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FIXED INVESTMENT AND THE COST OF RAISING FUNDS

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This paper attempts to make clear the interdependent mechanism determining the optimal level of fixed investment, the amount of funds by type to be raised and the marginal internal rate of return on investment, which is equal to the marginal cost of raising the composite fund. For this purpose, the composite cost-of-raising-funds schedule has been deduced by combining the marginal cost of raising fund schedules by type. The conditions for existence and stability of the equilibrium point has also been derived explicitly. The purpose of this paper is not a generalization of the theory of investment behavior, but a preliminary work for establishing an empirical hypothesis of the firm's behavior of fixed investment, which will be presented in a sequel.

I. INTRODUCTION

For these decades, the microeconomic theory of investment behavior has been developed intensively in close relation to empirical studies in this field. After Keynes' proposal of marginal efficiency of investment (or capital), Klein (1947) ingeniously presented fixed investment schedule in terms of Keynesian marginal efficiency concept. Despite this explicit specification, many empirical studies of investment behavior have been developed with their emphasis on intuitive formulations, rather than on explicit presentation of rational behavior of the firm.

Jorgenson (1967) has presented powerful criticism on these intuitive models of investment behavior developed in the field of econometric studies, and himself presented a theory of investment behavior based on the neoclassical theory of capital accumulation in the form of comparative dynamics of fixed investment behavior. However, he has neglected interdependency between internal rate of return on investment and costs of capital, particularly cost of raising funds, assuming the latter as given. In reality, we have various types of funds supplied to the firm through financial markets, and the optimal investment for the firm could never be determined without taking into account changes in the cost of raising

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funds including not only cash payments such as interests, clerical and other arrangement fees, but also imputed cost for the top management of the firm, in view of the tendency of separability of management and ownership of capital of modern corporate businesses in many developed countries.

Modigliani and Miller (1958) have proposed a new approach to the theory of investment behavior, and they have arrived at two important propositions.

The first is that the market value of a firm is independent of its capital structure, and can be obtained by capitalizing its expected return with the expected rate of return, common to the homogeneous group to which it belongs, and the second is that its expected return per share is equal to the sum of its expected rate of return on capital and premium relating to its financial risk.

The first proposition seems to be very important, in the context of the present paper: that is, the cost of capital to the firm is not affected by its financing policy at all. M–M (1963) have made an exception for the case of tax-exemptions of interest payments on external debts.

To arrive at these two propositions, M–M have made three basic assumptions: (i) it is possible to decompose all the firms into some "equilavent return" classes, (ii) any type of bond does make the same amount of yield per unit of time, and this yield is of certainty to all, individual or firm, and (iii) bond market is under perfect competition in the sense that the law of price-equality for one good holds everywhere and at anytime.

It should be noted that M–M have neglected an important effect on firms' behavior of the "separability" of the capital ownership and management in modern corporate business. In other words, M–M removed from consideration the role of the manager who should be responsible for holding stock price stable. Furthermore, they make a tacit use of the assumption of constant returns to scale within the group of homogeneous firms, and this seems to be indispensable for the first proposition to hold. However, if the expected rate of return on capital, common to "homogeneous" firms, is assumed to be constant, the independence of the cost of capital from financing policy of the firm may be meaningless from the viewpoint of the size of the firm. In reality, it may be the size of the firm that the cost of capital should be analyzed in its relation to financing policy.

It follows, from the considerations above, that (i) M–M neglect the reaction of the manager to changes in the stock price of his firm, caused by a change of the leverage ratio, and consequently by that of its net worth; (ii) the manager of firm will make adjustments of the size of the firm so as to retain its stock price at the level before a change in leverage ratio had taken place. This means that there is no "homogeneous" firm as defined by M–M. But, the first proposition by M–M could hold in the situation in which the expected rate of return on capital to industry k, ρ_k is to be realized when the leverage ratios of all the firms in industry k is the same with each other. In general, however, the expected rate of return on capital is to be determined in relation to the leverage ratio through financing policy of the firm simultaneously.

After M–M (1958), a large volume of literature, intending to extend the M–M theory, have appeared: for example, M–M (1961), Lintner (1962), Bierman and West (1966), Elton and Gruber (1968), Bierman (1970) and so forth have analyzed the effect of share repurchase or share-arbitration on the value of firm; and M–M (1963), Merrett and Sykes (1966), Baumol and Malkiel (1967), Lewellen (1969), and Davenport (1971) have considered tax effects on the M–M theory from theoretical and/or empirical view-points. However, as far as the effect of reaction by the manager of firm on the share-holders' arbitration is concerned, no remarkable discussions have been presented.

Duesenberry (1958) has developed cost of raising funds schedule which determines optimal volume of each fund from inside and outside the firm, internal rate of return on investment, and optimal level of investment simultaneously, in combination with marginal efficiency of investment schedule. His concept of the cost of raising funds includes those for internal funds (opportunity costs and imputed cost), borrowings from outside (interest payments, clerical and other arrangement fees, and imputed cost) and stock & bond issuings. Needless to say, it is characterized as an increasing function of the sum of funds to be raised, reflecting increasing imputed cost. This idea is certainly based on the existense of various sorts of pressure to the top management caused by falling ratio of dividends to profits, increasing new debts in the form of borrowings, and stock & bond issuings. It should also be noted that this type of cost is clearly one to the top management of the firm, but not to the owners of it. One difficult point is that Duesenberry's costs of raising funds schedule is not of marginal, but of average character so that it turns out to be rather ambiguous whether the intersection or equality of cost of raising funds schedule and marginal rate of return on investment schedule determines the equilibrium of the firm or not.

In this paper, the author will present a composite marginal cost of raising funds schedule and analyze its relation with marginal rate of return on investment schedule, which leads to the simultaneous determination of the volume of each fund to be raised, internal rate of return on investment, and optimal level of investment.

II. THEORETICAL FRAMEWORK

The modern top management of the firm may be considered to maximize the market value of the firm defined as the sum of the present values of expected net revenues which will be produced by the new fixed investment in cooperation with the initial stock of fixed capital. Given the initial stock of fixed capital and technological conditions for production activities, he estimates alternative investment projects, to discover one of them as the optimal investment project. Of course, he has to configurate, subjectively, the time-paths of prices of output, of material inputs, and of fixed investment goods; money wage rate; money interest rates;

and so forth, expected at the beginning of the planning period.

Since, in reality, these prices move from time to time, he has to redesign the investment project already in progress, fully or partly, in each (historical) time period.

He has also to make a plan to raise various types of long-term funds required for purchasing investment goods. He estimates costs of raising internal and external funds in cash and in kind. Costs of raising funds in cash may include payments for interests and other clerical arrangements, which could be marginally declining, reflecting economies of scale in debt-financing. Cost of raising funds in kind, as pointed out by Duesenberry, however, do not decline marginally, but it rather increases, as the amount of each fund increases.[1] This type of cost, called "imputed" cost, seems to play a very important role in determining the optimal size of investment.

