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AGRICULTURAL CAPITAL AND THE DEVELOPMENT OF A DUAL ECONOMY: AN APPLICATION TO GERMANY

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ABSTRACT

A dual economy model with agricultural capital is applied to Germany during 1850–1913. The predictive power of the model was found quite strong. Using a counterfactual analysis, the impact of agricultural capital on the development of the German economy was found not only raising the society's maximum sustainable rate of population growth, but also accelerating the growth of per capita income and industrialization. The results elucidate the importance of capital investment in agriculture for development planning.

I. INTRODUCTION

Planning models of development, such as those of Hornby (1968) and Dixit (1969), emphasize the importance of capital investment in agriculture. Conventional descriptive models of development,[1] however, such as those of Lewis (1954), Fei-Ranis (1964), Jorgenson (1961), Zarembka (1970), Sato-Niho (1971) and Marino (1975), do not include capital in the agricultural production function.[2] In these models, agricultural output was determined solely by the input of labor under a given state of technology. This may have been due to the fact that these models were intended to represent the traditional agricultural sector of contemporary developing countries. On the other hand, considerable historical evidence suggests that the application of capital inputs (such as fertilizer, insecticides, and machinery) in the agricultural sector was an important element in the successful

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[1] Conventional development models are classified into "classical" and "neoclassical", depending on their assumptions regarding whether disguised unemployment is existent or not and whether the real wage rate in agriculture is constant or variable. In this paper, however, we refer to both branches of existing theory as "conventional", in the sense that it is common in both theories to exclude capital from the agricultural production function and consequently to have productivity conditions in industry independent from the conditions of development. (One exception is the work of Zarembka, who discussed the case in which the price elasticity of the demand for food is not zero.)

[2] In their model of a dual economy, Kelley, Williamson and Cheetham (1972) introduced capital into agricultural production, but excluded land.

development of modern ecnomies such as those of Germany, Japan and the United States.[3]

Niho (1974) introduced agricultural capital into the conventional model of development in order to examine its effect on economic development. This model showed that the introduction of capital into agricultural production could significantly increase the society's maximum sustainable rate of population growth, thereby creating a greater possibility of development for the economy.[4] In addition, the model showed that the introduction of capital into agricultural production provides an important role for industrial technological progress in the industrialization of a dual economy.[5] This is a significant departure from the conventional models of development in which technological and production conditions in industry cannot play any role in assisting an economy to escape stagnation.[6] Furthermore, the framework provides a general model of growth and development in which both the conventional model of development and neoclassical two-sector growth model are included as special cases. If the production elasticity of capital in agriculture is zero, the model is reduced to the conventional model of development. If the production elasticity of land is zero so that the agricultural production function is subject to constant returns to scale, the model produces the results of the neoclassical two-sector growth model.

[3] Capital intensity in agriculture in three countries was: Germany, 2.95 in 1850 and 5.41 in 1913 (both per man and in thousand mark in 1913 prices); Japan, 155 in 1874 and 232 in 1936 (both per man and in 1934–36 yen); United States, .644 in 1889 and .926 in 1936 (both per man hour and in 1929 dollar). See Niho (1973) for details.

[4] There is a firmly established empirical association between a higher level of per capita income and a greater degree of industrialization (measured in terms of the proportion of the industrial workers to the total labor force or the proportion of the industrial output to the national product. See Jorgenson (1967).) A theoretical justification for this association is also provided by the present model. (See Footnote 11.) Hence, economic development in the sense of the sustained growth of per capita income and industrialization in the sense of continuous increase in the industrial component of the total labor force are equivalent.

[5] The conditions necessary for the sustained growth of the industrial sector (and equivalently of per capita income) in a dual economy have been well summarized by Jorgenson (1961) as the existence of a positive and growing agricultural surplus. Technological progress in agriculture always assists industrialization because it has a positive effect on agricultural surplus. Capital accumulation and technological progress in industry can also assist industrialization if (1) the the demand for food is affected by changes in the relative price of food or (2) capital is included in agricultural production as an input. In these cases, productivity increases in industry can contribute to agricultural surplus through their effect on the demand conditions of food (the first case), or through their effect on the production conditions of food (the second case). For more detailed discussion on the role of an agricultural surplus in industrialization, see Niho (1974b).

[6] This implication of conventional theories is based on the assumption that the price elasticity of demand for food is zero. If it is not zero, the productivity conditions in industry will affect the maximum sustainable rate of population growth even if capital is not introduced into agricultural production. The case in which the price elasticity of food is not zero was studied by Zarembka (1970) with a model in which the population growth was exogeneous and capital was not included in agricultural production.

The purpose of this paper is twofold. First, we apply this model to Germany[7] during 1850–1913[8] and examine the explanatory power of the model by comparing its predictions with the historical record; and secondly, we examine the impact of agricultural capital on the development of the German economy by employing a "counterfactual" analysis. That is, we examine what would have been the impact on German economic growth if capital had not been employed in agricultural production. Kelly and Williamson (1973) employed a counterfactual analysis to examine the sources of Japanese economic growth, and showed that this methodology can provide useful insights. A brief review of the model is presented in Part II. (The full mathematical model is provided in Appendix I.) Part III is devoted to an application of the model to Germany, while in Part IV the impact of agricultural capital on German economic growth is examined by a counterfactual analysis. Summary and conclusions are given in Part V. Data and their sources are included in Appendix II.

