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**THE EMPIRICAL CONTENT OF THE LABOR THEORY OF VALUE:  
The Transformation Problem Once Again\***

MURRAY WOLFSON

1. INTRODUCTION: WHAT DOES THE TRANSFORMATION PROBLEM MEAN?

Of all the industrial nations, Japan is probably the place where the schism between “modern” and “Marxian” economics is most acute. It is not surprising, therefore, that great attention has been paid in this country to Paul Samuelson’s critical articles [4 a–c] on the transformation of values into prices in the Marxian system, and Michio Morishima’s more sympathetic attempt [3] at a restatement aimed at a reconciliation of the two schools of thought.

Marx himself thought that the difficulties in the procedure by which values stated in terms of labor content could be transformed into market prices were mere technicalities. He felt that the outlines of a solution were presented in the third volume of *Capital*, and the details could be safely left to others. The essence of the commodity form, Marx argued, was developed in the strict labor theory of value in *Capital*, I. The amendments in *Capital*, III related these values to the observed market prices under competitive conditions of equality in rates of profit in industries with differing capital structures. It was a consideration of values, he felt, which served to “break the code;” it was *Capital*, I which exposed the underlying reality of class conflict, exploitation, and the historically transitory nature of capitalist society, hidden behind the apparent equality and static equilibrium of market exchange.

This paper will argue that the viewpoint just expressed could not be further from the truth. What is at stake in the transformation problem is not a technicality at all, but the very meaningfulness—or lack thereof—of the concept of value in economics. The question is whether it makes sense to speak of value as a “social crystal” of labor, a material reality which underlies the phenomenal form of everyday market prices analysed by “modern,” “bourgeois economists.” Is the labor theory of value a “meaningless detour,” as Samuelson has suggested? Should we employ the principle of Occam’s razor, and simply account for the observed prices by the usual supply and demand formulation in partial or general equilibrium form?

\* I wish to thank Professors Martin Bronfenbrenner, David Carlson, Leif Johansen, Tadao Horie, Hiroyasu Iida, Makoto Itoh, Stanislaw Gomulka, Michio Morishima, Paul Samuelson and Shigeru Tanese for their helpful comments. In particular, Professor Samuelson was most generous with his time in extended correspondence a few years ago. Anyone who knows the diverse views of these scholars will realize that since they disagree so completely with each other, they are not likely to agree with me either. Hence, they are neither responsible for my views nor for my errors.

Those persons that feel that values expressed in terms of congealed labor gives social insights not available in the direct price calculation, must face up to the transformation problem in explaining the market price phenomena in terms consistent with labor values. To do less, is to simply record labor flows, vacuously calling them “values,” but meaning nothing more than an *ex post* accounting record of no behavioral or operational significance. In some way, these labor flows must relate in a regular way to exchange, for the labor theory of value to have positive, scientific significance. Hence, the transformation problem is at the core of the question of the empirical content of the labor theory of value. If the labor theory of value is devoid of positive content, so is the class exploitation edifice built it unless another, more reasonable, viewpoint about the interaction of economic groups, can be advanced.

In our discussion of this issue, we start from the general axiom of science that the phenomena under investigation are themselves invariant under a change of units or coordinate systems established by the observer. Unless, in some sense, the very act of observation alters the reality of the objective world, the same physical relationships persist, no matter in which way the scientist chooses to formulate his analysis. Obviously some formulations are more convenient than others, producing simpler models more amenable to generalization or manipulation. Thus the Copernican revolution was a transformation of the astronomical record into a heliocentric coordinate system. This formulation was no more true than the geocentric model, but it proved to be simpler. The Copernicus-Kepler computations showed the invariance, opening the way to Newton’s generalization and synthesis. His results certainly hold, even if the truculent astronomer makes an inverse transformation to a geocentric framework, and even if the resulting computations should prove too cumbersome to complete in those terms.

The Marxian transformation problem likewise consists of change in units and a mapping from labor-value terms to competitive prices. We ask three questions: First, can we calculate what Marx called value, the labor embodied in a commodity even if it is produced with intermediate capital goods? Second, can the transformation be accomplished in an internally consistent way? Finally, are insights into the working of capitalism available as a result of labor calculations which will remain invariant upon a transformation into the “phenomenal world” of capitalist market prices? The first two questions are answered in the affirmative. The third is answered negatively in terms of positive economics, and some agnostic concerns are forthcoming if the labor theory of value is seen as a normative welfare proposition in disguise.

## 2. THE CONSISTENT CALCULATION OF THE LABOR DIRECTLY AND INDIRECTLY EMBODIED IN A COMMODITY

Let us examine the first question more closely. If one grants the homogeneity of labor, it certainly is possible to measure—at least conceptually—the current labor

content of commodities, including the indirect labor which Marx called “constant capital.” The possibility of such an accounting tautology without the infinite regress involved in reducing constant capital to labor units (even though production is actually always carried on with constant capital as well as the current labor of variable capital) was made evident by the Leontief linear general equilibrium system. The “input-output” formulation of an “open” transactions matrix treated labor as the primary input, and the labor directly and indirectly required to produce elements of final use could be computed.

