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Chapter 5

AN EMPIRICAL ANALYSIS OF LABOR DEMAND*

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INTRODUCTION

Some conspicuous changes emerged in labor market trends since about 1970 in the Japanese economy. These changes are, in short, substantial increases in the levels of output and of real capital stock on the one hand and the slow rate of growth in the number of employed persons and even a decline in the number of employees on the other. The purpose of this paper is to analyze these changes in the labor market.

In Section 1, we will introduce a model of labor demand which is built upon a production function.[1]

In Section 2, we will analyze reasons why these changes took place in the labor market on the basis of our empirical quantitative analysis of selected industries.

The production function on which our analysis will be based has the following four properties:

(1) The labor input consists of male and female labor which is explicitly divided in the model,[2]

(2) For a given level of output, the capital stock and the two kinds of labor demand are perfectly complementary with each other,

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[1] This study draws heavily on the specification of the production function developed by professor Kei-ichiro Obi [3]. Professor Obi has attempted a variety of alternative specifications of labor and capital input functions to analyze the data of various industries for the period of 1953 to 1969. I took advantage, in this study, of using the specifications which produced relatively better results in Professor Obi's research.

[2] There are two basic reasons why male and female labor inputs are explicitly distinguished in the model. One relates to the fact widely acknowledged ever since the pioneering research of late Professor Douglas [1] that the patterns of behavior and the roles of males and females are usually clearly different in determining the household's supply of labor services. The fact may be expressed, in short, as that the labor supply of principal earner of the household (usually the male household head) is inelastic with the market wage rate while that of other household members (notably the wife of the household head) is elastic with market wage rates and also closely and inversely related to the income of the principal earner. The other reason relates to the fact that male and female labor services are often quite different in quality from the viewpoint of the employer. For these reasons it is imperative to distinguish between male and female labor in constructing a general labor market model in which both labor demand and supply and their interactions are appropriately specified.

(3) For a given level of output, the combination of capital stock and the number of male and female workers depends on the ratio of male to female workers,

(4) For a given level of output, the extent to which male and female workers are substituted with each other depends upon the level of capital stock.

SECTION 1. MALE-FEMALE LABOR DEMAND MODEL FOR INDUSTRIES

This model is the base of the theory of firm and can be represented as

(1) $Q=f(L_m, L_f)$; Labor Input Function,

where Q is the level of capacity of plant, L_m is the input of male labor, also L_f is the input of female labor and f is a function that is generally assumed to be continously differential, so that the partial derivatives are continous. (1) represents the case of a firm in which male and female labor are substitutable for the given capacity.

(2) $K=g(L_f/L_m, Q)$; Capital Input Function,

where K is the amount of capital, (2) assums that capital stock varies with different combinations of female and male labor for the given capacity.

(3) $C=W_mL_m+W_fL_f+rK$; Total Cost Function,

where W_m and W_f are male and female wages respectively and r is the unit cost of capital. Here it is assumed that $\bar{W}_m$, $\bar{W}_f$, $\bar{r}$, and $\bar{Q}$ are exgoneous variables while L_m, L_f, K are endoneous variables. So the problem of the firm in this case of two labor inputs and one capital input is expressed as that of choosing three inputs L_m, L_f and K so as to minimize the cost (3) or $C=W_mL_m+W_fL_f+rK$ subject to (1) and (2), that is

$$\text{Min } \varphi=C+\lambda(\bar{Q}-f(L, L_f))=\bar{W}_mL_m+\bar{W}_fL_f+\bar{r}K+\lambda(\bar{Q}-f(L_m, L_f))$$
$$=\bar{W}_mL_m+\bar{W}_fL_f+\bar{r}g\left(\frac{L_m}{L_f}, \bar{Q}\right)+\lambda(\bar{Q}-f(L_m, L_f)).$$

The necessary (first-order) conditions for cost minimization are

$$\frac{\partial\varphi}{\partial L_m}=\bar{W}_m+\bar{r}\frac{\partial g}{\partial L_m}-\lambda\frac{\partial f}{\partial L_m}=0 \quad (4)$$

$$\frac{\partial\varphi}{\partial L_f}=\bar{W}_f+\bar{r}\frac{\partial g}{\partial L_f}-\lambda\frac{\partial f}{\partial L_f}=0 \quad (5)$$

These conditions require that

$$\frac{\partial f}{\partial L_m}\Big/\left(\bar{W}_m+\bar{r}\frac{\partial g}{\partial L_m}\right)=\frac{\partial f}{\partial L_f}\Big/\left(\bar{W}_f+\bar{r}\frac{\partial g}{\partial L_f}\right). \quad (6)$$

These conditions state that the marginal product of male or female labor input must equal not only its wage rate but also the unit cost of capital and that the value of equilibrium changes with changes in capital stock. One of the most widely used functions for empirical estimation of equation (1) is the Cobb-Douglas function of the form

(1)′ $$\bar{Q}=aL_m{}^b L_f{}^c$$

where a, b, c are positive parameters. And also (2) is specified of the form

(2)′ $$K=\alpha+\beta\frac{L_f}{L_m}+\gamma\bar{Q}.$$

So (6) is reduced to the form

(6)′ $$\frac{L_m\overline{W}_m}{L_f\overline{W}_f}=\left(\frac{b}{c}\right)+\left(\frac{b}{c}+1\right)\frac{\bar{r}}{L_m\overline{W}_f}.$$

Here let us summarize the parameters' conditions.

In (1)′ the following equation is derived with the definitive equation $L=L_m+L_f$.

$$Q=aL^{(b+c)}\left(\frac{L_m}{L_f}\right)^b\left(1+\frac{L_m}{L_f}\right)^{-(b+c)}$$

And the following equation is obtained as the partial derivative of Q with respect to the ratio of male to female labor.

$$\frac{\partial Q}{\partial(L_m/L_f)}=aL^{(b+c)}\left(\frac{L_m}{L_f}\right)^{(b-1)}\left(1+\frac{L_m}{L_f}\right)^{-(b+c)-1}\left(b-c\frac{L_m}{L_f}\right)$$

Here if

$$\frac{L_m}{L_f}<\frac{b}{c} \text{ then ① } \frac{\partial(L_m/L_f)}{\partial Q}>0$$

and otherwise

$$\frac{L_m}{L_f}\geq\frac{b}{c} \text{ then ② } \frac{\partial(L_m/L_f)}{\partial Q}\leq 0$$

And similarly the following equation is derived.

$$L=a^{1/(b+c)}\left(\frac{L_m}{L_f}\right)^{-b/(b+c)}\left(1+\frac{L_m}{L_f}\right)Q^{1/(b+c)}$$

And also the following equation is obtained as the partial derivative of L with respect to the ratio of male to female labor.

$$\frac{\partial L}{\partial(L_m/L_f)}=\frac{a^{-1/(b+c)}}{(b+c)}\left(-b+c\frac{L_m}{L_f}\right)Q^{1/(b+c)}$$

Here if

$$\frac{L_m}{L_f}<\frac{b}{c} \text{ then } \frac{\partial L}{\partial(L_m/L_f)}<0$$

and otherwise

$$\frac{L_m}{L_f}\geq\frac{b}{c} \text{ then } \frac{\partial L}{\partial(L_m/L_f)}\geq 0,$$

In (2)′ parameter γ is assumed to be fixed positive, and if β is positive we obtain the following relation:

$$\frac{\partial K}{\partial(L_f/L_m)}>0 \quad \text{or} \quad \frac{\partial K}{\partial(L_m/L_f)}<0,$$

that is, the amount of capital stock increases with an increase in the ratio of female to male labor input.