II. BRIEF REVIEW OF THE MODEL

1. *Notations*

z = per capita income
k = overall capital intensity
$\bar{k}$ = overall capital intensity in efficiency units
ϕ = growth rate of population
v = per capita food consumption
s = the proportion of agricultural workers to the total labor force
α = the rate of technological progress in agriculture
λ = the rate of technological progress in industry
β = the production elasticity of capital in agriculture
γ = the production elasticity of capital in industry
δ = the production elasticity of labor in agriculture
$1 - \beta - \gamma$ = the production elasticity of land in agriculture
ρ = the net saving ratio
ξ = the relative share of labor in agriculture
μ = the ratio of the agricultural to the industrial money wages

[7] The application of the model to the United States during 1889–1953 was not successful. Although our model could explain the behavior of the overall capital intensity well, it could not explain the behavior of per capita income. This may be due to the fact that while the relative shares in industry remained constant during the period, the relative shares in agriculture have been changing: my computation indicates that the relative share of capital in agriculture has been rising since 1929, from about .20 in 1929 to about .45 in 1953 (see Niho (1973)). Consequently, the specification of the agricutlural production function in a Cobb-Douglass function was not appropriate for the United States.

[8] W. G. Hoffmann's work provides an almost complete record of the development behavior of Germany since 1850. However, since there are interrupted periods due to World War I and World War II, and since the behavior during the Great Depression was abnormal (such as a negative capital's share), we shall confine our analysis to the period 1850–1913.

2. *Workings of the Model*

We assume that the agricultural production function is subject to diminishing returns to scale with respect to capital and labor, while the industrial production function exhibits constant returns to scale with respect to these two inputs.[9] The dynamic behavior of the model is described by a system of two non-linear differential equations in per capita income and over-all capital intensity in efficiency units as[10]

$$\text{(1)} \qquad \frac{\dot{z}}{z} = \frac{1}{M(z)}\left[\alpha + \beta\frac{\lambda}{1-\gamma} - (1-\beta-\delta)\phi(z) + \beta\frac{\dot{\bar{k}}}{\bar{k}}\right]$$

$$\text{(2)} \qquad \frac{\dot{\bar{k}}}{\bar{k}} = N(z)\bar{k}^{\gamma-1} - \phi(z) - \frac{\lambda}{1-\gamma}$$

where[11]

$$\bar{k} = e^{-\frac{\lambda}{1-\gamma}t}k$$

$$M(z) = \frac{v'(z)z}{v(z)} + \left[\delta + \frac{\beta}{1-(1-D)s(z)}\right][1-(1-C)s(z)]\left(1 - \frac{v'(z)z}{v(z)}\right)$$

$$N(z) = \frac{\rho[1-(1-C)s(z)]}{[1-(1-D)s(z)]^{\gamma}}$$

$$\text{(3)} \qquad s(z) = \frac{v(z)}{v(z) + C(z - v(z))}$$

$C = \mu(1-\gamma)/\xi$ and $D = \mu\beta(1-\gamma)/\gamma\xi$.

It can be shown that both $M(z)$ and $N(z)$ are positive for all the relevant values of z, $s(z)$, $v(z)$ and $v'(z)$.

The system (1) and (2) will terminate its motion at the point $(z^*, \bar{k}^*)$ where $\dot{z}/z = 0$ and $\dot{\bar{k}}/\bar{k} = 0$. At this point,

$$\text{(4)} \qquad \phi(z^*) = \frac{\alpha + \beta\lambda/(1-\gamma)}{1-\beta-\delta},$$

delineates the maximum sustainable rate of population growth for the model. Thus, the maximum sustainable rate in this model is determined as the ratio of the

[9] For the rationale behind these assumptions, see Niho (1973a, p. 1077).

[10] If the agricultural production function is subject to constant returns to scale with respect to capital and labor, i.e., $1 - \beta - \delta = 0$, the term $(1 - \beta - \delta)\phi(z)$ does not occur in the determination of the growth rate of per capita income, or in equation (1). In this case, the model reproduces the results of the neoclassical two sector growth model with different rates of technological progress.

[11] Equation (3) implies $0 < s < 1$ and also

$$s'(z) = \frac{-C(v - v'z)}{[v + C(z - v)]^2} < 0 .$$

Hence, the model supplies a theoretical justification for the secular decline of the agricultural sector in the course of economic development.

weighted sum of the rate of technological progress in agriculture and the rate of (Harrod neutral) technological progress in industry (weighted by the production elasticity of capital in agriculture), to the production elasticity of land in agriculture. As a special case of the present model, if the production elasticity of capital in agriculture, β, is zero, the model reproduces the result of conventional models; the maximum sustainable rate is determined as the ratio of the rate of technical progress in agriculture to the production elasticity of land in agriculture.

At the point where $\dot{z}/z = 0$ and $\dot{\bar{k}}/\bar{k} = 0$, denoted $(z^*, \bar{k}^*)$, per capita income ceases to increase. This point is called the stagnation point. As long as the actual growth rate of the population tends to exceed the maximum sustainable rate of population growth, so that the point of stagnation exists, the economy most likely will approach the point of stagnation from any initial position (see Niho (1974, p. 1082)). On the other hand, if the actual growth rate of the population never exceeds the maximum sustainable rate, so that the inequality

$$\phi(z) < \frac{\alpha + \beta\lambda/(1-\gamma)}{1-\beta-\gamma}$$

always holds, then z^* defined in (4) does not exist. In this case the behavior of the economy is characterized by steady development. Per capita income increases continuously and the industrial component of the labor force becomes more and more dominant. In the long run the economy will reach the equilibrium position at which the agricultural and industrial sectors grow in a balanced fashion and per capita income grows at a constant rate, which is similar to the long-run equilibrium position described by neoclassical growth theory (see Niho (1974a, p. 1083)).

As a special case of the model, if land is not a limitational factor in agricultural production so that the agricultural production function exhibits constant returns to scale with respect to capital and labor alone, the growth of the population never works as a pressure upon the growth of per capita income. In this case the model reproduces the results of neoclassical two-sector growth model with different rates of technological progress. The long-run equilibrium is always stable, since the elasticity of substitution is equal to unity in each of the sectors (see Sato (1969)).