This possibility has been noted by many authors including Okishio [1], Seton [2], Morishima [3], Samuelson [4] as well as the present author [5b]. To make the computation, define  $c_i$  = constant capital;  $v_i$  = variable capital;  $s_i$  = surplus value;  $\pi_i$  = value, all defined for a unit of the  $i$ -th commodity,  $i=1, 2, \dots, n$ ;  $\pi_i = c_i + v_i + s_i$ . New value added is  $v_i + s_i$ , the direct labor expended on the  $i$ -th good. Now consider the technical relations lying behind values in terms of the Dorfman-Samuelson-Solow notation for the Leontief system. Let  $a = (a_{ij})$ , the technology matrix;  $a_0 = (a_{0i})$ , the row vector of unit labor inputs; and let  $\pi$  be a row vector of Marxian values. Assume  $i, j=1, 2, \dots, n$ , so that  $a$  is a square matrix representing  $n$  commodities. Assume no joint products. Before the division of new value added between workers and capitalists  $\pi = \pi a + a_0$ . Then  $\pi = a_0(I - a)^{-1} = a_0(I + a + a^2 + \dots + a^n + \dots)$  where  $a$  is taken to meet the Hawkins-Simons conditions of productivity that insure convergence. Since  $\pi a = c$  and  $a_0 = v + s$ , the problem of the infinite regress of the labor content of constant capital is solved as the convergent sum of an infinite series.

We have yet to determine whether these labor quantities are values in the sense that they are related to exchange ratios of commodities. Indeed this is the bone of contention. Yet it is clear that if it were the case that the labor theory of value holds in the sense of determining exchange ratios, equilibrium in the labor market would make for an equalized rate of surplus value,  $s'_i = s_i/v_i$ , as workers migrate from job to job to minimize the work they do compared to the income they receive. Symbolically, labor market equilibrium under the conditions posited would be  $(v_i + s_i)/v_i = s'_i + 1 = s' + 1$  for all  $i$ . We return to this point below as the central issue in the debate, the foregoing conclusion being carefully stated in the subjunctive mood.

### 3. TRANSFORMING VALUES INTO PRICES

Having computed the labor content of commodities, the next question is what has this computation to do with the ratio in which commodities exchange? In what sense are they truly values as Marx originally claimed in *Capital, I*? Marx himself initiated this question by conceding in the third volume of *Capital* that goods do not actually exchange at their values in competitive markets. A transformation is required because of the effect of competition in equalizing the profit rate in the presence of unequal organic compositions of capital. Goods were sold at “prices of production” which equalized the rates of return on capital. Hence the transformation from labor

values to market prices was seen by Marx as something different from a simple identity relationship.

To see this, assume for convenience that the constant capital stock is equal to its flow. Define the rate of profit in the  $i$ -th industry as  $r_i = s_i / (v_i + c_i)$  before the transformation. Clearly  $r = s'_i (1 - q_i)$  where  $q_i = c_i / (c_i + v_i)$ . Competition assures that  $r_i = r$  for all  $i$ , but  $q_i$  in general will vary. Then either:  $s'_i$  are unequal, or goods are not generally sold at their values. Marx opted for the second alternative for reasons we shall study. He required that goods be sold at prices of production different from values while retaining equality in the rate of surplus value. Marx's prices of production required that the aggregate surplus value generated in all industries be redistributed among firms (industries) in proportion to the total capital outlay on  $c$  plus  $v$ . Hence industries with higher organic composition would be selling their goods at prices of production above values, and those with lower than average organic compositions would sell goods below values.

Marx's formula for the transformation was:

$$r = \sum_i s_i / \sum_i (c_i + v_i), \text{ and } p_i = (c_i + v_i)(1 + r), i = 1, 2, \dots, n.$$

Prices were equal to values if  $q_i$  were equal for all  $i$ .

This transformation was admittedly incorrect since the  $c_i$  and  $v_i$  which were to be "marked up" by the profit, should themselves be first transformed into prices, and the computation of  $r$  should be in price rather than value terms. In other words Marx's own solution was a jumble of value units and price units, whereas a consistent system ought to be stated in one or the other. Not both at once.

#### 4. THE BORTKIEWICZ-SETON CORRECT SOLUTION

After the appearance of *Capital*, III, there followed a very lengthy debate on the correction of Marx's price calculations. The literature is cited in the works we have already mentioned. The germ of a correct solution originated in the efforts of von Bortkiewicz, but the problem is best defined in the article by Francis Seton [2]. In their exposition the problem is made to appear a bit more difficult, and the solution less general, by their search for absolute prices instead of price ratios. Their search for a numéraire simultaneously with the transformation of relative values to relative prices led to a three industry model in which prices are stated in terms of luxury goods consumed by capitalists which therefore do not enter into interindustry relations. If we concern ourselves for the moment only with the determination of relative prices, we can confine our illustration to a two-industry model. In this we follow Marx's reproduction schema of *Capital*, II, in which the first industry (department) produces means of production and the second produces consumer goods. We can generalize the presentation to as many industries as we see fit by matrix methods.

Seton explains that Marx was searching for coefficients, let us call them  $y_1$  and  $y_2$ , which will adjust the value of labor embodied in commodities into prices of

production. That is,  $p_i = \pi_i y_i$ . (The symbols are slightly different from Seton, since we want to relate the results to Samuelson's presentation given below.) Then equality in the rate of profit will require that:

$$(1) \quad \begin{aligned} (c_1 y_1 + v_1 y_2)(1+r) &= y_1 w_1 \\ (c_2 y_1 + v_2 y_2)(1+r) &= y_2 w_2. \end{aligned}$$

Since we know the interindustry labor flows, there remain three unknowns,  $r$ ,  $y_1$  and  $y_2$ , in this system of two equations. But as long as we are willing to confine ourselves to determining relative prices from relative value, we only need to find  $y_1/y_2$  and  $r$ . The nature of the equation system permits us to solve for these two variables simultaneously, thus assuring consistency.\* Thus the transformation has been carried out.

## 5. SAMUELSON WIELDS OCCAM'S RAZOR

Samuelson [4a, b] studied the solution and observed what should have been obvious to everyone at the start, namely that the category of value in labor units had nothing at all to do with explaining relative exchange ratios. Rather these values were completely determined by the technology which underlay the production schema, Marx's interindustry table. It was the physical system of interindustry coefficients, reflecting the material inputs from each industry required to produce the others, that explained prices under capitalist competition. To be sure, these could be cast into labor units, but the calculation in effect required that this be done both for inputs and outputs, so that the labor units cancelled themselves out as soon as they had been inserted. In his characteristically provocative fashion, he remarked that this was an exercise in writing and then erasing the labor units in the calculation of the exchange ratios.

