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SECOND BEST OPTIMUM IN THE PRESENCE OF MONOPOLY

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I. INTRODUCTION

We consider an economy which includes industries subject to monopoly control, and suppose that the ratios of price to marginal cost in a certain industries are given. What can then be said about the optimum ratios of price to marginal cost in other industries? This is a typical problem of the thory of second best and a certain notable conclusions have been derived e.g., by Lipsey and Lancaster [2] and Green [1]. But their results are based on very restrictive assumptions as we shall explain below, and it is of some interest to analyse the problem under more general assumptions.

Lipsey and Lancaster, in Section VI of their paper [2], concentrate their attention on two industries, one of which is subject to monopoly control and the other is to determine the optimal pricing policy regarding the monopoly as one of the data. They assume that the transformation function of the economy is linear as well as that the objective function is logarithmic linear. In this particular model the optimal pricing policy for the second industry is to set its price higher than marginal cost but not so far above marginal cost as is the case in the monopolized industry.

It is the chief end of this paper to show that their conclusion for the above special case is true for more general situations. In particular we drop the hypothesis that the transformation function is linear as well as well as that the objective function is logarithmic linear and assume only that goods are substitutes in the Hicksian sense.

This result may be compared with that of Green [1] in the following way. On the one hand his result is more general than ours in that he considers the situation where the ratios of prices to marginal costs in *several* industries are given. On the other hand our result is more general in that we replaced their assumption of linearity of transformation function by that of decreasing marginal product.

In Section II basic assumptions of the model will be explained and in Section III our main theorem will be stated and proved for the case where there are only three commodities including labor. In Section IV we shall explain how the result will be modefied when there are more than three commodities.

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II. BASIC MODEL

Let y_i $(i = 1, 2)$ be the amount of the i-th good produced and L_i $(i = 1, 2)$ be the amount of labor employed in its production. We denote the production function of the i-th good by f_i $(i = 1, 2)$. Thus we have

$$y_i = f_i(L_i) \qquad (i = 1, 2)\,. \tag{1}$$

It is assumed that the marginal product of labor is positive and decreasing.

Let us assume that there is a single consumer in the economy and that his preference is represented by a utility function

$$U = U(x_1, x_2, a - L) \tag{2}$$

where x_i $(i = 1, 2)$ is the amount of the i-th good consumed, a is the total amount of labor available and L is the total amount of labor employed. Thus $a - L$ may naturally be interpreted as the consumption of leisure.

In equilibrium we must have

$$L = L_1 + L_2 \tag{3}$$

and

$$x_i = y_i \qquad (i = 1, 2)\,. \tag{4}$$

It is assumed that the first order derivatives of U are positive and that U is quasi-concave. More specifically, we shall assume that the bordered Hessian

$$H = \begin{bmatrix} 0 & U_2 & U_2 & U_3 \\ U_1 & U_{11} & U_{12} & U_{13} \\ U_2 & U_{21} & U_{22} & U_{23} \\ U_3 & U_{31} & U_{32} & U_{33} \end{bmatrix} \tag{5}$$

of U has a negative determinant and that the cofactor H_{ii} of U_{ii} in H is positive for each $i = 1, 2, 3$.

Let

$$k_i = \frac{p_i}{MC_i} \qquad (i = 1, 2) \tag{6}$$

be the ratio of price of the i-th good to its marginal cost. We assume that this ratio is given for $i = 1$.

We also assume that the labor market is competitive and the producers maximize profit. Thus we have

$$MC_i = \frac{w}{f_i'(L_i)} \qquad (i = 1, 2) \tag{7}$$

where w is the wage rate and $f_i'(L_i)$ is the marginal product of labor in the i-th industry.

So far as the consumer behaves as a price taker and maximizes utility, he equates marginal rate of substitution between a pair of commodities to the corresponding price ratio. Thus we have

$$\frac{p_i}{w} = \frac{U_i}{U_3} \qquad (i = 1, 2)\,. \tag{8}$$

Our problem is to find the ratio k_2 of price to marginal cost for the second industry which maximizes the utility function (2) subject to (1), (3), (4), (6), (7), and (8).