III. APPLICATION TO GERMANY

In applying this model to Germany, we first estimate the values of the parameters as well as the shapes of the population growth function $\phi(z)$ and the Engel function $\nu(z)$. These values and shapes reveal some features of the German economy which are responsible for their successful development. Secondly, we assign these values and shapes to the parameters and the functions respectively, and evaluate the explanatory power of the model by comparing its predictions with the historical data.

1. *Specifications of the Population Growth Function and the Engel Function*

Investigation of the data (see Table AI) suggests that a demographic population growth function and a Cobb-Douglas type Engel function may be appropriate. The results of the regression are:

$$(5) \qquad \phi(z) = .0197 - 3.869\frac{1}{z}, \qquad R = .600$$
$$(18.984) \quad (9.559)$$

and

$$(6) \qquad \log v(z) = 3.003 + .332 \log z, \ R^2 = .805$$
$$(23.936) \quad (16.04)$$

Equation (6) satisfies $0 < v'(z) < 1$ and $v''(z) < 0$ for all the relevant values of $z[v'(z) < 1 \text{ for } z > 17.0]$. The t-values shown in parentheses indicate that a significant relationship exists between the level of per capita income and the growth rate of the population, and also between per capita income and per capita consumption of agricultural products.

2. *Specification of the Production Elasticities and the Factor Shares*

γ: the production elasticity of capital (the relative share of capital's income) in industry: From the distribution of national product by sectors (see Hoffmann (1965, pp. 506–509)), we compute the relative share of capital's income in industry as $\gamma = .19$ (the average during the period). Since the industrial sector is assumed to be competitive, the production elasticity of capital is the same as the relative share of capital's income. The relative share of labor and the production elasticity of labor are, then identified as $1 - \gamma = .81$.

β: the production elasticity of capital (the relative share of capital's income) in agriculture: From the determination of the return to capital in each sector by its marginal productivity and the distribution of capital between the two sectors such that the returns are equalized (see equations (A5), (A6) and (A7)), the value of β can be determined as[12]

$$\beta = \gamma\left(\frac{y_m/q}{k_m}\right)\Big/\left(\frac{y_a}{k_a}\right) = .20 \text{ (the average during the period)},$$

where y_a and y_m are the average productivities of agricultural and industrial workers, k_a and k_m are capital intensities in agriculture and industry, and q is the terms of trade (the price of agricultural commodities in terms of industrial goods). The values of γ and β are shown in Table AI. Small variations in the values of γ and β (which are obtained independently of the production functions) indicate that our assumptions of Cobb-Douglass production functions are consistent with the historical record.

δ: the production elasticity of labor in agriculture: This parameter is obtained

[12] Our figure for β is different from Hoffmann's figure (1965, pp. 506–509). This is because we assume the existence of rent for land in agriculture, while Hoffmann assumes that the entire income in agriculture is distributed between capital and labor in agriculture.

by estimating the agricultural production function (see equation (A1)) with the value of β fixed at $\beta = .20$ as:[13]

$$\log y_a - .20 \log k_a = -.951 + .0094\, t + .054 \log\left(\frac{N}{L_a}\right), \quad R^2 = .919$$
$$(8.528) \quad (21.934) \quad (.830)$$

The estimated value of the production elasticity of land in Germany, $1 - \beta - \delta = .054$, is extremely small. Furthermore, the t-value indicates that the agricultural production function may be assumed to exhibit constant returns to scale with respect to capital and labor.[14] In the remaining analysis, however, we use .054 as the production elasticity of land.

The production elasticity of labor in agriculture is then identified as $\delta = .746$. We assume that the real wage rate in agriculture is determined by the marginal productivity of labor, i.e., $\delta = \xi$.

From the determination of the real wage rate in the two sectors (equations (A3) and (A4)) and the relationship between the two money wages (equation (A9)), we have

$$\mu = \frac{\xi}{1-\gamma} \frac{y_a}{y_m/q} .$$

Since the term $(y_m/q)/y_a$ remains almost constant at 1.54 throughout the period, the ratio of the agricultural to the industrial money wage must have remained constant[15] at $\mu = .6$.

3. *Estimation of the Technological Progress*

We obtain the rates of technological progress from the production functions as

$$\alpha = \frac{\dot{y}_a}{y_a} - \beta\frac{\dot{k}_a}{k_a} + (1 - \beta - \delta)\frac{\dot{L}_a}{L_a}$$

for agriculture, and

$$\lambda = \frac{\dot{y}_m}{y_m} - \gamma\frac{\dot{k}_m}{k_m}$$

for industry. The obtained rates are shown in Table AI.

The average annual rates of technological progress in Germany were .95 percent in agriculture and 1.05 percent in industry. As a weighted sum of the two sectors, it was $\alpha + \beta\cdot\lambda/(1-\gamma) = 1.21$ percent, which is greater than the averaeg growth rate of the population, 1.06 percent. This may have been an important factor

[13] The estimation of the agricultural production function without using the outside information, $\beta = .20$, was not successful, due to a multicollinearity problem.

[14] This implies that the growth of the German economy can be virtually explained by a neo-classical two-sector growth model.

[15] This implies that many German agricultural workers received compensation in kind: land use, potatoes, wood, grain, etc. I owe this to Professor Beckmann.

for successful development of the German economy. The small production elasticity of land in Germany may be another factor contributing to the successful development of the economy, since a small production elasticity of land reduces the forces of diminishing returns to labor caused by the population growth.

4. *Specification of the Savings Ratio and the Initial Value of the Technological Improvement Factor in Industry*

The savings ratio can be obtained from the behavioral equations governing society's savings and capital accumulation[16] (equations (A16) and (A17),

$$\dot{K} = \rho(Y_m + qY_a) .$$

The results are given in Table AI. For the purpose of our analysis, we assume that the savings ratio is determined exogeneously each year, i.e., $\rho = \rho(t)$.