This can be seen very clearly in terms of Seton's model. His equations can be written in a fashion which exposes the technology coefficients. Thus equation set (1) becomes:

$$(2) \quad \begin{aligned} (y_1 \pi_1 x_{11} + a_{01} X_1 m_1 \pi_2 y_2)(1+r) &= y_1 \pi_1 X_1 \\ (y_1 \pi_1 x_{12} + a_{02} X_2 m_2 \pi_2 y_2)(1+r) &= y_2 \pi_2 X_2 \end{aligned}$$

where  $\pi_i$  represents values in department  $i$ ,  $x_{ij}$  represents the physical flow of goods

\* More elegantly, if we divide through by  $w_1$  in the first equation and  $w_2$  in the second, we express the value of the inputs into each department as a ratio of the value of the output. Thus:

$$\begin{aligned} y_1(c_1/w_1) + y_2(v_1/w_1)(1+r) &= y_1 \\ y_1(c_2/w_2) + y_2(v_2/w_2)(1+r) &= y_2. \end{aligned}$$

Knowing these input output ratios (from the reproduction schema) we can find  $(1+r)$  as an eigenvalue. For non-trivial solutions the eigenvector  $(y_1, y_2)$  can be found up to a factor of proportionality. In a two equation system, this yields  $y_1/y_2$  as stated in the text.

from department  $i$  to department  $j$ ,\*  $a_{0i}$  is the amount of labor used in the  $i$ -th industry per unit physical output, and  $X_i$  is the physical output of each industry. Here  $m_i$  represents the amount of consumer goods used by workers in each industry per unit of labor performed.

Translating back into Marxian terms  $\pi_1 x_{11} = c_1$  and  $\pi_1 x_{12} = c_2$ ;  $a_{01} X_1$  equals the physical amount of direct labor performed in the first department and  $a_{02} X_2$  is the labor performed in the second. Measured in value units,  $m_1 \pi_2 = w_1$  is the wage rate in the first department, and  $m_2 \pi_2 = w_2$  for the second. Hence for the  $i$ -th industry,  $v_i = a_{0i} X_i m_i \pi_2$ . In price units, the wage rate is  $W_i = w_i y_2$ . The prices of production in the two departments are, as stated before,  $p_i = \pi_i y_i$ .

Now dividing through by  $X_1$  and  $X_2$  in their respective departments, and making the substitutions suggested in the preceding paragraph:

$$(3) \quad \begin{aligned} (p_1 a_{11} + a_{01} W_1)(1+r) &= p_1 \\ (p_1 a_{12} + a_{02} W_2)(1+r) &= p_2 \end{aligned}$$

where equalization of the wage rate in price units requires that  $W = W_1 = W_2 = m_1 p_2 = m_2 p_2$ . Substituting this wage rate into equation set (3):

$$(4) \quad \begin{aligned} (p_1 a_{11} + p_2 m_1 a_{01})(1+r) &= p_1 \\ (p_1 a_{12} + p_2 m_2 a_{02})(1+r) &= p_2 \end{aligned}$$

Hence given the technical coefficients,  $a_{ij}$  and  $a_{0i}$ , and the real wage in each department ("subsistence")  $m_i$ , we can solve for  $(1+r)$  and  $p_1/p_2$ , the profit rate and relative prices.

In matrix terms, direct capitalist equal profit rate pricing gives the vector of prices  $p$  as  $p = (1+r)(W a_0 + P a)$  which on solution becomes  $P = W a_0 (1+r)(I - a(1+r))^{-1}$ . Since  $W = P m$ , if real wages are known and equalized across industries,  $P = P(m a_0 + a)(1+r)$ , and matters become much simpler.  $P$  is found as the eigenvector of  $P(I - (m a_0 + a)(1+r)) = 0$  where  $(1+r)$  is found as the roots of the determinant of the matrix coefficients of  $P$ .

In Marx's simplified model, equality of the real wages implies that  $m_1 = m_2$ , but this is an artifact of the specialization of production mentioned in the previous note. If there were more than one industry producing consumer goods, or what amounts to the same, if department 1 produced consumer as well as capital goods, then  $m_{ij}$  would have to be written as the consumption of the workers in the  $j$ -th industry of products produced in the  $i$ -th. Equality of wage rates in price units would then yield:

$$W = m_{11} p_1 + m_{21} p_2 = m_{12} p_1 + m_{22} p_2.$$

From this it is clear that equal wage rates does not imply identical consumption by workers in each industry. Of course a similar result would obtain if wages were equalized in value units. One would then substitute  $w$  for  $W$ , and  $\pi_i$  for  $p_i$  and come

\* Note that in Marx's system, the departments are completely specialized so that the first department producing means of production does not also produce any items of consumption. The second department likewise does not produce any constant capital equipment. Therefore  $x_{21} = x_{22} = 0$ , unlike the more general Leontief system.

to the same conclusion. We belabor this obvious point because the differences in consumption in different industries becomes an issue later in the paper.

It is now clear that calculations in labor units have nothing at all to do with determining the exchange ratios of goods. As Seton showed the calculation can be carried on in those terms, but it is the technical relations which determine the labor flows between industries rather than the other way around, and it is the technology coefficients which underlie the prices. To insist on the superfluous value calculation would make one like the schoolboy who learns that  $3X=6$  implies that  $X=2$ , but who insists that it is really the case that  $15X=30$  which accounts for the value of  $X$ .