And if β is not positive, we gain also the following relation:

$$\frac{\partial K}{\partial(L_f/L_m)}<0 \quad \text{or} \quad \frac{\partial K}{\partial(L_m/L_f)}>0,$$

that is, the level of capacity decreases with an increase in the ratio of female to male labor input.

And the second-order conditions of cost minimum are:

$$\begin{vmatrix} \dfrac{\partial^2 C}{\partial L_m^2} & \dfrac{\partial^2 C}{\partial L_f \partial L_m} \\ \dfrac{\partial^2 C}{\partial L_m \partial L_f} & \dfrac{\partial^2 C}{\partial L_f^2} \end{vmatrix} >0, \quad \frac{\partial^2 C}{\partial L_m^2}>0 \quad \text{or} \quad \frac{\partial^2 C}{\partial L_f^2}>0$$

that is:

① $$\frac{\partial^2 C}{\partial L_m^2}=\left(\frac{b}{c}+1\right)\frac{L_f}{L_m^2}\left(\frac{b}{c}W_f+\left(\frac{b}{c}+2\right)\beta\gamma\frac{1}{L_m}\right)$$

② $$\frac{\partial^2 C}{\partial L_f^2}=\frac{c}{b}\left(\frac{c}{b}+1\right)\frac{1}{L_f}\left(\frac{W_m L_m}{L_f}+\beta\gamma\frac{1}{L_m}\right)$$

③ $$\frac{\partial^2 C}{\partial L_m^2}\cdot\frac{\partial^2 C}{\partial L_f^2}-\frac{\partial^2 C}{\partial L_f \partial L_m}\cdot\frac{\partial^2 C}{\partial L_m \partial L_f}$$
$$=\frac{1}{L_m}\left\{\frac{W_m W_f}{L_f}\left(\frac{c}{b}+\frac{b}{c}+1\right)+\frac{\beta^2\gamma^2}{L_m^3}\frac{2(b+c)}{b^2}\right\}$$
$$+\frac{\beta\gamma}{L_m^2}\frac{(b+c)}{b}\left\{\frac{W_m}{L_f}\left(\frac{2c}{b}+\frac{b}{c}+2\right)+\frac{L_f}{L_m}\right\}$$

Conditions ①, ② and ③ are satisfied if all parameters a, b, c, β, γ are positive. If β is negative and other parameters are positive, whether all conditions satisfied or not depends on the sign of $(b/c)W_f+(b/c+2)\beta\gamma(1/L_m)$ in ①, the sign of $W_m L_m/L_f+\beta\gamma(1/L_m)$ in ②, and also on the sign of equation ③.

Totally differentiating function (1)′, in which capacity is fixed, will give the following equation:

$$\frac{\partial Q}{\partial L_m}dL_m+\frac{\partial Q}{\partial L_f}dL_f=0$$

then we can get the marginal substitution ratio of female to male labor.

(7) $$-\frac{dL_f}{dL_m}=\frac{\partial Q/\partial L_m}{\partial Q/\partial L_f}=\frac{b}{c}\frac{Q/L_m}{Q/L_f}=\frac{b}{c}\frac{L_f}{L_m}$$

and in (2)′ we can get the elasticity of capital to the female-male labor ratio:

$$(8)\qquad \frac{\partial K/K}{\partial(L_f/L_m)\,/\,(L_f/L_m)}=\frac{\partial K}{\partial(L_f/L_m)}\cdot\frac{1}{K}\frac{L_f}{L_m}=\frac{\beta}{K}\cdot\frac{L_f}{L_m}$$

and also we can get the elasticity of capital to capacity.

$$(9)\qquad \frac{\partial K/K}{\partial Q/Q}=\frac{\partial K}{\partial Q}\frac{Q}{K}=r\frac{Q}{K}$$

Now the two first order conditions in (4), (5) and the labor input function in (1)′ form a system of three simultaneous equations that determine cost-minimizing inputs L_m, L_f, K. However, we can not solve this directly because it is a nonlinear system.

So we describe the reduce form as (10), (11), (12).

$$(10)\qquad L_m=\varphi(Q,\ W_m,\ W_f,\ r)$$

$$(11)\qquad L_f=\phi(Q,\ W_m,\ W_f,\ r)$$

$$(12)\qquad K=H(Q,\ W_m,\ W_f,\ r)$$

Totally differentiating these equations we obtain the following equations:

$$(13)\qquad dL_m=\frac{\partial\varphi}{\partial Q}dQ+\frac{\partial\varphi}{\partial W_m}dW_m+\frac{\partial\varphi}{\partial W_f}dW_f+\frac{\partial\varphi}{\partial r}dr$$

$$(14)\qquad dL_f=\frac{\partial\phi}{\partial Q}dQ+\frac{\partial\phi}{\partial W_m}dW_m+\frac{\partial\phi}{\partial W_f}dW_f+\frac{\partial\phi}{\partial r}dr$$

$$(15)\qquad dK=\frac{\partial H}{\partial Q}dQ+\frac{\partial H}{\partial W_m}dW_m+\frac{\partial H}{\partial W_f}dW_f+\frac{\partial H}{\partial r}dr$$

Here if parameters a, b, c, β, γ are positive, the sign of the partial derivatives are

$$\frac{\partial\varphi}{\partial Q}>0\quad \frac{\partial\varphi}{\partial W_m}<0\quad \frac{\partial\varphi}{\partial W_f}>0\quad \frac{\partial\varphi}{\partial r}>0$$

$$\frac{\partial\phi}{\partial Q}>0\quad \frac{\partial\phi}{\partial W_m}>0\quad \frac{\partial\phi}{\partial W_f}<0\quad \frac{\partial\phi}{\partial r}<0$$

$$\frac{\partial H}{\partial Q}>0\quad \frac{\partial H}{\partial W_m}>0\quad \frac{\partial H}{\partial W_f}<0\quad \frac{\partial H}{\partial r}<0$$

These state that demands for male and female labor and for capital depend upon not only on changes in the capacity of plant but also on wages and unit cost of capital.

(13) plus (14) represents the change of total demand labor, that is

$$\begin{aligned} dL&=dL_m+dL_f\\ &=\left(\frac{\partial\varphi}{\partial Q}+\frac{\partial\phi}{\partial Q}\right)dQ+\left(\frac{\partial\varphi}{\partial W_m}+\frac{\partial\phi}{\partial W_m}\right)dW_m+\left(\frac{\partial\varphi}{\partial W_f}+\frac{\partial\phi}{\partial W_f}\right)dW_f\\ &\quad+\left(\frac{\partial\varphi}{\partial r}+\frac{\partial\phi}{\partial r}\right)dr. \end{aligned}$$

In Section 3 we will see that in the process of rapid economic growth, changes in the total demand for labor depend mainly upon changes in productive capacity, but in the phase of slower growth changes in male-female wage differentials and changes in unit cost of capital play an important role in determining the total labor demand.

