TECHNOLOGICAL CHANGE AND PRODUCTION FUNCTION: The Case Of West Germany

SHINICHIRO NAKAMURA*

I. INTRODUCTION

The experiences of the last two centuries show the marked importance of technological change in generating the process of modern economic growth. This process is characterized by a high rate of increase in product per capita accompanied with a rapid structural change. Needless to say, Leontief's Input–Output (I–O) analysis is the most efficient and useful method to analyze technological change in a whole economic system. In the theoretical scheme of input–output analysis, the thorough understanding of the process of technological change in an economic system would be eventually made by elucidating the process of changes in input coefficients. This study is a step toward this goal.

In this paper, the author presents a theoretical model to explain changes in input coefficients in a dynamic process. The outline of this model is presented in Section II of this paper. Section III shows the results of a test performed to ascertain the empirical validity of the basic assumption made in the model. Section IV summarizes the results of the statistical estimation of the production function which plays a crucial role in the model. Concluding remarks are given in Section V.

Throughout this study time series data of the West German economy was used for the empirical testing of the theoretical model. We chose the West German economy for the analysis on the following two grounds:

(1) West Germany has completely comparable I–O tables covering a long period, data on production, capital stock and investment which can be directly combined with the I–O tables.

(2) Our model requires the economic data that describes a highly developed industrial economy.

II. THE THEORETICAL MODEL

1. The model begins with the basic assumption that all technologies are embodied in capital equipments. By introducing the concept of a plant, consisting of a complex of capital equipments, as an indivisible unit of production, technologies are supposed to be embodied in each plant. It is also assumed that once a plant has been built as a result of investment, no changes in its technology and production capacity take place so long as the plant remains in operation.[1] Therefore in the model

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[1] Hence in the model an increase in production capacity of the economic system is regarded not as an expansion in the capacity of the existing plants but as an addition of the capacity posessed by the newly built plants to that of the existing ones.

technological change represented by changes in input coefficients takes place only through the construction of new plants and scrapping of existing ones; input coefficients are constant in the short run. Thus in the present study investment is directly linked with the construction of new plants, and is treated as the vehicle of technological change.[2]

A number of plants producing a single type of commodity form a sector and we assume that the economic system is divided into n sectors; the number of the types of commodities being produced in the economic system is n and in each sector only one type of commodity is produced. In the present study we do not treat technological change represented by the creation of new commodities such as the $n+1$th commodity. Furthermore the model is designed to explain changes in input coefficients which would still be observed after the elimination of the product mix.

2. Under these assumptions the development process of the economic system, which has started its production in period 1, over s periods ($s>1$) with respect to production, consumption and investment is described as in eq. (1).[3] It is assumed that production capacity of the sectors increases annualy by period s and that the gestation period of investment is equal to one period. The following notations are used:

X_i = the vector of production capacity in period i,
A_i = the matrix of input coefficients observed in period i,
B_i = the matrix of capital coefficients observed in period i,
C_i = the vector of consumption in period i.

$$
\begin{aligned}
X_1 &= A_1X_1 + B_1(X_2 - X_1) + C_1 \\
X_2 &= A_2X_2 + B_2(X_3 - X_2) + C_2 \\
&\cdots\cdots\cdots \\
&\cdots\cdots\cdots \\
X_{s-1} &= A_{s-1}X_{s-1} + B_{s-1}(X_s - X_{s-1}) + C_{s-1} \\
X_s &= A_sx_s + C_s\,.
\end{aligned} \tag{1}
$$

In period t ($1 \leqq t \leqq s$) production capacity has increased by the amount $\Delta X_t = X_t - X_{t-1}$. If we write $x_t = \Delta X_t$, x_t represents the production capacity added in period t. With this notation eq. (1) can be rewritten as follows:

$$
\begin{aligned}
x_1 &= A_1x_1 + B_1x_2 + C_1 \\
x_1 + x_2 &= A_2(x_1 + x_2) + B_2x_3 + C_2 \\
&\cdots\cdots\cdots \\
&\cdots\cdots\cdots \\
x_1 + x_2 + \cdots + x_s &= A_s(x_1 + x_2 + \cdot + x_s) + C_s\,.
\end{aligned} \tag{2}
$$

3. From eq. (2) it is shown that in period t ($1<t\leqq s$) $X_t = \sum_{i=1}^{t} x_i$, i.e. production capacity attained in period t is the sum of the past additions in capacity resulted from

[2] An explicit treatment of gross investment as the vehicle of technological change was made by Salter (1960).

[3] A similar description of the development process is seen in Leontief (1969).

investment over the period 0 to $t-1$. The present production capacity exists as a result of capital accumulation in the past. Durability of plants determines the degree of this linkage of present production to past investment.

Production activity in each period is performed by operating the plants previously built. Each of these plants embodies a particular technology chosen in each period when investment was made. Therefore production activity in each period is carried out by employing various technologies simultaneously. From this reason in eq. (2) the input coefficients observed in any period except 1 are the averages of technologies simultaneously employed.[4]

4. To make a clear distinction between input coefficients observed and those which represent the technological input-output relationship of each plant, we express the former in a matrix form as $\hat{A}_i$ and the latter as A_i. Each column vector of A_i, for example $(a^i_{1j}, a^i_{2j}, \cdots, a^i_{nj})$, represents the technology chosen in period $i-1$ and installed in a new plant in period i with a production capacity x^i_j, and that of $\hat{A}_i$, for example $(\hat{a}^i_{1j}, \hat{a}^i_{2j}, \cdots, \hat{a}^i_{nj})$, expresses the input- output structure of a sector observed in period i. The following relationship exists between $\hat{a}^i_{ij}$ and a^i_{ij}:

(3)
$$\hat{a}^i_{ij} = \sum_{t=1}^{i} a^t_{ij} \frac{x^t_j}{\sum_{t=1}^{i} x^t_j} .$$

From eq. (3) it is seen that $\hat{A}_i = A_i$ is realized if and only if at least one of the following two conditions is fulfilled:

(4a)
$$A_1 = A_2 = \cdots = A_i ,$$

(4b)
$$x_1 = x_2 = \cdots = x_{i-1} = 0 .$$

(4a) is when all technologies chosen in each period are the same, or in other words in each sector only one technology has been chosen in the past. (4b) is the initial period when production of a commodity was started, or when the durability of all plants is equal to 1 period. Since conditions (4a) and (4b) are very unlikely to be met, what we can actually observe will be weighted averages of various technologies.

5. In contrast to that observed input coefficients refer to whole production capacity, X_i, observed capital coefficients refer, as is demonstrated in eq. (2), only to that part of production capacity which was newly added, x_i. Therefore while observed input coefficients reflect all technologies in use, capital coefficients only the technologies used in the newest part of prodution capacity.[5]

6. From eq. (3) the nature of the object of our observation and the causes that directly generate changes in input coefficients become clear. The changes are directly attributed to changes in technology chosen when investment is made, and to those in

[4] The importance of treating obtained input coefficients as averages is stressed in Leontief (1953), p. 23. Also with respect to the theoretical grounds of the coexistence of different technologies within a sector producing a single type of commodity, see Salter (1960), Chapter IV.

[5] This fundamental difference in character between input- and capital coefficients is explained in detail in Carter (1957).

the composition of the production capacity attached to each technology in the whole production capacity of the sector (hereafter this composition is called the capacity weight of technology).

Changes in technology chosen can be elucidated by determining the production function which represents possible technologies, and by analyzing the mechanism of the choice of technology on the production function. Each technology, which is expressed in its complete form as a set of input- and capital coefficients, can be interpreted as corresponding to a point on the production function.

This relationship between the production function and input coefficients would be clearly demonstrated by taking an example of a sector, for example sector j. The production function of sector j is written in its implicit form as:

$$f_j(x_j, x_{1j}, x_{2j}, \cdots x_{mj}, \cdots, x_{nj}, L_j) = 0 .$$

Here x_j stands for output, $x_{1j}, x_{2j}, \cdots, x_{nj}$ for inputs, and L_j for labor. Let the commodities $x_1, x_2, \cdots, x_m$ be used as flow inputs, i.e. as raw materials, parts and fuels, and the commodities $x_{m+1}, \cdots, x_n$ as stock inputs which compose a plant.[6] Building materials, metal constructions such as tanks and pipes, and machineries of various types are included in the latter group of commodities. Let a particular technology chosen on this production function be T_j^*. Then this technology would be expressed as:

$$T_j^*; (\bar{x}_j, \bar{x}_{1j}, \cdots, \bar{x}_{nj}, \bar{L}_j) .$$

Dividing $\bar{x}_{1j}, \bar{x}_{2j}, \cdots, \bar{x}_{nj}, \bar{L}_j$ by $\bar{x}_j$, we obtain a set of input- and capital coefficients representing this technology:

$$(a_{1j}, a_{2j}, \cdots, a_{mj}, b_{m+1j}, \cdots, b_{nj}, l_j) .$$

The explanation of changes in the capacity weight of technology can be made by analyzing factors which determine the scale of investment and durability (economic life) of plants.