We conclude from all this, that it is indeed possible to make a consistent accounting system of labor flows, and *ex post* relate them to exchange ratios. But as far as we have proceeded, this is merely an accounting tautology that tells us nothing about exchange in addition to the Leontief calculation of prices reflecting technology and the bourgeois search for maximum profits. It seems as if there is no such entity as value.

## 6. EXPLOITATION AND THE RATE OF EXPLOITATION

We are now faced with two systems of valuation which can be made consistent with the facts of exchange under competitive conditions. The empirical content of the labor theory is the same as the price theory—technical coefficients and equalization of the profit rate. No more. What other information could the labor theory of value provide that would account for some empirically observable facts not apparent in price calculations? What regularities could be determined by the labor theory of value which would be invariant upon a transformation into prices?

Sometimes Marx argues, along with some less dogmatic Marxists [10], that the labor theory of value is needed to explain where profits come from. It is certainly true that accounting in labor units shows that the laborers' income contains less labor than they expend. Marx speaks of having uncovered the key to the difference in applying the labor theory of value to labor power, and deriving surplus value as the unearned income of the capitalists. Okishio and Morishima also are at pains to make this point. Yet if this were all that were involved, opponents of Marx could easily point out that the choice of the primary input in the Leontief system is essentially a matter of convenience and pragmatic relevance. Labor need not be the only scarce input. When one expands the problem to the linear programming analysis of choice, there typically are many such primary inputs. Thus, *mutatis mutandis* one could construct a similar theory of exploitation of capital, land and so on. Indeed, in the present international context, it is useful to regard petroleum as the most significant scarce input determining relative prices. Following the Marxian lead, we should arrive at the conclusion that since Iran, Libya and associates have only accumulated \$50 billion, rather than having relieved the world of all its GNP, the world is exploiting the OPEC cartel rather than conversely.

In a world with many factors of production, *all* of them get less of their *own* input



than they throw on the market. But they also get the services of other factors. That is what exchange is all about, the exchange of services between factors of production implicit in the exchange of goods. Whether factors get the value of their marginal product or are the victims of monopolistic exploitation, and the well worn “adding up” problem of the value of marginal products to equal product price, are the relevant theoretical issues.

The question to which we are led in searching for the empirical relevance of Marx’s economics is not so much the existence of non-labor incomes, but their relative magnitude. Here we have something resembling a critical test between the two competing theories. The “modern” bourgeois economist’s distributive mechanism is merely an extension of product pricing into the factor markets. It was Marx’s historical view that the distribution of income depended on the state of the class struggle in defining the “historically determined” subsistence wage. The labor theory of value was an attempt to integrate that concept into competitive markets. If the labor theory of value can identify a datum which reflects such conflict, and which will bear the strain of empirical scrutiny, then it might well be argued that Marx has told us something that is not perceptible to “modern” economics.

The statistic in question is, of course, the rate of surplus value. It is the ratio of the time that workers spend on behalf of the capitalists compared to the labor effort that goes into producing their own subsistence. If, as Marx believed, this rate is uniform cross-section, then we have something to work on. Indeed it is Marx’s statement that this rate is equalized between industries that gives rise to the transformation problem in the first place: How can the profit rate be equalized with a uniform rate of surplus value if organic compositions differ and goods sell at their values? If the rate of exploitation were not equalized, then the transformation problem would not exist, prices would have no relation to values, and the labor theory of value would simply lapse into an accounting identity of no explanatory value. On the other hand, if the rate were equalized, then values could be computed, and it would be a perfectly legitimate question to ask what the real world relation is between prices and values, each of which would have a claim to empirical validity.

Symbolically, if the  $s'_i$  are different then we have the accounting tautology:

$$(\pi_1 \pi_2) = (\pi_1 \pi_2) \left[ a + \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} a_{01}(1+s'_1) & 0 \\ 0 & a_{02}(1+s'_2) \end{bmatrix} \right]$$

from which it is impossible to deduce either rates of surplus value or exchange values. *If* however, it were known that the rates of surplus value were equal, its magnitude as well as the value of commodities could be readily computed. Under such conditions we would have  $\pi = \pi a + \pi m a_0 + s' \pi m a_0$  and  $\pi(I - a - (1+s')m a_0) = 0$ . Non-trivial  $\pi$  requires that the matrix expression be singular. Then  $s'$  can be found from the roots of the polynomial  $\det(I - a - (1+s')m a_0) = 0$ , and values are given as eigenvectors corresponding to  $s'$ .

## 7. IS THE RATE OF SURPLUS VALUE EQUALIZED ACROSS INDUSTRIES?

Yet why should the rate of exploitation be equal in all industries? In what sense can one talk about *the* rate as if it exists in the same sense that an equilibrium profit rate results from competitive pressures? Now it certainly is true that *if* the labor theory of value holds for all goods, including items of labor consumption, a homogeneous labor supply implies a uniform rate of surplus value. That is to say, if  $a_{0i}$  is the new value added ( $v_i + s_i$ ), the labor directly expended in the  $i$ -th industry, and  $w$  is the wage rate in *value* units, then for the  $i$ -th industry the rate of surplus value is  $s'_i = (a_{0i} - wa_{0i})/wa_{0i}$ , and hence  $s'_i = (1 - w)/w$ . Thus if the wage rate equals the value (not the prices) of goods consumed by workers, and goods exchange at their values generally, then  $a_{0i}$  factors out and the uniformity of the wage rate for homogeneous labor implies a uniform rate of surplus value.

The present issue is, however, the empirical relevance of Marx's system, not its internal consistency. The question of the uniformity of the rate of surplus value is now seen to be a matter of fact: Are wage rates uniform when stated in terms of the *value* of the elements of workers' consumption? How might one affirm this in light of the inability of any of the economic agents to observe values? *Ex hypothesi* it is prices which are the phenomenal form to which workers, like everyone else, respond in a market economy. Unless some such link is found between prices and values, Marx's system is hopelessly entrapped in concepts of his own invention that have no relation to reality.