In reality, we have various types of external funds and the magnitude of imputed raising cost for the same amount of each external fund differs from each other, reflecting institutional and/or customary factors such as conditions of repayment or clearing-off, power of claimants by type of credit, the historical length of time in transactions of the firm with its creditors in the past, and so forth. This is one of the reasons why the composition of funds to be raised should be made clear explicitly. Moreover, this inevitably leads to an explicit introduction of the flow-balance of uses and sources of funds for investment expenditures, as a constraint for maximization of the sum of present values of net revenues expected during the planning periods. Taking account of these factors, we may formulate fixed investment behavior of modern top management of the firm in the simplest form as follows.

Modern top management's behavior of fixed investment is supposed to be based on six basic assumptions below:

(i) The entrepreneur, given the technological conditions, maximizes the sum of the present values of net revenues expected during the periods for an investment project.

(ii) The optimal size of an investment project is determined and realized, by top management of the firm, at the beginning of the planning period.

(iii) The stationary expectation hypothesis is adopted for the expected time paths of the price of output and those of inputs of productive factors during the planning horizon.[2]

(iv) The funds for fixed investment, raised from outside is classified into three portions: borrowings from private financial intermediaries, issuings of corporate bonds, and the "Others".[3]

[1] See J. S. Duesenberry (1958), pp. 87–99.

[2] It is possible to assume that for a further extension, these prices will move at certain exponential rate respectively.

[3] The "Others" includes stock-issuings, trade-credit received, net of advanced payments, and it may be quite easily to develop to the case of m different types of funds, without loss of generality.

(v) External funds is repaid, once and for all, at the end of the planning period.

(vi) The cost of raising fund of each type increases as its amount increases, reflecting increasing imputed cost.

Under the assumptions above, a hypothesis is presented to approximate the mechanism determining the optimal size of fixed investment and the amount of each external fund simultaneously. Let us define T as the length of planning horizon; $p^*(t, 1)$, $w(t, 1)$ and $q(t, 1)$ as net price of output, wage rate and prices of fixed investment goods expected at the beginning-of-period t respectively; $X(t, \tau)$, $L(t, \tau)$, and $p^*(t, 1)C(t, \tau)$ as amount of output, that of labor inputs, and costs of raising funds, evaluated in terms of product price, in the τth period expected at the beginning-of-period t; and r_t as a discount rate at time t. Then, the sum of the present values of net revenues, during T periods, expected at the beginning-of-period t, $\pi(T)$, is:

$$(1)\quad \pi(T) = \sum_{\tau=1}^{T} \{p^*(t, 1)X(t, \tau) - w(t, 1)L(t, \tau) - p^*(t, 1)C(t, \tau)\}(1+r_t)^{-\tau} - q(t, 1)I(t, 1),$$

where fixed investment $I(t, 1)$ is to be realized at the beginning-of-period t, and this is why investment expenditures $q(t, 1)\, I(t, 1)$ is not discounted at all. Of course, since part of the expense is composed of costs of raising funds such as interest payments that are to be paid in each period over the planning horizon, the sum of expenditures for appropriation of the investment goods is:

$$p^*(t, 1) \sum_{\tau=1}^{T} C(t, \tau)(1+r_t)^{-\tau} + q(t, 1)I(t, 1).$$

Production function is written as below:

$$(2)\qquad X(t, \tau) = G[L(t, \tau), \gamma I(t, 1) + K(t-1)], \quad \tau = 1, \ldots, T,$$

where $K(t-1)$ is stock in fixed capital at the beginning-of-period t, and γ is a constant, given subjectively by an entrepreneur, as a measure of the rate of productive efficiency in which investment goods newly acquired, come into operation during T periods. If $\gamma = 1$, the investment $I(t, 1)$ should have its 100 percent efficiency continuously through the planning periods. In reality, however, γ may vary over the range of $[0, 1]$; that is, it would rise at first, arrive at unity and then, go down gradually, but for the simplicity's sake, the entrepreneur is assumed to take γ as a constant on the average during the planning periods.

By assumption (ii), $X(t, \tau)$ and $L(t, \tau)$ are determined at time t and their levels remain unchanged throughout the planning periods, simply because, given the price of output $p^*(t, 1)$ and that of fixed investment goods $q(t, 1)$, and wage rate $w(t, 1)$, investment $I(t, 1)$ is determined and realized at the beginning-of-period of the planning horizon, so that the optimal levels of $X(t, \tau)$ and $L(t, \tau)$ are uniquely determined and fixed at the beginning-of-period t, so as to maximize the profit $\pi(T)$ subject to constraints (2) and an identity saying investment expenditures is equal to the sum of amounts of funds raised both from inside and outside the firm.

Production function (2) is a continuous function with continuous partial derivatives of the first and the second order. To satisfy the necessary and sufficient conditions for a maximum of $\pi(T)$, the signs of these derivatives should be, neglecting the subscripts, as below:

$$(3)\quad \frac{\partial X}{\partial L} > 0,\ \frac{\partial X}{\partial I} > 0,\ \frac{\partial^2 X}{\partial L^2} < 0,\ \frac{\partial^2 X}{\partial I^2} < 0,\ \frac{\partial^2 X}{\partial L \partial I} = \frac{\partial^2 X}{\partial I \partial L} > 0.$$

The costs of raising funds is assumed to include not only cash payments for interests and clerical arrangements, but also imputed costs for disutility of top management of the firm such as psychological pressures caused from various sorts of meddlings by the claiments. Marginal costs of raising funds in cash might be decreasing as the sum of funds increases because of economies of scale with respect to the size of funds to be raised.[4] Marginal imputed costs of raising each type of external fund could, however, be assumed to be an increasing function of its amount. Furthermore, the increasing imputed costs will offset economies of scale in costs of interests and clerical arrangements, so that marginal costs of raising funds, as the sum of costs in cash and otherwise, will increase as the amount of funds increases.

Let us define $BRW(t, 1)$, $BND(t, 1)$, and $OTH(t, 1)$ as funds to be raised in the form of borrowings from private financial intermediaries, bond-issuings, and the "other" external debts planned respectively at the beginning-of-period t, $F(t, 1)$ as internal funds available to top management of the firm at the beginning-of-period t, $RL(t)$ as interest rate for borrowings, $RB(t)$ as that of bonds to be issued newly, $RO(t)$ as that of the "other" external funds, and $RF(t)$ as rate of opportunity costs for internal funds to be raised. Then, total costs of raising funds at the beginning of the τth period of the planning periods starting at period t, $C(t, \tau)$, is written as below:

$$(4)\quad C(t, \tau) = C[RL(t), RB(t), RO(t), RF(t), BRW(t, 1), BND(t, 1), OTH(t, 1), F(t, 1), \tau]$$

$$\text{or} \quad = C[RL(t), RB(t), BRW(t, 1), BND(t, 1), \tau] + C_O,$$

where C_O is the sum of costs of raising the "other" external funds and opportunity costs of internal funds.[5,6] By choosing the second expression of equation (4), it will become easier to make clear how the interrelation between costs of raising

[4] See E. A. G. Robinson (1931), Chapt. VI.