This problem is simplified to:
maximize

$$U(f_1(L_1), f_2(L_2), a - L_1 - L_2) \tag{2$'$}$$

subject to

$$k_1 = \frac{U_1}{U_3} f_1'(L_1)\,. \tag{9}$$

We shall suppose that suitable regularity conditions are imposed on production functions and the utility function to ensure that the maximum is attained in the interiror of the domain.

III. THE MAIN THEOREM

Using Lagrangean method, the function to be maximized will be

$$U(f_1(L_1), f_2(L_2), a - L_1 - L_2) + \lambda\left(\frac{U_1}{U_3} - \frac{k_1}{f_1'(L_1)}\right). \tag{1}$$

The conditions that the expression (1) shall be at a maximum are:

$$U_1 f_1'(L_1) - U_3 + \lambda\left[\frac{U_3(U_{11}f_1'(L_1) - U_{13}) - U_1(U_{31}f_1'(L_1) - U_{33})}{U_3^2} + \frac{k_1 f_1''(L_1)}{(f_1'(L_1))^2}\right] = 0 \tag{2}$$

$$U_2 f_2'(L_2) - U_3 + \lambda\left[\frac{U_3(U_{12}f_2'(L_2) - U_{13}) - U_1(U_{32}f_2'(L_2) - U_{33})}{U_3^2}\right] = 0 \tag{3}$$

where the functions are evaluated at the second best optimum.

Using Eq. (9) in Section II and

$$k_2 = \frac{U_2}{U_3} f_2'(L_2) \tag{4}$$

to eliminate $f_1'(L_1)$ and $f_2'(L_2)$, we obtain

(2)′ $$(k_1 - 1)U_3 + \frac{\lambda}{U_1 U_3^2}\left[k_1 U_3^2 U_{11} + U_1^2 U_{33} - (k_1 + 1)U_1 U_3 U_{13} + \frac{1}{k_1} U_1^3 f_1''(L_1)\right] = 0$$

(3)′ $$(k_2 - 1)U_3 + \frac{\lambda}{U_2 U_3^2}[k_2 U_3^2 U_{12} + U_1 U_2 U_{33} - U_2 U_3 U_{13} - k_2 U_1 U_3 U_{32}] = 0\,.$$

Hence eliminating λ,

(5) $$(k_2 - 1)U_2\left[k_1 U_3^2 U_{11} + U_1^2 U_{33} - (k_1 + 1)U_1 U_3 U_{13} + \frac{1}{k_1} U_1^3 f_1''(L_1)\right] = (k_1 - 1)U_1[k_2 U_3^2 U_{12} + U_1 U_2 U_{33} - U_2 U_3 U_{13} - k_2 U_1 U_3 U_{32}]$$

and so

(6) $$k_2 = \frac{-k_1 U_2 H_{22} + \frac{1}{k_1} U_1^3 U_2 f_1''(L_1)}{(k_1 - 1)U_3 H_{23} - U_2 H_{22} + \frac{1}{k_1} U_1^3 U_2 f_1''(L_1)}$$

where H_{ij} is defined as the cofactor of U_{ij} in H introduced in the previous section.

Needless to say, we can not know the exact value of k_i from (6) alone, since the values of functions on the right hand side are unknown to us. But we may impose certain *a priori* informations about the signs of functions.

THEOREM 1: *Under the assumptions of Section II if all goods are substitutes in the Hicksian sense and* $p_1 > MC_1$ *then, at the second best optimum,* $1 < p_2/MC_2 < p_1/MC_1$.

Proof: We have to show that if $k_1 > 1$ then at the second best optimum $1 < k_2 < k_1$.

From (6) we drive

(7) $$k_1 - k_2 = \frac{k_1(k_1 - 1)U_3 H_{23} + \frac{k-1}{k_1} U_1^3 U_2 f_1''(L_1)}{(k_1 - 1)U_3 H_{23} - U_2 H_{22} + \frac{1}{k_1} U_1^3 U_2 f_1''(L_1)}\,.$$

Since we have $H_{22} > 0$, $f_1''(L_1) < 0$ and $H_{23} < 0$ if the second and the third goods are substitutes, it follows that $k_1 > k_2$.