Marx evidently had more than an inkling of the difficulties that he had made for himself. In the quotation given below, when Marx is explaining the equilibration of the rate of surplus value, he appears at first to qualify its uniformity only by short term market delays and frictions; but in the second paragraph, it is clear that there is a more fundamental misgiving in his mind.

If capitals employing unequal amounts of living labor are to produce unequal amounts of surplus-value, it must be assumed, at least to a certain degree, that the intensity of exploitation, or the rate of surplus value, are the same, or that any existing difference in them are balanced by real or imaginary (conventional) elements of compensation. This would presuppose a competition among the laborers and an equilibration by means of their continual emigration from one sphere of production to another. Such a general rate of surplus value—as a tendency, like all other economic laws—has been assumed by us for the sake of theoretical simplification. But in reality it is an actual premise of the capitalist mode of production, although it is more or less obstructed by practical frictions causing more or less considerable differences locally, such as the settlement laws for English farm laborers. But in theory it is the custom to assume that the laws of capitalist production evolve in their pure form. In reality, however, there is always but an approximation. Still, this approxi-

mation is so much greater to the extent that the capitalist mode of production is normally developed, and to the extent that its adulteration and amalgamation with remains of former economic conditions is outgrown.

The whole difficulty arises from the fact that commodities are not exchanged simply as commodities, but as products of capitals, which claim equal shares of the total amount of surplus value, if they are of equal magnitude, or shares proportional to their different magnitudes. And this claim is to be satisfied by the total price realized by a certain capital on the commodities produced by it within a certain space of time. This total price, again, is but the sum of the prices of the individual commodities produced by this capital. *Capital, III*, (8, p. 206)

“The whole difficulty,” Marx evidently realizes, is that there is no reason to believe that the rate will be equalized when, “commodities are not exchanged simply as commodities” (presumably at their values), but, “as products of capitals,” at prices different from values. If workers consume more than one good, then in general  $W = m_{11}p_1 + m_{21}p_2 = m_{12}p_1 + m_{22}p_2 \neq w = m_{11}\pi_1 + m_{21}\pi_2 = m_{12}\pi_1 + m_{22}\pi_2$ . The issue is as simple as that.

One way of assuring equality of the value of workers’ consumption even if goods are sold at their prices, is to follow Morishima in assuming that workers’ subsistence consists of the same collection of goods, the vector  $m$ , for all workers everywhere regardless of skill, occupation or consumer choice. Then if values equal the vector  $\pi$ ,  $w = \pi m$  and uniform wages implies uniform rates of surplus value. This view is certainly empirically meaningful, but it is palpably false from a factual point of view. Moreover, reducing the empirical content of the labor theory of value to a statement about the uniformity of consumption habits for all workers does too much. That is to say, if this assumption were to hold, then *any* system of valuation of wage goods would give rise to a uniformity of the wage rate. The labor theory of value, like any other theory of value simply would have nothing to do with wages. Thus the problem of valuation of workers’ consumption goods is simply swept under the rug by a too convenient assumption. Surely one ought not to solve problems in value theory by assuming away problems in valuation.\*

\* Yet the curious aspect of Morishima’s book is that he really realized that this is the case. At one point (p. 66) he shows that uniformity of the rate of surplus value depends on “homogeneity” of workers’ preferences, on one hand; and on the other hand, his algebra at this point indicates that if the homogeneity is the rule, uniformity of the rate of surplus value would result even if workers responded to price or value stimuli in choosing their consumption bundles. Indeed, as remarked above, the rate would be equalized regardless of the stimuli which led to the uniform consumption. Indeed in the last chapter of his book he concedes the restrictive nature of his assumptions. Nonetheless, despite the fact that he concludes that the labor theory of value output to be dropped. Professor Morishima spends the rest of his now famous book using such terms as “exploitation,” the rate of surplus value, and values as if they had empirical meaning.

If the drastic assumption about uniform consumption is dropped, then the labor theory of value depends on the uniformity of the rate of surplus value as a comparison of work done on behalf of capitalists with work done for laborers, measured in some system of valuation perceptible to the parties involved. There are three possibilities which we might profitably discuss.

(i) It might be that the rate of surplus value stated in labor units is *also* the ratio of the non-labor income in each industry evaluated in price units, compared to money wages as the price-weighted sum of labor consumption. This might occur if prices and values were identical due to some singular condition on the technology. Marx himself considered and rejected the equal organic composition condition because he thought that this value ratio mirrored technical coefficients. It turns out that equal technical coefficients are not a necessary condition for prices to equal values, even though it is sufficient. There are other sufficient conditions, but each one is more implausible than the next.\*

(ii) Still another rationalization for equality in  $s'$ , is that shares in new value added are the result of what Marx called class struggle, and which modern economists identify as “bilateral monopoly” between employers and employees. As we shall argue later, there is some sense in considering the trend of the ratio of factor shares from a macroeconomic, time-series, point of view. But the issue here is not the distribution of aggregate shares in price units *over time* but its *cross section uniformity in value units* at any given time.

Even if workers should compare their own earnings with those of their employers in the course of collective bargaining over shares in the “new value added” by production, the comparison cannot be made in labor units. Certainly workers know how long they work ( $a_{0i} = v$ ), but they do not perceive their own compensation in value units, since their real wages are computed in terms of the prices of goods they consume. *A fortiori*, the gross revenue which the employer receives is also perceptible only in price units.

(iii) The third alternative involves mixed units, hours of labor compared to money wages. Of course money wages are equalized compared to the marginal product evaluated in price units. But if prices are not equal to values as Marx himself tells us, the mere fact of money wage rates equalization tells us nothing about its equalization in value units and hence we have no information about the rate of surplus value.