[5] The amount of internal funds to be raised is assumed to be predetermined, but it is easily to go to the more general case in which the amount of internal funds is also assumed to be endogenously determined, following Duesenberry. See Duesenberry (1958), pp. 94–95.

[6] If it is assumed that repayment or pay-back is proportional to the debt payable in each period, the residual at the end of the τ th period is $(1 - \theta_1)^\tau BRW(t, 1)$ for borrowings, and $(1 - \theta_2)^\tau BND(t, 1)$ for bonds respectively, where constants θ_1 and θ_2 are ratios of pay-back to borrowings and bonds respectively. If we take the assumption above, it may not be very important whether to take assumption (v) or not.

funds schedule and internal rate of return on investment works so as to determine the amount of fixed investment and that of funds to be raised respectively.

As will be seen later, to satisfy the necessary and sufficient conditions of maximization, the signs of derivatives in (4) should be as follows:

$$(5)\quad \frac{\partial C}{\partial RL} = C_{RL} > 0,\ \frac{\partial C}{\partial RB} = C_{RB} > 0,\ \frac{\partial C}{\partial BRW} = C_1 > 0,\ \frac{\partial C}{\partial BND} = C_2 > 0,$$

$$\frac{\partial^2 C}{\partial BRW^2} = C_{11} > 0,\ \frac{\partial^2 C}{\partial BND^2} = C_{22} > 0,$$

$$\frac{\partial^2 C}{\partial BRW\,\partial BND} = C_{12} = \frac{\partial^2 C}{\partial BND\,\partial BRW} = C_{21} = 0,$$

where the two marginal costs of raising funds are assumed to be independent of each other, for the simplicity's sake. This assumption of 'separability' seems to be very useful in approximating costs of raising funds schedule. Needless to say, these signs indicate marginal costs of raising funds are all positive and increasing as the amount of each fund to be raised increases.

In equation (4), the effects of external debts already received on the cost of raising funds are not taken into account at all. They may have dual effects on the cost of raising funds: the one is positive, reflecting increasing imputed costs for the higher level of debts at the beginning-of-period t, and the other is negative, reflecting increasing customership or partnership between debtor and creditor in financial transactions. Let us define $SBRW(t-1)$, and $SBND(t-1)$ as the outstanding borrowings and bond-issuings at the beginning-of-period t respectively. Then the net effect of these two types of debts can be written as follows:

$$\frac{\partial^2 C}{\partial SBRW(t-1)\,\partial BRW(t,1)} \gtreqless 0;\quad \frac{\partial^2 C}{\partial SBND(t-1)\,\partial BND(t,1)} \gtreqless 0.$$

The signs of derivatives above should be established only through empirical studies, and this will be done in another paper.

Another constraint to the maximization of profit $\pi(T)$ is an identity saying that investment expenditures is equal to the sum of internal and external funds to be raised. This can be written as below:

$$(6)\quad \Omega = q(t,1)I(t,1) - F(t,1) - BRW(t,1) - BND(t,1) - OTH(t,1) = 0.$$

As a matter of fact, the "other" funds $OTH(t, 1)$ should be equal to the sum of other external funds (including new stock issues) minus advance-payments in the previous period for appropriation of investment goods in current period.

Thus, by assumption (i), top management of the firm is supposed to maximize the sum of the present values of net revenues during the planning horizon expected at the beginning-of-period t, subject to constraints (2) and (6). The function to be maximized under the constraint (6) can be written as below:

$$(7)\qquad \varphi = \pi(T) - \lambda\Omega,$$

where λ is a Lagrange's multiplier. The necessary conditions for the maximization are:

$$\frac{\partial \varphi}{\partial L} = \sum_{\tau=1}^{T} \left\{p^* \frac{\partial X}{\partial L} - w\right\}(1 + r_t)^{-\tau} = 0, \tag{8}$$

$$\frac{\partial \varphi}{\partial I} = \sum_{\tau=1}^{T} \left\{p^* \frac{\partial X}{\partial I}\right\}(1 + r_t)^{-\tau} - q(1 + \lambda) = 0, \tag{9}$$

$$\frac{\partial \varphi}{\partial BRW} = - p^* \sum_{\tau=1}^{T} \frac{\partial C}{\partial BRW}(1 + r_t)^{-\tau} + \lambda = 0, \tag{10}$$

$$\frac{\partial \varphi}{\partial BND} = - p^* \sum_{\tau=1}^{T} \frac{\partial C}{\partial BND}(1 + r_t)^{-\tau} + \lambda = 0, \tag{11}$$

$$\frac{\partial \varphi}{\partial \lambda} = - \Omega = 0, \tag{12}$$

As easily seen, λ is marginal internal rate of return on fixed investment expenditures qI. From (9), λ can be written as below:

$$\lambda = \sum_{\tau=1}^{T} \frac{p^*}{q} \frac{\partial X}{\partial I}(1 + r_t)^{-\tau} - 1 \tag{9)'$$

The optimal solutions for L, I, BRW, BND and λ have to satisfy these five equations above, most of which are nonlinear in relevant variables. To satisfy the sufficient conditions of maximization, the principal minors of bordered Hessian (13), denoted by H below, alternates in sign such that sign of $H_{ii} = (-1)^i$, where H_{ii} is the principal minor of order i.

$$H = \begin{vmatrix} \pi_{LL} & \pi_{LI} & \pi_{L\cdot BRW} & \pi_{L\cdot BND} & -\Omega_L \\ \pi_{IL} & \pi_{II} & \pi_{I\cdot BRW} & \pi_{I\cdot BND} & -\Omega_I \\ \pi_{BRW\cdot L} & \pi_{BRW\cdot I} & \pi_{BRWBRW} & \pi_{BRWBND} & -\Omega_{BRW} \\ \pi_{BND\cdot L} & \pi_{BND\cdot I} & \pi_{BNDBRW} & \pi_{BNDBND} & -\Omega_{BND} \\ -\Omega_L & -\Omega_I & -\Omega_{BRW} & -\Omega_{BND} & 0 \end{vmatrix} \tag{13}$$

$$= \begin{vmatrix} SG_{LL} & SG_{LI} & 0 & 0 & 0 \\ SG_{IL} & SG_{II} & 0 & 0 & -q \\ 0 & 0 & -SC_{11} & 0 & 1 \\ 0 & 0 & 0 & -SC_{22} & 1 \\ 0 & -q & 1 & 1 & 0 \end{vmatrix} > 0,$$

where,

$$\pi_{xy} = \frac{\partial^2 \pi}{\partial x \partial y}, \quad \Omega_x = \frac{\partial \Omega}{\partial x}, \quad S = p^* \sum_{\tau=1}^{T} (1 + r)^{-\tau}, \quad G_{xy} = \frac{\partial^2 G}{\partial x \partial y}$$

for $x, y = L, I$ and $C_{ij} = \partial^2 C / \partial i \partial j$ for $i, j = BRW, BND$.