SECTION 2. ESTIMATED RESULTS OF MALE-FEMALE LABOR DEMAND MODELS FOR DIFFERENT INDUSTRIES

Tables 1 and 2 presents the results of estimation using directly the multiple regression method. According to the results, we notice that in many industries the theoretical sign conditions $a>0$, $b>0$, $c>0$ are not satisfied and estimates are not statistically significant. In the next step, we therefore estimated a reduced form equation (3) in order to circumvent the problem of multicollinearity. The result is shown in Table 3.

Let us substitute $A=b/c$ and $B=(b/c+1)\ \beta$ in equation (6)′;

$$\frac{W_mL_m}{W_fL_f}=\left(\frac{b}{c}\right)+\left(\frac{b}{c}+1\right)\beta\frac{P_k(i+de)}{W_fL_m}.$$

The results presented in Table 3 indicate that parameters of A and B are statistically significant. The sign condition $A>0$ is also satisfied. The value of β which is obtained by the relation $\beta=B/(A+1)$ is negative for the sectors of oil and coal, and textile products and therefore casts doubt on whether the second order condition for cost minimization is satisfied. An examination of the second order tion reveals that the condition is satisfied for oil and coal industry while unsatisfied for textile industry for the period 1955 to 1960. This may be the consequence of changes in the product mix in textile industry.

The value of $A=b/c$ indicates the male-female labor mix for each industry. The low values for textile and monetary, insurance and real estate industries indicate that the weight of female labor in production for these industries is high, while the high values for mining, primary metals, transportation equipment, oil and coal, transportation, communication and public utilities, and construction suggest that the relative contribution of male labor is high for these industries. The results also show that the condition $L_m/L_f<b/c$ is satisfied for all industry sectors.

We then estimate equation (1)′ utilizing the estimated value of parameter A. We estimate a and b from equation $Q=a(L_mL_f^{(1/A)})^b$ which was derived from $Q=a(L_mL_f^{(c/b)})^b$ by setting $c/b=1/A$. The results of estimation are shown in Table 4.

The expected sign condition $b>0$ is satisfied and statistically significant in all sectors except mining. Thus we can estimate the values of a, b and c from the relation $c=b/A$. Using the estimates of A and B, we can compute the value for β. Further we can compute α and β using the relationship $K-\beta\cdot L_f/L_m=\alpha+\gamma Q$. The results are presented in Table 5.

TABLE 1. $Q_j = a_j L_m^{b_j} L_f^{c_j}$

	$\log a_j$	a_j	b_j	c_j	$\bar{r}$	D.W.
4. Mining	9.0022 (0.7232)	.8121E−04	0.1515 (0.4173)	−0.01684 (0.5431)	0.8753	0.9862
5. Food	−13.7808 (3.4907)	.1035E−05	2.9580 (0.8189)	0.6185 (0.3548)	0.9213	0.5854
6. Textile products	−14.2603 (2.9046)	.6140E−06	2.4677 (0.1527)	1.0375 (0.3749)	0.9687	1.6114
7. Pulp paper and products	−17.3931 (2.6207)	.2794E−07	5.8754 (1.0347)	−1.7098 (0.7371)	0.9486	0.7102
8. Chemical and related products	−26.1189 (8.4480)	.4536E−11	3.8333 (3.5086)	2.2045 (2.6862)	0.9392	0.3596
9. Petroleum and coal products	−12.1323 (1.8569)	.5383E−05	5.7507 (0.5943)	−0.8307 (0.6184)	0.9291	0.6896
10. Stone, Clay and glass products	−19.3888 (1.1964)	.3798E−08	6.9518 (0.8801)	−2.9634 (0.8598)	0.9887	1.6059
11. Iron and Steal	−25.7723 (4.7017)	.6416E−11	7.1152 (1.2354)	−2.6381 (0.7770)	0.9559	1.1497
12. Fablicated metal products	− 9.0193 (1.8890)	.1210E−03	2.2913 (0.8177)	0.2237 (0.6587)	0.9913	0.8145
13. Machinery	− 4.6248 (6.1202)	.9806E−02	1.2516 (1.9578)	0.8860 (1.4117)	0.9265	0.2704
14. Electrical machinery and equipments	− 4.0882 (1.8850)	.1677E−01	0.8361 (0.7174)	1.1295 (0.4609)	0.9904	0.6502
15. Transportation equipment	−11.7703 (2.3251)	.7731E−05	2.7412 (0.6837)	0.4832 (0.4772)	0.9966	1.6350
16. Precision Instruments	−12.4872 (2.2962)	.3774E−05	3.8667 (1.0018)	−0.1024 (0.6019)	0.9886	0.7156
17. Other Manufacturing industries	−16.0545 (2.9802)	.1066E−06	2.9843 (1.0221)	0.3255 (0.7072)	0.9640	0.3762
18. Transportation communication and public utilities	−15.7959 (0.7336)	.1380E−06	3.5557 (0.2221)	−0.6055 (0.2254)	0.9927	0.9001
19. Construction	− 3.5547 (1.5954)	.2859E−01	0.8026 (0.3384)	0.9776 (0.1830)	0.9935	0.9200
20. Wholesale and retail trade	−15.7819 (2.0178)	.1400E−06	2.3580 (0.6811)	0.5421 (0.5028)	0.9815	0.6570
21. Financial, insurance and real estate dealing	− 9.1062 (3.3808)	.1109E−03	2.8080 (0.9695)	−0.0662 (0.4534)	0.9814	0.5444

Note: Numbers in parentheses are standard errors.

It is found that all sectors satisfy the sign condition $\gamma > 0$ and are statistically significant. The value of α, however, has different signs for different sectors