Since the investment function is derived from the production function as a reduced form, an efficient analysis of technological change must first be made by determining the production function for each sector of the economic system.[7]

7. In the present section the theoretical model was presented, which provides a basic framework for the analysis of changes in input coefficients in a dynamic process. The main characteristic of this model consists in treating input coefficients as averages of technologies weighted by the production capacity attached to each. The model shows that changes in input coefficients can be explained by the analysis of changes in technology chosen when investment is made and by the analysis of changes in the capacity weight of technology. The possible range of changes in input coefficients is determined by the availability of technologies for the production of a

[6] The engineering grounds of this type of production function, i.e. the stock–flow production function, are given in Smith (1961), Chapter II.

[7] As an important example of the unification of production and investment theory see Smith (1961), especially Chapter XI.

commodity. The production function is used as an analytical tool to represent the possible range of changes in input coefficients. Each technology chosen is interpreted as corresponding to a point on the production function.

As would be clearly shown from the critical importance of the basic assumption (capital embodiment of technology) and production function in the model, any further theoretical development would not be possible without obtaining a positive confirmation on the empirical validity of the assumption and knowing the form and the values of the parameters of the production function for each sector.

III. THE TEST OF THE BASIC ASSUMPTION

1. The test on the empirical validity of the basic assumption that all technologies are embodied in capital equipments is preformed in this study by examining whether the high rate of technological change is always accompanied by a rapid change in the age structure of capital equipments. If all technologies are embodied in capital equipments, technological change must have been accompanied by the introduction of new equipments.[8] First an efficient measure of changes in input coefficients must be developed to assure the degree of technological change in each sector.

2. Measurements of changes in input coefficients were made by using the method of making the weighted distribution of relative changes in coefficients for each sector. This is the method originally developed by Leontief and used to analyze the structural change of the American economy during 1919–1939.[9] Let a particular input coefficient of sector j in period 0 be a_{ij}^0, and that in period 1 be a_{ij}^1, also the amount of input for each period be x_{ij}^0 and x_{ij}^1. Then the index of relative change $\bar{a}_{ij}$ and its weight w_{ij} are given by

$$\bar{a}_{ij} = \frac{2(a_{ij}^1 - a_{ij}^0)}{a_{ij}^1 + a_{ij}^0},$$

$$w_{ij} = \frac{x_{ij}^1 + x_{ij}^0}{2}.$$

The weighted distribution of changes in input coefficients is described by the use of each individual change index, $\bar{a}_{ij}$, and the weight, w_{ij}.

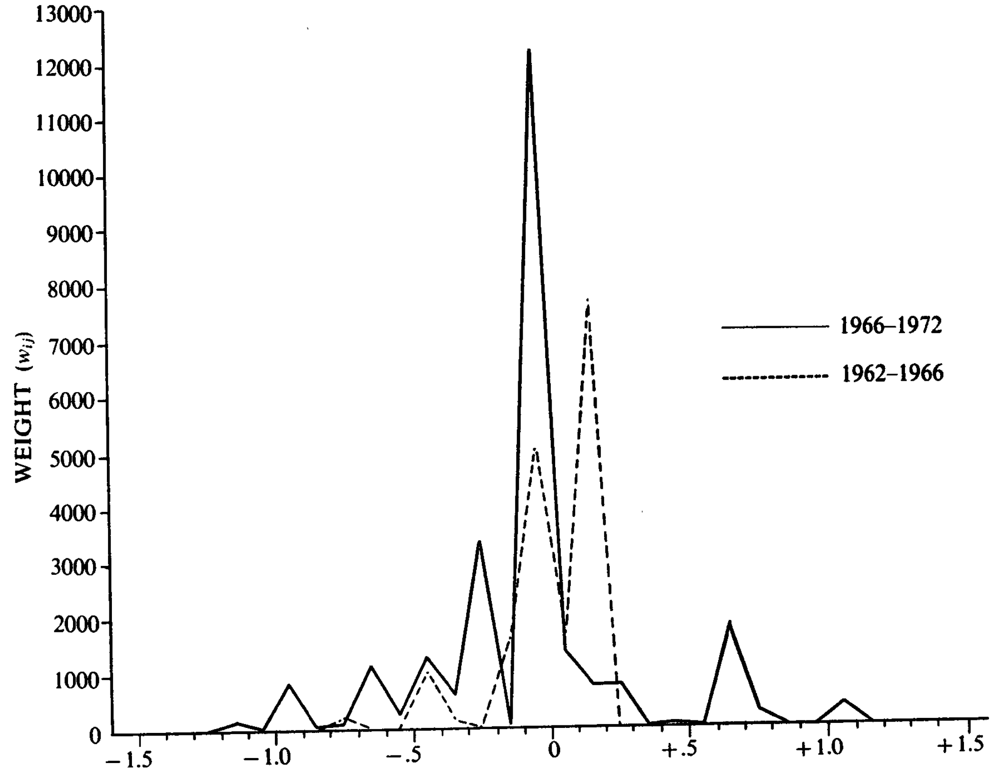
Leontief, in his study, used the mean value of the weighted distribution thus obtained as a convenient statistical measure of the magnitude of technological change on the grounds that the obtained distributions were nearly normal. However when we applied this method to the individual sectors of the West German input-output table, it was found that in general the form of the distribution is far from being normal.[10] This is demonstrated in Chart I. Consequently when analysis is

[8] Technological change may take place as a result of scrapping. Unless production is declining, however, this must be at least supplemented by replacement investment which means the installment of new equipments.

[9] See Leontief (1953), pp. 27–31.

[10] In this study the West German input-output tables for 1962, 1966 and 1972 published by Deutsches Institut für Wirtschaftsforschung were used. See Stäglin et al. (1973) and Pischner et al. (1975) for details of these tables. The tables were converted to competitive import type by the author.

(a) Chemical industry



(b) Vehicle construction

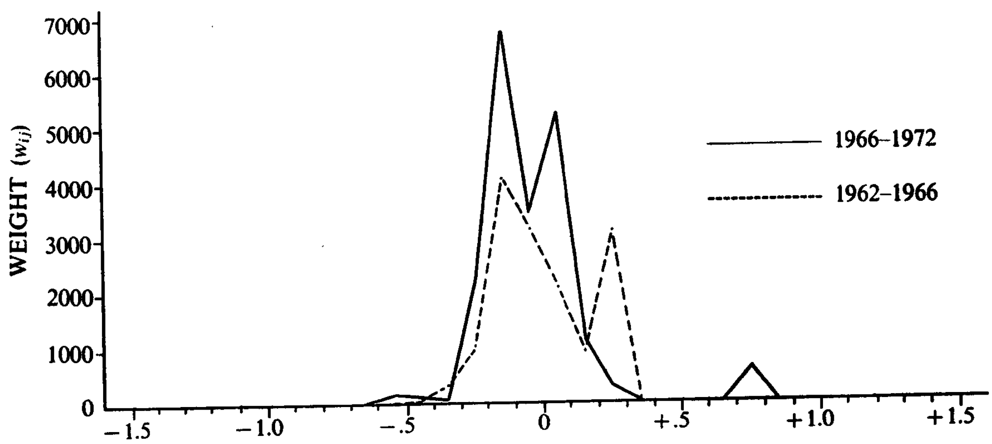
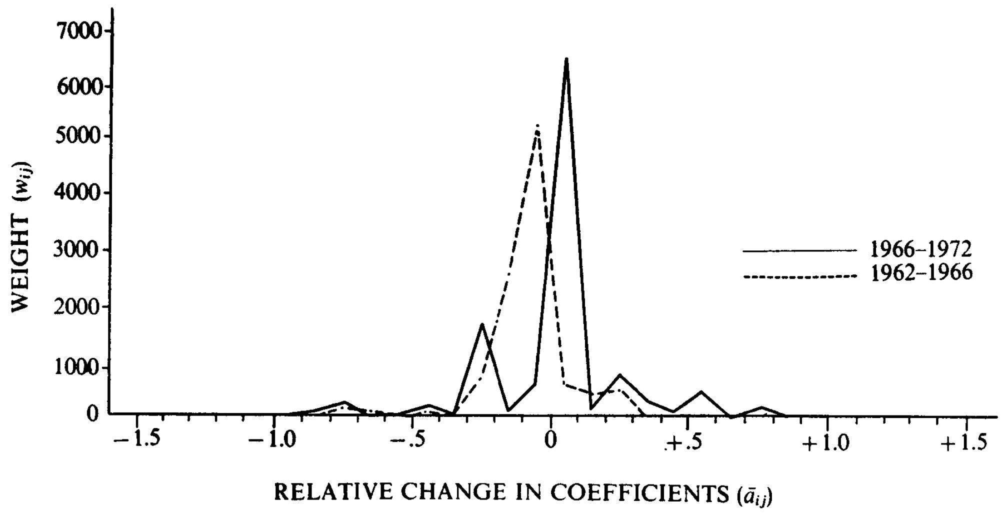


CHART I. WEIGHTED DISTRIBUTION OF RELATIVE CHANGES IN INPUT COEFFICIENTS
(Labor input coefficients are not included)