From the point of view of positive economics, therefore, we must conclude that empirical content of the labor theory of value boils down to the statement that

\* Suppose  $\pi = P$  where  $\pi(I - a - (1 + s')ma_0) = 0$  and  $P(I - (1 + r)(ma_0 + a)) = 0$ . Subtracting  $P((r - s')ma_0 + ra) = 0$ . This last matrix expression must be singular for non-trivial prices or values. One particular sufficient condition for this, is for the matrix itself to be 0, i.e., if  $(s' - r)ma_0 = ra$  (or  $(1 + r)a = s'ma_0$ ). This would happen if the labor composition of the matrix of workers' consumption were a model of the technology of the economy suitably scaled to maintain the equality. (One may play with this. If the scaling were accomplished so that  $a = ma_0$ , the  $1 + r = s'$ . Stated in terms of Marx's organic composition where  $r = s'(1 - q)$  a little algebra states the condition that  $s'q = 1$ . That would make  $s' = (c + v)/c$ , and the rate of profit  $r = s'(v/c + v)$  equal to  $v/c$ .) It is all too far fetched to be considered.

everybody eats the same breakfast-dinner-supper. Marx certainly seemed to think that this theory was not as particular in meaning as the foregoing. If a broader interpretation is given in terms of relative shares, we find ourselves making statements in terms of entities that are unobserved by any of the participants. We conclude that either the arguments for the labor theory of value have no operational significance on one hand, or advance a factually implausible statement about the nature of consumption on the other.

It is sometimes argued at this point that the labor theory of value as expressed in *Capital, I* was an admitted simplification designed to focus on essential social issues. The complications entailed by the interindustry model contained in the reproduction schema of Volume II, and the rate of profit-prices of production calculation in Volume III, were second order approximations, improving on the first fundamental truth. It should be clear from the foregoing that this view is incorrect. The technical relations entailed in the latter two volumes are sufficient to explain exchange, when stripped of the labor unit camouflage imported from the first volume. All that the labor calculations accomplish is to make it a bit more difficult to understand the underlying nature of the pricing process.

It seems to the author that the first volume of *Capital* is not a first approximation. It is simply wrong. The reason is not the method of simplification or abstraction. There is nothing wrong with such a procedure. The problem is that by positing a labor theory of value, Marx is led to ask a non-question: Where do non-labor incomes come from? His answer has to be exploitation of some sort, since the value of the product of labor (marginal or otherwise) clearly exceeds the wage of the single factor of production he considers. But if goods do not sell at their values, except in peculiar cases in which values equal prices, then labor content has no special claim to explaining prices, and there is no question to be asked.\*

\* Of course fixing worker consumption, and its equivalent, equality in the rate of surplus value, is "nice" from the point of view of constructing models of balanced growth because the whole closed system becomes homogeneous. Morishima shows that if output were the growth maximizing balanced growth vector, say  $y^*$  then the rate of surplus value and the rate of profit would be related by:

$$r = s'(\pi m a y^*) / \pi(a + m a_0) y^* = s'(v y^*) / (c + v) y^*.$$

Prices, Morishima asserts, would converge to the competitive capitalist ones. His theorem is: If  $M = a + m a_0$ , and  $M y^* = \lambda y^*$ , and  $y^*$  is associated with the largest positive eigenvalue of  $M$ , then  $r$  is as stated above. Proof: By definition  $M$  is square and non-negative so  $y^*$  is the column eigenvector associated with the largest positive eigenvalue of  $M$ . (Itself the largest eigenvalue in absolute value). Choosing  $r$  such that  $1 + r = 1/\lambda$ , it will assure the largest balanced growth rate. That is  $y^* = (1 + r) M y^*$  and  $m y^* = (1/(1 + r)) y^*$ . From the fact that  $\pi = \pi M + s' \pi m a_0$ ,  $\pi - \pi M = s' \pi m a_0$ . Postmultiplying by  $y^*$ ,  $\pi y^* - \pi M y^* = s' \pi m a_0 y^*$ . Recalling the definition of  $y^*$ , rearranging terms and premultiplying by  $\pi$ ,  $\pi y^* - \pi M y^* = r \pi M y^*$  so that  $s' \pi m a_0 y^* = r \pi M y^*$  and the conclusion follows. Morishima remarks that the correct transformation is not as in Marx where  $y^* = (1, 1, \dots)$ , but the weighting of the components by the balanced growth output. He goes on to show that this transformation, under certain assumptions regarding the structure of  $M$ , will cause any original set of prices to converge to capitalist prices by the repeated autoregressive process  $P_t = (1 + r) P_{t-1} M$ .

All this, of course, presupposes the uniformity of the rate of surplus value in the first instance. To identify with ultimate reality, situations in which a tolerable error is accepted to focus on the dynamic properties of homogeneous systems, is to search for some metaphysical unique "first cause" of value. It is

## 8. THE NORMATIVE VIEW: CAPITALIST PRICES OR COMMUNIST VALUES?

If there is no meaningful transformation in terms of a positive description of the workings of capitalism, we must examine the possibility that it constitutes a normative welfare economics [5b]. There is little doubt that Marx frequently thought in these terms, even though he expressed himself as if he were making a positive statement about the way things are, rather than as they ought to be. If currently expended labor were the only social cost which Marx's moral views could accept—that is to say if all labor were measured simply in terms of hours of effort regardless of the date at which the labor were performed—the equalization of the rate of surplus value and the pricing of goods at their values would constitute a reasonable goal for socialist planners. By implication, the capitalist pricing algorithm would be sub-optimal on Marx's norms. Equality of the rate of surplus value could be thought of as the result of a lagrangian calculation designed to minimize the labor required to produce a given product, and yet leave a certain amount of labor as a constraint to produce the needed workers' consumption goods. Perhaps, one might say, we *ought* to reckon in labor units, even though capitalism requires us to acknowledge property claims to income. Capitalist society forces us to calculate in price terms, thus equalizing the rate of profit on all capital advanced, constant as well as variable, rather than planning in terms of the rate of surplus value and current labor. It is worthwhile considering whether Marx ought to be viewed as criticizing a society which misallocates and squanders labor when seen from his class viewpoint.