On the other hand from (6) we also derive

(8) $$k_2 - 1 = \frac{-(k_1 - 1)(U_2 H_{22} + U_3 H_{23})}{(k_1 - 1)U_3 H_{23} - U_2 H_{22} + \frac{1}{k_1} U_1^3 U_2 f_1''(L_1)} = \frac{(k_1 - 1)U_1 H_{21}}{(k_1 - 1)U_3 H_{23} - U_2 H_{22} + \frac{1}{k_1} U_1^3 U_2 f_1''(L_1)}$$

Since the last expression is positive under our hypothesis, Therorem 1 is established.

IV. CONCLUDING REMARKS

In the previous sections we have assumed that there are three commodities and three agents in the economy. It is possible to broaden our framework in various directions. But we shall discuss only one of such possibilities in this section.

We suppose that there are m products and the same number of producers, and that there is a consumer who supplies the only primary factor of production, say, labor.

Our model, then, can be formulated by the following relationships:

$$y_i = f_i(L_i) \qquad (i = 1, \ldots, m) \tag{1}$$

$$U = U(x_1, \ldots, x_m, a - L) \tag{2}$$

$$L = \sum_{i=1}^{m} L_i \tag{3}$$

$$x_i = y_i \qquad (i = 1, \ldots, m)\,. \tag{4}$$

Eq. (1) expresses the production function of the i-th firm assumed to be concave and satisty standard reguralĭty conditions; and L_i is the amount of labor employed in the production of the i-th good ($i = 1, \ldots, m$). Eq. (2) represents the utility function of the consumer, $a - L$ representing that part of labor which is not used in production (cf. (3)), i.e., the consumption of leisure by the individual. We suppose that the total supply of labor a is exogeneously given. Eq. (4) expresses the equilibrium conditions in the product markets.

We assume, as before, that the consumer acts as a price taker and that factor markets are competitive. We also suppose that the ratio of price to marginal cost in the first industry is fixed at k_1. Our problem then is to find the optimum ratio k_i of price to marginal cost in the i-th industry ($i = 2, \ldots, m$).

As before this problem reduces to that of finding

$$k_i = \frac{U_i}{U_{m+1}} f_{in} \qquad (i = 2, \ldots, m) \tag{6}$$

that maximizes

$$U = U(f_1(L_1), \ldots, f_m(L_m), a - \sum_{i=1}^{m} L_i) \tag{7}$$

subject to

$$k_1 = \frac{U_1}{U_{m+1}} f_{1n} \tag{8}$$

where U_{m+1} represents the marginal utility of leisure and f_{in} represents the marginal product of labor in the production of the i-th good.

Let us now define a pair of goods r and s ($r \neq s$, $r, s = 1, i, n$) to be substitutes relative to the set of goods 1, i and n if $H_{rs} < 0$ where

$$H = \begin{bmatrix} 0 & U_1 & U_i & U_n \\ U_1 & U_{11} & U_{1i} & U_{1n} \\ U_i & U_{i1} & U_{ii} & U_{in} \\ U_n & U_{n1} & U_{ni} & U_{nn} \end{bmatrix}$$

and H_{rs} is the cofactor of U_{rs}in H.

This definition is closely related to, but not exactly the same as the Hicksian definition which characterizes the substitutability between a pair of goods with reference to the whole set of goods. But these two definitions coincide when there are exactly three commodities in the economy.

We can now state a generalized version of Theorem 1 in Section 3:

THEOREM 2: *Under the assumptions of this section, if each pair of three goods 1, i and n are relatively substitutes and* $p_1 > MC_1$ *then, at the second best optimum,* $1 < p_i/MC_i < p_1/MC_1$ *for each* $i = 2, \ldots, m$.

Since the proof is essentially the same as in the previous theorem, it may be omitted.

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