(c) Iron and steel industry



(d) Machinery construction

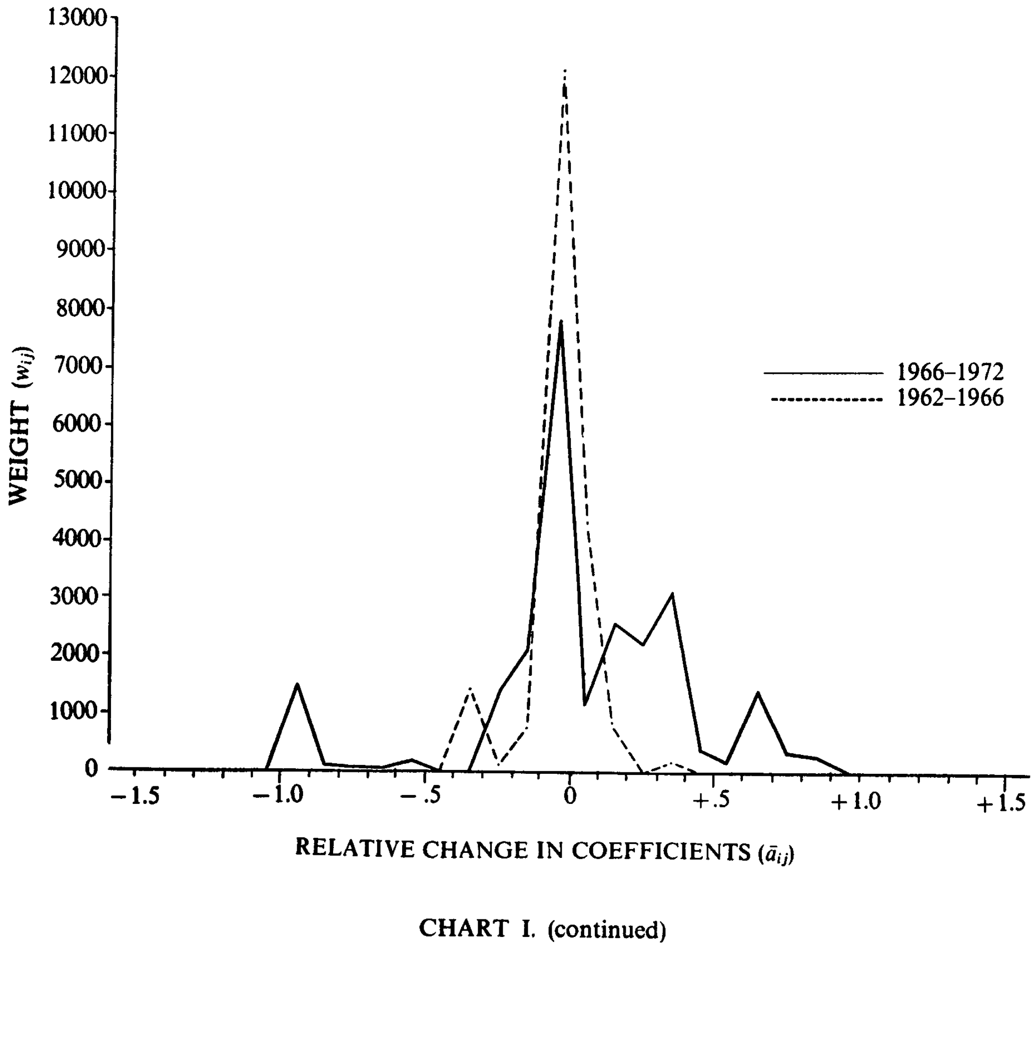


CHART I. (continued)

made on individual sectors, the mean value of the weighted distribution does not serve as an effective measure of technological change.[11]

Therefore in this study instead of the mean value the following index was used, which expresses changes in the absolute value (hereafter this index is called the absolute change index ACI).

$$ACI_j = \frac{\sum_{i=1}^{n} |\bar{a}_{ij}| w_{ij}}{\sum_{i=1}^{n} w_{ij}}, \qquad (j = 1, 2, \cdots, n) . \tag{5}$$

In this study this index was employed for intermediate inputs excluding labour input. As the index of changes in the latter, the rate of increase in the labour productivity $\Delta\left(\frac{X}{L}\right)$ was used.

3. The following index was employed as the measure of the degree of change in the age structure of equipments in each sector over the period $t-m$ to $t-1$. The gestation period of investment was supposed to be one period. Let I_{ij} and K_{ij} be investment and capital stock of sector j in period i.[12]

$$e_j = \frac{\sum_{i=t-m}^{t-1} I_{ij}}{K_{ij}}, \qquad (j = 1, 2, \cdots, n) . \tag{6}$$

Provided that the durability of equipments exceeds m periods, this index takes a value smaller than one, and indicates the percentage of equipments having been newly installed since period $t-m$ in the total volume of equipments in period t.

4. The test was undertaken by the use of these three indexes: the index of changes in intermediate input coefficients ACI_j, the rate of increase in labor productivity $\Delta(X/L)_j$, and the index of changes in the age structure of equipments e_j. First the observation period was divided into two parts, 1962–1966 and 1966–1972. Then these three indexes were measured for each of these periods. The results appear in Table 1.

The assumption would be positively confirmed if in each sector the period with relatively high rate of technological change coincides with the period showing a relatively rapid change in the age structure of equipments. In Table 1 the symbol in column A, B and C indicates the relative magnitude of these changes in each period. If the direction of the symbol in column A and B coincides with that in column C the empirical validity of the assumption would be positively confirmed.

The results in Table 1 indicate that the symbol coincides in direction except for a few sectors. In the case of intermediate inputs (22) aerospace is the only sector where this coincidence does not hold. In the case of labor inputs this does not hold in three sectors: (22) aerospace, (23) shipbuilding, and (31) musical instruments, etc..

[11] The measurements made by Leontief were limited to over all change.

[12] The data for capital stock and investment are from Krengel et al. (1972, 1976).

TABLE 1. RELATIONSHIP BETWEEN TECHNOLOGICAL CHANGE AND INVESTMENT IN WEST GERMAN MANUFACTURING INDUSTRY

	(1)			(2)			(3)		
Sector* I-O (DIW) Code Number	Changes in Intermediate Input Coefficients ACI_j			Changes in Labor Input Coefficients $\Delta(X/L)_j$			Changes in the Age Structure of Equipments e_j		
	1962–66	A	1966–72	1962–66	B	1966–72	1962–66	C	1966–72
22 Aerospace	.5012	>	.3719	−26.2	>	23.8	.16	<	.21
23 Shipbuilding	.0989	<	.2867	28.7	>	23.5	.44	<	.60
21 Vehicle construction	.1350	<	.1423	14.8	<	18.6	.37	<	.41
25 Precision engineering	.1557	<	.4181	20.8	<	29.8	.31	<	.39
19 Constructional steel	.0800	<	.4236	14.6	<	27.8	.31	<	.33
20 Machinery construction	.0851	<	.2636	10.8	<	21.4	.28	<	.36
24 Electrical equipment	.1034	<	.2702	18.6	<	34.5	.31	<	.37
27 Hardware and metal goods	.1047	<	.2746	16.9	<	26.3	.34	<	.40
26 Steel forging	.1134	<	.2900						
11 Iron & steel foundries	.1918	<	.2812	4.2	<	21.3	.26	<	.30
12 Steel drawing and cold rolling mills	.1161	<	.2833	24.6	≃	24.4	.25	<	.33
10 Iron & steel industry	.1291	<	.1733	13.4	<	27.4	.33	≃	.32
13 NF metals industry	.2492	<	.7316	2.7	<	46.2	.24	<	.39
42 Tobacco manufactures	.0949	<	1.0041						
41 Brewing & malting	.1631	<	.4181						
43 Industry of other foods & beverages	.0995	<	.2816	16.8	<	27.1	.27	<	.33
40 Sugar industry	.1888	<	.2024						
39 Edible oils & margarine industry	.1895	<	.4118						
38 Grain milling	.2195	<	.4707						
29 Glass industry	.1390	<	.4291	20.5	<	28.2	.38	<	.42
28 Fine ceramic industry	.1937	<	.5685	20.5	≃	19.2	.28	<	.36
9 Building materials	.1255	<	.4337	18.1	<	37.6	.38	≃	.37
37 Clothing industry	.1033	<	.3357	9.2	<	25.5	.30	<	.35
35 Leather industry	.1218	<	.4396	11.3	<	18.7	.24	<	.27
36 Textile industry	.1310	<	.2195	17.9	<	29.1	.24	<	.31
31 Musical instruments, etc.	.2157	<	.6330	19.1	>	14.7	.41	<	.48
30 Timber manufactures	.1575	<	.5105	14.6	<	26.1	.31	<	.39

TABLE 1. (continued)

	(1)			(2)			(3)		
Sector* I-O (DIW) Code Number	Changes in Intermediate Input Coefficients ACI_j			Changes in Labor Input Coefficients $\Delta(X/L)_j$			Changes in the Age Structure of Equipments e_j		
	1962–66	A	1966–72	1962–66	B	1966–72	1962–66	C	1966–72
33 Printing & duplicating	.1022	<	.4856	.2	<	27.2	.30	<	.37
32 Papar & board manufactures	.0830	<	.4124	14.6	<	26.1	.37	<	.44
18 Cellulose & paper processing	.1444	<	.5540	21.1	<	34.7	.24	<	.36
17 Saw-mills & timber processing	.1821	<	.5910	19.0	<	39.4	.26	<	.35
16 Rubber & asbestos manufactures	.0910	<	.4317	15.5	<	28.7	.32	<	.42
34 Plastics manufactures	.1564	<	.4372	28.0	<	34.8	.46	<	.52
14 Chemical industry	.1272	<	.2620	26.4	<	33.9	.28	<	.42
15 Oil refining	.4431	<	.5067	26.4	<	32.4	.32	<	.39

* In this table, the sectors are ordered following the hierarchy of the fundamental structure of production determined by the triangulation of the input-output tables.

