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# ABSENTEEISM, TRADE-UNIONS AND INPUT DEMAND FUNCTIONS 

R. W. Latham and D. A. Peel

## INTRODUCTION

A variety of socio-economic reasons have been provided to account for the formation of trade unions. These include, the securing of rights in industry such as holidays or defined rates of wages. (See e.g. S. and B. Webb [7], or Flanders [5], Bain and Elsheikh [1]. In a recent paper Reza [6] has provided a novel economic rationale for the formation of trades unions. His thesis depends crucially on the firms optimal response in terms of its input demands (e.g. men, hours and capital services) to changes in the rate of absenteeism. (Where absenteeism is broadly defined to include absence from work for any reason). Assuming that the production technology can be described by a Cobb-Douglas function, it is shown that a perfectly competitive firm will respond to an increase in the rate of absenteeism by ceteris paribus, reducing its demands for men on the payroll, capital services and men in attendance and increasing its demand for hours per man in attendance.

Given these input responses and the further assumption that the supply of hours per worker is a function of the real wage Reza offers a possible raison d'etre for the formation of trade unions. In order to clarify the subsequent analysis Reza's argument is briefly reviewed.

Consider a rise in the real wage rate which, for simplicity is assumed to be caused by a fall in the retail price level. At this higher real wage the representative employee is working fewer hours than he wishes but if he attempts to work longer hours the firm will respond (as outlined above) by reducing its demand for hours. Thus the gap between hours demanded and supplied is increased rather than reduced. Given that he would lose earnings and that he is already consuming more leisure than he wants it is unreasonable to expect any individual worker to increase his absenteeism in order to increase the firm's demand for hours. In this situation a possible solution is the formation of a union so that the benefits of higher universal absenteeism can be shared. Strikes are a good example. Upon returning to work after a strike all the employees obtain the benfits of the higher rate of absenteeism (the strike) in the form of longer working hours and hence higher total earnings. Thus a strike may yield additional income for the workers even if it is settled without an adjustment in the wage rate. Reza goes further to show that the institution of a seniority system may be a concomitant of unionization.

Clearly the firm's input responses to changes in absenteeism play an important role in Reza's thesis. Since they were derived using a particular form of production function it seems worthwhile to examine their sensitivity to alternative tecnological assumption. Our purpose in the remainder of the paper is to rigorously reanalyse

[^0]Reza's model for the case of a general production technology. We will also show how Reza's analysis provides interesting possibilities for the extension of utility maximising models of union behaviour.
In the following analysis it is assumed that:

$$
\begin{equation*}
Q=F(A, H, K) \tag{1}
\end{equation*}
$$

where
$Q=$ real output in period $t$,
where
$A=$ the number of employees in attendance in period $t$,
$H=$ the average number of hours worked per man in attendance in period $t$,
$K=$ the flow of capital services used by the firm in period $t$.
Before proceeding it is also worth noting that in Reza's analysis the output elasticity of men was constrained to be greater than that of hours and that this is at odds with the empirical results obtained for Cobb-Douglas production functions obtained by Feldstein [4] and Craine [2].

## SOME ANALYSIS

The firm is assumed to maximise

$$
\begin{equation*}
\pi=q F(A, H, K)-\frac{c A}{1-\beta}-w A H-r k \tag{2}
\end{equation*}
$$

where $\pi$ is profit per time period
$q$ is the price of the product assumed constant
$w$ is the hourly wage rate
$r$ is the price of capital services assumed constant
$\beta$ is the rate of absenteeism
$c$ is the user cost per man on the payroll ( $L$ ). This includes items such as training costs.
The number of men on the payroll is related to the rate of absenteeism and employees in attendance by $L=(1-\beta) A$.
Necessary and sufficient conditions for the firm to maximise profit are: where $F_{A}=\partial Q / \partial A, F_{A A}=\partial^{2} Q / \partial A^{2}$ etc.

