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## Chapter 6

### **A METHOD FOR MEASURING THE SCALE ELASTICITY OF PRODUCTION: AN EMPIRICAL STUDY OF JAPANESE MANUFACTURING INDUSTRIES FROM 1964 TO 1972**

Kanji YOSHIOKA

#### SUMMARY

The aim of this article is to present a method for measuring the scale elasticity of production, i.e. the 'passus' coefficient. We assume the form of production function to be as general as possible in order to avoid specification error that might otherwise be involved. This method is tested using the data of Japanese manufacturing industries. So long as this data set is used, this method proves efficient and reveals the existence of scale economies in more than a half of Japanese manufacturing industries.

#### 1. INTRODUCTION

Are there economies of scale in production? This is the essential question underlying my investigation. The decentralization of authority in the economic system, it has often been argued, could lead to efficient resource allocation by taking advantage of the competitive market mechanism. But if there were considerable scale economies in almost all production processes, the atomistic competition, which means many producers enter into a market and compete with one another, would lead to inefficient resource allocation, in the sense of producing less than the maximum amount of products for the given scarce resources. And in such conditions, the more perfectly competition works, the more rapidly the decentralized mechanism would change into monopolistic centralization of production. And finally this system would collapse of itself. Under these circumstances, we would have to seek for a new system, where consumer's sovereignty can coexists with the advantage of the scale economies. Arguments like this, often discussed, stress the importance of the empirical analysis 'how much the economy of scale exists in the real world of production'.

It has occasionally been emphasized by writers that there are economies of scale in real production processes. When Adam Smith [9] advocated the division of labor as the main cause of enriching wealth of nations, he called attention to the fact that extensive division of labor was adopted in large scale manufactures. Marshall [7] analyzed the chief advantage of large scale production, by pointing out such elements as economy of skill, economy of machinery and economy of materials. These are classics of research based on observations.

The theoretical concept of 'perfect competition' has been accepted for its simplicity as a model of frictionless and ideal economy. However, while describing the distribution by using the principle of marginal productivity under perfect competition, we cannot accept the assumption of monotonically increasing return to scale in production. If there were a finite optimal size in the production process, the neighborhood around the quantity of equilibrium would be approximated sufficiently by the constant return to scale. This theorem is, needless to say, a valuable inheritance of economics from the time of Wicksteed [11] and Wicksell [10].

But in view of the present undeveloped state of economics as empirical science, we cannot easily reject either of them. Contrary to the assertion of finite optimal size, it has been maintained that the property of monotonically increasing return to scale is one of the very characteristics of modern industries (Florence [3]). Therefore, whether or not economies of scale exist in real production, and if they do in which industry and how much need to be investigated empirically. Besides, we also have to be careful in evaluating the value of the scale elasticity of production depending upon the objective of discussion; whether it is the issue of distribution or the scale economy itself.<sup>1</sup>

Among recent research works of scale economies, Griliches and Ringstad [5] and Ozaki [8] are noteworthy for the coverage of industries and the sample size. Though these studies differ in data source and the specification of production function, i.e. the former used Norwegian manufacturing data and applied Cobb-Douglas and C.E.S. production functions and the latter used Japanese manufacturing data employing a non-homothetic limitational production function, the results commonly revealed the existence of scale economies in many sectors.

The purpose of this article is to attempt for another method. In order to avoid specification errors involved in formulation of production functions, we attempt to estimate scale elasticities directly from the original data instead of deriving from estimated production functions. And then this method is tested by the published data of *the Japanese Census of Manufactures*.

## 2. THEORETICAL FRAMEWORK

In this section we introduce the theoretical framework for the method of

<sup>1</sup> When the production function is adopted for the description of distribution, we tend to regard, for example, the value 1.02 as being not significantly different from the situation of constant return to scale, because the additional two percent is so small. But when the magnitude of scale economy is evaluated in some specific industry which consists not only of small establishments but also of huge ones, this value in fact is sufficiently large as an indication of existence of economy. This is because, for example, the quantity of products of a huge establishment, which uses hundred times as much inputs as does a small establishment, is 110 times greater than that of the small establishment. In the case of the value of 1.1, the output of the large firm becomes 160 percent as much as the output of the small one.

measuring scale economies in production. At first we are concerned with a production process where a single homogeneous product is made of  $n$  kinds of factor inputs. Let  $X$  be the quantity of the product and  $V$  the vector of the factor quantities under consideration. Besides we assume production function  $X=F(V)$  as representing input output relationship.

The elasticity of the product quantity with respect to the scale of factor inputs has been used as the measure of the scale economies in such a single-ware production function. Ragner Frisch [4] named this elasticity 'the passus coefficient' and defined it as the elasticity of the product quantity with respect to one of the factors, when all the factors vary proportionally.<sup>2</sup>

$$(1) \quad \text{The passus coefficient; } K = d^{pr}X / d^{pr}v_i \cdot v_i / X$$

(for any  $i$ -th factor input)

where  $d^{pr}v_i$  represents infinitesimal proportional factor increment and  $d^{pr}X$  indicates corresponding infinitesimal increment in the product quantity. The passus coefficient is derived as the sum of the marginal elasticities of inputs. That is to say, when all the factors increase infinitesimally with maintaining constant proportionality among them the increments of the product are derived in the form as,

$$dv_i/v_i = \text{constant for any } i\text{-th input,}$$

$$dX = F_1 dv_1 + \dots + F_n dv_n,$$

$$d^{pr}X = (F_1 v_1 + \dots + F_n v_n) \cdot d^{pr}v_i / v_i$$

and thus the passus coefficient is rewritten as

$$(2) \quad K = \frac{d^{pr}X/X}{d^{pr}v_i/v_i} = F_1 \frac{v_1}{X} + \dots + F_n \frac{v_n}{X}.$$

In estimating this coefficient based on observed economic data, we have usually assumed some specification about the production function and estimated several parameters which in turn are integrated to get the estimator of this coefficient. With regard to the Douglas type production function, for instance, this estimator corresponds to the total value of the estimated marginal elasticities of all inputs which appear in the above mentioned equation. Approaches like this have, needless to say, a great advantage of acquiring integrated information on the production structure under considerations, such as the marginal elasticity of substitutions, the marginal productivity of inputs and so on. However, from the viewpoint of analyzing scale economies, these approaches are not always the best. When analysing empirically a specific industry, we are often faced with the situation in which the observed data indicate the familiar relationship, i.e. the greater the quantity of any input, the greater the output. Because of these high correlations

<sup>2</sup> Although there are many names for this elasticity, we use the 'scale elasticity' or the 'passus coefficient' in this article. The names; 'elasticity of production' by Johnson, 'function coefficient' by Carlson and 'elasticity of productivity' by Allen are examples. [1], [2], [6].

among the factors, it is hard to know the isoquant curve of the product. This 'multi-collinearity' problem is one of the most difficult one. Because of this difficulty, we intend to seek for another method. Therefore, we concentrate our attention mainly on two points. The first is to estimate the passus coefficient directly from the original data and not through the estimated production functions. And the next is to specify the form of the function as generally as possible.

Although it is not quite feasible or theoretically warranted to estimate jointly all the relevant parameters of a production function in the case in which the data of inputs are mutually closely correlated, such situations may be in effect suitable for an analysis by which to evaluate scale economies. As the extreme case, for instance, if all the factors were kept strictly proportional and if the average amount of the product per factor for the case of the larger amount of the factor input is greater than the case of the smaller amount of the factor input, we would take this as evidence of economy of scale between these two cases. Frisch has given the following relation as the approximate estimator of the passus coefficient.

$$(3) \quad K = \frac{d^{pr} X / X}{d^{pr} v_i / v_i} \simeq \frac{\Delta \log X}{\Delta \log v_i} = \frac{\log X^{j+1} - \log X^j}{\log v_i^{j+1} - \log v_i^j}$$

where superscript  $j$  denotes the  $j$ -th observations. This equation is useful not only in the case in which product quantities are close to each other, but also in the case in which the quantities are far apart, because the value of  $K$  in this equation is in effect a kind of average value of the relevant passus coefficients.<sup>3</sup>

However the data, generated within the economic mechanism, seldom present such extreme cases. This is the matter of primary concern, and thus the theoretical framework is constructed with three assumptions: (1) convexity of isoquant surface of the product to the origin of the factor space, (2) rationality of the producer and (3) homogeneity of the production function. The third assumption is chosen for the sake of simplicity, that is, in order to make the passus coefficient always constant regardless of the factor quantities. The production function is given by

$$(4) \quad X = \lambda^{-k} F(\lambda V), \lambda > 0, V > 0, X > 0$$

where  $K$  means the degree of homogeneity as well as the passus coefficient, and  $\lambda$  is any positive constant.