From inequality (3) and (5),

$$G_{LL} < 0, \quad G_{II} < 0, \quad G_{LI} = G_{IL} > 0, \quad C_{11} > 0, \quad C_{22} > 0,$$

therefore, from the condition (13), a very important relationship can be deduced; that is,

$$\frac{1}{q^2}\left[G_{II} - \frac{G_{IL}^2}{G_{LL}}\right] < \frac{C_{11}C_{22}}{C_{11} + C_{22}}. \tag{14}$$

As will be shown later, the right hand side of this inequality is the slope of the composite curve of the marginal raising costs of *BRW* and *BND*, and the left hand side is that of the curve of the marginal internal rate of return on fixed investment expenditures qI, which is negative, equal to zero or positive, according as returns are decreasing, constant or increasing with respect to scale ($G_{II}G_{LL} - G_{IL}^2 \lesseqgtr 0$). This assures that an equilibrium point exists, which determines the optimal level of fixed investment, the optimal amount of borrowings, that of new issues of corporate bonds, and the marginal costs of raising funds that is equal to the marginal internal rate of return on fixed investment expenditures.

III. GRAPHICAL PRESENTATION

The mechanism determining the optimal solutions of the amounts of fixed investment, borrowings from private financial intermediaries, and bond-issuings can be shown more clearly by graphical presentation. Figure 1 shows the curves of marginal costs of raising funds on the assumption of separability ($C_{12} = 0$). The marginal cost of borrowings and bond-issuings are taken on the vertical axis and the amounts of funds, on the horizontal axis respectively.

Along the horizontal axis, $\overline{\text{OF}}$ is the sum of internal funds available to top management of the firm, and is assumed to be predetermined, the marginal cost of which is assumed to be also exogenous and shown as $\overline{\text{Of}_1}(= \overline{\text{Ff}_2})$.[7] $\overline{\text{FG}}$ is the sum of the "other" external funds to be raised, and its marginal raising costs is also given exogenously, and shown as $\overline{\text{Fg}_1}(=\overline{\text{Gg}_2})$ in the Figure. For instance, the effective stock holders might press the top management to issue new stocks in their debt-financing so as to devide part of the expected return on new investment to the stock holders. In Figure 1, the marginal raising costs $\overline{\text{Of}_1}$ and $\overline{\text{Fg}_1}$ is taken arbitrarily, so that those magnitudes should empirically be taken as datum.

Another vertical axis GG′ is drawn in as the measure for the costs of raising external funds. The curves aa′ and bb′ illustrate them. Let us call aa′ the curve for fund A, and bb′, the curve for fund B. We do not know, a priori, which of those two curves corresponds to the marginal cost of borrowings, (or the cost of bond-issuings). This may be one of those things that should be made clear through empirical studies following the theoretical considerations developped hereafter.

[7] This assumption is adopted only for the simplicity's sake, without loss of generality; that is, costs of raising internal funds can easily be dealt with just the same way as for those of external funds.

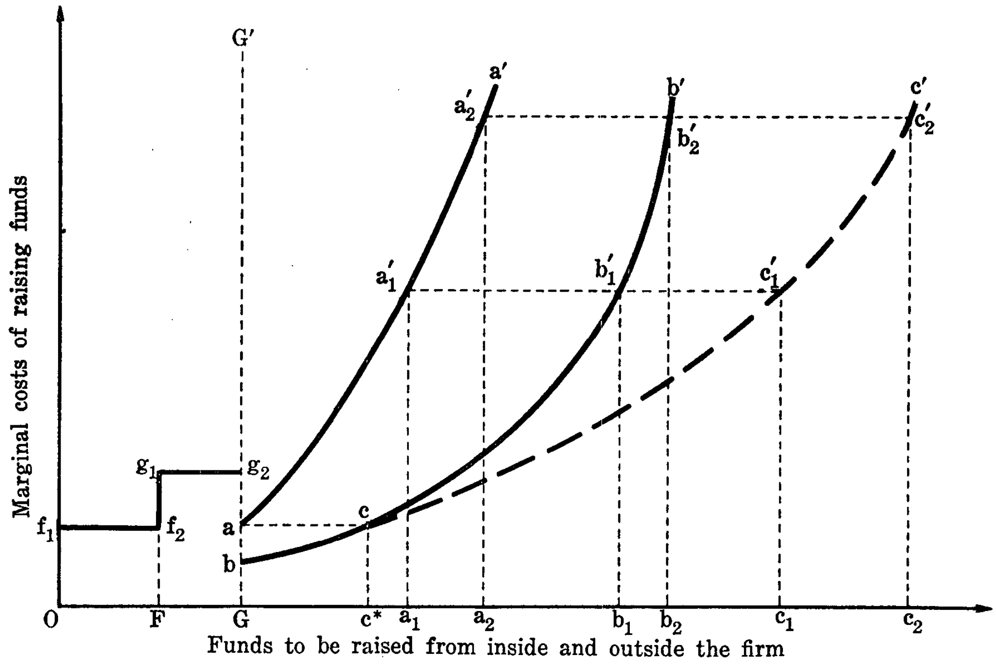

Fig. 1. The Composite Curve of Marginal Costs of Raising Funds

The curve cc′ shows the sum of external funds A and B to be raised at common marginal costs. For instance, $\overline{Ga}$ is the marginal cost of fund A at which the volume of fund A to be raised is unity, while $\overline{c^*c}$, which is of the same length as $\overline{Ga}$, is the marginal cost of fund B at which the volume of fund B to be raised is $\overline{Gc^*}$. Thus, the total of external funds that can be raised at marginal cost $\overline{Ga}(=\overline{c^*c})$ is $\overline{Gc^*}$, and the sum of funds to be raised from inside and outside the firm is $\overline{Oc^*}(=\overline{OF}+\overline{FG}+\overline{Gc^*})$, where the volume of fund A is zero. Next, $\overline{Ga_1}$ is the volume of fund A to be raised at the marginal cost of $\overline{a_1a_1'}$. This is identical to the marginal cost of $\overline{b_1b_1'}$ at which the volume of fund B can be raised by the amount of $\overline{Gb_1}$. Consequently, the total of external funds that can be raised at the marginal cost of $\overline{a_1a_1'}(=\overline{b_1b_1'}=\overline{c_1c_1'})$ is $\overline{Gc_1}$.[8] So, the final shape of the curve showing the total internal and external funds as against common marginal cost is the one going through the points ($of_1f_2g_1g_2bcc'$) in Figure 1. Let us call this the composite curve for the marginal cost of raising funds, or simply the composite curve.