$$
\frac{\partial \pi}{\partial A}=q F_{A}-\frac{c}{1-\beta}-w H=0
$$

(3)

$$
\begin{aligned}
& \frac{\partial \pi}{\partial H}=q F_{H}-w A=0 \\
& \frac{\partial \pi}{\partial K}=q F_{K}-r=0
\end{aligned}
$$

(4)

$$
\begin{gathered}
\left|\begin{array}{ll}
q F_{A A} & q F_{A H}^{-w} \\
q F_{H A}^{-w} & q F_{H H}
\end{array}\right|,\left|\begin{array}{ll}
F_{H H} & F_{H K} \\
F_{K H} & F_{K K}
\end{array}\right|,\left|\begin{array}{ll}
F_{A A} & F_{A K} \\
F_{K A} & F_{K K}
\end{array}\right|>0 \\
\Delta=\left|\begin{array}{lll}
q F_{A A} & q F_{A H}^{-w} & q F_{A K} \\
q F_{H A}^{-w} & q F_{H H} & q F_{H K} \\
q F_{K A} & q F_{H K} & q F_{K K}
\end{array}\right|<0
\end{gathered}
$$

The relevant comparative statics results are

$$
\frac{\partial A^{*}}{\partial \beta}=\frac{q^{2} c}{\Delta(1-\beta)^{2}}\left|\begin{array}{ll}
F_{H H} & F_{H K}  \tag{5}\\
F_{K H} & F_{K K}
\end{array}\right|<0
$$

$$
\frac{\partial H^{*}}{\partial \beta}=\frac{-c}{\Delta(1-\beta)^{2}}\left|\begin{array}{ll}
q F_{A H}^{-w} & q F_{H K}  \tag{6}\\
q F_{K A} & q F_{K K}
\end{array}\right|
$$

$$
\frac{\partial K^{*}}{\partial \beta}=\frac{c}{\Delta(1-\beta)^{2}}\left|\begin{array}{ll}
q F_{A H}^{-w} & q F_{H H}  \tag{7}\\
q F_{K A} & q F_{K H}
\end{array}\right|
$$

$$
\begin{equation*}
\frac{\partial M^{*}}{\partial \beta}=\frac{1}{(1-\beta)} \cdot \frac{\partial A^{*}}{\partial \beta}+\frac{A^{*}}{(1-\beta)^{2}} \tag{8}
\end{equation*}
$$

These responses to a change in the rate of absenteeism are quite different to those obtained in Reza's model i.e. $\partial H^{*} / \partial \beta, \partial K^{*} / \partial \beta$ and $\partial M^{*} / \partial \beta$ are ambiguous in sign. Similar results are obtained if a premium payment for overtime hours is introduced. In these cirumstances it is possible for the representative worker, acting as an individual in the situation described in the second paragraph, to reduce his absenteeism and in so doing increase the firm's demand for hours. Consequently the gap between hours demanded and supplied may be narrowed.

Thus Reza's deductions concerning union formation, strikes, and so on depend critically upon the character of the production function. Therefore if the model has any validity the above analysis demonstrates that the unionization or nonunionization of the workers in a particular firm or industry may depend, in part, on the nature of the technology. In those firms, for example, where the difference between hours demanded and supplied can be reduced by workers acting as individuals one would expect the incidence of unionization to be lower. However, it should be emphasized that this analysis is a long way from providing a complete theory of union formation, e.g. S. and B. Webb [6].

An extension of the model which reinforces the above results is to consider the effects of absenteeism on the firm's factor demands when absenteeism is assumed to have deleterious effects on efficiency. Absenteeism is particularly disruptive when, for example, the work-force is organized into teams. In these circumstances (1) is replaced by

$$
\begin{equation*}
Q=G(A, H, K, \beta), G_{\beta}<0 \tag{1}
\end{equation*}
$$

and is easy to show that $\partial A^{*} / \partial \beta, \partial H^{*} / \partial \beta, \partial K^{*} / \partial \beta$, and $\partial M^{*} / \partial \beta$ are all ambiguous in sign.