The model (see Appendix I), assumes that the initial value of the technological improvement factor in industry, B_0, is equal to unity. However, the values of y_m and k_m in the data are not adjusted for $B_0 = 1$. Thus, by assigning the values of the initial period (1850) to y_m and k_m in the industrial production function, $y_m = B_0 e^{\lambda t} k_m^{\gamma}$, and setting $t = 0$, we obtain $B_0 = .613$.

5. *Evaluation of the Predictive Power of Equations (1), (2) and (3)*

We can now examine the predictive power of equations (1), (2) and (3) by assigning the actual values of z and $\bar{k}$ in each year together with the specified values of the parameters and the specified shapes of the functions $\phi(z)$ and $v(z)$ obtained in the preceding sections. The comparison of the observed and the predicted values of s, $\dot{\bar{k}}/\bar{k}$ and $\dot{z}/z$ are shown in Figures 1, 2 and 3 and also in Tables AII and AIII.

The regression of the observed on the predicted values gives us:

for equation (3) $\quad s_{observed} = .074 + .841 \; s_{predicted} \qquad R^2 = .941$

for equation (2) $\quad \dfrac{\dot{\bar{k}}}{\bar{k}}_{observed} = -.001 + .756 \dfrac{\dot{\bar{k}}}{\bar{k}}_{predicted} \qquad R^2 = .774$

for equation (1) $\quad \dfrac{\dot{z}}{z}_{observed} = .011 + .376 \; \dfrac{\dot{z}}{z}_{predicted} \qquad R^2 = .639 .$

The above results indicate that the explanatory power of the model is quite strong; explaining 94 percent of the behavior of the proportion of the agricultural workers to the total labor force; 77 percent of the growth rate of the overall capital intensity in efficiency units, and 64 percent of the growth rate of per capita income[17]. While equation (2) overestimates the average growth rate of $\bar{k}$ (.40 percent per annum for the predicted and .23 percent per annum for the observed), equation (1) predicts

[16] Since data for depreciation is not available, we compute the net savings ratio.

[17] The fluctuations of the predicted values of $\dot{z}/z$ are larger than the actual values. This is reflected in a low value of the coefficient .376 for the regression of the observed on the predicted values of $\dot{z}/z$.

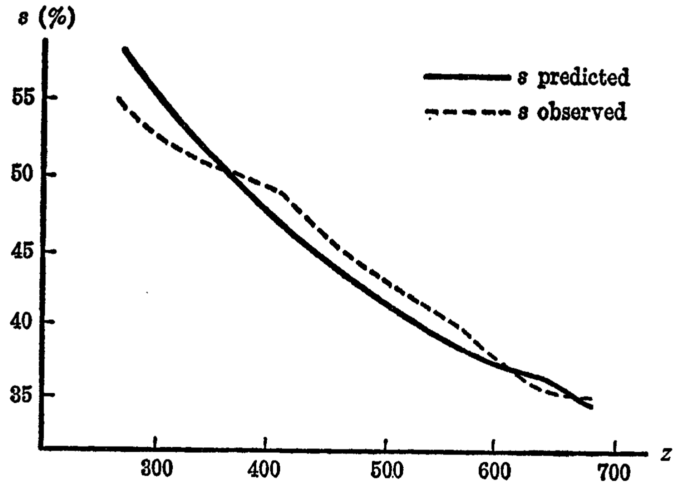

Fig. 1. Predicted and Observed Values of the Proportion of the Agricultural Workers to the Total Labor Force

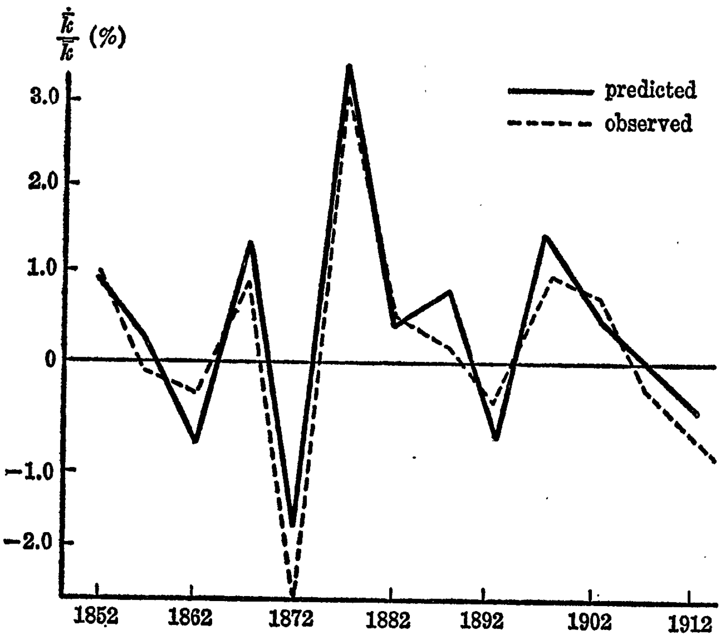

Fig. 2. Observed and Predicted Values of the Growth Rate of the Overall Capital Intensity in Efficiency Units

the average growth rate of per capita income (1.56 percent per annum) almost precisely.

6. *Simulated Time Path of the Economy*

As the last analysis of this part, we solve the system of differential equations (1) and (2) with the initial values of z and $\bar{k}$, in order to obtain the time paths of z and