Suppose there are two samples which contain different products,  $n$  types of factors and corresponding prices of these factors. Let  $(X^1, V^1, P^1)$ ,  $(X^2, V^2, P^2)$  be the two samples, where different superscripts indicate different sample sets. By using these observations, the factor space is drawn as Figure 1, where the dotted line A represents the ray on which all the factors are kept as proportional to  $V^2$ , and similarly the dotted line B is defined as the line which passes through the origin and point  $V^1$ . In addition  $\tilde{V}^1$  is the intersecting point of ray A on the cost

<sup>3</sup> Frisch [4], p. 72.

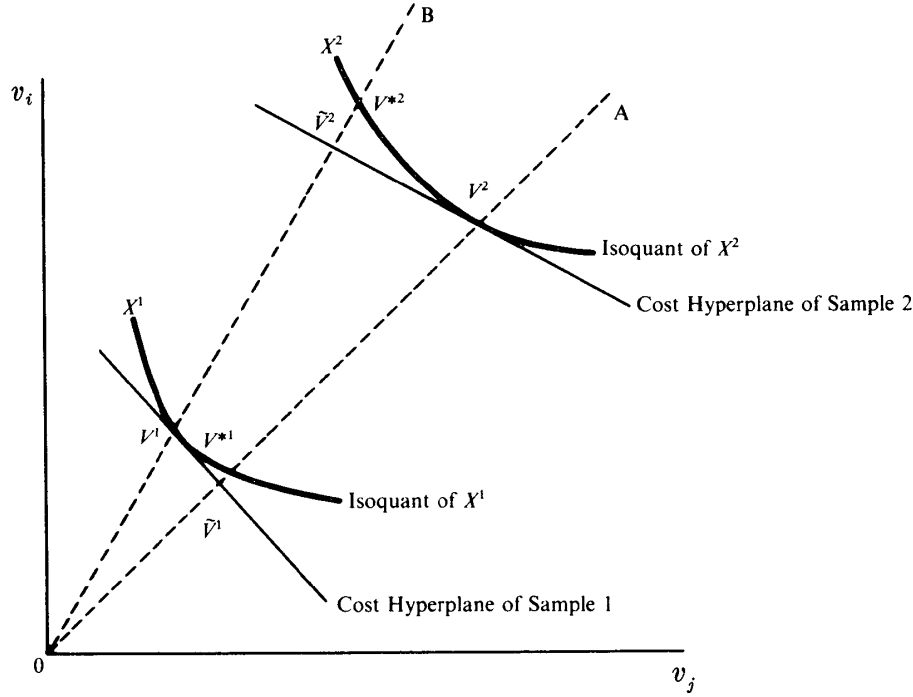


Fig. 1.

hyperplane of sample 1 and  $V^{*1}$  is that on the isoquant surface.  $\tilde{V}^2$  and  $V^{*2}$  are defined for the sample 2 in the same way. From equation (4), the following relation is reduced.

$$V^2 = \lambda V^{*1}, F(V^{*1}) = \lambda^{-k} F(\lambda V^{*1}) = \lambda^{-k} F(V^2).$$

Then we have

$$(5) \quad K = \frac{\log X^2 - \log X^1}{\log v_i^2 - \log v_i^{*1}}. \quad (i \text{ stands for any } i\text{-th input, } i=1, \dots, n)$$

and similarly,

$$(6) \quad K = \frac{\log X^2 - \log X^1}{\log v_i^{*2} - \log v_i^1}. \quad (i=1, \dots, n)$$

When we have no information about the values  $V^{*1}$  and  $V^{*2}$ ; we can only observe the region or the boundary of the passus coefficient. That is to say, assuming the convexity of the product isoquants and the producer's rationality, the factor point  $V^{*1}$  is placed on the region which has the upper limit  $\text{Max}[v_1^1/v_1^2 \cdots v_n^1/v_n^2] \cdot V^2$  and the lower limit  $\tilde{V}^1$  as shown by Figure 2.

$$\tilde{V}^1 \leq V^{*1} \leq \text{Max} \left[ \frac{v_1^1}{v_1^2} \cdots \frac{v_n^1}{v_n^2} \right] \cdot V^2, \quad \text{where } \tilde{V}^1 = \frac{P^1 V^1}{P^1 V^2} V^2.$$

Then the lower limit of the passus coefficient, defined as  $L$ , is obtained.

$$(7) \quad L = \frac{\log X^2 - \log X^1}{\log v_i^2 - \log \tilde{v}_i^1}. \quad (i=1, \dots, n)$$

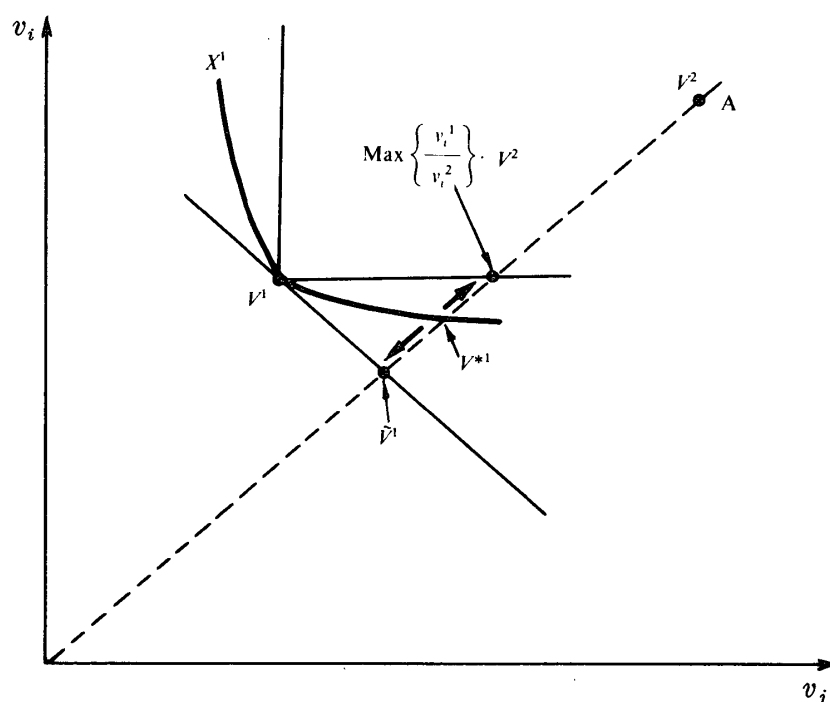


Fig. II.

And similarly with regard to the factor  $V^{*2}$  on ray B, the upper bound of the passus coefficient is restricted by  $U$ , which is defined as

$$(8) \quad U = \frac{\log X^2 - \log X^1}{\log \tilde{v}_i^2 - \log v_i^1} \quad (i=1, \dots, n)$$

where

$$\tilde{v}_i^2 = \frac{P^{2'} V^2}{P^{2'} V^1} v_i^1.$$

$$(9) \quad L \leq K \leq U$$

### 3. STOCHASTIC MODEL

Suppose there are  $m+1$  samples which show different output quantities. And then we denote the number of each sample in ascending order of its product quantity, such as

$$\begin{cases} X^1, X^2, \dots, X^j, X^{j+1}, \dots, X^m, X^{m+1} \\ V^1, V^2, \dots, V^j, V^{j+1}, \dots, V^m, V^{m+1} \\ P^1, P^2, \dots, P^j, P^{j+1}, \dots, P^m, P^{m+1} \end{cases}$$

The range of quantity of the product between the minimum  $X^1$  and the maximum  $X^{m+1}$  is divided into  $m$  intervals. Consequently we can observe the upper and lower limits of the passus coefficient within each interval.

$$(10) \quad L_j = \frac{\log X^{j+1} - \log X^j}{\log v_i^{j+1} - \log \tilde{v}_i^j}$$

$$(11) \quad U_j = \frac{\log X^{j+1} - \log X^j}{\log \tilde{v}_i^{j+1} - \log v_i^j} \quad (i=1, \dots, n)$$

where

$$\tilde{v}_i^j = \frac{P^{j'} V^j}{P^{j'} V^{j+1}} \cdot v_i^{j+1}$$

$$\tilde{v}_i^{j+1} = \frac{P^{j+1'} V^{j+1}}{P^{j+1'} V^j} \cdot v_i^j$$

These upper and lower limits are assumed as the stochastic variables, each of which has a mean and a variance. Although there are several reasons for stochastic variables, such as the shock in the approximation of homogeneous function, measurement errors and the bias associated with cost minimization, these are impossible to identify. Therefore, as the first approach we had better analyse them under the assumption which is easy to test.

When denoting  $K_L$  as the mean of the lower  $L_j$  and  $K_U$  as that of the upper  $U_j$ , it would be reasonable to adopt the assumption  $K_L < K < K_U$ . Under this assumption, the sample mean of  $L_j$  is the under-biased estimator of  $K$ , and that of  $U_j$  is the over-biased estimator.

$$(12) \quad E(\bar{L}) = (1/m) \sum_{j=1}^m E(L_j) = K_L < K$$

$$(13) \quad E(\bar{U}) = (1/m) \sum_{j=1}^m E(U_j) = K_U > K$$

where  $\bar{L}$  means the sample mean of  $L_j$ , and  $\bar{U}$  means that of  $U_j$ .