TABLE 2. $K_j=\alpha+\beta\cdot L_f/L_m+\gamma Q_j$

	α	β	r	$\bar{r}$	D.W.
4. Mining	−1,547.6214 (279.8103)	13,241.2930 (3,723.6813)	2.0085 (0.3120)	0.9404	0.9101
5. Food	−12.5456 (300.1715)	−1,419.0269 (428.1132)	0.4647 (0.0215)	0.9864	0.9604
6. Textile products	3,948.6041 (7073.3165)	−1,186.2100 (208.1842)	0.2238 (0.0869)	0.9812	1.4016
7. Pulp paper and products	155.8238 (134.5990)	−529.6629 (346.5565)	0.6091 (0.0165)	0.9942	0.9316
8. Chemical and related products	3,706.1711 (1,472.2915)	−0.000001 (4,687.4427)	0.7508 (0.0270)	0.9934	0.7955
9. Petroleum and coal products	83.3233 (83.9982)	−85.2410 (399.1196)	0.3205 (0.0103)	0.9940	1.0495
10. Stone, clay and glass products	1,130.7077 (722.5867)	−2,749.4750 (1,772.9133)	0.6469 (0.0279)	0.9856	0.7563
11. Iron and Steal	219.8267 (1,0530.0871)	−2,680.3100 (10,234.9880)	0.5703 (0.0385)	0.9756	0.9491
12. Fablicated metal products	520.6925 (371.1561)	−2,284.9870 (1,391.0353)	0.3927 (0.0331)	0.9718	0.6510
13. Machinery	446.6808 (240.5069)	−2,676.7080 (1,495.7259)	0.2190 (0.0121)	0.9872	0.9769
14. Electrical machinery and equipments	−153.6233 (300.0301)	643.4980 (595.4454)	0.2525 (0.0286)	0.9735	1.0019
15. Transportation equipment	570.4597 (751.0188)	−5,321.3900 (6,821.2751)	0.4436 (0.0374)	0.9859	0.7311
16. Precision instruments	65.0028 (17.4918)	−81.7728 (32.9967)	0.3203 (0.0082)	0.9975	2.3953
17. Other manufacturing industries	−531.9959 (759.7285)	399.2260 (1,292.7173)	0.3586 (0.0247)	0.9826	0.4429
18. Transportation communication and public utilities	752.0871 (935.5346)	−1,470.3200 (5,476.7214)	1.3790 (0.0241)	0.9981	1.2548
19. Construction	658.7935 (404.0401)	−10,828.7000 (3,688.7478)	0.3281 (0.0340)	0.9819	0.7051
20. Wholesale and retail trade	4,219.9558 (898.6190)	−5,453.2220 (1,498.6993)	0.6624 (0.0211)	0.9939	0.8093
21. Financial, insurance and real estate dealing	−575.4876 (152.0225)	−287.8740 (266.6752)	0.5834 (0.0221)	0.9973	0.8074

and often is not statistically significant. Table 6 presents the estimates of parameters a, b, c, α, β and γ which have been obtained by the procedure described above.

The parameters obtained from regression of equations (1)′ and (2)′ are not

TABLE 3. $\frac{W_{mj}L_{mj}}{W_{fj}L_{fj}}=\frac{b_j}{c_j}+\left(\frac{b_j}{c_j}+1\right)\beta_j\frac{P_k(i+de_j)}{W_{fj}L_{mj}}$

	b/c	$(b/c+1)\beta$	$\bar{r}$	D.W.
4. Mining	13.5488 (1.9340)	4,6445.390 (813.5570)	0.8063	0.8386
5. Food	1.8631 (0.1126)	591.9762 (66.7026)	0.9059	0.8999
6. Textile products	1.0795 (0.0381)	−66.3082 (16.3880)	−0.6891	0.4321
7. Pulp paper and products	4.3729 (0.1886)	282.6364 (35.8770)	0.8844	0.4426
8. Chemical and related products	5.3200 (0.0673)	440.3453 (26.1017)	0.9713	1.5035
9. Petroleum and coal products	12.1089 (0.6077)	−12.3509 (0.2815)	0	1.8262
10. Stone, clay and glass products	4.6912 (0.0510)	196.0050 (11.2092)	0.9732	2.0424
11. Iron and steal	12.9283 (0.5443)	2,851.3775 (335.3846)	0.8986	0.9817
12. Fablicated metal products	5.5650 (0.0894)	379.1801 (28.0671)	0.9562	1.4555
13. Machinery	8.1340 (0.1907)	1,611.0429 (101.7715)	0.9676	1.4076
14. Electrical machinery and equipments	2.2269 (0.0716)	472.6496 (21.3270)	0.9831	1.4215
15. Transportation equipment	12.5224 (0.3523)	1,926.0608 (188.4483)	0.9268	0.4549
16. Precision instruments	2.2234 (0.0753)	102.0622 (6.8468)	0.9637	0.9407
17. Other manufacturing industries	2.6684 (0.0693)	652.9274 (81.6386)	0.8874	1.6413
18. Transportation communication and utilities	9.3592 (0.2569)	1,551.6881 (615.3380)	0.4896	0.8317
19. Construction	8.7611 (0.3708)	5,318.4651 (442.7077)	0.9455	1.2510
20. Wholesale and retail trade	2.4520 (0.0517)	2,611.8312 (160.8511)	0.9691	0.8676
21. Financial, insurance and real estate dealing	1.4682 (0.1033)	471.3059 (41.5842)	0.9393	0.3273

stable. This is probably due to the effect of multicollinearity. However, we were able to obtain statistically significant parameters by the regression of equation (6)′ which is derived from equations (1)′ and (2)′ on the basis of the cost

TABLE 4. $Q_j = a_j(L_{mj} L_{fj}^{c_j/b_j})^{b_j}$

	log a	a	b	$\bar{r}$	D.W.
4. Mining	9.9637 (0.5513)	21,240.259	−0.6181 (0.0905)	−0.8537	0.5111
5. Food	−10.0757 (1.9376)	.4209E−04	1.9519 (0.2032)	0.9181	0.2723
6. Textile products	−21.7144 (2.3678)	.3711E−09	2.3748 (0.1904)	0.9492	0.9404
7. Pulp paper and products	−10.4950 (1.8141)	.2767E−04	2.6825 (0.2788)	0.9184	0.2012
8. Chemical and related products	−29.5443 (3.2162)	.1476E−12	5.3739 (0.4629)	0.9419	0.3617
9. Petroleum and coal products	−12.1234 (2.0293)	.5431E−05	5.0861 (0.5418)	0.9147	0.2403
10. Stone, clay and glass products	−15.7534 (1.2090)	.1440E−06	3.2435 (0.1738)	0.9764	0.3977
11. Iron and steal	−9.3042 (1.8174)	.9104E−04	2.6556 (0.2723)	0.9203	0.3713
12. Fablicated metal products	−8.5944 (0.4850)	.1851E−03	2.1011 (0.0654)	0.9919	0.8088
13. Machinery	−7.2592 (1.4412)	.7037E−03	2.1153 (0.2014)	0.9303	0.3026
14. Electrical machinery and equipments	−5.8524 (0.4562)	.2873E−02	1.5264 (0.0515)	0.9904	0.8016
15. Transportation equipment	−12.9065 (0.4080)	.2482E−05	3.0795 (0.0607)	0.9967	1.7029
16. Precision instruments	−8.5706 (0.5702)	.1896E−03	2.1150 (0.0833)	0.9870	0.3832
17. Other manufacturing industries	−14.1745 (1.4817)	.6983E−06	2.2449 (0.1475)	0.9651	0.2875
18. Transportation communication and public utilities	−14.7076 (0.9477)	.4098E−06	2.7310 (0.1123)	0.9859	0.4575
19. Construction	−9.6190 (0.7103)	.6646E−04	2.1388 (0.0831)	0.9874	0.6972
20. Wholesale and retail trade	−14.8748 (1.0794)	.3467E−06	1.9939 (0.0918)	0.9824	0.5863
21. Financial, insurance and real estate dealing	−3.1635 (0.5742)	.4228E−01	1.0844 (0.0549)	0.9789	0.2661

minimization principle. This model represented by equation (6)′, however, is not a reduced form in the strict sense of the word since this model treats the variables Q, W_m, W_f and r as exogeneous variables and variables L_m, L_f and K as endogeneous variables. Making the variable L_m which is in the right hand