The composite curve may shift as the curves f_1f_2, g_1g_2, aa′ and bb′ shift respectively or simultaneously. It is easy to point out factors making these curves shift,

[8] It should be noted that marginal costs $\overline{a_1a_1}$ or $\overline{c_1c_1'}$ is very different from actual effective costs of raising funds observed in that the former includes imputed costs, but the latter does not. It is also worthy to point out that the imputed costs diferentials would not be reflected in the actual interest rate differentials. So, the fact that funds with different interest rate are raised at the same time, could be taken this way; that is, though interest rates are different with each other, marginal costs of raising funds including the imputed costs is equal to each other.

and in equation (4), the only kind of shift variable introduced explicitly is interest rates. However, it should also be taken into account that the factors affecting psychological pressure of claimants on top management of the firm will make shift the composite curve. For example, in the postwar Japan, because of the dominance of indirect financing system, particularly through city and local banks, the downward rigidity of financial transactions between firm and banks is very important.[9] As is well known the debt-financing of Japanese busines firms has relied heavily on bank-borrowings for a long time in the past. And this tendency has produced a strong cooperation between the firm and the bank. From the viewpoint of the cost of raising funds schedule, this will have the following implications: the high degree of dependence on bank-borrowings in corporate financing is a result of a sort of 'Habit-Formation' as in the modern theory of consumer behavior,[10] and through its cumulative effects, the curve for marginal raising cost of borrowings from city banks shifts downwards. This will lead to a downward shift of the composite curve.

Next, the curve for the marginal rate of return on fixed investment can be drawn as curve $\lambda\lambda'$ in Figure 2. This curve can be derived by differentiating the necessary conditions (8) and (9) with respect to fixed investment expenditures qI, given the price of investment goods q:

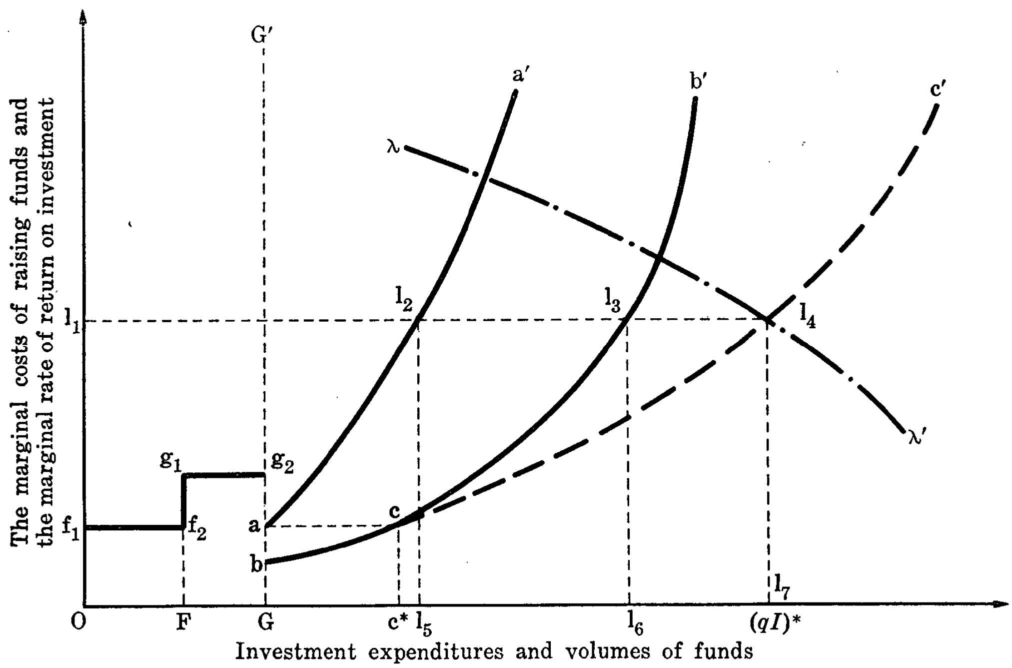

Fig. 2. The Optimal Level of Investment and the Demand for External Funds

[9] It is very suprising that Duesenberry has already pointed out the same fact in the United States as in Japan. See Duesenberry (1963), p. 11.

[10] See Tsujimura and Sato (1964).

$$\frac{d\lambda}{d(qI)} = \frac{S(G_{II}G_{LL} - G_{IL}^2)}{q^2 G_{LL}} \gtreqless 0.^{11} \tag{15}$$

Since it is assumed that production iso-quant curve is convex, the right hand side of equation (15) should be negative, positive or equal to zero, according to the assumptions of decreasing, increasing or constant return to scale. On the other hand, from (4) and (5), using simplified notation just shown above, the marginal costs of raising funds equations can be composed near at the equilibrium point as below:

$$\eta - \eta^* = SC_{11}(BRW - BRW^*)$$

$$\eta - \eta^* = SC_{22}(BND - BND^*),$$

where η is the marginal cost of raising funds composed of BRW and BND, and asterisks assign their equilibrium values. By solving these two equations with respect to $(BRW + BND)$, the equation for the composite curve can be written as below:

$$\eta = \frac{SC_{11}C_{22}}{C_{11} + C_{22}}[(BRW + BND) - (BRW^* + BND^*)] + \eta^*, \tag{16}$$

where the coefficient of $(BRW + BND)$ is, needless to say, the slope of the composite curve. From inequality (14), in combination with (15) and (16), the slope of the composite curve cc′ should be steeper than that of the marginal internal rate of return on fixed investment expenditures $(d\lambda/d(qI))$. In Figure 2, the intersection of the two curves cc′ and $\lambda\lambda'$ determines the equilibrium level of fixed investment expenditures $qI(=\overline{Ol_7})$, the optimal amounts of external funds $A(=\overline{Gl_6} = \overline{l_6l_7})$ and $B(=\overline{Gl_6})$ to be raised, and the marginal internal rate of return on fixed investment $(\overline{l_4l_7})$ simultaneously. As already pointed out, from (14), (15) and (16), this equilibrium point l_4 is assured to be stable.

IV. FACTORS AFFECTING THE EQUILIBRIUM POINT

The equilibrium point for fixed investment, external funds to be raised, labor inputs, and internal rate of return on investment λ, may shift through changes in money wage rate, the price of investment goods, netprice of output, the amount of internal funds available, interest rates for borrowings and bonds to be issued, and so forth. To see the influences of these factors, a conventional way is to differentiate equations (8)–(12) with respect to these predetermined variables respectively. Here, as in Sections 2 and 3, the assumption of separability between the two marginal costs of raising funds, $C_{12} = C_{21} = 0$, is adopted for the first approximation.