Through these inferences, the experimental framework is summarized. If the lower estimator  $\bar{L}$  of the passus coefficient indicates such a large value as to accept the hypothesis  $K_L > 1$ , the considered production process will express the scale economies. And the upper estimator  $\bar{U}$  will be a measure of the decreasing return to scale. When these estimators are so near as to accept the hypothesis  $K_L = K_U$ , this method is efficient for estimating the passus coefficient. But when  $\bar{U}$  is so much greater than  $\bar{L}$  that the hypothesis  $K_U > K_L$  may well be accepted, this approach is not so efficient for the estimation of the passus coefficient. On the contrary, if  $K_U < K_L$  is accepted, we should have serious doubt about the above-mentioned assumptions. Besides, if the observations show that  $L_j$  and  $U_j$  are dependent on the quantity of product, we must closely examine the homogeneity of the function under the considered production process.

#### 4. EMPIRICAL ANALYSIS OF JAPANESE MANUFACTURING INDUSTRIES

This empirical analysis is concerned with the Japanese manufacturing industries



from 1964 through 1972. The data are taken from 'The Statistical Table by Employment Size of Establishment' in *Census of Manufactures; Reported by Industries*.

Applying cross sectional analysis, we estimate the passus coefficient for each calender year for each two-dight industry classification according to the Japanese Standard Industrial Classification System except for Tobacco and Ordnance. Production functions for all industries are assumed to take the form

$$(14) \quad X = \lambda^{-K} F(\lambda v_1, \lambda v_2, \lambda v_3)$$

where the values of the following variables are the average values for establishments classified by the number of their employees:

$X$  ; value of products,

$v_1$  ; number of workers,

$v_2$  ; book value of fixed assets at the beginning of the year,

$v_3$  ; value of intermediate inputs including used materials, electricity and consignment fee.

And the prices of these three factors are:

$p_1$  ; payroll per employees in each employment size,

$p_2$  ; average depreciation ratio plus the average banking interest rates,

$p_3$  ; defined as unity.

These definitions and the data are not free from deficiencies: e.g. deficiencies associated with small samples and with heterogeneity of products and inputs etc. To take care of these deficiencies is beyond the scope of this study<sup>4</sup>.

The results are shown in the following diagram figure and by Appendix Table 1 of the end of this paper.

In this picture the horizontal axis indicates the value of the passus coefficient and the plots express the estimates of the coefficients for different industries for different each years. To be precise, when both estimates,  $\bar{L}$  and  $\bar{U}$ , are greater than 1, the lower estimate is plotted. And when both are smaller than 1, the upper is plotted. But when the lower is smaller than 1 and the upper is greater than 1, the result is not plotted. Cases in which the lower is greater than the upper that show up occasionally are also omitted. This figure thus indicates that industries which have many plots in the region of greater than 1 are strongly characterized by scale economies.

<sup>4</sup> Defining capital input is one of the most difficult problems. When the book value of asset is used to represent capital input, old and new equipments are mixed in it. The book value can be a useful instrumental variable for capital input, free from the effects of inflation or innovation, only when the weight of the new investment to the book value is constant regardless of the establishment size. In this data set, such weights are uncorrelated with the establishment size from 1964 to 1972. In addition let us remind the fact that the period of observation of this study is the period of rapid growth of the Japanese economy when the capital equipment has been fully utilized. Under such a circumstance, the use of book value of fixed assets as an instrumental variable for the variable of capital input may be theoretically justified. Yoshioka [12].

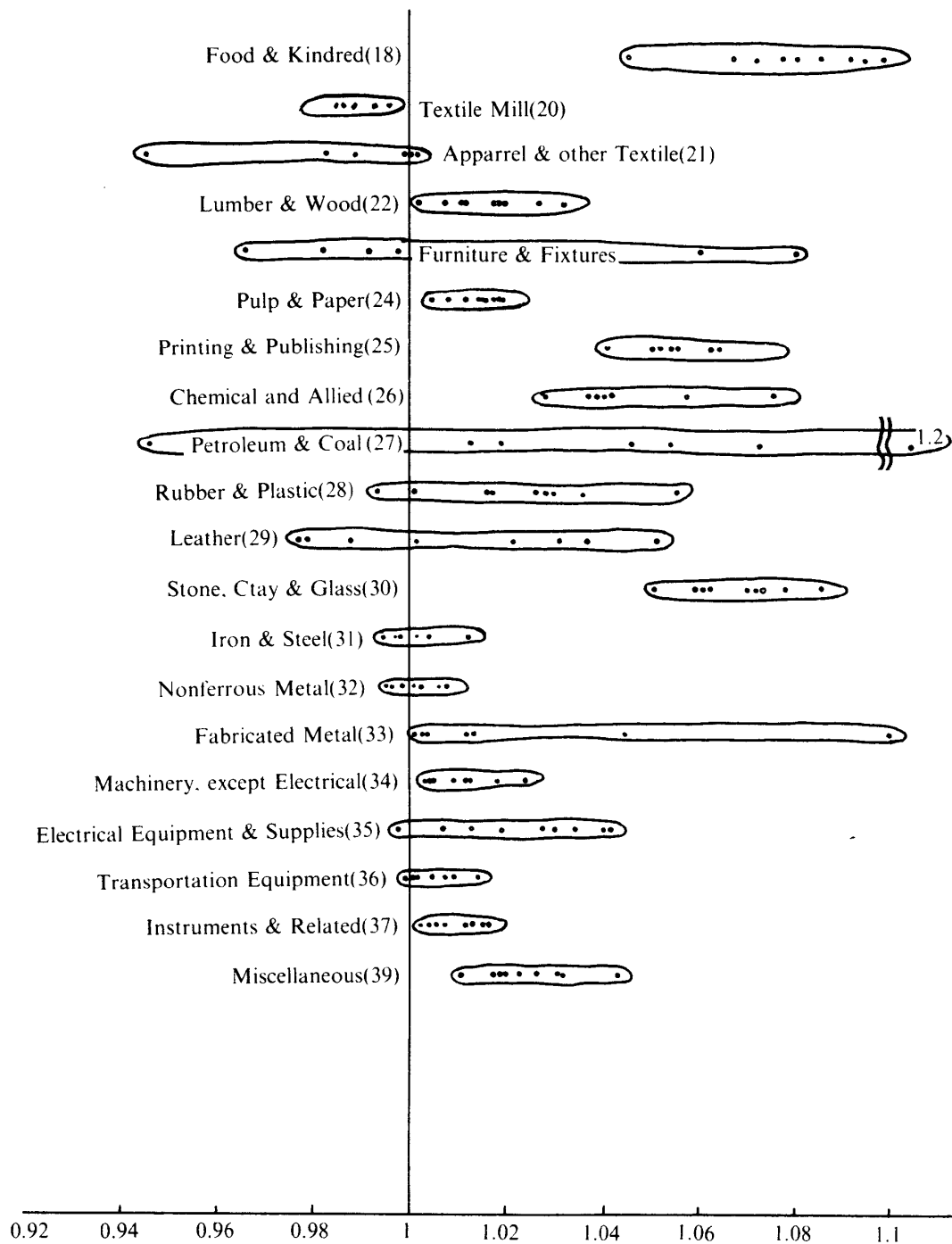


Fig. III. The Estimates of the Passus Coefficient.

This result shows that there are thirteen industries, the lower estimates of which are greater than 1 for almost all years. Table 1 lists those industries. In contrast the Textile Mill Products industry is the only industry in which the upper estimates are smaller than 1 for all years. Other six 2-digit industries of manufac-

TABLE 1. INDUSTRIES WHICH INDICATE THE SCALE ECONOMIES

No.	Name of Industry	Range of Lower Estimates
18	Food and Kindred Products	1.0459-1.0993
22	Lumber and Wood Products	1.0023-1.0326
24	Pulp and Paper Products	1.0049-1.0188
25	Printing and Publishing	1.0410-1.0744
26	Chemical and Allied Products	1.0280-1.0423
28	Rubber and Plastic Products N.E.C.	0.9937-1.0557 ( <sup>67</sup> Exception)
30	Stone, Clay, and Glass Products	1.0511-1.0865
33	Fabricated Metal Products	0.9999-1.1093 ( <sup>67</sup> Exception)
34	Machinery, except Electrical	1.0039-1.0248
35	Electrical Equipment and Supplies	0.9970-1.0416 ( <sup>64</sup> Exception)
36	Transportation Equipment	0.9998-1.0145 ( <sup>68</sup> Exception)
37	Instruments and Related Products	0.9982-1.0164 ( <sup>65</sup> Exception)
39	Miscellaneous Manufacturing	1.0117-1.0442

turing, we may useful be classified into two groups. The first is the group of industries whose estimates are located around 1; e.g. Iron & Steel Foundries and Nonferrous Metal. The second group is the others whose estimates are so dispersed from year to year that we cannot characterise them with any particular feature: Apparel & Other Textile Products, Furniture & Fixtures, Petroleum & Coal Products, and Leather & Leather Products. The result, summarized as above, reveals that more than half of the industries were enjoying scale economies for the years of our analysis.