TABLE 5. $K_j - \beta \cdot L_{fj}/L_{mj} = \alpha + \gamma Q_j$

	α	γ	$\bar{\gamma}$	D.W.
4. Mining	−688.3782 (169.4371)	2.6738 (0.3200)	0.8944	0.2742
5. Food	−1,091.3182 (131.15247)	0.4160 (0.00233)	0.9742	0.4344
6. Textile products	60.5420 (156.7071)	0.6679 (0.0569)	0.9431	0.5406
7. Pulp, paper and products	−67.0263 (24.1677)	0.6017 (0.0167)	0.9935	0.8731
8. Chemical and related products	−7.6691 (89.2511)	0.7046 (0.0229)	0.9911	0.8616
9. Petroleum and coal products	65.8671 (14.5471)	0.3217 (0.0083)	0.9944	1.0404
10. Stone, clay and glass products	−2.4079 (39.1378)	0.6559 (0.0285)	0.9843	0.7382
11. Iron and steal	−71.3480 (198.8539)	0.5638 (0.0298)	0.9770	0.9306
12. Fablicated metal products	−101.0541 (40.4851)	0.3492 (0.0218)	0.9683	0.4903
13. Machinery	−7.3577 (37.0946)	0.2017 (0.0086)	0.9849	0.9971
14. Electrical machinery and equipments	91.1865 (62.6555)	0.2724 (0.0157)	0.9729	0.8940
15. Transportation equipment	−0.0028 (76.9257)	0.4170 (0.0170)	0.9862	0.7610
16. Precision Instruments	5.9485 (4.2705)	0.2984 (0.0067)	0.9957	0.1099
17. Other manufacturing industries	−403.0072 (93.8729)	0.3617 (0.0162)	0.9834	0.4456
18. Transportation communication and public utilities	477.4335 (111.6259)	1.3826 (0.0202)	0.9982	1.2468
19. Construction	−559.1880 (105.0502)	0.2287 (0.0136)	0.9711	0.5880
20. Wholesale and retail trade	528.6606 (166.9978)	0.6089 (0.0236)	0.9875	0.5882
21. Financial, insurance and real estate dealing	−835.4502 (49.4719)	0.5481 (0.0107)	0.9967	0.6667

side of equation (6)′ as an exogeneous variable, however, will make the equation non-linear and will make the least squares regression method infeasible. Therefore, the parameters obtained through the procedure as described above should be regarded only as the first approximation.

TABLE 6. ESTIMATED PARAMETERS

	a_j	b_j	c_j	α_j	β_j	γ_j
5. Food	$.42089468\times10^{-4}$	$.19518852\times10$	$.10476442\times10$	$-.10913182\times10^{4}$	$.20675925\times10^{3}$	$.41597271$
6. Textile products	$.31776255\times10^{-9}$	$.23748045\times10$	$.21999573\times10$	$.60542000\times10^{2}$	$-.31886946\times10^{2}$	$.66790879$
7. Pulp, paper and products	$.27673784\times10^{-4}$	$.26825436\times10$	$.61344838$	$.67026310\times10^{2}$	$.52604149\times10^{2}$	$.60167872$
8. Chemical and related products	$.14759165\times10^{-12}$	$.53738992\times10$	$.10101395\times10$	$-.76691000\times10$	$.69675363\times10^{2}$	$.70461269$
9. Petroleum and coal products	$.54310395\times10^{-5}$	$.50860971\times10$	$.42002812$	$.65867080\times10^{2}$	$-.94217532\times10^{0}$	$.32171318$
10. Stone, clay and glass products	$.14400814\times10^{-6}$	$.32435064\times10$	$.69140543$	$-.24078500\times10$	$.34440143\times10^{2}$	$.65585083$
11. Iron and steal	$.91039780\times10^{-4}$	$.26556160\times10$	$.20541056$	$-.71348000\times10^{2}$	$.30831987\times10^{4}$	$.56382026$
12. Fablicated metal products	$.18513134\times10^{-5}$	$.21010768\times10$	$.37755350$	$-.10105407\times10^{3}$	$.57758018\times10^{2}$	$.34916957$
13. Machinery	$.70370311\times10^{-3}$	$.21152539\times10$	$.2598958$	$-.73577400\times10$	$.17626313\times10^{3}$	$.20165162$
14. Electrical machinery and equipments	$.28729411\times10^{-2}$	$.15264332\times10$	$.68544101$	$.9118654\times10^{2}$	$.14647008\times10^{3}$	$.27238897$
15. Transportation equipment	$.24817663\times10^{-5}$	$.30795352\times10$	$.24592186$	$-.27867700\times10^{2}$	$.14243469\times10^{3}$	$.41697827$
16. Precision instruments	$.18959696\times10^{-3}$	$.21150425\times10$	$.95127154$	$.59485400\times10$	$.31663056\times10^{2}$	$.29844708$
17. Other manufacturing industries	$.69834960\times10^{-6}$	$.22449181\times10$	$.84131178$	$-.40300720\times10^{3}$	$.17798918\times10^{3}$	$.36174211$
18. Transportation, communication and public utilities	$.40978344\times10^{-6}$	$.27310479\times10$	$.29180220$	$.47743350\times10^{3}$	$.14978778\times10^{3}$	$.13826070\times10$
19. Construction	$.66455301\times10^{-4}$	$.21388225\times10$	$.24412724$	$-.55918800\times10^{3}$	$.54486346\times10^{3}$	$.22869668$
20. Wholesale and retail trade	$.34668786\times10^{-6}$	$.19938545\times10$	$.81315101$	$.52866060\times10^{3}$	$.75661169\times10^{3}$	$.60887421$
21. Financial, insurance and real estate dealing	$.42279370\times10^{-1}$	$.10843859\times10$	$.73857834$	$-.83545020\times10^{3}$	$.19095072\times10^{3}$	$.54810072$

We will therefore try to estimate the parameters which will minimize the objective

$$OB=(L_m-\hat{L}_m)^2+(L_f-\hat{L}_f)^2+(K-\hat{K})^2$$

by using the values of L_m, L_f, and K which are computed from the parameters obtained above as the initial values.

As a first step, we can obtain the following equation:

$$\left(\frac{Q}{aL_m{}^b}\right)^{1/c}=\frac{W_m}{W_f}\left[L_m\Big/\left\{\left(\frac{b}{c}\right)+\left(\frac{b}{c}+1\right)\beta\frac{P_k(i+de)}{W_fL_m}\right\}\right]$$

by solving equations (6)′ and (1)′ with respect to L_f. Since this equation is non-linear, we went through the following procedure: denoting the right hand side as XX and the left hand side as YY, we computed the value of L_m which minimizes the value $ZZ=(XX-YY)^2$, and solved for L_f using equation (1)′ and then obtain K by substitute into equation (2)′ to obtain K.