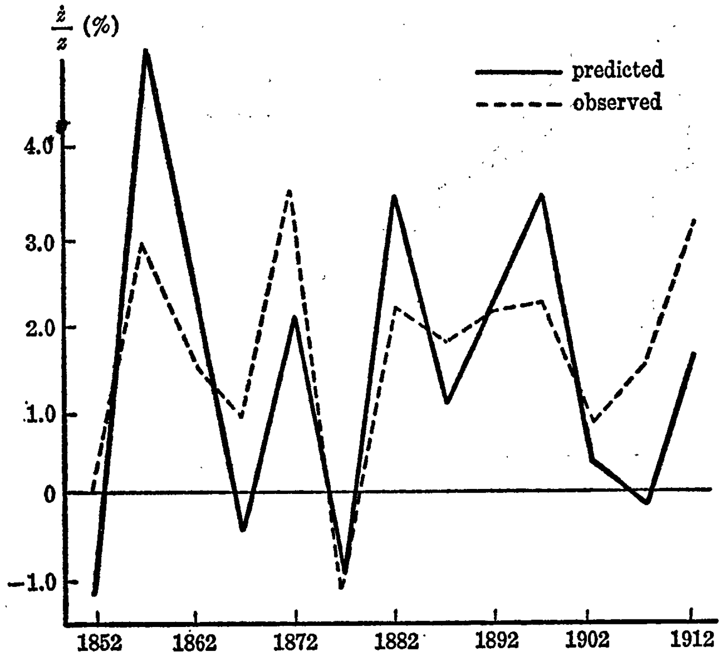

Fig. 3. Observed and Predicted Values of the Growth Rate of Per Capita Income

$\bar{k}$ implied by our model.[18] (The obtained time paths of z and $\bar{k}$ are shown in Table AII.) We have solved for $\bar{k}$ and z when the parameters α, λ and ρ take their estimated values in each year, and when these parameters are fixed at their average values. The former solution represents the time path of $\bar{k}$ and z along which the fluctuations may be reflected. This time path is supposed to follow the actual time path closely. The latter solution represents the long-run trend of the economy from which the fluctuations are eliminated. This time path indicates whether the structure of the German economy, represented by the specified values of the parameters, is such that steady development can be generated. $\bar{k}_{\underset{\theta=\theta(t)}{\mathrm{sim}}}$ and $z_{\underset{\theta=\theta(t)}{\mathrm{sim}}}$ denote the former time path, and $k_{\underset{\theta=\bar{\theta}}{\mathrm{sim}}}$ and $z_{\underset{\theta=\bar{\theta}}{\mathrm{sim}}}$ denote the latter time path.

As pointed out previously, our model tends to overestimate the growth of $\bar{k}$, and consequently the simulated time path of $\bar{k}$ shows a greater growth than actual $\bar{k}$. Although equation (2) predicts the average growth rate of per capita income precisely, the simulated time path of z underestimates the actual growth of per capita income. This is because overestimation of $\bar{k}$ has a negative effect on the growth rate of per capita income as

[18] As a method for solving a system of non-linear ordinary differential equations, the Runge-Kutta method is available, which approximates the solution by linearizing the system at each point. However, since we are interested in the discrete time paths of variables, we treat equations (1) and (2) as the difference equations, i.e.,

$$z(t+1) - z(t) = \frac{1}{2}(z(t+1) + (z(t))H(z(t), \bar{k}(t); \alpha(t), \lambda(t), \rho(t))$$

$$\bar{k}(t+1) - \bar{k}(t) = \frac{1}{2}(\bar{k}(t+1) + \bar{k}(t))\, G(z(t), \bar{k}(t); \lambda(t), \rho(t))\,.$$

$$\frac{\partial(\dot{z}/z)}{\partial\bar{k}} = \frac{-\beta(1-\gamma)N(z)\bar{k}^{\gamma-2}}{M(z)} < 0 .$$

Figure 4 illustrates the simulated time path of z and $\bar{k}$, and Figure 5 is the phase diagram. For the specified values of the parameters, $\dot{z}/z$ remains always positive, so that $\dot{z}/z = 0$ line does not exist in Figure 5. Consequently, the point of stagnation (at which the $\dot{z}/z = 0$ line and the $\dot{\bar{k}}/\bar{k} = 0$ line intersect) does not

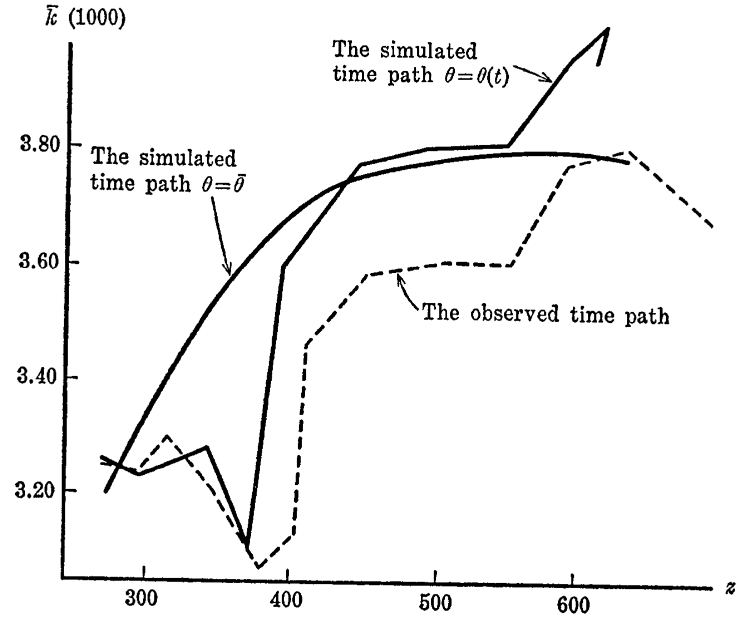

Fig. 4. Observed and Simulated Time Path of $(z, \bar{k})$ in Germany (1913 Mark)