Whenever statistical inference is based on the data of small samples, it has several limitations. The above mentioned estimation and the test for it are not exceptions. According to the Appendix Table I, the standard deviations of  $L_j$  and  $U_j$  are large compared with their sample means. Therefore we set the  $t$ -Test for the null hypothesis of  $K_L=1$  and  $K_U=1$  for each industry for each year, by assuming (1)  $L_j(j=1, \dots, m)$  are random samples obtained from a normal distribution with mean  $K_L$  and (2)  $U_j(j=1, \dots, m)$  are random samples taken from a normal distribution with mean  $K_U$ .<sup>5</sup> Under these assumptions  $t_L$  and  $t_U$

<sup>5</sup> The test of the normal distribution is valuable not only for this test, but for the above mentioned point estimation. Because, if it is accepted,  $\bar{L}$  and  $\bar{U}$  will become the best estimators of  $K_L$  and  $K_U$ . But this test is omitted for the lack of the sample size. With regard to random sampling, the test of the serial correlation is tried and then the sample correlation coefficients between  $L_j$  and  $L_{j-1}$ , and between  $U_j$  and  $U_{j-1}$  are calculated. Appendix Table II shows this result. According to this result and  $t$ -Test, the cases where the hypothesis of zero correlation is rejected with the 25% critical region count 82 among 360 cases. The number 82 is less than 25% for all cases. Then we may consider them as having no serial correlations as a whole.

$$(15) \quad t_L = \frac{\bar{L} - K_L}{S_L / \sqrt{m}}, \quad t_U = \frac{\bar{U} - K_U}{S_U / \sqrt{m}}$$

respectively have a  $t$ -distribution with  $m-1$  degrees of freedom, where  $S_L$  indicates the sample standard deviation of  $L_j$  and  $S_U$  does that of  $U_j$ . The tests are based on these relations and their critical region is 25 percent in both sides. The result is shown in the Appendix Table I, where we use the symbol 0 to indicate acceptance of the hypothesis  $K_L=1$  or  $K_U=1$ , and the symbol of inequality for rejection of it. Though this result shows the tendency of scale economies in several industries, such as Printing & Publishing, Chemical & Allied Products and Stone, Clay & Glass Products, we cannot reject the hypothesis of the constant return to scale in almost all the other industries for which we have found scale economies above by means of point estimation. The main cause of these situations are perhaps due to the largeness of their standard deviations relative to their sample means. Therefore we must be careful in evaluating the above mentioned estimation of the passus coefficients.

The next test is conducted to examine the efficiency of this method itself. As we explained in Section 3, if  $K_L > K_U$  is accepted, we should suspect the validity of some or all of the three assumptions. This method is efficient only when  $\bar{L}$  is so near to  $\bar{U}$  as to accept the hypothesis of  $K_L = K_U$ . When observing the result in the Appendix Table I with such a viewpoint, we can find the three properties such that, (1) the upper estimates  $\bar{U}$  are fairly close to the lower  $\bar{L}$  in each case, (2) almost all cases indicate that  $\bar{U}$  is greater than  $\bar{L}$ , and (3) few cases indicate  $\bar{U}$  is smaller than  $\bar{L}$  and vice versa. Considering these three findings, set the hypothesis  $K_L = K_U$ . Moreover, for the simplification of the test, we have to presuppose that the variance of  $L$  is equal to that of  $U$ . This presupposition is meaningful, only when the sample standard deviation  $S_L$  is remarkably close to  $S_U$  in any case. In order to find the relation between  $S_L$  and  $S_U$ , the next three figures are drawn as follows:

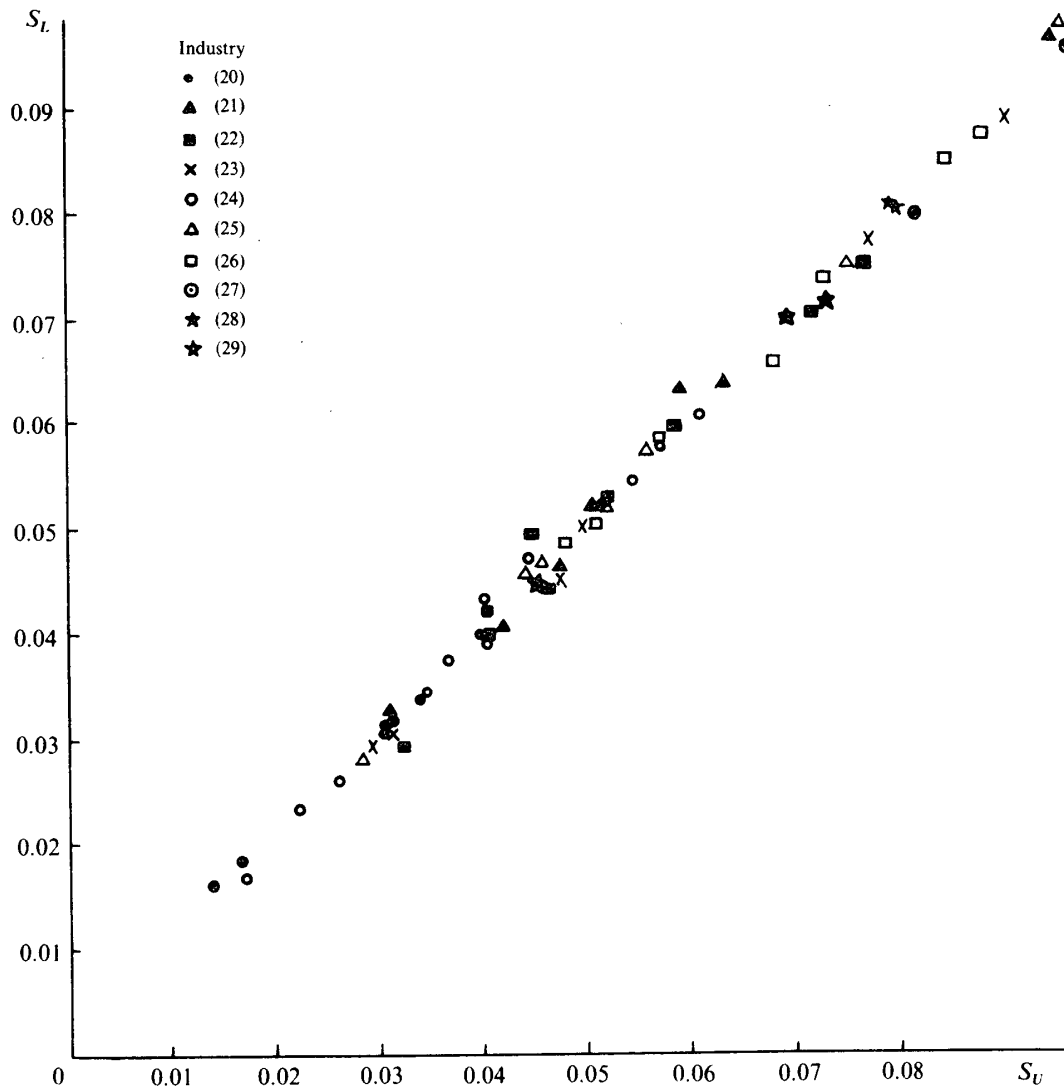


Fig. IV.