In the second step, we tried to obtain more precise parameters using the above mentioned method of minimizing the value of OB by means of changing the values of parameters a, b, c, α, β, and γ by the infinitesimal amount $\Delta\varepsilon$ about the theoretically predicted values of L_m, L_f and K. Since we have found through this method that the values of these parameters did not change significantly from their initial values, we decided to regard the obtained values for the parameters presented in Table 6 as the final reliable results.

SECTION 3. MALE-FEMALE LABOR DEMAND IN METAL PRODUCTS INDUSTRY

On the basis of the parameters estimated thus far, we will try to simulate in this section the following four kinds of effects by altering such variables as output X, male wages W_m, female wages W_f and unit capital cost r.

Case 1: The effect of an increase in output on labor demand. This effect may be examined by fixing r, W_m and W_f at the levels of this year while changing to X the level of next year.

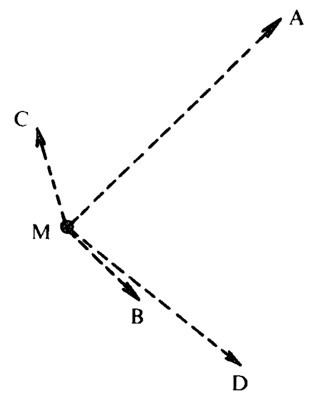
Case 2: The effect of change in unit capital cost on labor demand. This effect may be measured by changing r at the level of next year while fixing X, W_m and W_f at the levels of this year.

Case 3: The effect of an increase in male wage rate on substitution of female for male workers. This effect may be measured by changing W_m to the level of next year while fixing X, r and W_f at the levels of this year.

Case 4: The effect of an increase in female wage rate on substitution of male female workers. This effect may be measured by changing W_f to the level of next year while fixing X, r and W_m at the levels of this year.

These four types of effects correspond to the components of the following equation:

$$
\begin{aligned}
dL &= dL_m + dL_f \\
&= \left(\frac{\partial\varphi}{\partial Q} + \frac{\partial\psi}{\partial Q}\right) dQ && \text{Case 1} \\
&\quad + \left(\frac{\partial\varphi}{\partial r} + \frac{\partial\psi}{\partial r}\right) dr && \text{Case 2} \\
&\quad + \left(\frac{\partial\varphi}{\partial W_m} + \frac{\partial\psi}{\partial W_m}\right) dW_m && \text{Case 3} \\
&\quad + \left(\frac{\partial\varphi}{\partial W_f} + \frac{\partial\psi}{\partial W_m}\right) dW_m. && \text{Case 4}
\end{aligned}
$$



$\overrightarrow{MA}$ Case 1

$\overrightarrow{MB}$ Case 2

$\overrightarrow{MC}$ Case 3

$\overrightarrow{MD}$ Case 4

Fig. 1.

Figure 1 shows the effects on labor demand of an increase in male wage, female wage or unit capital cost in the form of a vector, on the basis of sign conditions $a>0$, $b>0$, $c>0$, $\beta>0$ and $\gamma>0$. The cases C, B and D are the cases in which the level of output is held unchanged. The changes in the aggregate amount of labor demand can be represented by male-female labor demand which depend on the size of vectors $\overrightarrow{MA}$, $\overrightarrow{MB}$, $\overrightarrow{MC}$ and $\overrightarrow{MD}$.

Table 7 presents the computed magnitudes of the vectors for 1960 and 1961 when the output increased rapidly for the metal product industry. Figure 2 shows the locations of A, B, C and D on isoquants. We can see from Figure 2 that the magnitude of $\overrightarrow{MA}$ was greater than the magnitude of $\overrightarrow{MB}+\overrightarrow{MC}+\overrightarrow{MD}$ when the

TABLE 7. FABLICATED METAL PRODUCTS 1960, 1961

1960

	OB.	ES.	Case 1 X : 1961	2 r : 1961	3 W_m : 1961	4 W_f : 1961
LT	672.0	657.2	740.8	657.3	671.9	649.6
$\triangle LT$			83.5825	0.0739	14.6281	−7.6353
L_m	523.8	509.9	571.3	509.8	491.9	527.1
$\triangle L_m$			61.4065	−0.1209	−18.0056	17.1872
L_f	148.2	147.3	169.5	147.5	180.0	122.5
$\triangle L_f$			22.1761	0.1949	32.6338	−24.8225
K	391.9	123.5	194.4	123.5	127.9	120.2
$\triangle K$			70.8960	0.0260	4.4429	−3.2643

1961

	OB.	ES.	Case 1 X : 1962	2 r : 1962	3 W_m : 1962	4 W_f : 1962
LT	742.4	740.8	802.3	740.9	749.6	733.8
$\triangle LT$			61.4491	0.0583	8.7461	−7.0292
L_m	562.5	571.3	671.3	571.2	559.9	584.5
$\triangle L_m$			45.9909	−0.0891	−11.4258	13.1837
L_f	179.9	169.5	185.0	169.7	189.7	149.3
$\triangle L_f$			15.4582	0.1474	20.1719	−20.2130
K	439.7	194.4	254.7	194.4	196.8	192.0
$\triangle K$			60.3061	0.0176	2.4308	−2.3841

output increased rapidly. Similarly, Table 8 presents the computed values for the vectors for years 1969, 1970 and 1971, and the corresponding locations of the vectors are shown in Figure 3.

In the case of the metal product industry, the rate of increase in output has declined from 13.5 percent for 1969 to 1970 down to 2.27 percent 2.3 percent for 1970 to 1971, and in contrast, the rate of increase in female wages has increased from 17.1 percent for 1969 to 1970 up to 19.8 percent for 1970 to 1971. It is for these reasons that the demand for female labor force has declined since the effect of $\overrightarrow{\text{MD}}$ was greater than the effect of $\overrightarrow{\text{MA}}$.

TABLE 8. Fablicated Metal Products 1969, 1970, 1971

1969

	OB.	ES.	Case 1 X : 1970	2 r : 1970	3 W_m : 1970	4 W_f : 1970
LT	1,239.8	1,250.9	1,317.0	1,250.6	1,271.7	1,237.4
$\triangle LT$			66.1272	−0.2618	20.8583	−13.4506
L_m	946.7	954.1	1,003.5	954.5	930.4	976.3
$\triangle L_m$			49.3845	0.35939	−23.6675	22.18411
L_f	293.1	296.8	313.5	296.2	341.3	261.1
$\triangle L_f$			16.74276	−0.62114	44.52585	−35.63469
K	781.3	927.0	1,063.5	926.9	930.2	924.5
$\triangle K$			136.5742	−0.0444	3.221	−2.5165