(A) Influences of Money Wage Rate

To see the influences of money wage rate on fixed investment, labor inputs,

[11] See Appendix B to this paper.

borrowings, bond-issuings and the internal rate of return on fixed investment, differentiate equations (8)–(12) with respect to money wage rate w, then we have,

$$SG_{LL}\frac{\partial L}{\partial w} + SG_{LI}\frac{\partial I}{\partial w} - \zeta(T) = 0 \tag{17}$$

$$SG_{IL}\frac{\partial L}{\partial w} + SG_{II}\frac{\partial I}{\partial w} - q\frac{\partial \lambda}{\partial w} = 0 \tag{18}$$

$$-SC_{11}\frac{\partial BRW}{\partial w} - SC_{12}\frac{\partial BND}{\partial w} + \frac{\partial \lambda}{\partial w} = 0 \tag{19}$$

$$-SC_{21}\frac{\partial BRW}{\partial w} - SC_{22}\frac{\partial BND}{\partial w} + \frac{\partial \lambda}{\partial w} = 0 \tag{20}$$

$$-q\frac{\partial I}{\partial w} + \frac{\partial BRW}{\partial w} + \frac{\partial BND}{\partial w} = 0 \tag{21}$$

where $\zeta(T) = \sum_{\tau=1}^{T} (1 + r)^{-\tau}$. Equations (17)–(21) can be solved with respect to $\partial L/\partial w$, $\partial I/\partial w$, $\partial BRW/\partial w$, $\partial BND/\partial w$ and $\partial \lambda/\partial w$, in matrix expression, as below:

$$\begin{pmatrix} \frac{\partial L}{\partial w} \\ \frac{\partial I}{\partial w} \\ \frac{\partial BRW}{\partial w} \\ \frac{\partial BND}{\partial w} \\ \frac{\partial \lambda}{\partial w} \end{pmatrix} = \phi^{-1} \begin{pmatrix} \zeta(T) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{22}$$

where,

$$\phi = \begin{bmatrix} SG_{LL} & SG_{LI} & 0 & 0 & 0 \\ SG_{IL} & SG_{II} & 0 & 0 & -q \\ 0 & 0 & -SC_{11} & -SC_{12} & 1 \\ 0 & 0 & -SC_{21} & -SC_{22} & 1 \\ 0 & -q & 1 & 1 & 0 \end{bmatrix}, \tag{23}$$

and the value of the determinant of matrix ϕ is identical to that of (13); that is,

$$|\phi| = S^3[(G_{II}G_{LL} - G_{IL}^2)(C_{11} + C_{22}) - q^2 G_{LL}C_{11}C_{22}] > 0,$$

where $C_{11} = C_{21} = 0$. The inverse of matrix ϕ is presented in Mathematical Appendix C. Making use of $|\phi|^{-1}$, the solution (22) can be written explicitly as below:

$$(24)\quad \begin{pmatrix} \dfrac{\partial L}{\partial w} \\ \dfrac{\partial I}{\partial w} \\ \dfrac{\partial BRW}{\partial w} \\ \dfrac{\partial BND}{\partial w} \\ \dfrac{\partial \lambda}{\partial w} \end{pmatrix} = |\phi|^{-1} \begin{pmatrix} \zeta S^2\{G_{II}(C_{11} + C_{22}) - q^2 C_{11}C_{22}\} \\ -\zeta S^2 G_{IL}(C_{11} + C_{22}) \\ -\zeta S^2 q G_{IL} C_{22} \\ -\zeta S^2 q G_{IL} C_{11} \\ -\zeta S^3 q G_{IL} C_{11} C_{22} \end{pmatrix} < \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

The reason why the effects of money wage rate on fixed investment is negative, is that the net price of output is fixed together with other prices and rates, so that the optimal level of output turns out to be reduced. Inequality (24) implies that since wage rate does not have any influence on the cost of raising funds schedule at all, downward-shift of the curve for marginal internal rate of return on investment, caused by wage increase, will move the intersection of these two curves.

(B) Influences of the Price of Fixed Investment Goods

The same calculations can be applicable to arrive at the effects of price of fixed investment goods on labor inputs, fixed investment, borrowings, bond-issuings and the internal rate of return on fixed investment; that is,

$$(25)\quad \begin{pmatrix} \dfrac{\partial L}{\partial q} \\ \dfrac{\partial I}{\partial q} \\ \dfrac{\partial BRW}{\partial q} \\ \dfrac{\partial BND}{\partial q} \\ \dfrac{\partial \lambda}{\partial q} \end{pmatrix} = |\phi|^{-1} \begin{pmatrix} -S^2 G_{IL}\{(1+\lambda)(C_{11} + C_{22}) + SqIG_{IL}C_{11}C_{22}\} \\ S^2 G_{LL}\{(1+\lambda)(C_{11} + C_{22}) + SqIC_{11}C_{22}\} \\ S^2 C_{22}\{SI(G_{LL}G_{II} - G_{IL}^2) + qG_{LL}(1+\lambda)\} \\ S^2 C_{11}\{SI(G_{LL}G_{II} - G_{IL}^2) + qG_{LL}(1+\lambda)\} \\ S^3 C_{11}C_{22}\{qG_{LL}(1+\lambda) + SI(G_{LL}G_{II} - G_{IL}^2)\} \end{pmatrix} \begin{matrix} < 0 \\ < 0 \\ \lesseqqgtr 0 \\ \lesseqqgtr 0 \\ \lesseqqgtr 0 \end{matrix}$$

The reason why the signs of the effects on borrowings, bond-issuings, and the internal rate of return on investment cannot be assured, is that they depend on the relative scale of reduction of investment and increase in the price of investment goods; that is, given the levels of internal funds and other funds available,

$$\frac{\partial}{\partial q}(qI) = q\frac{\partial I}{\partial q} + I \lesseqqgtr 0,$$

and it also depends on the assumption of decreasing, increasing or constant return to scale $G_{II}G_{LL} - G_{IL}^2 \lesseqqgtr 0$.

(C) Influences of Interest Rates

In this analysis, there are three rates that have different economic implications from each other; that is, interest rates for borrowings and bond-issuings reflect upon themselves complex institutional factors respectively. Furthermore, the marginal internal rate of return on investment does not coincide with the discount rate which may be a very important factor in the conventional theory of investment behavior, since the optimization behavior of an entrepreneur or top management, in this analysis, leads to equalization of the marginal internal rate of return on fixed investment and the marginal costs of raising funds, including not only cash-payments for interests, but also the imputed cost to top-management of the firm.