From these findings it is concluded that the assumption of capital embodied technological change is in good accord with reality. In addition the assumption seems to apply more rigidly in intermediate inputs than in labor inputs.

The finding that in most of the sectors investigated the symbol in column A and B coincides in direction suggests the simultaneity of changes in intermediate- and labor input coefficients. This gives an empirical support to expressing technology as a set of intermediate- and labor input coefficients. This finding also indicates the inappropriateness of treating changes in intermediate input coefficients and those in labor input coefficients as taking place independent of each other, and of assuming separability between them.

IV. THE STATISTICAL DETERMINATION OF THE PRODUCTION FUNCTION

1. The production function determining the technological input–output relationship in each sector has to be estimated in order to carry out the analysis on the process of technological change in each sector.

It may be emphasized here that the production function in this study is of a long-run type indicating possible technologies from which a choice has to be made when a new plant is to be built. Choice of technology on this production function is possible only prior to the act of investment.

2. In the present study the production function is specified by the two following forms on the basis of whether factor substitutability is present or not:[13]

(7) $X = AK^{r_k} L^{r_L}$ (Factor substitutable generalized Cobb-Douglas type),

$$\left.\begin{aligned} &(8a)\quad K = a_K X^{b_K} \\ &(8b)\quad L = a_L X^{b_L} \end{aligned}\right\}$$ (Factor limitational generalized Leontief type).[14]

Here L stands for the quantity of labour input, K for capital equipment input, X for production scale.

3. The procedure stated below was taken in order to determine the production function for each sector of the West German manufacturing industry.

(i) *The test of factor substitutability* (1)

The follwing demand functions for capital equipment and labour are derived from the condition of cost minimization under the constraint of the Cobb-Douglas type production function:

$$(9a)\qquad K = \left[\frac{1}{A}\left(\frac{r_K}{r_L}\right)^{r_L}\right]^{1/(r_K+r_L)} \left(\frac{P_L}{P_K}\right)^{r_L/(r_K+r_L)} X^{1/(r_K+r_L)}$$

or

$$K = a_0\left(\frac{P_L}{P_K}\right)^{a_1} X^{a_2},$$

$$(9b)\qquad L = \left[\frac{1}{A}\left(\frac{r_L}{r_K}\right)^{r_K}\right]^{1/(r_K+r_L)} \left(\frac{P_K}{P_L}\right)^{r_K/(r_K+r_L)} X^{1/(r_K+r_L)}$$

or

$$L = b_0\left(\frac{P_K}{P_L}\right)^{b_1} X^{b_2}.$$

Here P_K and P_L stand for the prices of capital equipment and labour. If the true form of the production function is the Cobb-Douglas type, the following conditions have to be fulfilled between the estimates of (9a) and (9b):

$$(9c)\qquad \begin{aligned} &\hat{a}_1 + \hat{b}_1 \simeq 1.0\,, \\ &\hat{a}_2 \simeq \hat{b}_2\,. \end{aligned}$$

If (9c) holds then the empirical validity of this production function would be confirmed and its structural parameters can be derived from these reduced form parameters. However this stringent test is only possible when the data corresponds to theoretical requirements, i.e. the data consists of only newly built plants.[15]

[13] Ultimately all inputs including materials and energy should be included in the production function, and capital input should be disassembled into its component parts (see II–6). In this study inputs are limited to the most basic ones, capital equipment (capital stock) and labor. Further analysis on energy input structure and attempt to disassemble capital equipment into its component parts are now under way.

[14] On the concept of the factor limitational production function see Ozaki (1969).

[15] See Appendix A. For examples of the studies performed by the use of data for newly built plants, see Komiya (1962) and Dhryms and Kurz (1964).

Even if the data does not fulfill the theoretical requirement to undertake this stringent test, as is the case in this study, the estimates of (9a) and (9b) are useful to examine the validity of factor substitutability. In order for factor substitutability to be valid, at least the estimates of a_1 and b_1 in (9a) and (9b) must be significant, and have positive signs.

(ii) *The test of factor substitutability* (2)

By rewriting (7) in the following form, the Cobb-Douglas type production function was directly estimated for each sector.

(10a)
$$\frac{X}{L} = AL^{r_K + r_L - 1}\left(\frac{K}{L}\right)^{r_K}$$
or
$$\frac{X}{L} = a_0 L^{a_1}\left(\frac{K}{L}\right)^{a_2}.$$

Since both r_K and r_L have to be positive, the estimates of a_1 and a_2 must fulfill the following condition if the production function is really of the Cobb-Douglas type.

(10b)
$$\hat{a}_1 + 1 > \hat{a}_2 .$$

For the sectors in which this condition holds, (10c) was estimated to obtain the estimates of r_L independently.

(10c)
$$\frac{X}{K} = AK^{r_K + r_L - 1}\left(\frac{L}{K}\right)^{r_L}$$
or
$$\frac{X}{K} = b_0 K^{b_1}\left(\frac{L}{K}\right)^{b_2}.$$

For the estimates of b_1 and b_2 which were significant, the theoretical consistency was checked on the grounds of if they fulfill the condition such as (10b).

Then the sectors where factor substitutability was rejected in this test were compared with those in test (1). Comparing the results in both tests the validity or invalidity of factor substitutable production function was finally determined for each sector.

(iii) *The estimation of the factor limitational production function*

The estimates of the parameters in (8a), (8b) were obtained for all the sectors. The validity of this type of production function was examined on the basis of the statistical significance of the estimates and of the explanatory power of the model. Of all the sectors in which the factor limitational model applies statistically, only those sectors where factor substitutability was rejected in both tests (1) and (2) or in either one were determined to have the production function of this type.

Combined with the results in tests (1) and (2) the production function was finally determined for each sector.

4. In this study all measurements were made by using time series data for the period 1960 to 1972 on West German manufacturing sectors published by Deutsches Institut für Wirtschaftsforschung and Statistisches Bundesamt. All data except the number of plants were taken from the publishments of DIW.[16] The data for production and capital equipment (Brutto-Anlagevermögen) are at 1962 constant prices.

In order to narrow the gap between the data required by the theoretical model and that actually available, two modifications were made: (1) The data for production, capital equipment and labour were divided by the number of plants; theoretically required data for individual plants were approximated by the data for average plants. (2) The data for capital equipment were adjusted by the ratio of utilization to convert the actual data into estimates of the equilibrium capital equipment requirements corresponding to the observed outputs and labour inputs.[17]

With our limited information the average plant data adjusted by the ratio of utilization was the best modification of the available data to narrow the gap between them and those required by the theoretical model.[18]

Theoretically it is desirable to use the data for gross output as the measure of production scale. As these data were not available, those for net output were employed in proxy of the former. As the gross output data are available for the years 1962, 1966 and 1972 in the I–O tables, a calculation was made to ascertain the degree of approximation of gross output by net output. The rates of increase between 1962 and 1972 in both concepts of production were calculated for each sector. Spearman's rank correlation coefficient and the ordinal correlation coefficient were calculated. Spearman's coefficient turned out to be .7359 and the correlation coefficient to be .9093. Hence the degree of approximation can be regarded as being satisfactory and the possible error generated by this approximation being negligible.

Labor was measured in number of employees (annual averages).

As the price of labor, the index of total labor costs (wages and salaries) per employee was used. As the price of capital equipment the index of the prices of investment goods was employed.

5. *The Test of Factor Substitutability* (1): *the Estimation of the Factor Demand Function*

The equations actually used for the estimation are

$$\text{(9a)}' \qquad \ln\left(\frac{uK}{N}\right) = a_0 + a_1 \ln\left(\frac{P_L}{P_K}\right) + a_2 \ln\left(\frac{X}{N}\right),$$

$$\text{(9b)}' \qquad \ln\left(\frac{L}{N}\right) = b_0 + b_1 \ln\left(\frac{P_K}{P_L}\right) + b_2 \ln\left(\frac{X}{N}\right),$$

[16] Krengel et al. (1972, 1976). The data on number of plants (Betriebe) are from Statistisches Jahrbuch für die Bundesrepublik Deutschland.

[17] For details of the ratio of utilization prepared by DIW, see Krengel (1970).

[18] The simulation experiment in Appendix A indicates that we can obtain satisfactory estimates of the parameters of the production function from the aggregate time series data when the data are divided by the number of plants.