The nature of the choice between the two criteria is clearly a choice between equal profit rate-capitalist maximization of output, and constructing a equal rate of surplus value minimum labor cost society system that produces less but in some Marx-laboristic sense is more fair. In the previous little book [5b] I looked at this problem in terms of static optimization. Let us now review the issues in an intertemporal sense where we now find ourselves reviewing the history of economic thought. For, to the classic formulation of labor cost, we must report the Austrian reply: *When* is the labor performed? Is labor expended in the past to be valued at the same level as labor performed now? Is labor expended in the past to be valued at the same level as labor performed now? Defenders of Marx such as Wolfstetter [7], and his critics such as Samuelson and Weizsäcker [4a], have coined the term "synchronized labor" to indicate that in the presence of economic growth—either in the number of workers or their productivity—labor expended earlier will be more scarce and hence should be valued more highly than present labor. It is only another was of expressing Böhm-Bawerk's remarks about the interest premium on more

just this sort of transcendental extension of our reason beyond the range specified by the empirical observations that Kant warned about two centuries ago. Concretely, suppose there were a country in which it so happened that organic compositions of capital were equal at some moment in history so that the problems we have been discussing did not arise, and prices happened to equal values. Would it follow that labor was exploited in that country and not in others? Surely Ricardo went through all this, and we are merely retracing his steps in matrix form.

productive roundabout methods of production. Samuelson and Weizsäcker specifically use the grape-juice-versus-wine-in-the cellar as an illustration of the theorems they prove in more sophisticated matrix terms.

Granting for the sake of argument the Marxian ethical priority of labor, they show that if a dating of labor is undertaken the value of commodities to society will amount to the ordinary bourgeois pricing mechanism based on equal rates of profit. This is optimal *provided* that the goal of society is maximum steady state growth. In practice, this is precisely the goal and, though unwillingly, the method of many socialist economies. The soviet economy is increasingly being driven along this line as it moves from extensive growth to considerations of more intensive, resource-husbanding paths of economic expansion.

But even though this is the trend in Soviet practice, there is no reason why it must be what Marx had in mind. It should be noted that he vehemently and dramatically rejected such an Austrian formulation when intimations of it were suggested in the course of his analysis. For instance, in the *Grundrisse* [8b, p. 518], where we can see his mind at work, he found himself at one stage including circulation time in the duration of labor, and hence in value. Then he wrote "False!" after this passage. Numerous other examples could be cited, such as his criticisms of both "abstinence" and "waiting" theories of profit. Why? The simple and obvious answer is probably the correct one. He saw the implication of such a dating of labor, and realized that it was contrary to his theory of class struggle. Profit as an economic category has nothing to do with the private—or public—ownership of the means of production, rather it reflects the need to take time into consideration in allocating resources in production as well as the usual risk-uncertainty factors discussed by such authors as Frank Knight. The choice whether to include profit as a cost of production by economic entrepreneurs or central planners amounts to a decision whether they wish to maximize output or whether they have other social norms in mind which prompt them to forego using the full productive potential of the available factors of production. The existence of profit, therefore, does not mean that workers are exploited. Far from being an historical relative entity, profit calculations probably preceded capitalism in disguised institutional form, and more than likely, would survive it.

## 9. SOCIAL CLASSES AND GROWTH PATHS

There is another way of putting this issue which does not really relieve Marx of the error of confusing his positive and normative judgements, but which makes his position a bit more understandable. We have spoken of capital (or dated labor) as if it were a disembodied factor of production. Yet in capitalist society this factor is the property of a particular social class of individuals, and so their growth decisions have distributive implications as well as other social consequences. Given the technology of an economy, Samuelson long ago showed the existence of a "factor price frontier" which, in modern terms, states the Ricardian theorem of a generally inverse relationship between wage and profit rates. Furthermore, we know from von

Neumann that the profit rate is the maximum steady state rate of growth of the economy. Consequently, to the extent that Marx was asking who makes the profit-wage-growth decisions in capitalist society, he was on reasonable ground to complain that the choices were unsatisfactory from labor's point of view.

Let us look at this a bit more closely. Holding the technological alternatives as given, as a finite knowable set, the remaining parameters of the system, the three variables mentioned, remain to be determined, one by the other. If labor is really in infinitely elastic supply at "subsistence wage," then the optimality of capitalist pricing for the purpose of maximum growth is obvious and no further discussion is required. This is the spirit of Ricardian economics, at least as it has been traditionally interpreted. If, however, subsistence is rather loosely defined, as Marx sometimes does, by speaking of the level of necessary worker consumption as "historically determined," the wage rate (to be sure in price units) may well reflect "class antagonistic" relations over what is considered normal consumption. It is then possible to discuss alternative optimal growth paths corresponding to alternative points on the factor price frontier.

Which path ought to be chosen? Under early Nineteenth Century conditions in England, the plentiful supply of labor made it possible for capitalists to enforce a high profit-high growth-low wage expansion path. The weak state of labor organization made it inevitable that workers would have to acquiesce. Marx may well have argued that this was not the universal optimum, but only the capitalist's optimum. Of course he made his point in a grotesquely exaggerated way. His labor theory of value and equal rate of surplus value are not really descriptions of the workings of capitalism. Nevertheless, it seems reasonable for us to consider that there is more than a little sense in arguing that low market wages and high profits are not always socially desirable, at least from the point of view of those who have to bear the burden of rapid growth. One could make the same argument for the divergence between internal and external costs in labor pricing as are presently being made for natural resources, public goods and the like. Indeed, the passages in *Capital* which describe the considerations surrounding the passage of the English Factory Acts suggest just this line of argument.