Note: The vertical axis indicates the value of standard deviation of  $L_j$ . Horizontal axis does that of  $U_j$ . The Numbers indicate those of two-digit industries.

where the vertical axis indicates the magnitude of  $S_L$  and the horizontal axis does that of  $S_U$ . A simple regression fitted to this data set provides the result,

$$(16) \quad S_L = -0.00012 + 0.99685 S_U \quad \bar{r} = 0.9906$$

$$(t = 0.114) \quad (t = 96.95) \quad d.f = 178$$

where  $t$  is the  $t$ -value of the above regression coefficient. According to these figures and the result of the regression, it will be supported that the variance of  $U$  is approximately equal to that of  $L$  as a whole. In such conditions,

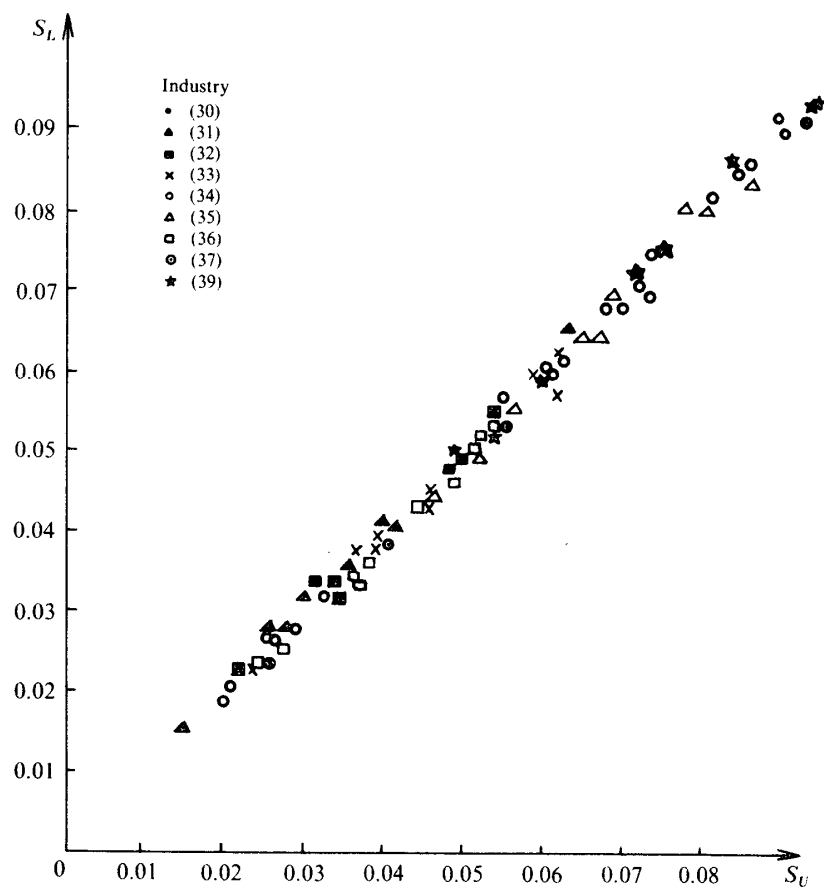


Fig. V.

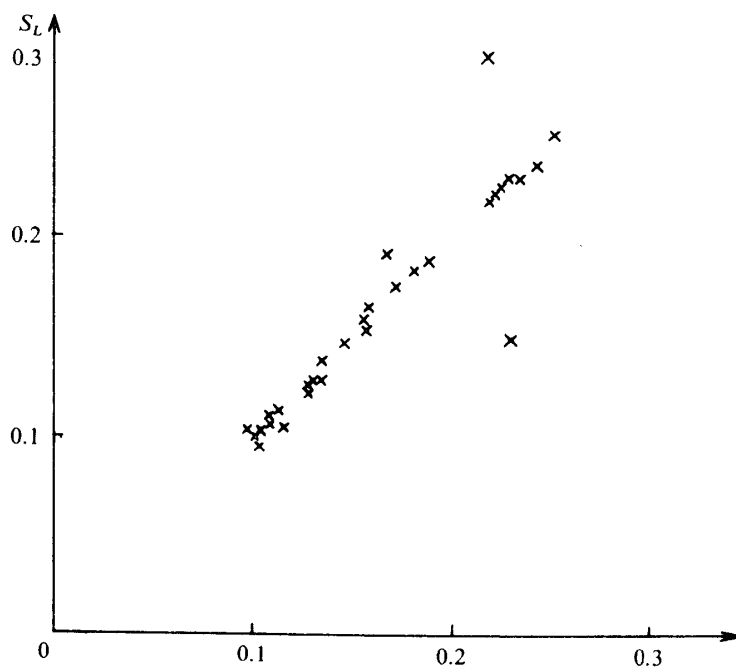


Fig. VI.

Note: This figure is drawn about the large values of standard deviation.

$$(17) \quad t = \frac{\bar{L} - \bar{U}}{\sqrt{(m-1)S_L^2 + (m-1)S_U^2}} \sqrt{\frac{m^2(2m-2)}{2m}}$$

has a  $t$ -distribution with  $2m-2$  degrees of freedom. By using this relation, some tests are tried. And the result is also shown in the Appendix Table I, where we also use the symbol 0 to indicate acceptance of the hypothesis  $K_L = K_U$ . This result suggests that in any case  $\bar{L}$  is so close to  $\bar{U}$  that we cannot reject the hypothesis  $K_L = K_U$  under the 25 percent significance level. Therefore, this method for estimating the passus coefficient is quite efficient so far as this data source is concerned.

#### CONCLUSION AND LIMITATION

We have presented a method for measuring the scale elasticity of production function. This method was tested by the published data of the Japanese Census of Manufactures from 1964 through 1972. So long as this data set was concerned, more than half of the manufacturing industries were found to enjoy scale economies.

However, this empirical study should be considered as the only experimental case, because the published data that we used have the shortage of small samples and the lack of homogeneity of products and inputs. Additional studies based on the original questionnaires themselves will be useful to take care of these weaknesses.

With regard to the methodological aspect, we found this method quite efficient for the case of the collinearity between mutual factor inputs.

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APPENDIX TABLE I. THE DETAILS OF RESULT

Year	$L$	$U$	$S_L$	$S_U$	Null-Hypothesis			Sample Size
					$K_L=1$	$K_U=1$	$K_L=K_U$	
18. Food & Kindred Products								
1964	1.0993	1.1003	.2285	.2286	○	○	○	8
1965	1.0736	1.0749	.2219	.2228	○	○	○	8
1966	1.0780	1.0796	.2245	.2242	○	○	○	8
1967	1.0680	1.0699	.1571	.1561	○	$K_U>1$	○	8
1968	1.0459	1.0451	.2180	.2202	○	○	○	7
1969	1.0919	1.0918	.2499	.2508	○	○	○	7
1970	1.0869	1.0862	.1559	.1565	$K_U>1$	$K_U>1$	○	7
1971	1.0954	1.0939	.1870	.1889	$K_U>1$	$K_U>1$	○	7
1972	1.0810	1.0782	.2287	.2343	○	○	○	7
20. Textile Mill Products								
1964	.9947	.9955	.0302	.0305	○	○	○	8
1965	.9845	.9861	.0161	.0144	$K_L<1$	$K_U<1$	○	8
1966	.9867	.9882	.0186	.0169	$K_L<1$	$K_U<1$	○	8
1967	.9881	.9888	.0319	.0314	○	○	○	8
1968	.9943	.9945	.0342	.0341	○	○	○	7
1969	.9856	.9853	.0309	.0307	○	○	○	7
1970	.9802	.9801	.0343	.0343	$K_L<1$	$K_U<1$	○	7
1971	.9852	.9855	.0397	.0399	○	○	○	7
1972	.9957	.9959	.0522	.0517	○	○	○	7
21. Apparel & other Textile Products								
1964	.9888	.9894	.0468	.0474	○	○	○	7
1965	.9983	.9999	.0520	.0511	○	○	○	7
1966	1.0016	1.0019	.0631	.0592	○	○	○	7
1967	.9981	.9983	.0455	.0441	○	○	○	7
1968	.9062	.9435	.2898	.2162	○	○	○	7
1969	1.0014	1.0031	.0402	.0422	○	○	○	5
1970	1.0161	1.0182	.0639	.0635	○	○	○	5
1971	.9800	.9832	.0327	.0310	$K_L<1$	○	○	5
1972	1.0040	1.0062	.0974	.0949	○	○	○	5
22. Lumber & Wood Products								
1964	1.0110	1.0113	.0704	.0717	○	○	○	8
1965	1.0195	1.0226	.0293	.0323	$K_L>1$	$K_U>1$	○	8
1966	1.0122	1.0140	.0447	.0462	○	○	○	8
1967	1.0023	1.0051	.0526	.0520	○	○	○	8
1968	1.0079	1.0109	.0494	.0447	○	○	○	7
1969	1.0184	1.0182	.0422	.0404	○	○	○	7
1970	1.0326	1.0335	.0593	.0584	$K_L>1$	$K_U>1$	○	7
1971	1.0271	1.0293	.0750	.0768	○	○	○	7
1972	1.0187	1.0198	.0506	.0508	○	○	○	7



APPENDIX TABLE I. (Continued)