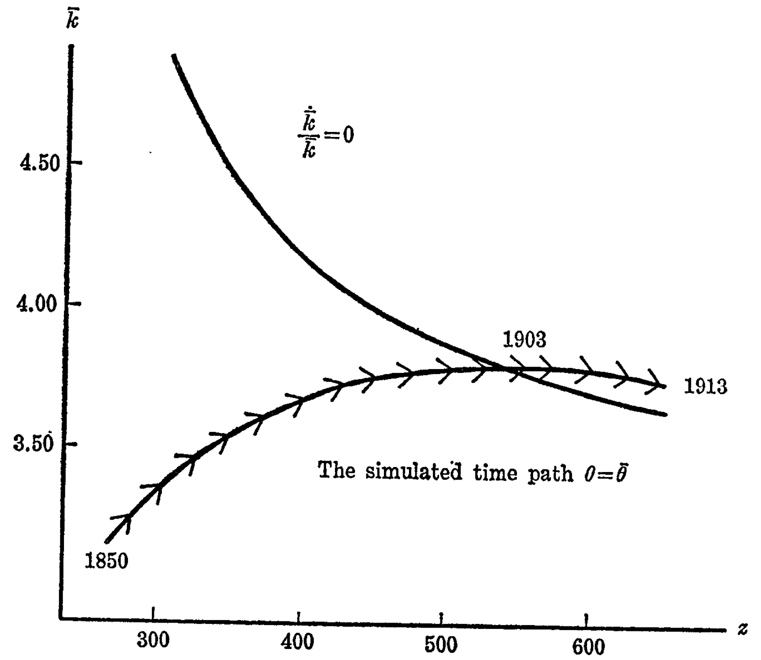

Fig. 5. The Phase Diagram for Germany

exist, indicating that a steady development path would be generated from any initial position.

IV. IMPACT OF AGRICULTURAL CAPITAL ON ECONOMIC DEVELOPMENT: A COUNTERFACTUAL ANALYSIS

We now examine the impact of agricultural capital on the growth of the Germany economy by a counterfactual analysis. Namely, we solve for the time paths of z, $\bar{k}$ and s that would have existed if capital were not included in the agricultural production function. As has been noted previously, without agricultural capital our model is reduced to the conventional model (by Sato-Niho). Hence, we derive the time paths of z, $\bar{k}$ and s implies by the conventional model by simulating[19]

$$\text{(7)} \qquad \frac{\dot{z}}{z} = \frac{\alpha - (1-\delta)\phi(z)}{\frac{v'(z)z}{v(z)} + \delta[1 - (1-C)s(z)]\left(1 - \frac{v'(z)z}{v(z)}\right)}$$

$$\text{(8)} \qquad \frac{\dot{\bar{k}}}{\bar{k}} = B_0\rho\bar{k}^{\gamma-1} + \frac{\dot{s}}{1-s} - \phi(z) - \frac{\lambda}{1-\gamma}$$

and

$$s = \frac{v(z)}{v(z) + C(z - v(z))}\,.$$

(For the derivation of these equations, see Sato and Niho (1971)). We have used $\delta = .746$ (the same figures as in Part III) as the production elasticity of agricultural labor, and $1 - \delta = .254$ (the sum of the production elasticities of land and capital in agriculture) as the production elasticity of land.

Without agricultural capital, the maximum sustainable rate of population growth is determined by $\alpha/(1-\delta)$. With the average rate of technological progress in agriculture $\alpha = .95$ percent per annum, the maximum sustainable rate $\alpha/(1-\delta)$ still would have been above the average growth rate of the actual population. Thus, had capital not been employed in the agricultural production, the Germany economy still would have shown steady growth of per capita income. However, the main impact would have been a much slower growth of per capita income and a retarded industrialization. As shown in Table I, had capital not been employed in the agricultural production function, the growth of per capita income would have been only 49 percent of the actual growth during the period, and the decline in the proportion of agricultural workers to the total labor force would have been only 65 percent of what actually occurred.

Our analysis reveals that the inclusion of agricultural capital in economic develop-

[19] If capital is not included in the agricultural production function, the system becomes decomposable; the time path of z is determined by equation (7) alone; then, given the time path of z, equation (8) determines the time path of $\bar{k}$.

TABLE I. COMPARISON OF THE ACTUAL AND THE COUNTERFACTUAL DEVELOPMENT (Five-Year Average)

Year	Proportion of Agricultural Workers (percent)		Per Capita Income (1913 mark)		Overall Capital Intensity in Efficiency Units (thousands of 1913 mark)	
	Actual	Counter-factual	Actual	Counter-factual	Actual	Counter-factual
1850–1854	55.1	58.5	267.0	267.6	3.254	3.157
1865–1869	51.4	50.3	340.1	353.2	3.230	2.760
1880–1884	48.3	52.2	409.2	328.9	3.548	3.613
1895–1899	40.2	44.9	552.7	433.1	3.604	3.436
1910–1913	35.2	42.6	692.8	476.4	3.700	3.964

ment not only raises the maximum sustainable rate of population growth, thereby enhancing the possibility of steady development, but also accelerates the growth of per capita income and industrialization. This second role of agricultural capital elucidates the importance of capital investment in agriculture, especially for development planning. For, from the planner's viewpoint, very slow development is almost as undesirable as the impossibility of development.

V. SUMMARY AND CONCLUSION

The main findings in the applications of our model to Germany during 1850–1913 are as follows:

(1) Both the Malthusian effect on population growth and Engel's effect on per capita food consumption were at work.

(2) The production elasticities in both the agricultural and industrial sectors remained more or less constant, validating the assumptions of the Cobb-Douglas production functions for both sectors.

(3) Our model was able to explain the developmental behavior of the German economy very well.

(4) The successful development of the German economy during this period is attributable to: (i) the moderate population growth, 1.06 percent as the average annual growth rate; (ii) the sufficiently high rate of technical progress, $\alpha + \beta \cdot \lambda/(1-\gamma) = 1.21$ percent as the weighted sum of the two sectors, and (iii) the small production elasticity of land which minimizes the forces of diminishing returns to labor caused by population growth. These three factors make the maximum sustainable rate of population growth so high that the point of stagnation does not exist in the phase diagram, guaranteeing that a steady development path would be generated from any initial position.

In addition, our counterfactual analysis suggests that the introduction of capital into the agricultural production function is important not only for raising the society's maximum sustainable rate of population growth, but also for accelerating the growth of per capita income and industrialization. Had capital not been employed in agricultural production, the growth of per capita income would have been less than 50 percent of what was actually attained. This points to the importance of capital investment in agriculture for development planning.

