the bottom-level elasticities of substitution in consumption are doubled individual 1 in particular gains substantially more than in Table 4; in fact he gains by more than 2 per cent except when tariffs or all taxes are reduced to less than 50 per cent of the base levels. When top-level elasticities of substitution are doubled individual 1 again gains more than in Table 4; in fact, he gains by about 1 per cent except when tariffs or all taxes are reduced to less than 50 per cent of the base levels. Individual 1's share of tax revenue was raised to 67 per cent so that the ratio of his share to that of individual 2 is double that of the base level. With the base tax set individual 1 now gains 1.0 per cent from the aggregate economic growth. Conversely, when his share is reduced to 33 per cent, his consumption gain for this tax set falls to 0.5 per cent, compared to the 0.76 per cent gain in Table 4 with an equal tax share. Of course, as tax rates are reduced these tax distribution parameters have less effect. These results show that the level of tax rates in association with the distribution of tax revenues largely determines the inter-household allocation of consumption gains.

TABLE 4. GROWTH RATES UNDER DIFFERENT TAX REGIMES—COUNTRY 1

	Malmquist Index for			Hicks Real Expenditure Index	Paasche Real Output Index
	Individual 1	Individual 2	Aggregate		
Base Tax Set	0.81	3.85	2.70	2.54	6.22
Proportional Tax Reductions					
(i) Input Taxes					
70 per cent	0.76	3.89	2.71	2.55	6.22
50 ″	0.72	3.91	2.72	2.56	6.22
30 ″	0.67	3.95	2.73	2.56	6.22
0 ″	0.58	4.00	2.74	2.57	6.19
(ii) Tariffs					
70 per cent	0.67	4.07	2.69	2.53	6.24
50 ″	0.56	3.93	2.69	2.52	6.25
30 ″	0.43	3.96	2.68	2.50	6.26
0 ″	0.20	4.03	2.67	2.48	6.28
(iii) Excise					
70 per cent	0.80	3.87	2.72	2.56	6.27
50 ″	0.80	3.88	2.73	2.56	6.29
30 ″	0.78	3.90	2.74	2.57	6.32
0 ″	0.75	3.94	2.76	2.58	6.35
(iv) All Taxes					
70 per cent	0.62	3.94	2.72	2.55	6.28
50 ″	0.46	4.02	2.74	2.55	6.31
30 ″	0.27	4.11	2.76	2.55	6.33
0 ″	−0.23	4.27	2.79	2.55	6.38

IV

How do the gains from increasing production capabilities compare with gains from other alternative actions, such as domestic or foreign tax changes? Table 5 compares the gains in consumption due to the growth in production capabilities as described in the previous Section with the gains from three other actions: the elimination of distortions in the home country (country 1), the removal in country 2 of tariffs which restrict imports from country 1 and of other taxes in country 2, and a rate of capital formation and technological change in country 2 equal to that of country 1 in Section III. In all three cases we start with the base pre-growth situation so that the magnitude of the separate gains to country 1 from growth in production capabilities can be compared directly with the magnitudes of growth in

TABLE 5. GROWTH RATES UNDER DIFFERENT STRATEGIES

	Percentage Rates of Growth of Malmquist Indices for		
	Individual 1	Individual 2	Country 1
Growth in Country 1 Only	+ 0.81	+ 3.85	+ 2.70
Tax Changes in Country 1 Only			
Zero input taxes	− 0.93	+ 2.88	+ 1.44
Zero tariffs	− 8.38	+ 0.61	+ 2.78
Zero excise taxes	+ 0.16	+ 3.95	+ 2.52
Zero tax distortions	− 9.88	+ 7.12	+ 0.71
Growth in Country 2 Only	+ 3.47	+ 3.49	+ 3.48
Tax Changes in Country 2 Only			
Zero input taxes	+ 1.67	+ 0.89	+ 1.18
Zero tariffs	+18.75	+19.46	+19.19
Zero excise taxes	− 5.48	− 5.86	− 5.72
Zero tax distortions	+15.27	+14.31	+14.67
All Combined (i.e. equal growth and zero tax distortion in countries 1 and 2)	+ 6.73	+31.29	+22.02

consumption due to these other causes.[12]

Comparing first the effects on aggregate consumption of growth in the production sets of country 1 with the effects of "tax reforms" in this country, one finds that the former exceed the gains from any of the tax changes considered. This is true even though some of the tax changes entail the elimination of some highly distortionary sets of taxes. The gains from eliminating excise taxes come closest to the gains from growth in these simulations. There are also quite significant gains from the elimination of distortionary input taxes which create inefficiencies in production. The losses from eliminating the home country's tariffs result from adverse movements in the terms of trade. This movement of the terms of trade plays an important part in all of the tax changes and reduces the benefits from all strategies of tax reforms because part of the gains from improved allocation of production resources and consumption are transmitted to the second country via the increases in the demand for imported substitutes.

In three of the four cases of tax changes in country 1 individual 2 gains while individual 1 loses, indicating a social conflict in the implementation of these policies. In these cases individual 2 is the gainer because he is relatively well

[12] The whole gains or losses do not equal the sum of the components in Table 4 and in general for both individuals 1 and 2 and for the economy. This is because of interactions between strategies when they are carried out simultaneously. However, the decomposition obtained by starting each strategy from the initial or base pre-growth situation is the appropriate one for comparing the alternatives.

endowed with capital, the input whose relative price rises and because he receives a lower proportion of his gross income initially from tax revenues which fall as the tax rates fall.