Year	$L$	$U$	$S_L$	$S_U$	Null-Hypothesis			Sample Size
					$K_L=1$	$K_U=1$	$K_L=K_U$	
23. Furniture & Fixtures								
1964	.9974	.9988	.0304	.0311	○	○	○	7
1965	.9919	.9922	.0500	.0497	○	○	○	7
1966	.9783	.9820	.1368	.1345	○	○	○	7
1967	1.0808	1.0778	.2999	.2890	○	○	○	7
1968	1.0297	1.0287	.0291	.0295	$K_L>1$	$K_U>1$	○	5
1969	1.0218	1.0217	.0398	.0405	○	○	○	5
1970	1.0380	1.0382	.0456	.0469	$K_L>1$	$K_U>1$	○	6
1971	.9633	.9669	.1810	.1824	○	○	○	6
1972	1.0608	1.0621	.0889	.0907	$K_L>1$	$K_U>1$	○	6
24. Pulp & Paper Products								
1964	1.0188	1.0215	.0397	.0404	$K_L>1$	$K_U>1$	○	8
1965	1.0165	1.0192	.0580	.0569	○	○	○	8
1966	1.0147	1.0178	.0374	.0366	○	○	○	8
1967	1.0121	1.0153	.0432	.0402	○	○	○	8
1968	1.0171	1.0190	.0169	.0173	$K_L>1$	$K_U>1$	○	7
1969	1.0156	1.0182	.0261	.0264	$K_L>1$	$K_U>1$	○	7
1970	1.0177	1.0220	.0391	.0408	○	○	○	7
1971	1.0049	1.0074	.0607	.0611	○	○	○	7
1972	1.0079	1.0118	.0234	.0224	$K_L>1$	$K_U>1$	○	7
25. Printing & Publishing								
1964	1.0536	1.0612	.0573	.0566	$K_L>1$	$K_U>1$	○	8
1965	1.0410	1.0446	.0279	.0282	$K_L>1$	$K_U>1$	○	8
1966	1.0527	1.1068	.1499	.2296	○	$K_U>1$	○	8
1967	1.0547	1.0585	.1066	.1086	$K_L>1$	$K_U>1$	○	8
1968	1.0518	1.0547	.1282	.1307	○	○	○	7
1969	1.0644	1.0963	.1757	.1709	○	○	○	7
1970	1.0541	1.0577	.0516	.0517	$K_L>1$	$K_U>1$	○	7
1971	1.0638	1.0670	.0750	.0752	$K_L>1$	$K_U>1$	○	7
1972	1.0744	1.0779	.0985	.0963	$K_L>1$	$K_U>1$	○	7
26. Chemical & Allied products								
1964	1.0395	1.0399	.0470	.0447	$K_L>1$	$K_U>1$	○	8
1965	1.0281	1.0284	.1074	.1078	○	○	○	8
1966	1.0280	1.0285	.0489	.0478	$K_L>1$	$K_U>1$	○	8
1967	1.0377	1.0381	.0464	.0547	$K_L>1$	$K_U>1$	○	8
1968	1.0423	1.0432	.0656	.0680	$K_L>1$	$K_U>1$	○	7
1969	1.0581	1.0580	.0582	.0572	$L_L>1$	$K_U>1$	○	7
1970	1.0766	1.0769	.0742	.0727	$K_L>1$	$K_U>1$	○	7
1971	1.0402	1.0413	.0850	.0844	$K_L>1$	$K_U>1$	○	7
1972	1.0402	1.0407	.0873	.0879	$K_L>1$	$K_U>1$	○	7

APPENDIX TABLE I. (Continued)

Year	$L$	$U$	$S_L$	$S_U$	Null-Hypothesis			Sample Size
					$K_L=1$	$K_U=1$	$K_L=K_U$	
27. Petroleum & Coal Products								
1964	1.1995	1.2068	.5426	.5423	○	○	○	8
1965	1.0470	1.0584	.0958	.0969	$K_L>1$	$K_U>1$	○	7
1966	1.0135	1.0183	.2034	.2060	○	○	○	7
1967	1.0735	1.0695	.1459	.1462	○	○	○	6
1968	1.0198	1.0320	.0796	.0818	○	○	○	5
1969	.9992	1.0233	.1221	.1296	○	○	○	5
1970	1.0545	1.0543	.1044	.0960	○	○	○	5
1971	.9501	.9460	.0995	.1185	○	○	○	5
1972	.9969	.9987	.2308	.2301	○	○	○	7
28. Rubber & Plastic Products								
1964	1.0176	1.0187	.0462	.0458	○	○	○	8
1965	1.0013	1.0021	.0774	.0773	○	○	○	8
1966	1.0170	1.0182	.0445	.0453	○	○	○	8
1967	.9937	.9904	.1045	.1169	○	○	○	7
1968	1.0293	1.0304	.0635	.0644	○	○	○	7
1969	1.0266	1.0250	.0802	.0797	○	○	○	7
1970	1.0300	1.0314	.0717	.0727	○	○	○	7
1971	1.0362	1.0367	.0700	.0688	$K_L>1$	$K_U>1$	○	7
1972	1.0557	1.0622	.0629	.0665	$K_L>1$	$K_U>1$	○	7
29. Leather & Leather Products								
1964	.9784	.9788	.1095	.1084	○	○	○	7
1965	1.0369	1.0289	.1900	.1669	○	○	○	7
1966	.9988	1.0024	.0808	.0789	○	○	○	7
1967	1.0520	1.0519	.1647	.1590	○	○	○	7
1968	1.0220	1.0393	.0648	.0840	○	○	○	5
1969	1.0017	1.0076	.1285	.1341	○	○	○	5
1970	.9725	.9771	.1243	.1272	○	○	○	5
1971	.9799	.9889	.0985	.1123	○	○	○	5
1972	1.0316	1.0330	.1024	.1024	○	○	○	5
30. Stone, Clay, & Glass Products								
1964	1.0603	1.0668	.0847	.0859	$K_L>1$	$K_U>1$	○	8
1965	1.0612	1.0688	.1142	.1129	$K_L>1$	$K_U>1$	○	8
1966	1.0511	1.0565	.0751	.0737	$K_L>1$	$K_U>1$	○	8
1967	1.0637	1.0698	.0617	.0624	$K_L>1$	$K_U>1$	○	8
1968	1.0739	1.0786	.0682	.0677	$L_L>1$	$K_U>1$	○	7
1969	1.0865	1.0928	.0901	.0907	$K_L>1$	$K_U>1$	○	7
1970	1.0783	1.0812	.0817	.0812	$K_L>1$	$K_U>1$	○	7
1971	1.0724	1.0747	.0918	.0896	$K_L>1$	$K_U>1$	○	7
1972	1.0714	1.0754	.0862	.0861	$K_L>1$	$K_U>1$	○	7

APPENDIX TABLE I. (Continued)

Year	$L$	$U$	$S_L$	$S_U$	Null-Hypothesis			Sample Size
					$K_L=1$	$K_U=1$	$K_L=K_U$	
31. Iron & Steel								
1964	.9963	.9977	.0302	.0300	○	○	○	8
1965	.9937	.9952	.0157	.0145	○	○	○	8
1966	.9979	.9990	.0277	.0278	○	○	○	8
1967	.9972	.9986	.0274	.0265	○	○	○	8
1968	1.0047	1.0059	.0652	.0631	○	○	○	7
1969	1.0130	1.0138	.0801	.0778	○	○	○	7
1970	1.0002	1.0014	.0414	.0398	○	○	○	7
1971	.9953	1.0017	.0411	.0414	○	○	○	7
1972	.9999	1.0021	.0354	.0354	○	○	○	7
32. Nonferrous Metal								
1964	1.0085	1.0094	.0230	.0222	○	○	○	8
1965	.9957	.9977	.0431	.0440	○	○	○	8
1966	.9991	1.0018	.0495	.0496	○	○	○	8
1967	.9973	.9989	.0399	.0401	○	○	○	8
1968	1.0083	1.0093	.0342	.0342	○	○	○	7
1969	1.0067	1.0070	.0555	.0536	○	○	○	7
1970	1.0035	1.0053	.0481	.0484	○	○	○	5
1971	.9967	.9973	.0339	.0334	○	○	○	7
1972	1.0126	1.0128	.0342	.0321	○	○	○	7
33. Fabricated Metal Products								
1964	1.0130	1.0155	.0629	.0617	○	○	○	8
1965	1.0040	1.0066	.0452	.0458	○	○	○	8
1966	1.0017	1.0036	.0595	.0587	○	○	○	8
1967	.9999	1.0020	.0380	.0367	○	○	○	8
1968	1.0027	1.0037	.0433	.0448	○	○	○	7
1969	1.0127	1.0142	.0379	.0389	○	○	○	7
1970	1.0131	1.0141	.0391	.0388	○	○	○	7
1971	1.0415	1.0453	.0570	.0618	$K_L>1$	$K_U>1$	○	7
1972	1.1093	1.1142	.2353	.2431	○	○	○	7
34. Machinery, except Electrical								
1964	1.0120	1.0137	.0268	.0265	$K_L>1$	$K_U>1$	○	8
1965	1.0039	1.0055	.0269	.0208	○	○	○	8
1966	1.0186	1.0217	.0566	.0550	○	○	○	8
1967	1.0054	1.0068	.0268	.0260	○	○	○	8
1968	1.0102	1.0114	.0191	.0201	$K_L>1$	$K_U>1$	○	7
1969	1.0099	1.0111	.0710	.0717	○	○	○	7
1970	1.0054	1.0064	.0322	.0330	○	○	○	7
1971	1.0130	1.0139	.0279	.0285	○	○	○	7
1972	1.0248	1.0258	.0604	.0602	○	○	○	7