1970

	OB.	ES.	Case 1 X : 1971	2 r : 1971	3 W_m : 1971	4 W_f : 1971
LT	1,300.0	1,319.3	1,331.5	1,317.3	1,338.8	1,302.9
$\triangle LT$			12.1419	−2.0157	19.4289	−16.4099
L_m	980.5	1,000.4	1,009.3	1,003.1	979.0	1,026.4
$\triangle L_m$			8.9011	2.6563	−21.4485	25.9542
L_f	319.5	318.9	322.1	314.2	359.8	276.5
$\triangle L_f$			3.2408	−4.67193	40.87741	−42.36407
K	975.2	1.063.9	1,089.9	1,063.6	1,066.7	1,061.1
$\triangle K$			26.0301	−0.3177	2.8152	−2.8495

1971

	OB.	ES.	Case 1 X : 1972	2 r : 1972	3 W_m : 1972	4 W_f : 1972
LT	1,303.8	1,327.0	1,347.6	1,328.7	1,349.4	1,314.9
$\triangle LT$			20.672	1.7685	22.4515	−12.0542
L_m	984.7	1,015.4	1,026.8	1,012.9	989.3	1,035.4
$\triangle L_m$			11.4449	−2.4515	−26.0615	20.006
L_f	319.1	311.6	320.8	315.8	360.1	279.5
$\triangle L_f$			9.22709	4.22	48.51285	−32.06017
K	119.8	1,089.2	1,130.8	1,089.5	1,092.5	1,087.1
$\triangle K$			41.5621	0.2836	3.2992	−2.1309

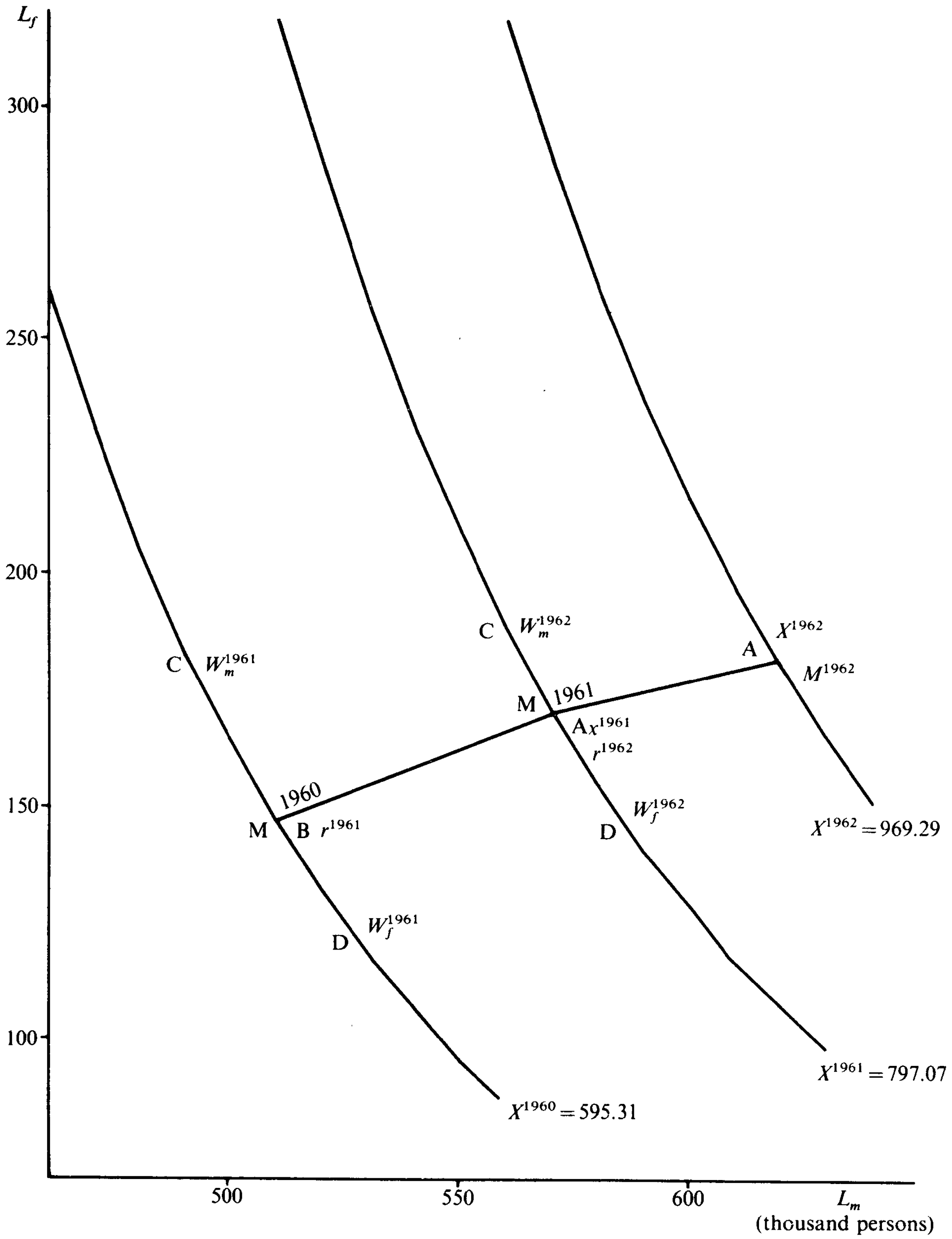


Fig. 2. Fabricated metal products.