Taking into consideration these things, any direct connection between these rates and the marginal internal rate of return on investment is not introduced explicitly, except for assuming the existence of implicit term-structure of interest rates; that is,

$$\text{(26)}\qquad \frac{d\zeta}{dr}=\frac{d}{dr}\left[\sum_{\tau=1}^{T}(1+r)^{-\tau}\right]=-\sum_{\tau=1}^{T}\tau(1+r)^{-(1+\tau)}=\zeta_r<0$$

$$\text{(27)}\qquad \frac{dr}{dRL}=r_{RL}>0,$$

$$\text{(28)}\qquad \frac{dr}{dRB}=r_{RB}>0.$$

The effects of interest rate for borrowings RL on the five endogenous variables are as below:

$$\text{(29)}\qquad \begin{pmatrix} \dfrac{\partial L}{\partial RL} \\ \dfrac{\partial I}{\partial RL} \\ \dfrac{\partial BRW}{\partial RL} \\ \dfrac{\partial BND}{\partial RL} \\ \dfrac{\partial \lambda}{\partial RL} \end{pmatrix} = |\phi|^{-1} \begin{pmatrix} Sp^*q\{\zeta_r r_{RL}(C_{11}+C_{22})-S\zeta C_{22}C_{1\cdot RL}\}G_{IL} \\ -Sp^*q\{\zeta_r r_{RL}(C_{11}+C_{22})-S\zeta C_{22}C_{1\cdot RL}\}G_{LL} \\ -Spq^2C_{22}\{\zeta_r r_{RL}-S\zeta C_{1\cdot RL}\}G_{LL} \\ -Sp^*\{q^2C_{11}G_{LL}\zeta_r r_{RL}-S\zeta C_{1\cdot RL}(G_{LL}G_{II}-G_{IL}^2)\} \\ -S^2p^*[\{q^2(1+\lambda)G_{LL}C_{11}C_{22}-\lambda(C_{11}+C_{22}) \\ \times(G_{LL}G_{II}-G_{IL}^2)\}\zeta_r r_{RL} \\ -S\zeta C_{22}C_{1\cdot RL}(G_{LL}G_{II}-G_{IL}^2)] \end{pmatrix} \begin{matrix} <0 \\ <0 \\ <0 \\ \gtreqless 0 \\ \\ >0 \\ \\ \end{matrix}$$

where, the necessary conditions are used to simplify the expression above; that is from the necessary conditions (2), (3) and (4),

$$SG_I-q(1+SC_1)=0$$

$$SG_I-q(1+SC_2)=0,$$

consequently,

$$G_I - qC_1 = G_I - qC_2 = q/S > 0.$$

The sign of $\partial BRW/\partial RL$ can be made clear, using the sufficient condition (14); that is,

$$\frac{G_{LL}G_{II} - G_{IL}^2}{q^2G_{LL}} < \frac{C_{11}C_{22}}{C_{11} + C_{22}} \cdot \frac{1+\lambda}{\lambda},$$

where, needless to say, $(1+\lambda)/\lambda > 1$. However, the sign of $\partial BND/\partial RL$ cannot be assured and it depends on an inequality:

$$-\frac{G_{LL}G_{II} - G_{IL}^2}{q^2G_{LL}} \gtreqless \frac{\zeta_r r_{RL}C_{11}}{S\zeta C_{1\cdot RL}}.$$

The effects of interest rate for bond-issuings RB on the five endogenous variables are as below:

$$(30)\quad \begin{pmatrix} \dfrac{\partial L}{\partial RB} \\ \dfrac{\partial I}{\partial RB} \\ \dfrac{\partial BRW}{\partial RB} \\ \dfrac{\partial BND}{\partial RB} \\ \dfrac{\partial \lambda}{\partial RB} \end{pmatrix} = |\phi|^{-1} \begin{pmatrix} Sp^*q\{\zeta_r r_{RB}(C_{11} + C_{22}) - S\zeta C_{11}C_{2\cdot RB}\}G_{IL} \\ -Sp^*q\{\zeta_r r_{RB}(C_{11} + C_{12}) - S\zeta C_{11}C_{2\cdot RB}\}G_{LL} \\ -Sp^*\{q^2C_{22}G_{LL}\zeta_r r_{RB} - S\zeta C_{2\cdot RB}(G_{LL}G_{II} - G_{IL}^2)\} \\ -Sp^*\{q^2C_{11}G_{LL}(\zeta_r r_{RB} - S\zeta C_{2\cdot RB}) - S\zeta C_{2\cdot RB}(G_{LL}G_{II} - G_{IL}^2)\} \\ -S^2p^*[\{q^2(1+\lambda)G_{LL}C_{11}C_{22} - \lambda(C_{11} + C_{22})\cdot(G_{LL}G_{II} - G_{IL}^2)\}\zeta_r r_{RB} - S\zeta C_{11}C_{2\cdot RB}(G_{LL}G_{II} - G_{IL}^2)] \end{pmatrix} \begin{matrix} < 0 \\ < 0 \\ \gtreqless 0 \\ < 0 \\ > 0 \end{matrix}$$

where the sign of $\partial BRW/\partial RB$ depends on an inequality:

$$-\frac{G_{LL}G_{II} - G_{IL}^2}{q^2G_{LL}} \lesseqgtr \frac{\zeta_r r_{RB}C_{22}}{S\zeta C_{2\cdot RB}}.$$

(D) Influences of Other Variables

The effects of the net price of output and of internal funds raised, among others, can be written as follows:

$$(31)\quad \begin{pmatrix} \dfrac{\partial L}{\partial p^*} \\ \dfrac{\partial I}{\partial p^*} \\ \dfrac{\partial BRW}{\partial p^*} \\ \dfrac{\partial BND}{\partial p^*} \\ \dfrac{\partial \lambda}{\partial p^*} \end{pmatrix} = |\phi|^{-1} \begin{pmatrix} -\dfrac{S^2}{p^*}[(C_{11}+C_{22})\{\zeta wG_{II} - qG_{IL}\} - \zeta wq^2C_{11}C_{12}] \\ \dfrac{S^2}{p^*}[(C_{11} + C_{22})\{\zeta wG_{IL} - qG_{LL}\}] \\ \dfrac{S^2q}{p^*}C_{22}(\zeta wG_{IL} - qG_{LL}) \\ \dfrac{S^2q}{p^*}C_{11}(\zeta wG_{IL} - qG_{LL}) \\ \dfrac{S^3}{p^*}[qC_{11}C_{22}\{\zeta wG_{IL} - q(1+\lambda)G_{LL}\} + \lambda(C_{11}+C_{22})(G_{II}G_{LL} - G_{IL}^2)] \end{pmatrix} > \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$(32)\quad \begin{pmatrix} \dfrac{\partial L}{\partial F} \\ \dfrac{\partial I}{\partial F} \\ \dfrac{\partial BRW}{\partial F} \\ \dfrac{\partial BND}{\partial F} \\ \dfrac{\partial \lambda}{\partial F} \end{pmatrix} = |\phi|^{-1} \begin{pmatrix} S^3 q G_{IL} C_{11} C_{22} \\ -S^3 q G_{LL} C_{11} C_{22} \\ -S^3 C_{22}(G_{LL}G_{II} - G_{IL}^2) \\ -S^3 C_{11}(G_{LL}G_{II} - G_{IL}^2) \\ -S^4 C_{11} C_{22}(G_{LL}G_{II} - G_{IL}^2) \end{pmatrix} \begin{matrix} > 0 \\ > 0 \\ < 0 \\ < 0 \\ < 0 \end{matrix} .$$