APPENDIX TABLE I. (Continued)

Year	$\bar{L}$	$\bar{U}$	$S_L$	$S_U$	Null-Hypothesis			Sample Size
					$K_L=1$	$K_U=1$	$K_L=K_U$	
35. Electrical Equipment & Supplies								
1964	.9970	.9980	.1007	.1021	○	○	○	8
1965	1.0139	1.0152	.0496	.0506	○	○	○	8
1966	1.0076	1.0691	.0648	.0664	○	○	○	8
1967	1.0196	1.0207	.0690	.0684	○	○	○	8
1968	1.0351	1.0363	.0646	.0645	○	○	○	7
1969	1.0412	1.0428	.0554	.0557	$K_L>1$	$K_U>1$	○	7
1970	1.0305	1.0321	.0802	.0800	○	○	○	7
1971	1.0284	1.0298	.0448	.0461	$K_L>1$	$K_U>1$	○	7
1972	1.0416	1.0437	.0836	.0856	$K_L>1$	$K_U>1$	○	7
36. Transportation Equipment								
1964	1.0077	1.0102	.0363	.0380	○	○	○	8
1965	1.0090	1.0122	.0528	.0527	○	○	○	8
1966	1.0044	1.0073	.0496	.0499	○	○	○	8
1967	1.0008	1.0032	.0348	.0358	○	○	○	8
1968	.9998	1.0015	.0467	.0488	○	○	○	7
1969	1.0076	1.0091	.0254	.0267	○	○	○	7
1970	1.0043	1.0064	.0340	.0362	○	○	○	7
1971	1.0016	1.0034	.0233	.0246	○	○	○	7
1972	1.0145	1.0162	.0511	.0516	○	○	○	7
37. Instrument & Related Products								
1964	1.0040	1.0069	.0919	.0946	○	○	○	8
1965	.9982	1.0011	.0163	.1087	○	○	○	8
1966	1.0028	1.0072	.0680	.0737	○	○	○	8
1967	1.0059	1.0093	.1102	.1121	○	○	○	8
1968	1.0166	1.0177	.0237	.0257	$L_L>1$	$K_U>1$	○	7
1969	1.0164	1.0187	.0599	.0605	○	○	○	7
1970	1.0123	1.0142	.0386	.0405	○	○	○	7
1971	1.0134	1.0162	.0533	.0551	○	○	○	7
1972	1.0071	1.0087	.0679	.0698	○	○	○	7
39. Miscellaneous Manufacturing Industries								
1964	1.0180	1.0197	.0521	.0527	○	○	○	8
1965	1.0318	1.0329	.0869	.0834	○	○	○	8
1966	1.0272	1.0284	.0501	.0485	$K_L>1$	$K_U>1$	○	8
1967	1.0206	1.0216	.0966	.0956	○	○	○	8
1968	1.0329	1.0357	.0591	.0599	$K_L>1$	$K_U>1$	○	7
1969	1.0442	1.0459	.0968	.0965	○	○	○	7
1970	1.0233	1.0245	.0753	.0748	○	○	○	7
1971	1.0117	1.0134	.0714	.0714	○	○	○	7
1972	1.0196	1.0217	.1426	.1419	○	○	○	7

APPENDIX TABLE II. CORRELATION COEFFICIENT OF  $L_j$  AND  $L_{j-1}$  AND THAT OF  $U_j$  AND  $U_{j-1}$ 

	1964	1965	1966	1967	1968	1969	1970	1971	1972
18. Food & Kindred Products									
$R_{L,L-1}$	[ 0.22] .0960(7)	[-0.18] -.0810(7)	[-0.85] -.3567(7)	[-1.00] -.4085(7)	★[-1.38] -.5682(6)	[-1.27] -.5368(6)	[-1.10] -.4809(6)	[-1.00] -.4463(6)	★[-1.64] -.6335(6)
$R_{U,U-1}$	[ 0.20] .0891	[-0.20] -.0902	[-0.86] -.3602	[-1.00] -.4093	○[-1.42] -.5789	[-1.31] -.5469	[-1.16] -.5023	[-1.09] -.4769	★[-1.76] -.6598
20. Textile Mill Products									
$R_{L,L-1}$	[-0.69] -.2941(7)	[ 0.15] -.0647(7)	[-0.51] -.2221(7)	-0.003 -.0015(7)	[ 0.06] .0285(6)	[-0.96] -.4344(6)	[-0.53] -.2544(6)	[ 0.26] .1269(6)	[-0.44] -.2162(6)
$R_{U,U-1}$	[-0.58] -.2525	[-0.11] .0507	[-0.72] -.3077	[-0.01] -.0052	[ 0.09] .0443	[-0.91] -.4128	[-0.40] -.1960	[ 0.27] .1340	[-0.41] -.1994
21. Apparel & other Textile Products									
$R_{L,L-1}$	★[-1.97] -.7026(6)	[-0.71] -.3328(6)	[-0.14] -.0704(6)	[ 1.13] .4926(6)	[-0.54] -.2591(6)	[-0.13] -.0902(4)	[-0.30] -.2071(4)	[-0.89] -.5319(4)	[-1.07] -.6025(4)
$R_{U,U-1}$	★[-2.12] -.7273	[-0.72] -.3401	[-0.07] -.0357	[ 0.86] .3953	[-0.70] -.3290	[-0.02] -.0108	[-0.25] -.1757	[-1.02] -.5846	[-1.03] -.5880
22. Lumber & Wood Products									
$R_{L,L-1}$	★[-2.17] -.6969(7)	★[ 1.45] .5437(7)	[-1.17] -.4637(7)	[-1.10] -.4405(7)	★[-2.09] -.7223(6)	[-1.21] -.5178(6)	★[-1.59] -.6216(6)	[ 1.01] .4494(6)	★[-3.58] -.8731(6)
$R_{U,U-1}$	★[-2.11] -.6868	☆[ 1.53] .5639	[-1.12] -.4470	[-1.10] -.4418	★[-1.88] -.6854	[-1.21] -.5182	★[-1.51] -.6030	[ 1.04] .4622	★[-3.51] -.8688
23. Furniture & Fixtures									
$R_{L,L-1}$	[ 0.58] 2.793(6)	[-0.32] -.1603(6)	[-0.97] -.4370(6)	[-0.90] -.4099(6)	[ 0.76] .4734(4)	[ 0.10] .0680(4)	★[ 1.70] .7013(5)	★[-6.41] -.9654(5)	[ 0.65] .3494(5)
$R_{U,U-1}$	[ 0.68] .3199	[-0.25] -.1227	[-1.04] -.4613	[-0.91] -.4157	[ 0.60] .3922	[ 0.07] .0514	★[ 1.54] .6654	★[-6.14] -.9624	[ 0.60] .3290

Note: Each number in ( ) is the sample size. And figures in [ ] are t-values, where  $t = (R\sqrt{n-2})/\sqrt{1-R^2}$ . The symbol (★) means the case in which the hypothesis  $H_0: R=0$  cannot be rejected under 25% significance level.

Table II. (Continued)