TABLE 2. ESTIMATES OF THE FACTOR DEMAND FUNCTIONS

	$L/N=a_0(P_K/P_L)^{a_1}\ (X/N)^{a_2}$			$uK/N=b_0(P_L/P_K)^{b_1}\ (X/N)^{b_2}$		
Sector	a_1	a_2	$\bar{R}^2$ DW	b_1	b_2	$\bar{R}^2$ DW
11 Industry of building materials	.8169 (.1053)	.7864 (.0998)	.833 .99	.7132 (.1380)	1.0357 (.1309)	.997 1.09
13 Iron & steel industry	.3428 (.1381)	* .0667 (.1604)	.672 1.56	.8029 (.1114)	.7751 (.1293)	.988 1.31
14 Iron and steel foundries	.1891 (.0717)	.3953 (.1064)	.502 1.11	.7195 (.0481)	.8674 (.0713)	.993 2.74
⊙15 Steel drawing & cold rolling mills	.3933 (.1365)	.4022 (.1010)	.669 2.08	* .0051 (.0433)	.9549 (.0321)	.999 1.26
⊙16 NF metals industry	.3085 (.1004)	.4377 (.1052)	.617 1.67	−.0721 (.0083)	.9975 (.0087)	.999 2.60
⊙19 Chemical industry	.5811 (.1782)	.4845 (.0919)	.912 2.28	−.4227 (.0642)	.8462 (.0342)	.999 1.21
⊙20 Oil refining	* −.4574 (.2959)	* −.3465 (.2219)	.035 1.14	* .0602 (.1707)	−1.0903 (.2488)	.935 .67
21 Rubber and asbestos manufactures	.4882 (.0492)	.7266 (.0484)	.953 2.43	.3085 (.0513)	1.0878 (.0504)	.995 1.38
22 Saw mills and timber processing	.7908 (.0523)	.5882 (.0419)	.95 1.85	.7396 (.1372)	.9106 (.1099)	.985 1.47
⊙23 Wood-ship, cellulose, paper and board industry	* .1304 (.1928)	* .2174 (.1403)	.692 1.19	* .3970 (.2522)	.9482 (.1835)	.993 1.42
25 Constrctional steel	.4712 (.0332)	.4961 (.0955)	.943 1.73	.8088 (.0852)	.9377 (.0245)	.999 1.69
26 Machinery construction	.2731 (.0359)	.5385 (.0796)	.835 1.15	.4961 (.0445)	1.1305 (.0987)	.985 2.05
⊙27 Vehicle construction	* 1889 (.1562)	.6871 (.1109)	.939 1.38	−.5135 (.1700)	−.3021 (.0926)	.974 1.47
28 Shipbuilding	.6346 (.0561)	.6941 (.0520)	.938 1.51	.2916 (.1188)	1.0654 (.1102)	.987 1.51
⊙29 Aerospace	−.8032 (.1983)	.3724 (.1098)	.935 1.52	1.3264 (.3889)	.9110 (.1907)	.954 1.29
30 Electlical equipment	.4866 (.0805)	.4508 (.0734)	.751 1.30	.0918 (.0166)	1.0046 (.0152)	.999 1.59
⊙31 Precision engineering & optical industry	* .0842 (.0707)	* .1729 (.0841)	.156 .82	1.0851 (.0916)	.4674 (.1090)	.964 1.73
32 Hardware and metal goods incl. steel forging	.4273 (.0309)	.4434 (.0394)	.941 2.25	.6379 (.0425)	1.0298 (.0543)	.997 1.75
36 Fine ceramic industry	.4784 (.1066)	.4698 (.1439)	.690 1.29	.4859 (.0801)	1.1767 (.1082)	.995 2.12

TABLE 2. (continued)

Sector	$L/N=a_0(P_K/P_L)^{a_1}(X/N)^{a_2}$			$uK/N=b_0(P_L/P_K)^{b_1}(X/N)^{b_2}$		
	a_1	a_2	$\bar{R}^2$ DW	b_1	b_2	$\bar{R}^2$ DW
⊙37 Glass industry	* .2164 (.2242)	.3826 (.1505)	.892 1.15	.4731 (.0802)	*1.0718 (.0538)	.999 1.39
38 Timber manufactures	.5561 (.0391)	.5370 (.0292)	.969 2.29	.2874 (.0412)	1.0199 (.0307)	.998 1.62
39 Musical instruments, toys, and sport articles	.2322 (.0503)	.3146 (.0669)	.629 1.79	.6161 (.1344)	1.3771 (.1788)	.991 1.89
⊙40 Paper and board manufactures	* .2289 (.1111)	.4178 (.0985)	.815 1.76	.6316 (.0817)	1.3657 (.0724)	.998 1.54
⊙41 Printing and duplicating	*−.1206 (.1097)	* .0614 (.1085)	.845 1.61	.3727 (.0396)	1.0197 (.0392)	.999 1.41
⊙42 Plastics manufactures	.5438 (.2328)	.3353 (.1360)	.256 1.42	*−.2411 (.0641)	1.0975 (.0374)	.998 1.32
43 Leather industry	.5961 (.0853)	*−.0483 (.1830)	.836 1.48	.6954 (.0812)	1.3722 (.1743)	.953 1.39
47 Textile industry	.6325 (.0908)	.4804 (.0738)	.795 1.38	.4239 (.0964)	.9193 (.0785)	.995 1.60
48 Clothing industry	.5471 (.0357)	.5897 (.0498)	.951 2.17	.7044 (.0379)	1.1167 (.0529)	.997 2.05
49 Industry of food, beverages and tobacco	.5355 (.0668)	.7319 (.0446)	.994 2.08	.2011 (.0286)	1.0659 (.0191)	.999 1.23

Numbers in parentheses are standard deviation of corresponding estimates.
$\bar{R}^2$ = adjusted coefficient of determination.
* – not significant at 5% level.
Degrees of freedom = 10.

where $u=$ the ratio of capacity utilization, and $N=$ the number of plants.

The results are reported in Table 2. They indicate that the conditions in (9c) are not fulfilled. As was stated in 3.(i) (see also Appendix A), however, upon consideration of the characteristics of the data used, to perform a stringent test on this condition is not possible. Therefore it may be too hasty to conclude only from these results that the assumption of cost minimization is false, or that the production function is misspecified.

The results are, however, still useful to perform a test of factor substitutability. In Table 2, ⊙ on the left side of the table indicates the sectors where factor substitutability is rejected on the grounds that the regression coefficient on the relative price term is not significant or shows a wrong sign.

TABLE 3. ESTIMATES OF THE COBB-DOUGLAS PRODUCTION FUNCTION

Sector	$(X/L)=a_0(L/N)^{a_1}(uK/L)^{a_2}$		
	a_1	a_2	$\bar{R}^2$ DW
11 Industry of building materials	.0891 (.0656)	.5798 (.0655)	.999 2.20
13 Iron & steel industry	−.0347 (.3916)	.6400 (.0771)	.962 1.59
14 Iron & steel foundries	.3940 (.2239)	.4603 (.0400)	.922 1.00
⊙15 Steel drawing & cold rolling mills	−.0247 (.0653)	1.0522 (.0136)	.999 1.12
⊙16 NF metals industry	−.0240 (.0405)	1.0952 (.0113)	.999 1.59
⊙19 Chemical industry	.2654 (.2084)	1.7262 (.0911)	.993 1.02
⊙20 Oil refining	.5792 (.6934)	1.8712 (.1264)	.919 1.56
21 Rubber & asbestos manufactures	.0568 (.0384)	.6435 (.0149)	.995 1.34
22 Saw-mills & timber processing	.2652 (.1361)	.6744 (.0273)	.981 1.66
⊙23 Wood-ship, cellulose, paper & board industry	−.4508 (.3339)	.8339 (.0434)	.989 1.33
25 Constructional steel	.3997 (.1221)	.5290 (.0416)	.973 1.70
26 Machinery construction	.1436 (.1516)	.4589 (.0348)	.939 1.29
27 Vehicle construction	−.0913 (.0979)	.6589 (.0752)	.962 1.38
28 Shipbuilding	.0473 (.1159)	.7142 (.0276)	.984 1.56
29 Aerospace	−.2441 (.1509)	.4998 (.1400)	.505 1.40
30 Electrical equipment	.0277 (.0436)	.9191 (.0661)	.999 1.22
31 Precision engineering & optical industry	.5258 (.7426)	.4169 (.1201)	.479 .85
32 Hardware & metal goods incl. steel forging	.4295 (.0822)	.5881 (.0113)	.996 2.13
36 Fine ceramic industry	.0363 (.1159)	.5911 (.0173)	.994 1.68
37 Glass industry	−.1209 (.1218)	.6878 (.0626)	.997 1.93

TABLE 3. (continued)

Sector	$(X/L)=a_0\,(L/N)^{a_1}\,(uK/L)^{a_2}$		
	a_1	a_2	$\bar{R}^2$ DW
38 Timber manufactures	.1549 (.0538)	.7718 (.0112)	.998 1.78
39 Musical instruments, etc.	.5146 (.1775)	.4512 (.0104)	.994 1.61
40 Paper & board manufactures	−.0099 (.1538)	.4617 (.0232)	.989 2.02
⊙41 Printing & duplicating	−.4372 (.2166)	.7457 (.0346)	.995 1.54
⊙42 Plastics manufactures	.0311 (.1109)	1.0387 (.0155)	.997 1.37
43 Leather industry	.4062 (.1646)	.6813 (.0613)	.983 2.04
47 Textile industry	.1347 (.1619)	.7983 (.0212)	.992 1.30
48 Clothing industry	.1778 (.0689)	.5286 (.0147)	.993 2.14
49 Industry of food, beverages and tobacco	.0032 (.0331)	.7562 (.0156)	.999 1.48

Degrees of freedom = 10.

6. *The Test of Factor Substitutability* (2): *the Direct Estimation of the Cobb-Douglas Type Production Function*

First the following equation is estimated:

$$\text{(10a)}' \qquad \ln\left(\frac{X}{L}\right) = a_0 + a_1 \ln\left(\frac{L}{N}\right) + a_2 \ln\left(\frac{uK}{L}\right).$$

The results appear in Table 3. The 7 sectors which do not satisfy the theoretical condition of (10b) are marked by ⊙ in the table.