V. CONCLUDING REMARKS

In this study, the cost of raising funds schedules by type have been introduced explicitly, and their workings have also been analyzed in relation to the marginal internal rate of return on fixed investment. The mechanism determining the amounts of funds by type to be raised was also presented, in terms of the composition of funds by type, in relation to the optimal level of fixed investment. Furthermore, the conditions of existence and stability of the equilibrium point or the intersection of the curve of marginal internal rate of return on fixed investment and that of marginal raising cost of the composite funds to be raised, have been obtained explicitly; that is, the slope of the composite curve of the marginal costs of raising funds schedules should be the reciprocal of the sum of reciprocals of the second partial derivatives of the cost-of-raising-funds function, which should be positive, and steeper than that of the curve of marginal internal rate of return on fixed investment.

Next, the influences of money wage rate, the price of fixed investment goods, interest rate for borrowings from private financial intermediaries, interest rate for new issuings of corporate bonds, the net price of output, and the amount of internal funds available for fixed investment expenditures, on the equilibrium level of fixed investment, labor inputs, the optimal amounts of borrowings and bond-issuings, and the marginal internal rate of return on fixed investment, have also been analyzed. This will throw some lights on investigation of the reason why actual interest rate differentials are not reflected in the composition of funds raised for investment expenditures in corporate businesses, and consequently, the demands for various types of long-term funds do not appear to be uniquely dependent on interest rates in reality.

It is possible to deal with the amount of internal funds as an endogenous variable in the same way as for the case of two types of external funds (*BRW* and *BND*). It is also easy to extend this analysis to the case of *m* types of external funds. In this case, the slope of the composite curve is $[\sum_{i=1}^{m} C_{ii}^{-1}]^{-1}$, where C_{ii} is the coefficient for the second partial derivative of the costs-of-raising-funds function C, with respect to the ith type of fund, and $C_{ii} > 0$ for all i.

The effects of the outstanding external debts on the five endogenous variables in question, which could be almost equivalent to the inequality (30) with the opposite sign, may be very interesting to see the effect of the customership or partnership between the debtors and the creditors. The effect of deterioration of fixed capital has been neglected here, but this does not make any crucial change in the conclusions. Finally, it should be noted that the theory presented here, is possible to be statistically tested. A stochastic model and its estimation will be developed in another paper.

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MATHEMATICAL APPENDIX A.

The value of bordered Hessian determinant concerning the sufficient condition of maximization of profits is obtained as below:
From (15),

$$\begin{vmatrix} SG_{LL} & SG_{LI} & 0 & 0 & 0 \\ SG_{IL} & SG_{II} & 0 & 0 & -q \\ 0 & 0 & -SC_{11} & 0 & 1 \\ 0 & 0 & 0 & -SC_{22} & 1 \\ 0 & -q & 1 & 1 & 0 \end{vmatrix}$$

$$= S^3[(C_{11} + C_{22})(G_{LL}G_{II} - G_{IL}^2) - q^2G_{LL}C_{11}C_{22}] > 0,$$

Following the sign-conditions (3) and (5),

$$C_{11} > 0,\ C_{22} > 0,\ G_{IL} > 0,\ G_{LL} < 0,\ G_{II} < 0, \text{ and } S = p^* \sum_{\tau=1}^{T} (1 + r)^{-\tau},$$

the value of this determinant should be positive. The inequality above can be rewritten as below:

$$\frac{1}{q^2}\left[G_{II} - \frac{G_{IL}^2}{G_{LL}}\right] < \frac{C_{11}C_{22}}{C_{11} + C_{22}}.$$

This is the inequality (14).

MATHEMATICAL APPENDIX B.

The slope of marginal internal rate of return on fixed investment expenditures can be derived as followings:
Equations (8) and (9) can be rewritten in terms of simplified notation as below:

(8)′ $$SG_L - \zeta(T)w = 0$$

(9)′ $$SG_I - q(1 + \lambda) = 0.$$

By differentiating these two equations with respect to qI, given q as an arbitrary value,

$$S\left(G_{LL}\frac{dL}{d(qI)} + \frac{G_{IL}}{q}\right) = 0$$

$$S\left(\frac{G_{II}}{q} + G_{IL}\frac{dL}{d(qI)}\right) - q\frac{d\lambda}{d(qI)} = 0.$$

Using matrix expression, these two equations can be rewritten as next:

$$\begin{bmatrix} SG_{LL} & 0 \\ SG_{IL} & -q \end{bmatrix} \begin{bmatrix} \dfrac{dL}{d(qI)} \\ \dfrac{d\lambda}{d(qI)} \end{bmatrix} = \begin{bmatrix} -\dfrac{SG_{IL}}{q} \\ -\dfrac{SG_{II}}{q} \end{bmatrix},$$

and consequently,

$$\frac{d\lambda}{d(qI)} = \frac{S(G_{LL}G_{II} - G_{IL}^2)}{q^2 G_{LL}}.$$

MATHEMATICAL APPENDIX C.

The inverse matrix ϕ^{-1} can be shown in full form:

$$\phi^{-1} =$$

$$\begin{bmatrix} S^2(G_{II}C_S - q^2C_M) & -S^2G_{IL}C_S & -S^2qG_{IL}C_{22} & -S^2qG_{IL}C_{11} & -S^3qG_{IL}C_M \\ -S^2G_{IL}C_S & S^2C_{LL}C_S & S^2qG_{LL}C_{22} & S^2qG_{LL}C_{11} & S^3qG_{LL}C_M \\ -S^2qG_{IL}C_{12} & S^2qG_{LL}C_{22} & S^2(q^2G_{LL}C_{22} - G_X) & S^2G_X & S^3C_{22}G_X \\ -S^2qG_{IL}C_{11} & S^2qG_{LL}C_{11} & S^2G_X & S^2(q^2G_{LL}C_{11} - G_X) & S^3C_{11}G_X \\ -S^3qG_{IL}C_M & S^3qG_{LL}C_M & S^3C_{22}G_X & S^3C_{11}G_X & S^4C_MG_X \end{bmatrix} \times \varDelta^{-1}$$

where $C_S = C_{11} + C_{22}$, $C_M = C_{11}C_{22}$, $G_X = G_{LL}G_{II} - G_{IL}^2$, and

$\varDelta = S^3[(C_{11} + C_{22})(G_{LL}G_{II} - G_{IL}^2) - q^2G_{LL}C_{11}C_{22}] > 0.$