	1964	1965	1966	1967	1968	1969	1970	1971	1972
<b>24. Pulp &amp; Paper Products</b>									
$R_{L,L-1}$	[−0.65] −.2790(7)	★[−1.66] −.5969(7)	[ 0.27] .1218(7)	[−0.88] −.3655(7)	[ 0.05] .2646(6)	[ 0.55] −.1036(6)	[−0.21] −.1036(6)	[−0.93] −.4224(6)	★[ 1.44] .5855(6)
$R_{U,U-1}$	[−0.60] −.2587	★[ 1.76] −.6193	[ 0.20] .0870	[−0.89] −.3703	[−0.002] −.0012	[ 0.70] .3314	[−0.11] −.0542	★[−0.97] −.4379	★[ 1.77] .6631
<b>25. Printing &amp; Publishing</b>									
$R_{L,L-1}$	★[−1.97] −.6610(7)	★[ 1.80] .6264(7)	★[−2.27] −.7220(7)	[−0.85] −.3558(7)	[−0.40] −.1973(6)	★[−1.65] −.6375(6)	[−0.88] −.4020(6)	[−0.49] −.2371(6)	[−0.81] −.3753(6)
$R_{U,U-1}$	★[−1.49] −.5540	★[ 1.36] .5202	★[−1.34] −.5129	[−0.84] −.3530	[−0.49] −.2369	★[−2.47] −.7770	[−0.82] −.3781	[−0.44] −.2127	[−0.78] −.3647
<b>26. Chemical &amp; Allied Products</b>									
$R_{L,L-1}$	[ 0.40] .1776(7)	★[−1.74] −.6137(7)	[−0.27] −.1197(7)	[−0.07] −.0335(7)	★[−3.14] −.8431(6)	[−1.07] −.4730(6)	★[−2.25] −.7929(5)	[ 0.10] .0513(6)	[ 0.05] .0230(6)
$R_{U,U-1}$	[ 0.53] .2325	★[−1.88] −.6443	[−0.35] −.1530	[−0.12] −.0555	★[−3.19] −.8476	[−1.05] −.4636	★[−2.19] −.7848	[ 0.10] .0520	[ 0.02] .0108
<b>27. Petroleum &amp; Coal Products</b>									
$R_{L,L-1}$	[−0.21] −.0949(7)	[−0.12] −.0603(6)	★[−1.40] −.5735(6)	[−0.90] −.4601(5)	[−0.40] −.2722(4)	★[−1.74] −.7761(4)	[−0.26] −.1830(4)	[ 0.11] .0804(4)	[−0.55] −.2668(6)
$R_{U,U-1}$	[−0.28] −.1263	[−0.05] −.0261	★[−1.40] −.5743	[−0.91] −.4646	[−0.30] −.2073	★[−4.56] −.9551	[−0.27] −.1900	[ 0.06] .0456	[−0.59] −.2831
<b>28. Rubber &amp; Plastic Products</b>									
$R_{L,L-1}$	★[−1.35] −.5162(7)	[−0.04] −.0176(7)	[−0.10] −.0440(7)	[ 0.01] .0055(6)	[−1.17] −.5061(6)	[−0.56] −.2704	[−0.87] −.4001(6)	[ 0.31] .1511(6)	[ 0.39] .1937(6)
$R_{U,U-1}$	★[−1.41] −.5325	[−0.05] −.0208	[−0.15] −.0673	[−0.05] −.0255	[−1.23] −.5235	[−0.71] −.3328	[−0.88] −.4041	[ 0.22] .1098	[ 0.39] .1896

Table II. (Continued)

	1964	1965	1966	1967	1968	1969	1970	1971	1972
<b>29. Leather &amp; Leather Products</b>									
$R_{L,L-1}$	[−0.85] −.3915(6)	[−1.06] −.4671(6)	★[−2.03] −.7129(6)	[ 0.76] .3540(6)	[ 0.50] .3352(4)	[−1.16] −.6331(4)	★[ 2.68] .8841(4)	[−0.25] −.1730(4)	[−0.93] −.5491(4)
$R_{U,U-1}$	[−0.84] −.3881	[−1.19] −.5120	★[−2.03] −.7126	[ 0.74] .3475	[ 0.44] .2961	[−1.19] −.6442	★[ 2.60] .8783	[−0.19] −.1355	[−1.09] −.6115
<b>30. Stone, Clay, &amp; Glass Products</b>									
$R_{L,L-1}$	★[ 1.46] .5467(7)	[−1.10] −.4410(7)	★[−2.56] −.7526(7)	[ 1.25] .4872(7)	[−0.39] −.1891(6)	[−0.02] −.0081(6)	[−0.77] −.3574(6)	[−0.42] −.2066(6)	[−0.49] −.2383(6)
$R_{U,U-1}$	★[ 1.57] .5755	★[−1.34] −.5127	★[−3.10] −.8112	[ 0.87] .3641	[−0.26] −.1248	[ 0.06] .0285	[−0.72] −.3369	[−0.36] −.1770	[−0.40] −.1963
<b>31. Iron &amp; Steel</b>									
$R_{L,L-1}$	★[−2.25] −.7098(7)	[−0.47] −.2066(7)	[−0.80] −.3378(7)	[−0.95] −.3904(7)	[−1.07] −.4723(6)	[−0.92] −.4195(6)	[−0.97] −.4371(6)	−0.27 −.1356(6)	[−1.34] −.5564(6)
$R_{U,U-1}$	★[−2.30] −.7172	[−0.79] −.3319	[−0.83] −.3481	[−1.05] −.4254	[−1.10] −.4829	[−0.90] −.4090	[−0.95] −.4283	[−0.57] −.2759	★[−1.49] −.5971
<b>32. Nonferrous Metal</b>									
$R_{L,L-1}$	[−1.17] −.4644(7)	[−0.99] −.4035(7)	[−0.96] −.3944(7)	[−1.23] −.4808(7)	★[−2.18] −.7333(6)	[ 0.007] .0036(6)	[−0.98] −.5697(4)	[ 0.20] .0973	[−1.17] −.5047(6)
$R_{U,U-1}$	[−1.00] −.4074	[−1.03] −.4177	[−0.93] −.3843	[−1.22] −.4789	★[−2.13] −.7289	[ 0.04] .0197	[−0.92] −.5439	[ 0.26] .1280	[−1.17] −.5044
<b>33. Fabricated Metal Products</b>									
$R_{L,L-1}$	★[−2.55] −.7518(7)	[−0.78] −.3285(7)	[−0.76] −.3216(7)	[−0.06] −.0290(7)	★[−2.90] −.8236(6)	[−0.37] −.1832(6)	[−0.44] −.2131(6)	★[−1.41] −.5752(6)	[−0.02] −.0101(6)
$R_{U,U-1}$	★[−2.20] −.7013	[−0.85] −.3554	[−0.68] −.2892	[−0.05] −.0236	★[−2.77] −.8103	[−0.30] −.1470	[−0.33] −.1616	[−1.13] −.4928	[−0.04] −.0188

Table II. (Continued)

	1964	1965	1966	1967	1968	1969	1970	1971	1972
<b>34. Machinery, except Electrical</b>									
$R_{L,L-1}$	[−0.19] −.0851(7)	[−0.07] −.0296(7)	[−0.83] −.3473(7)	[ 0.01] .0052(7)	[−0.05] −.0257(6)	★[−1.67] −.6415(6)	[−0.78] −.3N35(6)	[ 0.93] .4202(6)	[−1.19] −.5125(6)
$R_{U,U-1}$	[−0.16] −.0719	[−0.10] −.0441	[−0.84] −.3522	[ 0.07] .0306	[−0.07] −.0370	★[−1.66] −.6380	[−0.66] −.3130	[ 1.04] .4604	[−1.19] −.5128
<b>35. Electrical Equipment &amp; Supplies</b>									
$R_{L,L-1}$	[−1.32] −.5104(7)	[−0.02] −.0093(7)	[−0.50] −.2199(7)	[−0.35] −.1542(7)	[ 0.53] .2551(6)	[ 0.59] .2827(6)	[−0.16] −.0790(6)	★[ 1.43] .5826(6)	[−0.64] −.3050(6)
$R_{U,U-1}$	[−1.33] −.5114	[−0.02] −.0076	[−0.52] −.2249	[−0.34] −.1485	[ 0.62] .2962	[ 0.65] .3070	[−0.06] −.0290	★[ 1.43] .5810	[−0.63] −.2990
<b>36. Transportation Equipment</b>									
$R_{L,L-1}$	[ 0.82] .3460(7)	[−0.03] −.0148(7)	[−0.57] −.2485(7)	★[ 1.67] .5994(7)	[−0.40] −.1982(6)	[ 0.79] .3685(6)	★[ 1.62] .6293(6)	[−0.05] −.0236(6)	[−0.57] −.2750(6)
$R_{U,U-1}$	[ 0.90] .3728	[ 0.16] .0708	[−0.47] −.2048	★[ 1.78] .6228	[−0.31] −.1545	[ 0.97] .4354	★[ 1.74] .6558	[ 0.05] .0251	[−0.51] −.2485
<b>37. Instrument &amp; Related Products</b>									
$R_{I,L-1}$	★[−1.67] −.5985(7)	[−0.61] −.2643(7)	[ 0.18] .0795(7)	[−1.06] −.4288(7)	[−0.07] −.0374	★[−1.68] −.6425(6)	[−0.11] −.0560(6)	[−0.53] −.2550(6)	[−0.29] −.1457(6)
$R_{U,U-1}$	★[−1.87] −.6419	[−0.63] −.2727	[ 0.11] .0484	[−1.01] −.4108	[ 0.03] .0131	★[−1.55] −.6114	[−0.02] −.0097	[−0.66] −.3153	[−0.27] −.1341
<b>39. Miscellaneous Manufacturing Industries</b>									
$R_{L,L-1}$	★[−2.71] −.7709(7)	[−1.07] −.4317(7)	[ 0.33] .1449(7)	[−1.52] −.5618	[−1.03] −.4566(6)	[−1.26] −.7606(6)	★[−2.34] −.7606(6)	★[−3.02] −.8339(6)	★[−4.50] −.9137(6)
$R_{U,U-1}$	★[−2.79] −.7805	[−1.09] −.4368	[ 0.33] .1462	[−1.59] −.1595	[−1.12] −.4876	[−1.21] −.5188	★[−2.27] −.7502	★[−2.86] −.8196	★[−4.37] −.9092