Secondly the following equation is estimated for the 22 sectors excluding these 7 sectors to obtain the estimate of r_L:

$$\text{(10c)}' \qquad \ln\left(\frac{X}{uK}\right) = b_0 + b_1 \ln\left(\frac{uK}{N}\right) + b_2 \ln\left(\frac{L}{uK}\right).$$

Table 4 presents the results. As in Table 3, ⊙ on the left side of the table indicates the sectors in which the theoretical condition is not fulfilled.

Comparing the results shown in Tables 2, 3, and 4 reveals that for the 11 sectors presented in Table 5 below factor substitutability is rejected in both tests (1) and (2). It can be concluded that to these 11 sectors the substitutable production function is not applicable.

In (14) iron & steel industry, (40) paper & board manufactures, and (47) textile

TABLE 4. ESTIMATES OF THE COBB-DOUGLAS PRODUCTION FUNCTION

	$(X/uK) = b_0 (uK/N)^{b_1} (L/uK)^{b_2}$		
Sector	b_1	b_2	$\bar{R}^2$ DW
11 Industry of building materials	.0890 (.0656)	.5092 (.0657)	.997 2.20
⊙13 Iron & steel industry	−.0347 (.3916)	.3253 (.3261)	.882 1.59
14 Iron & steel foundries	.3940 (.2239)	.9337 (.2323)	.938 1.00
21 Rubber & asbestos manufactures	.0568 (.0384)	.4133 (.0466)	.982 1.34
22 Saw-mills & timber processing	.2652 (.1361)	.5908 (.1311)	.934 1.66
25 Constructional steel	.3997 (.1221)	.8706 (.0878)	.987 1.70
26 Machinery construction	.1436 (.1517)	.6846 (.1444)	.959 1.29
⊙27 Vehicle construction	−.0913 (.0979)	.2498 (.1685)	.919 1.38
28 Shipbuilding	.0472 (.1159)	.3331 (.1259)	.903 1.56
⊙29 Aerospace	−.2440 (.1509)	.2561 (.2683)	.786 1.40
30 Electrical equipment	.0277 (.0436)	.1085 (.0661)	.925 1.22
⊙31 Precision engineering & optical industry	.5258 (.7426)	1.1088 (.7613)	.644 .86
32 Hardware & metal goods incl. steel forging	.4295 (.0822)	.8414 (.0775)	.994 2.13
36 Fine ceramic industry	.0363 (.1159)	.4453 (.1052)	.988 1.68
⊙37 Glass industry	−.1209 (.1218)	.1913 (.1467)	.989 1.93
38 Timber manufactures	.1549 (.0538)	.3832 (.0602)	.976 1.78
39 Musical instruments, etc.	.5146 (.1775)	1.0634 (.1778)	.996 1.61
40 Paper & board manufactures	−.0099 (.1538)	.5284 (.1731)	.992 2.02
43 Leather industry	.4062 (.1646)	.7249 (.1089)	.977 2.04
⊙47 Textile industry	.1347 (.1619)	.3365 (.1578)	.892 1.30

TABLE 4. (continued)

	$(X/uK) = b_0\,(uK/N)^{b_1}\,(L/uK)^{b_2}$		
Sector	b_1	b_2	$\bar{R}^2$ DW
48 Clothing industry	.1778 (.0689)	.6491 (.0627)	.992 2.14
49 Industry of food, beverages and tobacco	.0032 (.0331)	.2469 (.0482)	.996 1.48

TABLE 5. THE 11 SECTORS WHERE FACTOR SUBSTITUTABILITY IS REJECTED IN BOTH TESTS (1) AND (2)

(15) Steel drawing and cold rolling mills	(27) Vehicle construction
(16) Non-ferrous metals industry	(29) Aerospace
(19) Chemical industry	(31) Precision engineering and optical industry
(20) Oil refining	(37) Glass industry
(23) Wood-ship, cellulose, paper and board industry	(41) Printing and duplicating
	(42) Plastics manufactures

TABLE 6. THE 15 SECTORS WHERE FACTOR SUBSTITUTABILITY IS NOT REJECTED IN EITHER TESTS (1) AND (2)

(11) Industry of building materials	(32) Hardware and metal goods incl. steel forging
(14) Iron and steel foundries	(36) Fine ceramic industry
(21) Rubber and asbestos manufactures	(38) Timber manufactures
(22) Saw-mills and timber processing	(39) Musical instruments, etc.
(25) Constructional steel	(43) Leather industry
(26) Machinery construction	(48) Clothing industry
(28) Shipbuilding	(49) Industry of food, beverages and tobacco
(30) Electrical equipment	

industry, the validity of factor substitutability is rejected either in test (1) or test (2). The determination of the production function for these sectors is postponed until the validity of factor limitationality is examined.

The 15 sectors where factor substitutability is not rejected in either tests (1) and (2) are shown in Table 6. For these 15 sectors the production function is determined to be of the substitutable type.

7. *Estimation of the Factor Limitational Production Function*

The following equations are employed for the estimation of the factor limitational production function:

$$\text{(8a)}' \qquad \ln(uK/N) = a_K + b_K \ln(X/N)\,,$$

$$\text{(8b)}' \qquad \ln(L/N) = a_L + b_L \ln(X/N)\,.$$

The results are summarized in Table 7. It is observed that while the capital input function gives the significant estimates and a good fit, the labour input function shows the significant estimates for only 13 out of 29 sectors. In Table 7 these 13 sectors are marked by ⊙. In 10 out of these 13 sectors, excluding (21) rubber and asbestos, (38) timber manufactures, and (49) foods and beverages, factor substitutability is rejected in both tests (1) and (2) or in either one.

Of the 11 sectors where factor substitutability is rejected in both tests (1) and (2) (see Table 5) (20) oil refining, (31) precision engineering, and (42) plastics manufactures give the significant estimates and a good fit for the labor input function when the data which are not divided by the number of plants are used (see Table 8).[19]

It is concluded that in 13 out of the 14 sectors, where factor substitutability is rejected in both tests (1) and (2) or in either one, the factor limitational production function gives the significant estimates and a satisfactory fit. In 12 out of the 15 sectors where factor substitutability is not rejected in either tests (1) and (2) (see Table 6) the factor limitational model gives neither significant estimates nor a good fit.

Consequently the final determination of the production function type for each sector of the West German manufacturing industry was carried out.[20] Table 9 summarizes the results of this final determination.

8. *Some Economic Implications of the Findings*

It is observed that in most of the key (basic) sectors of the economy such as basic metals and chemicals the production function is of the factor limitational type. In contrast to this in most of the consumption goods sectors the production function is of the substitutable type. The sectors with the limitatational model are characterized by the high capital intensity, and those with the substitutable model are relatively labor intensive: the average value of K/L is 47,900 DM in the former and 24,200 DM in the latter. These findings are in good agreement with those made by Ozaki (1976) using Japanese data in the period 1955 to 1968. According to Ozaki (1976) $K/L = 3.05$ in the sectors with the limitational model, and $K/L = .83$ in those with the substitutable model. This will suggest the international similarity (or commonness) of the production function.[21]

In the sectors with the substitutable production function the sum of $\hat{r}_K$ and $\hat{r}_L$ exceeds one, and hence these sectors seem to exhibit economies of scale. Applying a t test reveals that $\hat{r}_K + \hat{r}_L$ is significantly greater than one (at a 5% level) in (25) constructional steel, (32) hardware and metal goods, (38) timber manufactures, (39) musical instruments, etc., (43) leather industry, and (48) clothing industry.

In the sectors with the factor limitational type production function the estimates of b_L are significantly smaller than one. These sectors exhibit marked economies of scale with respect to the labor input. With respect to the capital equipment input, approximately constant returns to scale $b_K \simeq 1.0$ and diseconomies of scale $b_K > 1.0$ are observed to exist in these sectors with the exception of (19) chemical industry,

[19] The poor results derived from the average plants data seem to be attributable to the forms of capacity distributions in these sectors. For further analysis cross section data are required.

[20] The determination of the production function for (47) textile industry is reserved until more detailed data for this sector become available.

[21] Appendix B makes a comparison of the form of the production function between West Germany and Japan.