Suppose, for any technology and labor supply, the marginal productivity of labor gives the wage rate at some low level. Capitalist pricing will decree high profits and rapid growth which might, some time in the future, result in high wages when labor will be either less relatively abundant compared to capital, or when it accumulates "human capital" and raises its marginal productivity. But suppose labor is "impatient." Or, suppose labor suffers from rapid population growth or labor saving innovations, and never becomes more scarce. Then matters are more difficult. The alternatives involve a trade off between an "optimal" balanced growth path, and an unbalanced one involving slower growth, lower overall efficiency, but higher immediate returns to labor. Recent history, particularly in Britain, makes one somewhat unsympathetic to the practical wisdom of such an approach. Nevertheless, even though the problem of expressing this alternative in suitable



analytic form "taxes present ingenuity" (to quote Samuelson again), it is a meaningful line of criticism of capitalist pricing procedures.

An alternative to such a sub-Pareto-optimal program, would involve a program of Millian redistribution of stocks of wealth through such devices as inheritance and wealth taxes. It might thus be possible to provide wider access to non-labor incomes without unbearable dynamic efficiency loss. This is obviously a very complex question which ultimately has to deal with the motivational impact of wealth distribution on one hand, and the growing importance of human capital along with its distribution among social groups defined by class and race.

#### 10. FETISHISM OF COMMODITIES, TECHNOLOGY AND DISTRIBUTION

One last line of speculation remains to be explored. Implicit in the discussion of the supply side of price formation by bourgeois economists has been the existence of a bounded set of production alternatives (production function) which are known and over which some sort of maximization occurs. It is this frontier which defines marginal products and hence competitive distribution in the market. The feasible production set is defined by the supply of factors and the exogenously given state of technical knowledge. Even when these are subject to dynamic change (e.g. capital accumulation, labor force growth, or technical progress) it is assumed that the coefficients regulating their rate of change are themselves known or at least approximated by some iterative search process. After all, how else could maximization take place?

This technical relationship is the sense behind Samuelson's insistence that we go behind labor accounting and examine the underlying technology. In the present discussion we are involved in a world of fixed coefficients chosen so as to open communication with those holding Marxian preconceptions by voiding the law of diminishing returns and the consequent influence of demand in choosing among supply alternatives. Yet Marx did not think in those terms, preferring labor flows which seemed to him reflective of the aggregate distribution of income in the macroeconomic context. Indeed the labor theory of value may well be more charitably understood as a confusion between the microeconomic structure Marx constructed on the basis of the cross sectional uniformity of the rate of surplus value, and the macroeconomic observation of the relative constancy of labor's share over time.

Of course the rate in question cannot do double duty, but discussing Marx in macroeconomic terms opens questions as to the existence and meaningfulness of a production function as a means of explaining relative shares in the aggregate. We do not intend to discuss the conditions for the aggregation of production functions, but rather the question of the logical priority of microeconomic production frontiers in determining the marginal product of factors to each firm or industry. Is there such a technically given bound on feasible activities, or does it reflect social as well as

engineering data? Does its ascription to technical alternatives constitute a “fetishism of commodities”?

In this connection it is certainly hard to resist the logic of the “irreversibility” postulate which reasons, from the inability of production factors to reproduce their inputs without some sort of entropy, to the statement that the set of production possibilities is bounded away from a vector of outputs that equal inputs [9, pp. 62–69]. Nevertheless, the question which a reading of Marx raises is whether the set in question is in fact an exogenously given datum, which, given factor supplies, determines the distribution of income. Or, whether in some sense, technology is much more flexible over time, so that aggregate factor shares are determined in some rather messy way by class-political-union bargaining, and the technology set which entrepreneurs consider evolves to suit.

It seems hard to accept the autonomous technology assumption as simply given by the state of the arts. This typewriter upon which I work has been produced by engineers studying alternatives in their handbooks. Is that the state of the art or does it lie in the physics texts? But these are not the ultimate knowledge that can be brought to bear in making this machine. Perhaps the mathematicians can teach the physicists a thing or two. Do we really mean that the state of the arts only consists in the most immediate application available? And what is that? Is it not equally plausible that one could optimize production processes in terms of utilizing higher level knowledge not presently on line? If one attempts to avoid the infinite regress which seems implicit in the definition of the production boundary by establishing an innovation frontier, a production possibility frontier for new knowledge, the problem is only set back one step. What determines that new frontier being considered by managers of research and development activities? Only if one were really willing to deny that new knowledge ever is created, making that Aristotelian judgement that it exists *in potentia* and is known to the parties involved, would the concept appear to be well defined.

If, on the other hand, one were to speak of technical possibilities as those presently being considered by economic agents in the position to implement the ideas, then we would be discussing a behavioral relationship rather than a purely technical one. This is the sense, of course, behind Schumpeter’s distinction between invention and innovation. The consequence for the present discussion is that there may well be room for distributive effects to determine the technical alternatives under review as well as conversely.

This is an agnostic position guaranteed to please nobody—especially the author. It suggests that perhaps there is more room for bargaining over relative shares than neoclassical economics would admit. At the same time it does not really say how much scope there is to define different efficient growth paths, some favored by workers and some by capitalists (and other social groupings as well). How difficult is the choice between a sub-optimal growth path that might meet distributive norms and an efficient one which might hold the promise for greater income at some later date? In plain words, this position does not tell us if, and when, it would be possible

to admit greater labor claims to the national product without sacrificing long term productivity of the system.

But conceding that we do not have the answers to these questions only prompts the consideration that the present tidy solutions are a bit too secure. Perhaps in his own faulty and obdurate fashion, Marx has raised issues which must still exercise us today, and to which definitive solutions have yet to be advanced.

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