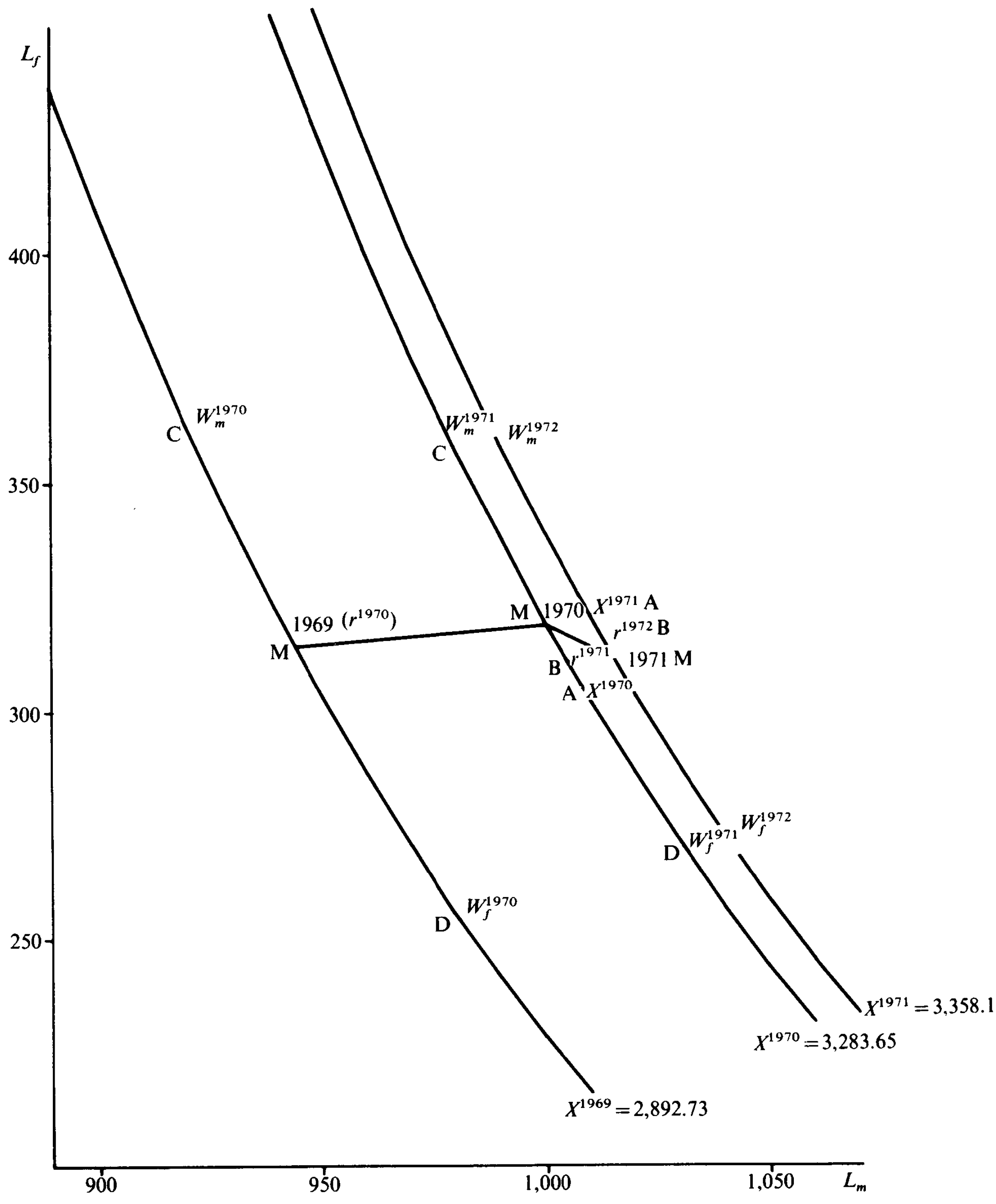


Fig. 3. Fabricated metal products.

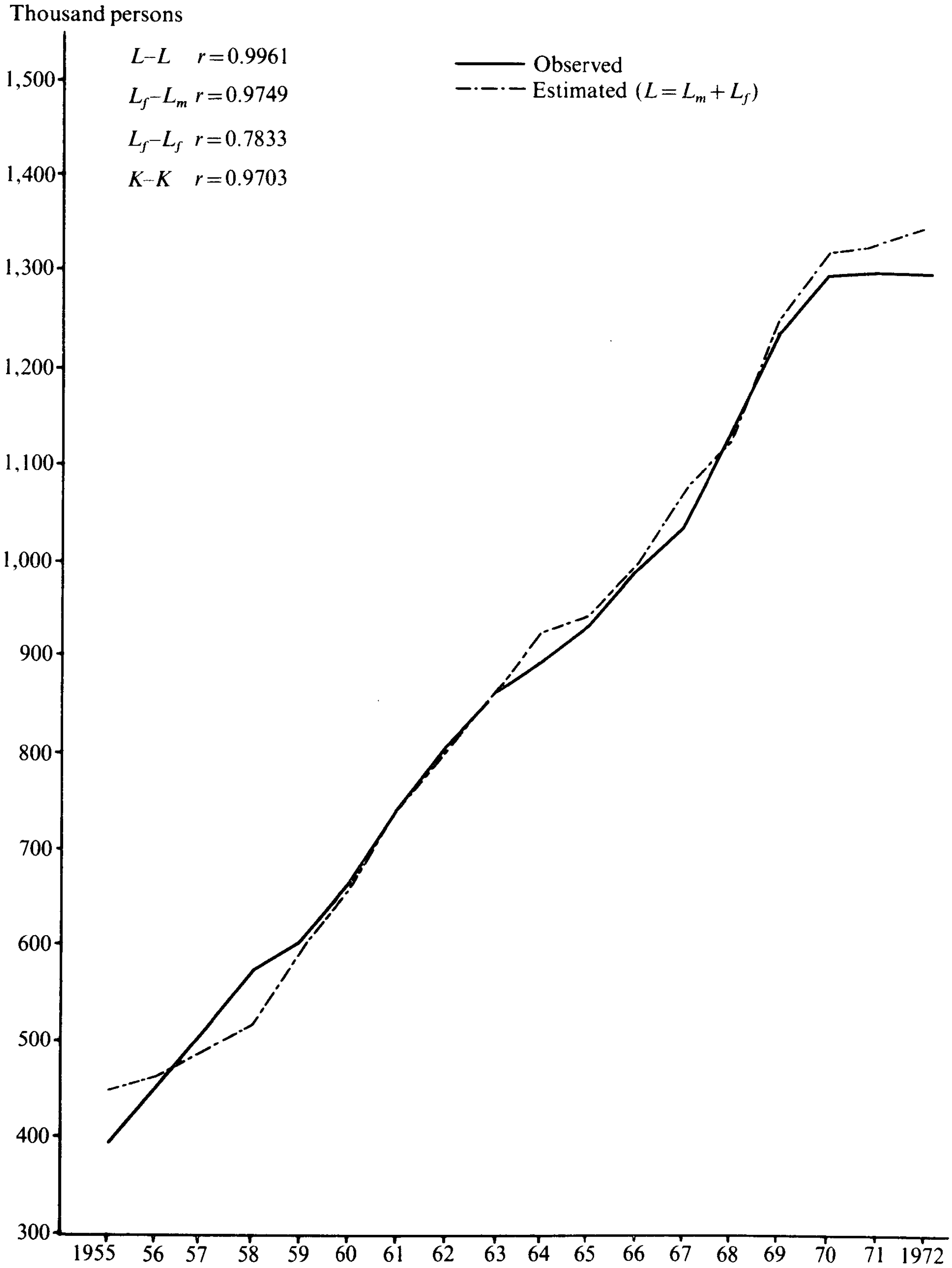


Fig. 4. Fablicated metal products.

CONCLUSION

Our analysis has yielded the following findings:

(1) With given levels of male wages, female wages and unit capital cost, the demand for male and female labor and capital stock increase with an increase in output,

(2) The marginal rate of substitution of female labor for male labor is greater than unity for a given level of output. In other words, an increase in the female–male ratio tends to increase the total demand for labor and vice versa,

(3) An increase in the female–male ratio for a given level of output will require an increased quantity of capital equipment,

(4) An increase in male wage rate, other things being equal, will induce a substitution of female labor for male labor, and the number of increased female workers will be greater than the number of male workers replaced by the female workers,

(5) An increase in female wage rate, other things being equal, will induce a substitution of male labor for female labor, and the number of increased male workers will be fewer than the number of female workers replaced by the male workers,

(6) An increase in the unit capital cost, other things being equal, will induce a substitution of male labor for female labor and reduce capital equipment.

The findings (1) to (6) suggest that the demand for male and female labor mix depends not only on the level of output but also on male and female wages and unit capital cost. The fact that the marginal rate of substitution of female for male labor is greater than unity plays an important role in determining the level of labor demand which changes in response to changes in relative wage rates for male and female labor.

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