We compare the effects of growth of the production sets in country 1 with the effects of growth in its trading partner, country 2, assuming in each case a 10 per cent increase in the capital stock and a two per cent Hicks-neutral technological improvement. We find that equal percentage growth in country 2 benefits country 1 to a greater extent than its own growth. This is a reflection of the very much greater size of country 2.

Last, we consider various tax changes in country 2. We find that the greatest gains of all of the cases considered come from country 1's obtaining completely free access to the markets of country 2, the "Rest of the World". This again reflects the relative size of the two countries together with the very high level of the initial barriers to country 1's exports (see Table 1). Free access to the other country allows country 1 to capture a much larger share of the global gains from trade which also increase themselves. In fact, country 2's aggregate consumption actually falls in this case. Conversely, lowering excise taxes in country 2 harms country 1. Lowering input taxes in country 2 increases productive efficiency and some of the increase in incomes and consumption are transmitted to the households of country 1.

Thus, the gains from growth in production capabilities in the home country are quite small as a proportion of the total gains from all of these events combined. But it should be remembered that, ignoring possible effects on the rate of capital formation, the gains from tax changes have a once-for-all effect whereas the growth in production sets is a continual process. Section III showed that the growth of production capabilities yields increases in aggregate consumption for almost all of the tax structures considered. Even relatively highly differentiated tax structures did not induce immiserising growth. Various changes in the tax structure had only small effect on the rate of growth of aggregate consumption in the economy though some tax changes and the distribution of tax revenues had much greater effect on the distribution of these gains between the individual households in the economy.

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REFERENCES

Bertrand, T. J. and F. Flatters (1971). "Tariffs, capital accumulation and immiserizing growth," *Journal of International Economics*, 1, 453–60.

Bhagwati, J. (1958). "Immiserizing growth: a geometrical note," *Review of Economic Studies*, 25, 201–205.

———, (1968). "Distortions and immiserizing growth: a generalisation," *Review of Economic Studies*, 35, 481–485.

Burmeister, E. and A. R. Dobell (1970). *Mathematical theories and economic growth* (Macmillan, London).

Chipman, J. and J. C. Moore (1971). "The Compensation Principle in Welfare Economics," A. Zarley, ed., *Papers in quantitative economics*, II (University of Kansas Press, Lawrence).

———, (1973). "Aggregate demand, real income and the compensation principle," *International Economic Review*, 14, 153–181.

———, (1978). "The New Welfare Economics 1939–1974," *International Economic Review*, 19, 547–584.

Corden, W. M. (1974). *Trade policy and economic welfare*, (Oxford).

Diewert, W. E. (1976). "Harberger's welfare indicator and revealed preference theory," *American Economic Review*, 66, 143–152.

Fei, J. C. H. and G. Ranis (1974). "Income inequality by additive factor components," Yale University Economic Growth Center, Center Discussion Paper No. 207, June 1974.

Foster, E. and H. Sonnenschein (1970). "Price distortion and economic welfare," *Econometrica*, 38, 281–97.

Hicks, J. R. (1932). *The theory of wages* (Macmillan, London).

Johnson, H. G. (1967). "The possibility of income losses from increased efficiency or factor accumulation in the presence of tariffs," *Economic Journal*, 77, 151–154.

Koopmans, T. C. (1951). *Activity analysis of production and allocation* (Wiley, New York).

Magee, S. P. (1973). "Factor market distortions, production, and trade: a survey," *Oxford Economic Papers*, 25, 1–42.

Malmquist, S., 1953. "Index Numbers and Indifference Surfaces," *Trabajos de Estadistica*, 4, 209–42.

Ohyama, M., 1972. "Trade and Welfare in General Equilibrium," *Keio Economic Studies*, 2, 37–73.

Ranis, G., 1973. "Industrial Sector Labor Absorption," *Economic Development and Culture Change*, 21, 387–408.

Sato, K., 1968. "A Two-Level Constant-Elasticity-of-Substitution Production Function," *The Review of Economic Studies*, 34, 201–218.

Scarf, H., 1967. "On the Computation of Equilibrium Prices," in Fellner, W. et al (eds.), *Ten Essays in Honour of Irving Fisher* (Wiley, New York).

———, 1973. *The Computation of Economic Equilibria*, Cowles Foundation Monograph 24 (Yale University Press, New Haven).

Shephard, R. W., 1970. *Theory of Cost and Production Functions* (Princeton University Press, Princeton, N. J.).

Shoven, J. B. and J. Whalley, 1972. "A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the U.S.," *Journal of Public Economics*, 1, 281–321.

———, 1973. "General Equilibrium with Taxes: A Computational Procedure and an Existence Proof," *The Review of Economic Studies*, 40, 475–89.

———, 1974. "On the Computation of Competitive Equilibrium on International Markets with Tariffs," *Journal of International Economics*, 4, 341–354.

Whalley, J., 1975. "A General Equilibrium Assessment of the 1973 United Kingdom Tax Reform," *Economica*, N.S., 27, 139–161.

———, 1976. "Some General Equilibrium Analysis applied to Fiscal Harmonization in the European Community," *European Economic Review*, October, 1976, 291–312.