TABLE 7. ESTIMATES OF THE FACTOR LIMITATIONAL PRODUCTION FUNCTION

Sector	$L/N=a_L(X/N)^{b_L}$		$uK/N=a_K(X/N)^{b_K}$	
	b_L	$\bar{R}^2$ DW	b_K	$\bar{R}^2$ DW
11 Industry of building materials	* .0272	.000	1.6986	.990
	(.0506)	.57	(.0479)	.76
⊙13 Iron & steel industry	−.2966	.519	1.6261	.933
	(.0794)	1.39	(.1253)	1.39
14 Iron & steel foundries	* .1723	.232	1.7157	.859
	(.0801)	.97	(.1994)	.71
⊙15 Steel drawing & cold rolling mills	.1234	.449	.9586	.999
	(.0376)	1.10	(.0088)	1.26
⊙16 Non-ferrous metals industry	.1388	.325	.9275	.999
	(.0534)	1.34	(.0093)	1.29
⊙19 Chemical industry	.1804	.829	.6250	.994
	(.0234)	1.29	(.0137)	.68
20 Oil refining	*−.0078	.000	.4959	.922
	(.0375)	.59	(.0414)	1.73
⊙21 Rubber and asbestos manufactures	.3291	.539	1.3391	.978
	(.0849)	.56	(.0578)	.69
22 Saw-mills and timber processing	*.0292	.000	1.4334	947
	(.0919)	.56	(.0975)	.81
⊙23 Wood-ship, cellulose, paper & board industry	.1239	.707	1.2330	.992
	(.0226)	1.13	(.0323)	1.28
25 Constrctional steel	* .0587	.000	1.6834	.311
	(.3962)	.17	(.66407)	.14
26 Machinery construction	* .1251	.000	1.8813	.819
	(.1447)	.94	(.2553)	.55
⊙27 Vehicle construction	.5631	.937	1.2939	.991
	(.0422)	1.37	(.0354)	1.33
28 Shipbuilding	* .1547	.219	1.3133	.981
	(.0740)	.49	(.0534)	.97
⊙29 Aerospace	.7319	.814	1.4753	.911
	(.1002)	1.48	(.1329)	1.05
30 Electrical equipment	* .0282	.000	1.0841	.324
	(.0483)	1.19	(.0093)	1.05
31 Precision engineering & optical industry	* .1144	.125	1.2210	.519
	(.0696)	.85	(.3271)	.52
32 Hardware & metal goods including steel forging	*−.0390	.000	1.7501	.949
	(.0781)	.76	(.1162)	.82
36 Fine ceramic industry	*−.1397	.150	1.7859	.979
	(.0790)	1.29	(.0740)	1.68

TABLE 7. (continued)

	$L/N=a_L(X/N)^{b_L}$		$uK/N=a_K(X/N)^{b_K}$	
Sector	b_L	$\bar{R}^2$ DW	b_K	$\bar{R}^2$ DW
⊙37 Glass industry	.2392 (.0238)	.893 1.15	1.3852 (.0172)	.998 1.86
⊙38 Timber manufactures	.1552 (.0499)	.419 .44	1.2172 (.0276)	.994 .70
39 Musical instrumts, toys & sport articles	* .0234 (.0376)	.000 1.53	2.1497 (.0999)	.975 1.68
⊙40 Paper & board manufactures	.2258 (.0361)	.761 1.15	1.8957 (.0587)	.989 1.36
⊙41 Printing and duplicating	.1783 (.0220)	.843 1.45	1.3807 (.0236)	.997 1.36
42 Plastics manufactures	* .0271 (.0390)	.000 1.30	.9608 (.0134)	.998 1.38
43 Leather industry	* −.6234 (.3781)	.125 .54	2.0429 (.4284)	.644 .59
47 Textile industry	* −.0107 (.0509)	.000 .78	1.2484 (.0382)	.989 .92
48 Clothing industry	* −.0234 (.1405)	.000 .46	1.9059 (.1797)	.903 .71
⊙49 Industry of food, beverages and tobacco	.3806 (.0220)	.961 .42	1.1979 (.0085)	.999 .54

* — not significant at 5% level.
Degrees of freedom = 11.

and (20) oil refining where the estimates of b_K are significantly smaller than one. The findings that these 2 sectors exhibit economies of scale with respect to the capital input (investment) are supported by engineering informations. In chemicals and oil refining a large portion of capital equipment consists of relatively simple capital goods such as tanks, gas holders and columns; for these capital goods the so called .6 rule between cost and capacity is known to hold in engineering practices.[22]

From the finding that $b_K > b_L$ in the sectors with the limitational model, it is suggested that an increase in plant scale leads to a rise in the capital intensity in these sectors.

[22] For details of the .6 rule, see Moore (1959) and Haldi and Whitcomb (1967).

TABLE 8. ESTIMATES OF THE FACTOR LIMITATIONAL PRODUCTION FUNCTION (The data for production, capital and labour are not divided by the number of plants.)

Sector	$L=a_L X^{b_L}$		$uK=a_K X^{b_K}$	
	b_L	$\bar{R}^2$ DW	b_K	$\bar{R}^2$ DW
20 Oil refining	.1348 (.0227)	.748 .86	.5703 (.0368)	.952 1.66
31 Precision engineering & optical industry	.1265 (.0402)	.426 .90	1.2576 (.3130)	.558 .56
42 Plastics manufactures	.4547 (.0102)	.994 1.44	.9782 (.0075)	.999 1.35

TABLE 9–a. SECTORS WITH THE FACTOR SUBSTITUTABLE PRODUCTION FUNCTION*

Sector	$X=AK^{r_K} L^{r_L}$			K/L
	r_K	r_L	r_K+r_L	1960,65,70 Average
11 Industry of building materials	.5798	.5092	1.0890	45.6
14 Iron & steel foundries	.4603	.9337	1.3940	28.3
21 Rubber and asbestos manufactures	.6435	.4133	1.0568	24.3
22 Saw-mills and timber processing	.6744	.5908	1.2652	36.6
25 Constructional steel	.5290	.8706	1.3996	14.3
26 Machinery construction	.4589	.6846	1.1435	21.2
28 Shipbuilding	.7142	.3331	1.0473	30.7
30 Electrical equipment	.9191	.1085	1.0276	17.9
32 Industry of hardware and metal goods including steel forging	.5881	.8414	1.4295	17.9
36 Fine ceramic industry	.5911	.4453	1.0364	18.3
38 Timber manufactures	.7718	.3832	1.1540	16.8
39 Musical instruments, toys jewelry, and sport articles	.4512	1.0634	1.5146	10.4
43 Leather industry	.6813	.7249	1.4062	13.1
48 Clothing industry	.5286	.6491	1.1777	8.5
49 Industry of food, beverages and tobacco	.7562	.2470	1.0032	56.9
				24.0**

* All the estimates are significant at 5% level.
** Total average.

TABLE 9–b. SECTORS WITH THE FACTOR LIMITATIONAL PRODUCTION FUNCTION*

Sector	$L=a_L X^{b_L}$ b_L	$K=a_K X^{b_K}$ b_K	K/L 1960,65,70 Average
13 Iron & steel industry	−.2966	1.6261	58.4
15 Steel drawing and cold rolling mills	.1234	.9586	37.0
16 Non-ferrous metals industry	.1388	.9275	43.6
19 Chemical industry	.1804	.6250	66.2
20 Oil refining	.1348	.5703	202.5
23 Wood-ship, cellulose, paper and board industry	.1239	1.2330	63.9
27 Vehicle construction	.5631	1.2939	36.9
29 Aerospace	.7319	1.4753	13.7
31 Precision engineering and optical industry	.1265	1.2576	14.8
37 Glass industry	.2392	1.3852	22.9
40 Paper and board manufactures	.2258	1.8957	18.8
41 Printing and duplicating	.1783	1.3807	25.5
42 Plastics manufactures	.4547	.9782	19.1
			47.9 **

* All the estimates are significant at 5% level.
** Total average.

V. CONCLUDING REMARKS

The explanatory power of the model on technological change presented in the present study depends on the empirical validity of the fundamental assumption (capital embodiment of technology) and on the possibility of finding out the stable production function. The empirical validity of the fundamental assumption has been positively confirmed in III. We have succeeded in determining the production function in IV. Therefore further analyses based on this model seem most promising.

In the estimation of the production function it was found that the production function is of the limitational type in most of the key (basic) sectors characterized by the high capital intensity. Hence in these sectors changes in labor- and capital input coefficients may take place mainly through changes in plant scale. In contrast to this, the production function is of the substitutable type in sectors which are relatively labor intensive: in most of the consumption goods sectors the production function is of this type. From the findings that these sectors exhibit economies of scale, it is suggested that in these sectors too changes in plant scale (in addition to changes in relative prices) have effects on labor- and capital input coefficients.

Therefore it is concluded that in the analysis on choice of technologies with respect to labor- and capital equipment inputs, more emphasis should be placed on the determination process of production scale than that of relative prices.

Keio University

REFERENCES

[1] Carter, A. P. (1957). "Capital Coefficients as Economic Parameters: The problem of Instability," in Problems of Capital Formation, Studies in Income and Wealth, Vol. 19. Princeton University Press, 287–310.

[2] Dhrymes, P. J. and Kurz, M. (1964). "Technology and Scale in Electricity Generation," Econometrica, Vol. 32, No. 3, 287–315.

[3] Haldi, J. and Whitcomb, D. (1967). "Economies of Scale in Industrial Plants," Journal of Political Economy, August, 373–385.

[4] Komiya, R. (1962). "Technological Progress and the Production Function in the United States Steam Power Industry," The Review of Economics and Statistics, 44, May, 156–166.

[5] Krengel, R. (1970). "Die Berechnung des industriellen Produktions-potentials in der Bundesrepublik Deutschland mit Hilfe von Zeitreihen des Brutto-Anlagevermögens," Beiträge zur Strukturforschung, Heft 10, Deutsches Institut für Wirtschaftsforschung, 40–44.

[6] Krengel, R., Baumgart, E., Boneß, A., Piscner, R. und Droege, K. (1972). Produktionsvolumen und-potential, Produktionsfaktoren der Industrie im Gebiet der Bundesrepublik Deutschland, Statistische Kennziffern, 12. Folge 1960–1971, Deutsches Institut für Wirtschaftsforschung.