Throughout the above analysis it was assumed that the wage rate and absenteeism were outside the firm's control. In other words they may be either market or union determined. If it is the latter it is interesting to consider the derivation of optimal wage and absenteeism rates when the union is assumed to maximize a utility function. The notion that unions maximize a utility function was first suggested by Dunlop [3]. Consider the case of the firm's labour force being organized into a single union whose utility function has the number of men on the firm's payroll (i.e. its membership) and average earnings as its arguments i.e.

$$
\begin{equation*}
U=U(M, w H) \tag{9}
\end{equation*}
$$

The union maximizes (9) subject to the firm's demand functions for men and hours ${ }^{2}$ i.e.

$$
\begin{gather*}
M^{*}=\frac{A^{*}}{(1-\beta)}=\frac{A^{*}(\beta, q, c, w, r)}{(1-\beta)},  \tag{10}\\
H^{*}=H^{*}(\beta, q, c, w, r) \tag{11}
\end{gather*}
$$

Therefore the union's problem is to

$$
\begin{equation*}
\max _{\{\beta, w\}} U=U\left(\frac{A^{*}(\beta, q, c, w, r),}{(1-\beta)} w H^{*}(\beta, q, c, w, r)\right) \tag{12}
\end{equation*}
$$

This yields

$$
\begin{equation*}
\frac{\partial U}{\partial \beta}=U_{1}\left[\frac{1}{(1-\beta)} \frac{\partial A^{*}}{\partial \beta}+\frac{A^{*}}{(1-\beta)^{2}}\right]+U_{2} w \frac{\partial H^{*}}{\partial \beta}=0 \tag{13}
\end{equation*}
$$

[^1]\[

$$
\begin{equation*}
\frac{\partial U}{\partial w}=\frac{U_{1}}{(1-\beta)} \frac{\partial A^{*}}{\partial w}+U_{2}\left[H^{*}+w \frac{\partial H^{*}}{\partial w}\right]=0 \tag{14}
\end{equation*}
$$

\]

The second order conditions for a local maximum are

$$
\frac{\partial^{2} U}{\partial \beta^{2}}, \frac{\partial^{2} U}{\partial w^{2}}<0
$$

$$
\begin{equation*}
\Delta^{1}=\left[\frac{\partial^{2} U}{\partial \beta^{2}} \cdot \frac{\partial^{2} U}{\partial w^{2}}-\left(\frac{\partial^{2} U}{\partial \beta \partial w}\right)^{2}\right]>0 \tag{15}
\end{equation*}
$$

Assuming that the implicit function theorem is satisfied (13) and (14) can be solved to obtain

$$
\begin{align*}
& w^{*}=w^{*}(q, c, r),  \tag{16}\\
& \beta^{*}=\beta^{*}(q, c, r),
\end{align*}
$$

but the effects of changes on $q, c$, and $r$ on $w^{*}$ and $\beta^{*}$ are all ambiguous. ${ }^{3}$

## CONCLUSIONS

The purpose of this paper has been to illustrate two important points. First, when Reza's model is analyzed for production technology other than Cobb-Douglas, many of the firm's responses to changes in workers' absenteeism are ambiguous. This is contrary to the results obtained in Reza's paper. Moreover, under certain circumstances we can derive implications as to the rationale of union formation that are opposite to Reza's thesis. Secondly, we showed how it is possible to formulate a


It is interesting to note that if the firm has a Cobb-Douglas technology as in Reza's model and the union has a Cobb-Douglas utility function, e.g. $U=a m^{\alpha 1}(w H)^{\alpha 1}$, then (13) and (14) cannot be solved to obtain $w^{*}$ and $\beta^{*}$.
utility maximising model of union behaviour in which optimal wages and absenteeism are determined. However, this illustration did not go much beyond simply touching formally upon one of the possible courses of theoretical development raised by Reza's provocative article.

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[^0]:    ${ }^{1}$ We are grateful to two referees for comments on an earlier draft of this paper.

[^1]:    ${ }^{2}$ Note that previous models of this type, e.g. Dunlop's in [2], have only included the stock constraint i.e. men.