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APPENDIX

I. *The Full Mathematical Model*

Notations. (Those defined in Part II are omitted.)

Y_a = agricultural output

L_a = labor input in agriculture

K_a = capital input in agriculture

L = the total labor force

N = the amount of arable land

w_a = the agricultural wage rate in terms of industrial goods

r_a = the return to capital in agriculture in terms of industrial goods

q = the terms of trade (the price of agricultural commodities in terms of industrial goods)

Y_m = industrial output

L_m = labor input in industry

K_m = capital input in industry

K = the total capital stock

P = the total population

w_m = the industrial wage rate

r_m = the return to capital in industry

S = the total savings

θ = the participation ratio

Agricultural Production Function:

(A1) $$Y_a = A_0 e^{\alpha t} K_a^{\beta} L_a^{\delta}\,, \qquad \beta + \delta < 1$$

Industrial Production Function:

(A2) $$Y_m = B_0 e^{\lambda t} K_m^{\gamma} L_m^{1-\gamma}$$

Determination of the Agricultural Real Wage Rate:

(A3) $$\frac{w_a}{q} = \xi \frac{Y_a}{L_a}$$

Determination of the Industrial Real Wage Rate:

(A4) $$w_m = (1-\gamma)\frac{Y_m}{L_m}$$

Determination of the Real Return to Capital in Agriculture:

(A5) $$\frac{r_a}{q} = \beta \frac{Y_a}{K_a}$$

Determination of the Real Return to Capital in Industry:

(A6) $$r_m = \gamma \frac{Y_m}{K_m}$$

Distribution of Capital Between Two Sectors:

(A7) $$K = K_a + K_m$$

Equality of the Returns to Capital in Two Sectors:

(A8) $$r_a = r_m$$

Relationship Between the Two Money Wages:

(A9) $$w_a = \mu w_m$$

Supply of the Total Labor Force:

(A10) $$L = \theta P$$

Distribution of the Total Labor Force Between Two Sectors:

(A11) $$L = L_a + L_m$$

Definition of Per Capita Income:

(A12) $$z = \frac{Y_a + Y_m/q}{P}$$

Population Growth Functions:

(A13) $$\frac{\dot{P}}{P} = \phi(z),\ \phi'(z) > 0$$

Definition of Per Capita Food Consumption:

(A14) $$v = \frac{Y_a}{p}$$

Engel Function:

(A15) $$v = v(z)\,, \qquad 0 < v'(z) < 1\,, \qquad v''(z) < 0$$

Determination of Savings:[20]

(A16) $$S = \rho[Y_m + qY_a]$$

Determination of Capital Accumulation:[21]

(A17) $$\dot{K} = S$$

[20] With this savings function, per capita consumption expressed in agricultural units becomes $(1 - \rho)[(Y_m/q)/P + v]$. Hence, the marginal propensity to consume agricultural output, $(1 - \rho)v'(z)$, diminishes with per capita income. This, of course, does not imply that the marginal propensity to save agricultural output increases with per capita income. I owe this to the referee of this Journal.

[21] With depreciation ratio, η, the equation for capital accumulation becomes $\dot{K} + \eta K = S$. As has been noted previously, since data for the depreciation ratio is not available, we use (A-17) instead. Hence, S represents the net savings ratio.

II. *Source of Data*

The source of data for raw variables are as follows:

P: Average population in Hoffmann (Table 1).

L: Total employment in Hoffmann (Table 20).

L_a: Employment in primary industry in Hoffmann (Table 20).[22]

L_m: Computed from $L - L_a$.

K: Total capital stock excluding land in Hoffmann (Table 39).

K_a: Agricultural capital stock exluding land in Hoffmann (Table 39).

K_m: Computed from $K - K_a$.

N: Agricultural land in Hoffmann (Table 30) deflated by the price index with 1913 price = 1.00. The price index is computed from Tables 39 and 40 in Hoffmann.

$Y_a + Y_m/q$: Total value added (in 1913 prices) in Hoffmann (Table 103).

Y_a: Value added (in 1913 prices) in primary industry in Hoffmann (Table 103).

Y_m/q: Computed from $(Y_a + Y_m/q) - Y_a$.

Ratios are computed according to the respective definitions, and growth rates are computed as:[23]

$$\frac{\dot{x}}{x} = \frac{x(t+1) - x(t)}{\frac{1}{2}[x(t) + x(t+1)]}$$

$\bar{k}_{\text{obs}}$ is computed as:

$$\frac{\bar{k}(0)(2 + \dot{\bar{k}}/\bar{k}_{\text{obs}})}{2 - \dot{\bar{k}}/\bar{k}_{\text{obs}}}$$

with $\bar{k}(0) = k(0) =$ the observed overall capital intensity in the intial period.

[22] Data for L_a may not include labor inputs of farm household members. Since farm household members constituted an important component of labor input in the agricutlural production of Germany, their exclusion from data would imply a rather serious misrepresentation of the agricultural production function that actually existed. However, since data for L constitutes the sum of L_a and employment in secondary and tertiary industries, the exclusion of farm household members from L_a does not imply that they are included in L_m. I owe this to the referee of this Journal.

[23] The growth rate computed from this formula is smaller than the growth rate computed from $[x(t+1) - x(t)]/x(t)$.