[7] ———, (1976). Produktionsvolumen und -potential, Produktionsfaktoren der Industrie im Gebiet der Bundesrepublik Deutschland, Statistische Kennziffern, 18. Folge Neuberechnung 1970–1975, Deutsches Institut für Wirtschaftsforschung.

[8] Leontief, W. W. (1953). "Structural Change," in Wasily Leontief et al., Studies in the Strucure of the American Economy, Oxford University Press, 17–52.

[9] ———, (1969). "The Dynamic Inverse," in A. P. Carter and A. Brody (eds.) Contributions to Input-Output Analysis. North-Holland Publishing Company, 17–45.

[10] Moore, F. T. (1959). "Economies of Scale: Some statistical Evidence," Quarterly Journal of Economics, Vol. 73, May, 232–245.

[11] Ozaki, I. (1969). "Economies of Scale and Input-Output Coefficients," in A. P. Carter and A. Brody (eds.) Applications of Input-Output Analysis, North Holland Publishing Co., 280–302.

[12] ———, (1976). "The Effects of Technological Changes on the Economic Growth of Japan, 1955–1970," in Polenske ed. Advances in Input-Output Analysis, Ballinger Publishing Co., 93–111.

[13] Pischner, R. Stäglin, R. und Wessels, H. (1975). Input-Output Rechnung für die Bundesrepublik Deutschland 1972, Deutsches Institut für Wirtschaftsforschung, Beiträge zur Strukturforschung, Heft 38.

[14] Salter, W. E. G. (1960). Productivity and Technical Change, Cambridge University Press.

[15] Smith, V. L. (1961). Investment and Production, Harvard University Press.

[16] Stäglin, R. und Wessels, H. (1973). Input-Output Rechnung für die Bundesrepublik Deutschland 1954, 1958, 1962, 1966, Deutsches Institut für Wirtschaftsforschung, Beiträge zur Strukturforschung, Heft 27.

APPENDIX A. ESTIMATES OF THE PRODUCTION FUNCTION DERIVED FROM THE AGGREGATE TIME SERIES DATA: A SIMULATION EXPERIMENT

1. In the present study, due to the lack of data for individual plants, we used as a substitute data which are aggregates over plants (aggregate time series data). As a modification to narrow the gap between them the latter were converted into plant average data (see IV. 4.). In the following a simulation experiment is performed to examine whether we can estimate the production function from this kind of data (aggregate time series data divided by the number of plants). First we set up the production function with hypothetical values of the parameters. Secondly the aggregate time series data are generated from this production function. Then we estimate the production function from this data and compare the results with the true values of the parameters.

2. We assume the production function that we want to estimate to be:

$$x = 10k^{.7}l^{.5}, \quad \text{for sector 1 and,} \tag{1}$$

$$k = x^{1.2},\ l = 10x^{.5}, \quad \text{for sector 2,} \tag{2}$$

where $x=$ plant scale, $k=$ capital equipment input, and $l=$ labor input.
The capital equipment- and labor demand functions for sector 1 are given by

$$\begin{aligned} k &= .1689\left(\frac{P_l}{P_k}\right)^{.4167} x^{.8333}, \\ l &= .1276\left(\frac{P_k}{P_l}\right)^{.5833} x^{.8333}. \end{aligned} \tag{3}$$

Supposing that the scale of plant increases at a % annually and that the relative price (P_l/P_k) raises at b %, the aggregate time series data (X, K, L) are generated as follows[1] (on next page):

Assuming $x_0=200$, $(P_l/P_k)_0=3.0$, $a=0.2$, $b=0.07$, and $t=13$, we estimate the parameters of the following equations from the data thus generated:

for sector 1

$$\ln\left(\frac{X}{N}\right) = a_0 + a_1 \ln\left(\frac{K}{N}\right) + a_2 \ln\left(\frac{L}{N}\right), \tag{4}$$

$$\ln X = a_0' + a_1' \ln K + a_2' \ln L, \tag{4$'$}$$

$$\ln\left(\frac{K}{N}\right) = b_0 + b_1 \ln\left(\frac{P_l}{P_k}\right) + b_2 \ln\left(\frac{X}{N}\right), \tag{5a}$$

$$\ln\left(\frac{L}{N}\right) = c_0 + c_1 \ln\left(\frac{P_k}{P_l}\right) + c_2 \ln\left(\frac{X}{N}\right), \tag{5b}$$

[1] It is supposed that only one plant is built annually and the durability of a plant is longer than t years.

SECTOR 1

Year	X	K	L	N (number of plants)
0	x_0	$.1689\left(\frac{P_l}{P_k}\right)_0^{.4167} x_0^{.8333}$	$.1276\left(\frac{P_k}{P_l}\right)_0^{.5833} x_0^{.8333}$	1
⋮	⋮	⋮	⋮	⋮
t	$\sum_{i=0}^{t}(1+a)^i x_0$	$.1689\left[\left((1+b)^i\left(\frac{P_l}{P_k}\right)_0\right)^{.4167}((1+a)^i x_0)^{.8333}\right]$	$.1276\left[\left((1+b)^{-i}\left(\frac{P_k}{P_l}\right)_0\right)^{.5833}((1+a)^i x_0)^{.8333}\right]$	t

SECTOR 2

Year	X	K	L	N (number of plants)
0	x_0	$x_0^{1.2}$	$10x_0^{.5}$	1
1	$x_0+(1+a)x_0$	$x_0^{1.2}+((1+a)x_0)^{1.2}$	$10(x_0^{.5}+((1+a)x_0)^{.5})$	2
⋮	.	⋮	⋮	⋮
t	$\sum_{i=0}^{t}(1+a)^i x_0$	$\sum_{i=0}^{t}((1+a)^i x_0)^{1.2}$	$10\sum_{i=0}^{t}((1+a)^i x_0)^{.5}$	t

for sector 2

(6a) $$\ln\left(\frac{K}{N}\right) = d_0 + d_1 \ln\left(\frac{X}{N}\right),$$

(6b) $$\ln\left(\frac{L}{N}\right) = e_0 + e_1 \ln\left(\frac{X}{N}\right),$$

(6a)′ $$\ln K = d_0' + d_1' \ln X ,$$

(6b)′ $$\ln L = e_0' + e_1' \ln X .$$

The results of the estimation appear in Table A.1. They indicate that the aggregate time series data give a satisfactory approximation to the true production function when they are divided by the number of plants, and that the reduced form estimation is not a useful means to obtain the structural parameters. Therefore our design of experiments can be regarded as adequate.

TABLE A-1. THE RESULTS OF THE NUMERICAL EXPERIMENT*

Sector 1	Estimates of the Production Function $x = 10k^{.7}l^{.5}$ Coefficients of			
	$\ln (K/N)$	$\ln (L/N)$	$\ln K$	$\ln L$
	.7492 (25.5)	.3975 (5.6)	.8539 (256.8)	.1458 (37.3)
	Estimates of the Factor Demand Function $k=.1689(P_l/P_k)^{.4167}x^{.8333}$, $l=.1276(P_k/P_l)^{.5833}x^{.8333}$ Coefficients of			
	$\ln (P_l/P_k)$	$\ln (X/N)$	$\ln (P_k/P_l)$	$\ln (X/N)$
	−.0116 (−.29)	1.1065 (26.5)	.1370 (2.8)	.3079 (5.9)
Sector 2	Estimates of the Production Function $k=x^{1.2}$, $l=10x^{.5}$ Coefficients of			
	$\ln (X/N)$	$\ln X$	$\ln (X/N)$	$\ln X$
	1.2268 (812.4)	1.0539 (214.9)	.4747 (215.7)	.8746 (79.0)

* The expressions in parentheses are the t ratios.

APPENDIX B. INTERNATIONAL COMPARISON OF THE FORM OF THE PRODUCTION FUNCTION: WEST GERMANY AND JAPAN

A comparison of the form of the production function is made in Table A.2 below for the 19 sectors (industries) which seem to be roughly comparable between West Germany and Japan (Ozaki (1976), Tables 5–1, 5–2 and 5–3).[1]

It is seen that in 13 out of the 19 industries compared the form of the production function is the same for West Germany and Japan. This will suggest the similarity (or commonness) of the production function in countries.

TABLE A–2. INTERNATIONAL COMPARISON OF THE FORM OF THE PRODUCTION FUNCTION*

Industry	West Germany	Japan
⊙ Iron & steel	L	L
⊙ Non-ferrous metals	L	L
⊙ Chemicals	L	L
⊙ Oil refining	L	L
⊙ Rubber products	S	S
Wood milling (saw-mills & timber processing)	S	L
⊙ Pulp (wood-ship, celluslose, paper and board)	L	L
Machinery	S	L
⊙ Motor vehicles	L	L
Shipbuilding	S	L
Electrical equipment	S	L
⊙ Precision engineering	L	L
⊙ Metal products (steel forging & hardware)	S	S
⊙ Fine ceramic (pottery, china & earthenware)	S	S
Glass products	L	S
⊙ Furniture (timber manufactures)	S	S
⊙ Paper products	L	L
Printing & publishing	L	S
⊙ Leather products	S	S

* L –the factor limitational type production function.
S –the factor substitutable type production function.
⊙ indicates the industries where the form of the production function is the same for these two countries

[1] In Ozaki (1976) the data (time sevies data 1955–68) are not divided by the number of plants and the determination of the functional form is made based on the statistical properties only.