TABLE AI. GROWTH RATE OF THE POPULATION, LEVEL OF PER CAPITA CONSUMPTION OF AGRICULTURAL PRODUCTS, RELATIVE SHARES OF CAPITAL'S INCOME, RATES OF TECHNOLOGICAL PROGRESS, AND NET SAVINGS RATIO IN GERMANY
(Five-Year Average)

Year	$\dot{p}/p$ (Annual %)	v (1913 mark)	γ	β	α (per annum %)	λ (per annum %)	ρ (%)
1850–1854	.46	120.9	.213	.271	−1.14	.09	8.3
1855–1859	.80	128.4	.209	.260	4.25	.82	9.6
1860–1864	1.01	141.8	.187	.219	1.41	2.22	12.6
1865–1869	.63	143.1	.179	.209	− .62	.06	11.5
1870–1874	.82	144.2	.186	.226	1.40	3.47	15.1
1875–1879	1.17	149.5	.158	.172	−1.21	−1.42	12.4
1880–1884	.70	148.3	.155	.152	2.75	.86	13.0
1885–1889	1.06	159.5	.160	.145	.54	.77	15.3
1890–1894	1.09	158.2	.167	.153	1.52	2.00	14.9
1895–1899	1.50	169.6	.193	.163	2.74	.72	19.0
1900–1904	1.47	170.7	.218	.174	0.4	.82	16.3
1905–1905	1.36	166.0	.225	.186	− .42	1.53	17.3
1910–1913	1.22	162.4	.228	.199	1.12	2.05	18.4

Note: For the computations of z and v, see Appendix I and II. For γ, β, α, λ and ρ, see Part III of the main text.

TABLE AII. GROWTH RATES OF OVERALL CAPITAL INTENSITY AND EFFICIENCY OF INDUSTRIAL LABOR, OBSERVED AND PREDICTED GROWTH RATES OF PER CAPITA INCOME ANO OVERALL CAPITAL INTENSITY IN EFFICENCY UNITS
(Five-Year Average)

Year	$\dot{k}/k$	$\lambda/(1-\gamma)$	$\dot{z}/z_{\text{obs}}$	$\dot{z}/z_{\text{pred}}$	$\dot{\bar{k}}/\bar{k}_{\text{obs}}$	$\dot{\bar{k}}/\bar{k}_{\text{pred}}$
1850–1854	1.10	.11	− .03	−1.11	.99	.95
1855–1859	.91	1.02	2.83	5.17	− .11	− .21
1860–1864	2.32	2.74	1.57	1.98	− .42	−1.04
1865–1869	.93	.07	.84	− .43	.87	1.38
1870–1874	1.64	4.29	3.57	2.05	−2.65	−2.04
1875–1879	1.43	−1.75	−1.15	−1.08	3.18	3.29
1880–1884	1.50	1.06	2.18	3.39	.44	.40
1885–1889	1.21	.95	1.83	.94	.26	.81
1890–1894	1.90	2.46	2.07	2.00	− .56	− .84
1895–1899	1.89	.89	2.16	3.51	1.00	1.47
1900–1904	1.77	1.01	.82	.35	.76	.67
1905–1909	1.58	1.88	1.53	− .15	− .30	− .04
1910–1913	1.67	2.53	2.79	1.62	− .80	− .44

Note: 1. The figures are in annual percentage rate.
2. For the computation of $\dot{k}/k$ see Appendix I and II. For $\lambda/(1-\gamma)$ see Part III of the main text.

TABLE AIII. COMPARISON OF OBSERVED, SIMULATED AND COUNTERFACTUAL FIGURES (Five-Year Average)

Year	s_{obs}	s_{pred}	$s_{c.f.}$	$\bar{k}_{obs}$	$\bar{k}_{sim}$ $\theta=\theta(t)$	$\bar{k}_{c.f.}$ $\theta=\theta(t)$
1850–1854	55.1	58.6	58.5	3.254	3.267	3.157
1855–1859	53.5	56.1	61.2	3.241	3.242	3.576
1860–1864	51.7	53.5	52.7	3.299	3.286	2.977
1865–1869	51.4	51.4	50.3	3.230	3.171	2.760
1870–1874	49.3	48.5	51.9	3.070	3.098	3.148
1875–1879	49.2	46.5	49.7	3.147	3.301	2.834
1880–1884	48.3	46.4	52.3	3.548	3.620	3.613
1885–1889	45.5	44.0	47.6	3.584	3.784	3.442
1890–1894	42.5	41.8	47.1	3.608	3.810	3.646
1895–1899	40.2	39.1	44.9	3.604	3.819	3.436
1900–1904	38.0	37.7	41.0	3.768	3.984	3.567
1905–1909	35.8	36.0	41.3	3.809	4.028	3.811
1910–1913	35.2	34.3	42.6	3.700	3.955	3.964

Note: s is in percent. $\bar{k}$ is in thousands of 1913 mark.

(TABLE AIII Continued)

Year	$\bar{k}_{sim}$ $\theta=\bar{\theta}$	$\bar{k}_{c.f.}$ $\theta=\bar{\theta}$	z_{obs}	z_{sim} $\theta=\theta(t)$	$z_{c.f.}$ $\theta=\theta(t)$	z_{sim} $\theta=\bar{\theta}$	$z_{c.f}$ $\theta=\bar{\theta}$
1850–1854	3.217	3.157	267.0	263.9	267.6	276.3	267.6
1855–1859	3.353	3.267	288.9	291.9	245.4	298.7	282.8
1860–1864	3.467	3.364	315.9	343.2	325.2	322.0	298.5
1865–1869	3.560	3.451	340.1	361.6	353.2	346.4	314.6
1870–1874	3.635	3.526	379.2	366.0	333.9	372.0	331.1
1875–1879	3.694	3.592	407.2	383.2	361.8	398.9	348.1
1880–1884	3.738	3.648	409.2	393.8	328.9	427.0	365.6
1885–1889	3.771	3.696	450.6	447.3	390.1	456.6	383.5
1890–1894	3.792	3.736	492.5	475.9	397.9	487.8	401.9
1895–1899	3.804	3.769	552.7	549.9	433.1	520.6	420.8
1900–1904	3.809	3.795	588.8	595.2	509.5	555.2	440.3
1905–1909	3.807	3.816	637.1	617.2	501.1	591.6	460.3
1910–1913	3.801	3.829	692.8	610.7	476.4	626.1	476.6

Note: z is in 1913 mark.

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